

Analyzing Location-Based Advertising for Vehicle Service Providers Using Effective Resistances

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Background

- Vehicle service providers have recently shown ads in vehicles.
- **This talk:** Understand how in-vehicle ads impacts service prices.

In-Vehicle Advertising: Example

Example 1 (Taxis): Curb Inc. installs tablets in taxis in 10 major cities in the U.S.



- Passengers can watch TV programs.
- Taxi companies can generate extra revenue by displaying ads.

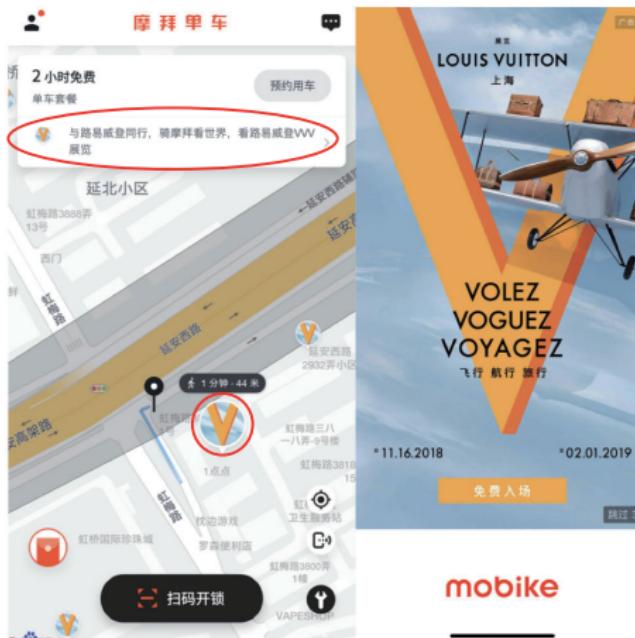
In-Vehicle Advertising

Example 2 (Ride-Sharing Systems): VUGO Inc. installs tablets for Uber & Lyft drivers, displays ads based on origins & destinations, and shares ad revenue with drivers.



In-Vehicle Advertising

Example 3 (Bike-Sharing Systems): Mobike Inc. recently tested location-based advertising in Shanghai.



Problem

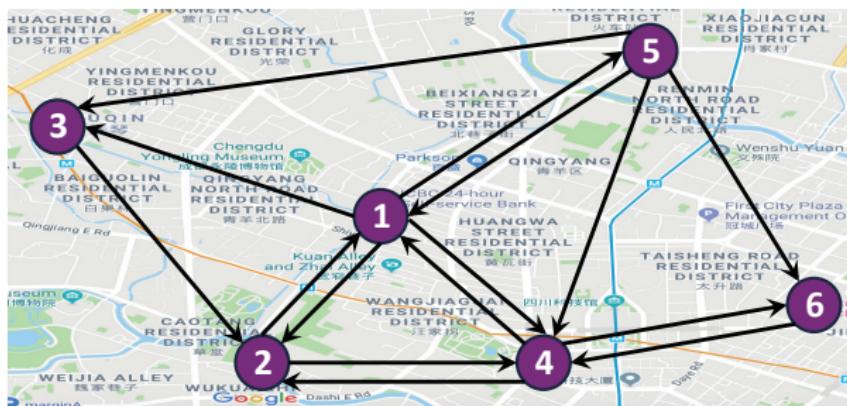
- First, we describe basic settings (e.g., traffic graph and prices).
- Second, we raise one key question about in-vehicle advertising.

Problem Description (Traffic Graph and Prices)

- We focus on a vehicle service provider who **owns** vehicles.
 - Traffic graph (**node**: location, **link**: traffic demand)
-
- Provider sets different vehicle service prices for different links.
Let p_{ij} be the price for link (i, j) (i : origin; j : destination).
 - e.g., $p_{13} = \$1/\text{minute}$.
 - Can be converted to **\$/mile** based on vehicle velocity.

