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Worksheet 1
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Algorithms for Scientific Computing

1 Project: Interpolation of the Trajectory of the Asteroid Pallas[1]

Motivated by the discovery of the asteroids Ceres (1801) and Pallas (1802), Carl Friedrich Gauss studied the computation of planet trajectories in the beginning of the 19th century. There, he was faced with the following problem of trigonometric interpolation.

Interpolation of the Asteroid's Trajectory

The following data of the trajectory have been available to Gauss:

Ascension θ (in degrees)	0	30	60	90	120	150
Declination X (in minutes)	408	89	-66	10	338	807
Ascension θ (in degrees)	180	210	240	270	300	330
Declination X (in minutes)	1238	1511	1583	1462	1183	804

Since the declination X is periodic with regard to θ, the given trajectory data should be interpolated by the following trigonometric function:

$$X(\theta) = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{2\pi k\theta}{360}\right) + b_k \sin\left(\frac{2\pi k\theta}{360}\right) \right) + a_6 \cos\left(\frac{2\pi \cdot 6\theta}{360}\right) \tag{1}$$

The data X_l and $\theta_l = 30l$ have to satisfy $X(\theta_l) = X_l$ for all $l = 0, \ldots, 11$. Thus,

$$X_l = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{\pi kl}{6}\right) + b_k \sin\left(\frac{\pi kl}{6}\right) \right) + a_6 \cos(\pi l)\,. \tag{2}$$

Python Demo

Create a Python script (using Jupyter Notebook) to compute the coefficients a_k and b_k. Plot the graph of the interpolated trajectory.

[1] Project idea and data are taken from: W. L. Briggs, Van Emden Henson, *The DFT – An Owner's Manual for the Discrete Fourier Transform*, SIAM, 1995

Note: There will be a respective introduction to Jupyter Notebook in the first tutorials.

Exercise 1

Show that the interpolation problem in equation (2) is equivalent to

$$X_l = \sum_{k=-5}^{6} c_k e^{i2\pi kl/12}, \tag{3}$$

if a_k and b_k are chosen as $a_k = 2\,\mathrm{Re}(c_k)$ and $b_k = -2\,\mathrm{Im}(c_k)$ for $k = 1, \ldots, 5$, while $c_0 = a_0$ and $c_6 = a_6$. Use the special property that all $X_l \in \mathbb{R}$ and therefore $c_{-k} = c_k^*$.

Python Demo

Equation (3) also results from an interpolation problem with the complex interpolation function

$$C(x) = \sum_{k=-5}^{6} c_k e^{ikx} \tag{4}$$

and the supporting points $x_n = 2\pi n/N$.

Use Python to compute and plot the interpolation function $C(x)$. Use the a_k and b_k from Exercise 1 and construct the C_k for all $k = -\frac{N}{2} + 1, \ldots, \frac{N}{2}$. Can $C(x)$ be used to describe the asteroid's trajectory?

Small Dictionary of Astronomy

Declination: The angle between the celestial object and the celestial equator (projection of the Earth's equator onto the celestial sphere).

(Right) Ascension: The angle between the First Point of Aries (the point where the ecliptic intersects the celestial equator) and the intersection point of the meridian of a celestial object and the celestial equator. It is equvalent to the geographical longitude but is measured to the east on the celestial equator. The units are usually hours, minutes and seconds, where 24 hours are equal to 360°.

Exercise 2

The functions $\cos$ and $\sin$ are axially respectively point symmetric to the ascension of 180 degrees. What can be found for the coefficients a_k and b_k from the previous exercise, if the following conditions hold:

$$\begin{aligned} X_l = X(\theta_l) &= X(360 - \theta_l) = X_{12-l} \qquad \text{respectively} \\ X_l = X(\theta_l) &= -X(360 - \theta_l) = -X_{12-l} \end{aligned}$$

Hint: Which values are allowed for X_0 and X_6 in the case $X_l = -X_{12-l}$?