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Worksheet 6
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Algorithms for Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Numerical Quadrature for One-dimensional Functions

This week we focus on the question how to determine the definite integral

$$F(f,a,b) = \int_a^b f(x)\,dx \quad \text{for functions } f:[a,b]\to\mathbb{R}.$$

It is not always possible to integrate the function f analytically, and so we use numerical quadrature. For the implementation of our algorithms we will use the Python language.

Excercise 1: Analytical Integration

Consider the functions

$$f(x) = -4\,x\,(x-1) \tag{1}$$

$$g(x) = \frac{8}{9}\cdot(-16\,x^4 + 40\,x^3 - 35\,x^2 + 11\,x). \tag{2}$$

Compute the antiderivatives and evaluate the integrals.

Hint: If not specified otherwise, the domain considered from now on is the unit interval $\Omega = [0,1]$.

Excercise 2: Composite Trapezoidal Rule

Write a function that approximates the integral via the Composite Trapezoidal Rule.

Can we use the hierarchical idea to reduce the overhead of adding new points?

Optional: Composite Simpson Rule

Do the same as in Exercise 2, only use the Composite Simpson Rule now.

Excercise 3: Archimedes' Hierarchical Approach

Again we focus on the question how to determine the definite integral

$$F(f,a,b) = \int_a^b f(x)\,dx \quad \text{for functions } f:[a,b] \to \mathbb{R}.$$

In this exercise we will use Archimedes' approach to approximate the integral.

Let $\vec{u} = [u_0,\ldots,u_n]^T \in \mathbb{R}^n, n = 2^l - 1, l \in \mathbb{N}$ a vector of function values with $u_i = f(x_i = \frac{i+1}{2^l})$.

a) Write a function that transforms a given vector $\vec{u} \in \mathbb{R}^n$ to a similar vector $\vec{v} \in \mathbb{R}^n$ containing the hierarchical coefficients needed for Archimedes' quadrature approach. **Hint:** Later in the lecture we will officially call this process "hierarchization", thus the function name.

b) Having computed the vector $\vec{v}$ with the hierarchical coefficients, implement a function evaluating the integral.

c) Write a function "dehierarchize1d" similar to "hierarchize1d" that computes the inverse of the transformation above.

Excercise 4: Thoughts about Adaptivity

Discuss how the previous methods could be extended in order to improve their approximation quality.