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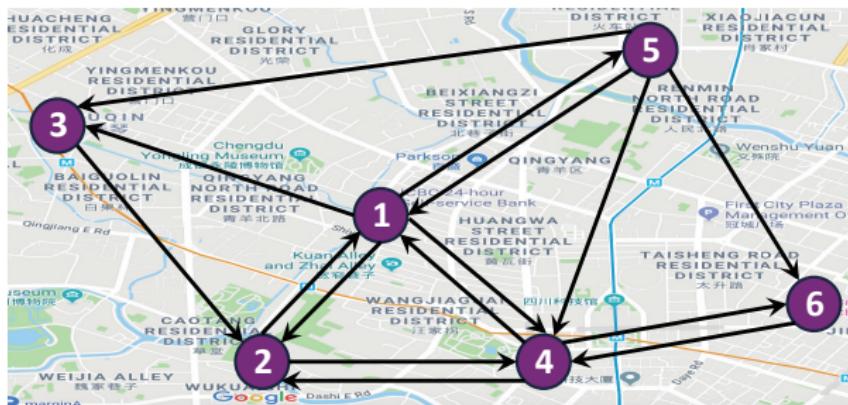
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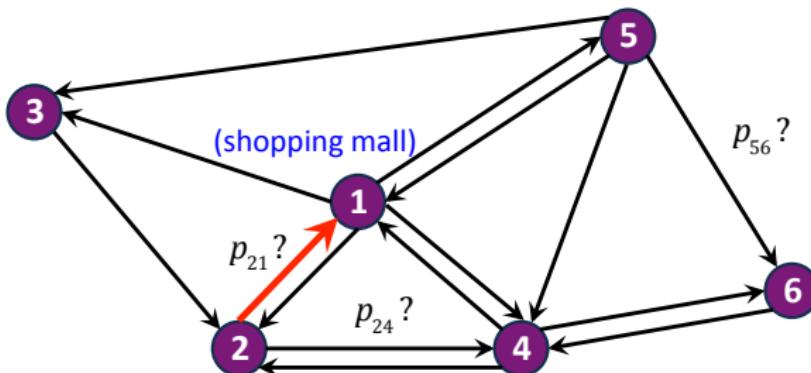
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Q: How Does Advertising Impact Prices in Network?

- Example: Provider collaborates with an advertiser (**shopping mall**) and gets ad revenues by displaying ads to riders on (2, 1).
- How should the provider change its service prices?
 - Reduce p_{21} to increase the number of riders on (2, 1)?
 - Increase p_{24} to save vehicles on (2, 4)?
 - How about p_{56} ? Non-negligible impact?
- This talk: (i) Derive expressions of prices (e.g., p_{21}, p_{24}, p_{56});
(ii) Analyze advertising's impact on prices.

