Algorithms for Scientific Computing

Grammars for Space-Filling Curves

Exercise 1: Grammars for Peano curves

Figure 1 shows the construction principle of two examples of the so-called Peano curve, namely the switch-back curve and the meander curve.
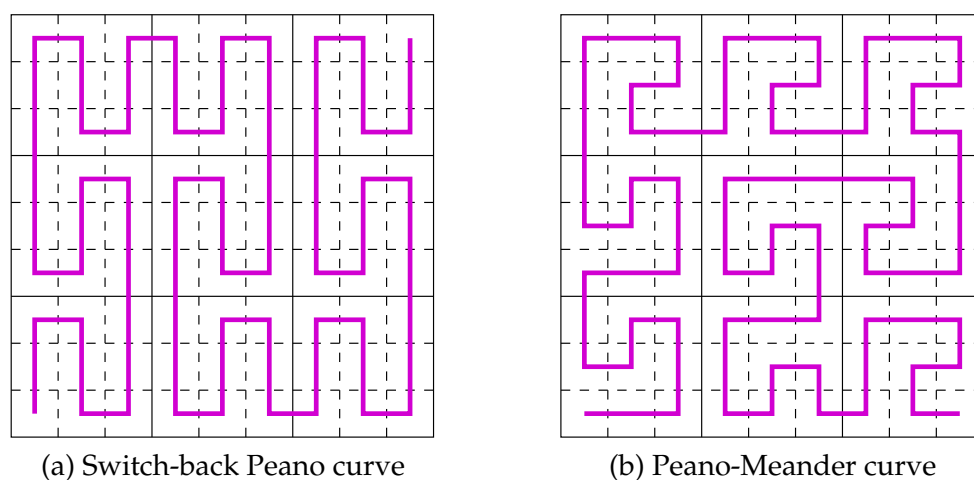

(a) Switch-back Peano curve   (b) Peano-Meander curve

Figure 1: Two distinct Peano-type space filling curves

a) Derive a grammar for each of the two curves as shown in the lecture.

b) Implement both grammars in an appropriate Python program. A code skeleton is attached.

Exercise 2: Real Turtle for the Hilbert Curve

The type of grammar which has been created in Exercise 1 can be used to derive an algorithm which defines the travel direction of the curve in a global coordinate system.

In this exercise we will derive a "real" turtle graphics algorithm for the Hilbert Curve, which only uses the following commands:

- Go one step ahead.

- Turn the travel direction by 90° to the right.

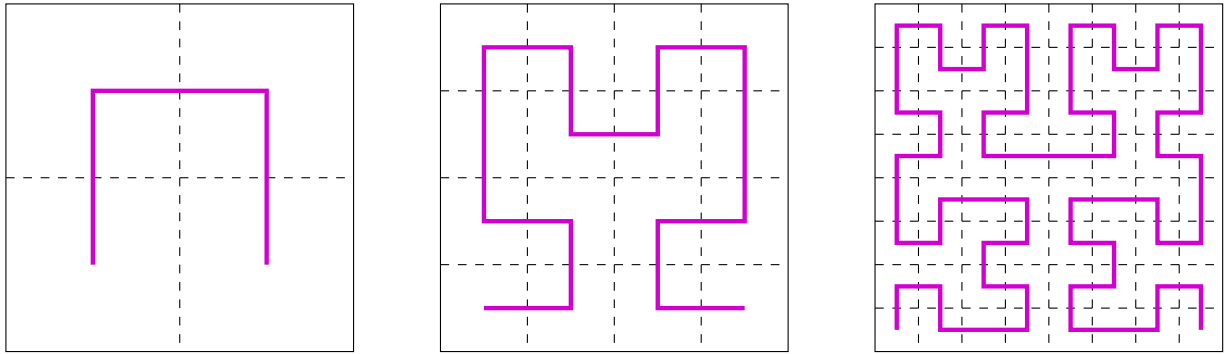- Turn the travel direction by 90° to the left.

Figure 2: Construction of the Hilbert curve

Try to find an algorithm for which the turtle turns at most once after doing a step (so it shouldn't turn more than once by 90° at the same spot).Implement the grammar. Attached you find a Python code skeleton.

Hint: You can, for example, consider where the curve enters and exits a sub-square. Try to think like the turtle: The next sub-square is always in front of you ...