

Algorithms for Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Haar Wavelets and Wavelet Transform

Exercise 3: Discrete Wavelet Transform

Compute the DWT for the Haar wavelets for the signal $s = [8, 4, -1, 1, 0, 4, 1, 7, -\frac{5}{2}, -\frac{3}{2}, 0, -4, -2, -2, 1, -5]$ using the Pyramidal Algorithm. Discuss the computation complexity of this method.

Solution:

Let H_l be the high-pass filters and L_l the low-pass filters. d denote the output vector.

Step 1: $d_3 = H_4 \cdot s = [2, -1, -2, -3, -\frac{1}{2}, 2, 0, 3]$, and
 $c_3 = L_4 \cdot s = [6, 0, 2, 4, -2, -2, -2, -2]$

Step 2: $d_2 = H_3 \cdot c_3 = [3, -1, 0, 0]$, and
 $c_2 = L_3 \cdot c_3 = [3, 3, -2, -2]$

Step 3: $d_1 = H_2 \cdot c_2 = [0, 0]$, and
 $c_1 = L_2 \cdot c_2 = [3, -2]$

Step 3: $d_0 = H_1 \cdot c_1 = [\frac{5}{2}]$, and
 $c_0 = L_1 \cdot c_1 = [\frac{1}{2}]$

$$d = [c_0, d_0, d_1, d_2, d_3] = [\frac{1}{2}, \frac{5}{2}, 0, 0, 3, -1, 0, 0, 2, -1, -2, -3, -\frac{1}{2}, 2, 0, 3]$$