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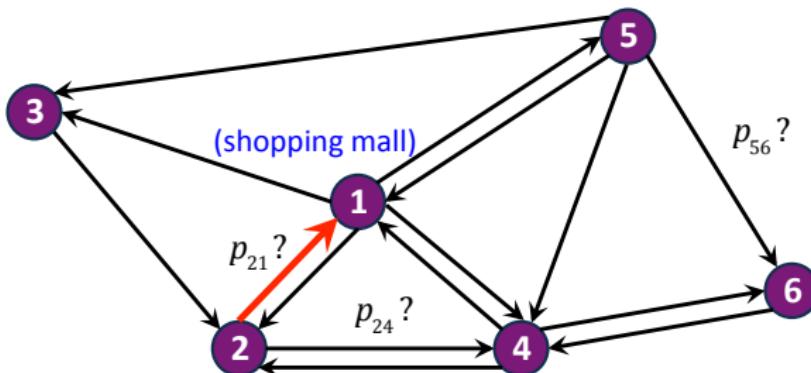
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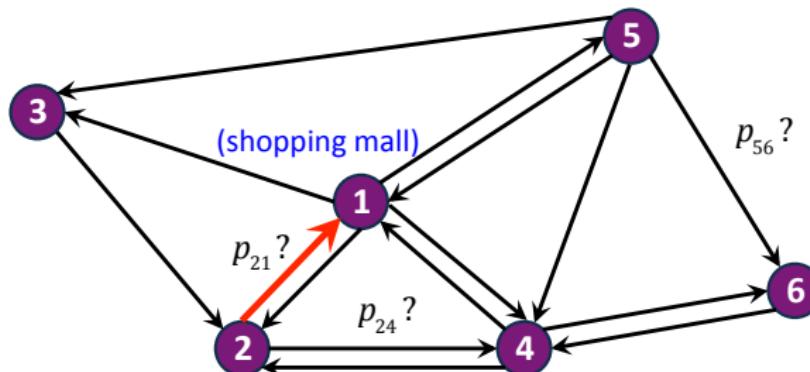
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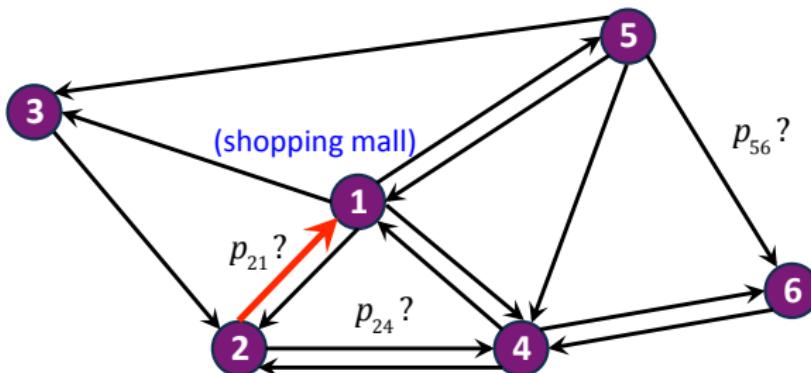
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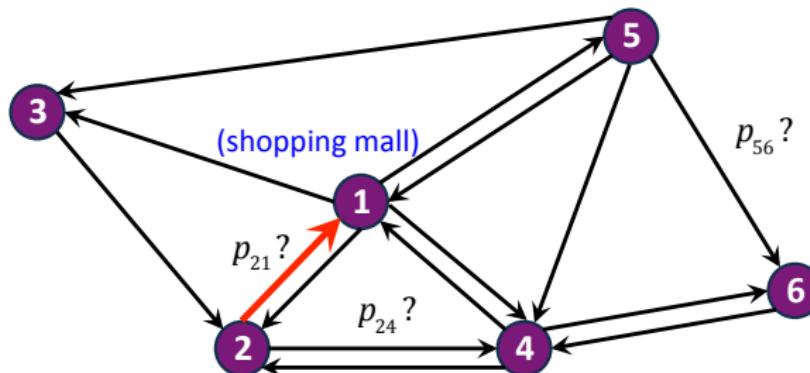
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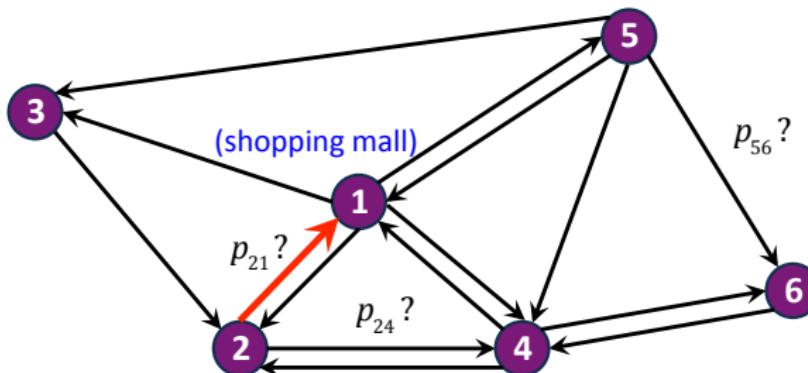
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Related Work

- Some prior work on advertising's impact on service prices:
 - **Media service:** [Kaiser and Wright 2006], [Peitz and Valletti 2008], [Godes *et al.* 2009], [Anderson and Jullien 2015]
 - **Location-based service:** [Yu *et al.* 2017]
 - **Mobile app service:** [Guo *et al.* 2018]
- Our work focuses on **vehicle service**.
 - There are **multiple prices** (each link in network has a price).
 - The advertising's impact is affected by the **network topology**.

