

Our goal is to do the transformation:

$c^{(J)}$

$c^{(J-1)}$	$d^{(J-1)}$
-------------	-------------

$c^{(J-2)}$	$d^{(J-2)}$	$d^{(J-1)}$
-------------	-------------	-------------

$c^{(J-3)}$	$d^{(J-3)}$	$d^{(J-2)}$	$d^{(J-1)}$
-------------	-------------	-------------	-------------

Starting with $c^{(3)} = s = [1, 2, 3, -1, 1, -4, -2, 4]^T$ as input

Remember that the transformation from a finer level to a coarser one can be splitted in low and high pass filters:

Low-pass Filter

$$R_1 = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$R_2 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_3 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

High-pass Filter

$$H_1 = \frac{1}{2} \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$H_2 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$H_3 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$c^{(3)} = 5$$

$$d^{(2)} = H_3 c^{(3)} = \left[-\frac{1}{2}, 2, \frac{5}{2}, -3 \right]^T$$

$$c^{(2)} = h_3 c^{(3)} = \left[\frac{3}{2}, 1, -\frac{3}{2}, 1 \right]^T$$

$$d^{(1)} = H_2 c^{(2)} = \left[\frac{1}{4}, -\frac{5}{4} \right]^T$$

$$c^{(1)} = h_2 c^{(2)} = \left[\frac{5}{4}, -\frac{1}{4} \right]^T$$

$$d^{(0)} = H_1 c^{(1)} = \frac{3}{4}$$

$$c^{(0)} = h_1 c^{(1)} = \frac{1}{2}$$

Output vector: $\left[c^{(0)}, d^{(0)}, d^{(1)}, d^{(2)} \right]^T$

$$= \left[\frac{1}{2}, \frac{3}{4}, \underbrace{\frac{1}{4}, -\frac{5}{4}}_{d^{(1)}}, \underbrace{-\frac{1}{2}, 2, \frac{5}{2}, -3}_{d^{(2)}} \right]^T$$

$\downarrow \quad \downarrow$
 $c^{(0)} \quad d^{(0)}$