MECE 5397

Assignment 3P

Due Wednesday February 8 at class time; if you prefer to submit it via Blackboard that is fine, but the deadline is the same

To do this assignment you should refer to the lecture notes for Week 1 and Week 3 posted on Blackboard and to the file `Tridiagonal_system.pdf` (from the book *Applied Numerical Methods with MATLAB for Engineers and Scientists*, Third Edition, by S. C. Chapra) also posted on Blackboard.

Assignment: Discretize the interval $0 \leq x \leq L$ using N equally spaced points and write a code using the tri-diagonal algorithm to solve the Helmholtz equation

$$\frac{d^2u}{dx^2} - \Lambda u(x) = 0 \,,$$

in the range $0 \leq x \leq L$. Consider two cases in which the boundary conditions are:

1. $u(x=0) = U_0$, $u(x=L) = 0$; the exact solution in this case is $u(x) = U_0 \sinh[\sqrt{\Lambda}\,(L-x)]/\sinh[\sqrt{\Lambda}\,L]$;
2. $du/dx|_{x=0} = v$, $u(x=L) = 0$; the exact solution is $u(x) = -(v/\sqrt{\Lambda})\sinh[\sqrt{\Lambda}\,(L-x)]/\cosh[\sqrt{\Lambda}\,L]$.

Do not start with a very large number of nodes N. Start with a reasonable number (e.g. $N = 10$) and then increase it if necessary. A good way to see if more nodes are needed is to use a certain value of N, and then repeat the calculation with $2N$ nodes. If the results of the two calculations are different, it means that N nodes are too few. In this case you should compare the results for $2N$ and $4N$ and so forth until the results of two successive discretizations are about the same; this is called a *grid convergence study* and is an essential procedure for reliable computations. We have given you the exact solutions so that, *after* the grid convergence study, you can check whether your results are indeed good.

For numerical purposes use two values of Λ, namely $\Lambda = 100$ and $\Lambda = 10,000$ and, for each one of them, solve both problems 1 and 2. For all cases you can take $L = 1$, $U_0 = 1$, $v = 1$.