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Model

Formulate provider's pricing problem considering ad revenues.

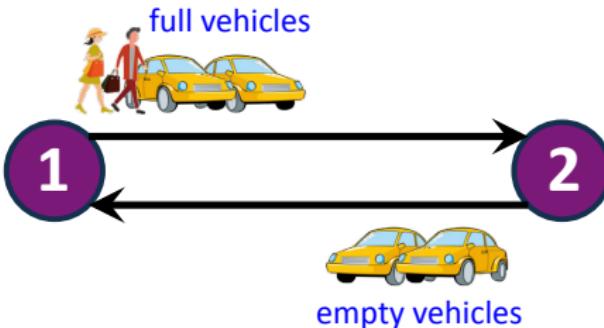
Model (Notations)

- Traffic network's parameters (constants)
 - $\theta_{ij} \geq 0$: number of users **considering** taking vehicle service on (i, j) **in each time slot** (e.g., one time slot = one minute).
 - $\xi_{ij} > 0$: travel time from i to j (measured by number of slots).
- Provider's decision variables
 - p_{ij} : service price for (i, j) (\$ per time slot).
 - Routing **full vehicles** (carrying users), **empty vehicles** (no users)
 - $q_{ij}^{\text{full}} \geq 0$: **full vehicles' departure rate** for (i, j) .
 - $q_{ij}^{\text{empty}} \geq 0$: **empty vehicles' departure rate** for (i, j) .

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a toy example:

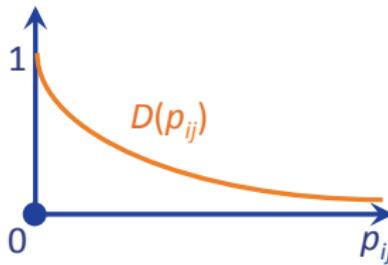


Model (Constraints)

- Demand constraint (decision variables are in blue)

$$q_{ij}^{\text{full}} \leq \theta_{ij} D(p_{ij}), \forall i, j.$$

- $D(p_{ij})$: fraction of users accepting p_{ij} and taking vehicle service.



Parameter: θ_{ij} : number of users considering service per slot.
Decisions: q_{ij}^{full} : full vehicle routing; p_{ij} : vehicle service price.

Model (Constraints)

- Vehicle flow balance constraint

$$\underbrace{\sum_j \left(q_{ij}^{\text{full}} + q_{ij}^{\text{empty}} \right)}_{\text{rate of vehicles departing from } i} = \underbrace{\sum_j \left(q_{ji}^{\text{full}} + q_{ji}^{\text{empty}} \right)}_{\text{rate of vehicles arriving at } i}, \forall i.$$

This couples the provider's decisions for different links.

Decisions: $q_{ij}^{\text{full}}, q_{ij}^{\text{empty}}$: full/empty vehicle routing.

Model (Objective)

- **Objective** (time-average profit from all links)

$$\max \sum_{(i,j)} \left(\underbrace{\xi_{ij} q_{ij}^{\text{full}}}_{\text{number of full vehicles running on } (i,j) \text{ in any slot}} \left(p_{ij} + a_{ij} - c^{\text{full}} \right) - \xi_{ij} q_{ij}^{\text{empty}} c^{\text{empty}} \right)$$

- Some new parameters:

- $a_{ij} \geq 0$: ad revenue per full vehicle per time slot on (i, j) .
- $c^{\text{full}}, c^{\text{empty}} > 0$: a (full/empty) vehicle's operation cost per slot.

