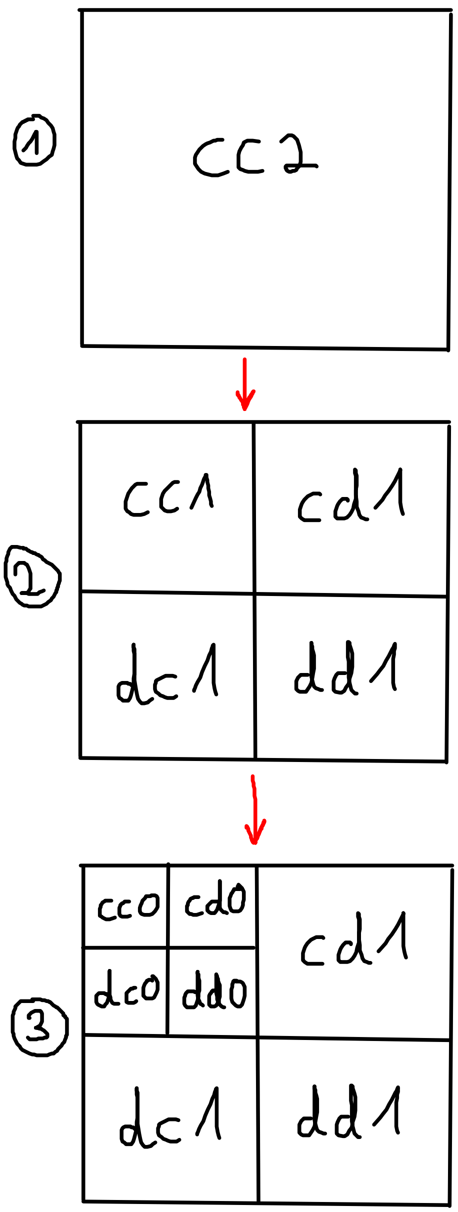
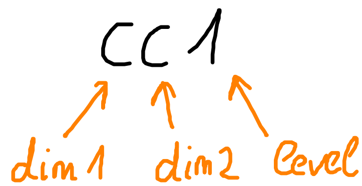
Our goal:

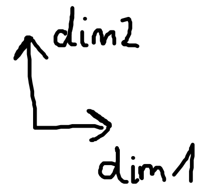


Notation

c : low filter applied

d : high filter applied





High Filters

$$H_1 = \frac{1}{2}\begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$H_2 = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Low Filters

$$h_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$h_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Note: The wavelets have not been scaled

①

$$CC2 = S = \begin{bmatrix} 4 & 2 & 3 & 5 \\ 1 & -7 & 0 & 8 \\ -1 & -3 & 9 & -3 \\ 6 & -2 & -1 & 1 \end{bmatrix}$$

$$\begin{tikzcd} \text{dim } 2 \\ {} \arrow[u] \arrow[r] & \text{dim} 1 \end{tikzcd}$$

②

$$\begin{tikzcd}[column sep=large]
& & {}^{**}\begin{bmatrix} -3/2 & 3/2 \\ -3/2 & 7/2 \end{bmatrix} \; \color{orange}{dd1} \\
& {}^{*}\begin{bmatrix} 1 & -1 \\ 4 & -4 \\ 1 & 6 \\ 4 & -1 \end{bmatrix} \arrow[ur, "\text{dim}2"', "H_2"] \arrow[r, "\text{dim}2", "h_2"'] & \begin{bmatrix} 5/2 & -5/2 \\ 5/2 & 5/2 \end{bmatrix} \; \color{orange}{dc1} \\
\begin{bmatrix} 4 & 2 & 3 & 5 \\ 1 & -7 & 0 & 8 \\ -1 & -3 & 9 & -3 \\ 6 & -2 & -1 & 1 \end{bmatrix} \arrow[ur, "\text{dim } 1", "H_2"'] \arrow[dr, "\text{dim } 1"', "h_2"] & & \begin{bmatrix} 3 & 0 \\ -2 & 3/2 \end{bmatrix} \; \color{orange}{cd1} \\
& \begin{bmatrix} 3 & 4 \\ -3 & 4 \\ -2 & 3 \\ 2 & 0 \end{bmatrix} \arrow[ur, "\text{dim}2", "H_2"'] \arrow[r, "\text{dim}2", "h_2"'] & \begin{bmatrix} 0 & 4 \\ 0 & 3/2 \end{bmatrix} \; \color{orange}{cc1}
\end{tikzcd}$$

$*$ Ex: $\frac{1}{2}\begin{bmatrix} 4 & 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$**$ Ex: $\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -3/2 \\ -3/2 \end{bmatrix}$

③ $$cc1 = \begin{bmatrix} 0 & 4 \\ 0 & 3/2 \end{bmatrix}$$

$$\begin{tikzcd}
 & & & {\begin{bmatrix} -5/8 \end{bmatrix}} \; dd0 \\
 & {\begin{bmatrix} -2 \\ -3/4 \end{bmatrix}} \arrow[urr, "\dim 2"', "H_1"] \arrow[drr, "\dim 2"', "h_1"] & & \\
 & & & {\begin{bmatrix} -11/8 \end{bmatrix}} \; dc0 \\
cc1 \arrow[uur, "\dim 1"', "H_1"] \arrow[ddr, "\dim 1"', "h_1"] & & & {\begin{bmatrix} 5/8 \end{bmatrix}} \; cd0 \\
 & & & \\
 & {\begin{bmatrix} 2 \\ 3/4 \end{bmatrix}} \arrow[uurr, "\dim 2"', "H_1"] \arrow[drr, "\dim 2"', "h_1"] & & \\
 & & & {\begin{bmatrix} 11/8 \end{bmatrix}} \; cc0
\end{tikzcd}$$

Final Result

$$\begin{bmatrix} 11/8 & 5/8 & 3 & 0 \\ -11/8 & -5/8 & -2 & 3/2 \\ 5/2 & -5/2 & -3/2 & 3/2 \\ 5/2 & 5/2 & -3/2 & 7/2 \end{bmatrix}$$