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Worksheet 2
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Algorithms for Scientific Computing

Exercise 1

In the last worksheet we showed that the a_k and b_k can be computed by

$$c_k = \frac{1}{12} \sum_{l=0}^{11} X_l e^{-i2\pi kl/12}, \tag{1}$$

i.e. by a DFT.

Use the idea of the Fast Fourier Transformation, to reduce this DFT of length 12 to the computation of some DFTs of length 6 or 3, respectively.

Use the fact that all $X_l \in \mathbb{R}$.

Draw a diagram, that shows the needed computation steps or write an appropriate program (for example in Python).

Exercise 2: DFT of Mirrored Data

Assume a periodic dataset $f \in \mathbb{C}^N$. What is the difference of the Fourier coefficients of this dataset and the "mirrored" dataset $\widetilde{f_n} := f_{N-n}$ for $n = 1, \ldots, N-1$ and $\widetilde{f_0} := f_0 \equiv f_N$?

Exercise 3: DFT and "Padding"

A dataset $f \in \mathbb{C}^N$ is extended by "zeros", which gives the dataset $\hat{f} \in \mathbb{C}^M$, $(M \geq N)$, with

$$\hat{f}_n := \begin{cases} f_n & \text{if} \quad n \leq N-1 \\ 0 & \text{if} \quad N \leq n \leq M-1 \end{cases}$$

Describe how the Fourier coefficients F_k of the original dataset f_n differ from the Fourier coefficients $\hat{F}_k$ of the extended set $\hat{f}_n$.

Exercise 4: Circular Convolution Theorem[1]

Let $f, g, h \in \mathbb{C}^N$ be periodic datasets so that h is the result of the circular convolution of f and g, i.e.:

$$h_l = \sum_{n=0}^{N-1} f_n g_{l-n}$$

and F, G and H be their Fourier coefficients.

Show that

$$H_k = N F_k G_k$$

How could this be used to compute h more efficiently?

[1] https://en.wikipedia.org/wiki/Discrete_Fourier_transform#Circular_convolution_theorem_and_cross-correlation_theorem