Parameter: ξ_{ij} : travel time.

Decisions: $q_{ij}^{\text{full}}, q_{ij}^{\text{empty}}$: full/empty vehicle routing; p_{ij} : vehicle service price.

Problem Formulation

- The provider's problem:

$$\max \sum_{\text{each link } (i,j)} \left(\xi_{ij} q_{ij}^{\text{full}} (p_{ij} + a_{ij} - c^{\text{full}}) - \xi_{ij} q_{ij}^{\text{empty}} c^{\text{empty}} \right)$$

s.t. $q_{ij}^{\text{full}} \leq \theta_{ij} D(p_{ij}), \forall i, j,$ (demand constraint)

$$\sum_j (q_{ij}^{\text{full}} + q_{ij}^{\text{empty}}) = \sum_j (q_{ji}^{\text{full}} + q_{ji}^{\text{empty}}), \forall i, \text{ (flow balance)}$$

var. $q_{ij}^{\text{full}}, q_{ij}^{\text{empty}} \geq 0, p_{ij}, \forall i, j.$

- Question:** What is $\frac{\partial p_{ij}^*}{\partial a_{xy}}$ (p_{ij}^* is optimal price; a_{xy} is ad revenue)?
 - Hard to directly compute p_{ij}^* : non-convex problem in general.
- Rest of talk**
 - [Solution] design p_{ij}^ϕ and analyze $\frac{\partial p_{ij}^\phi}{\partial a_{xy}};$
 - [Performance] study p_{ij}^ϕ 's optimality theoretically & numerically.
 - p_{ij}^ϕ achieves close-to-optimal profit $\implies \frac{\partial p_{ij}^\phi}{\partial a_{xy}}$ gives insights.

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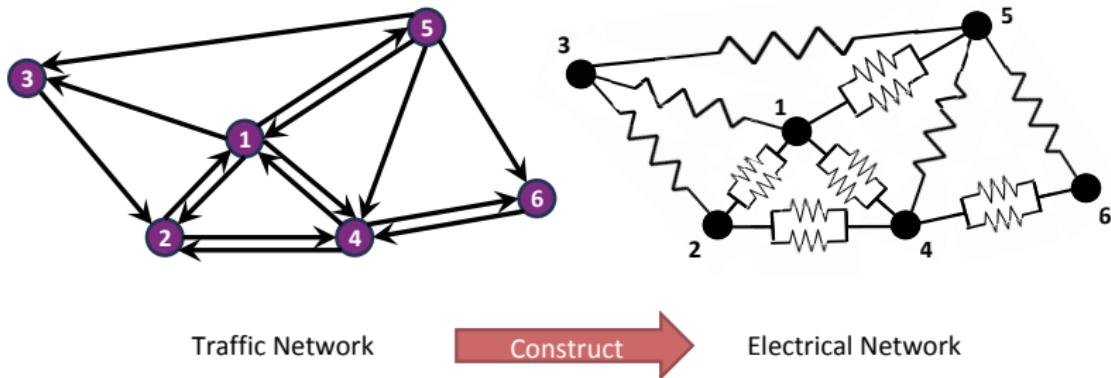
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Solution

- We propose an innovative design of p_{ij}^ϕ .
 - Vehicle networks are similar to electrical networks (e.g., keeping vehicle/current flow balance at each node).
 - We will borrow notions from electrical networks to design p_{ij}^ϕ .

Construction of Electrical Network

- Construct an **electrical network** based on traffic network.

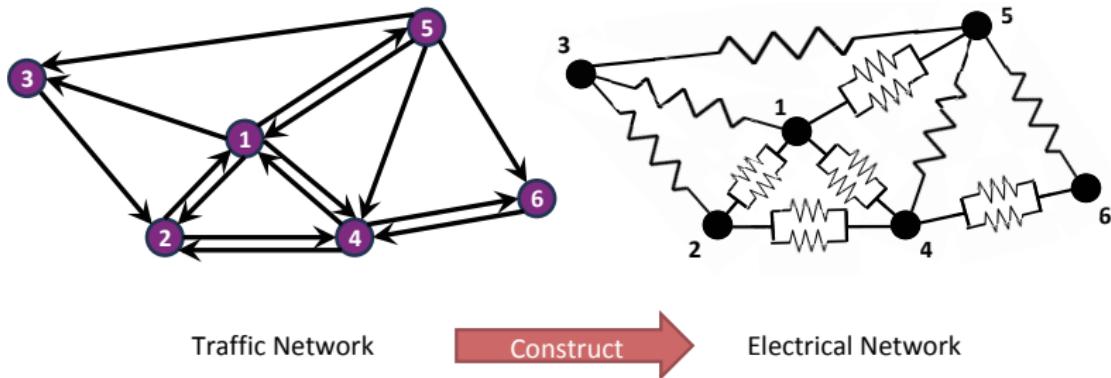


- link (i, j) \implies a resistor with resistance $r_{ij} = \frac{\xi_{ij}}{\theta_{ij}}$.

Parameters: θ_{ij} : number of users considering service per slot; ξ_{ij} : travel time.

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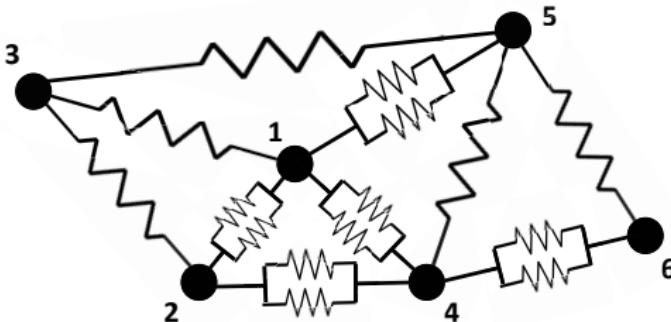


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Effective Resistances

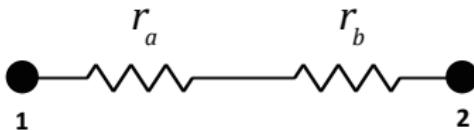
- Given an electrical network described by $\{r_{ij}\}_{i,j}$, we can compute *Effective Resistance* between any two nodes i and j .



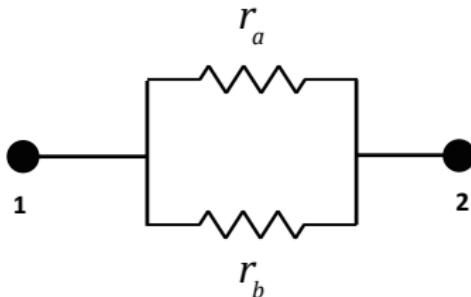
Effective Resistances

- Examples of computing *Effective Resistance*.

effective resistance
between 1 and 2



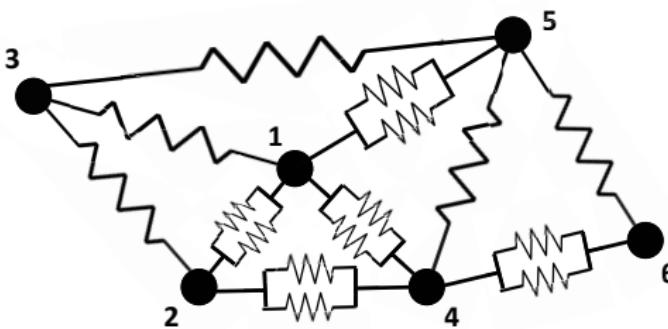
$$R_{12} = r_a + r_b$$



$$R_{12} = \frac{r_a r_b}{r_a + r_b}$$

Effective Resistances

- Let $R_{ij}(\theta, \xi)$ denote the **effective resistance** between i and j (can be computed in polynomial time).
 - e.g., $R_{34}(\theta, \xi)$'s value depends on all resistors.

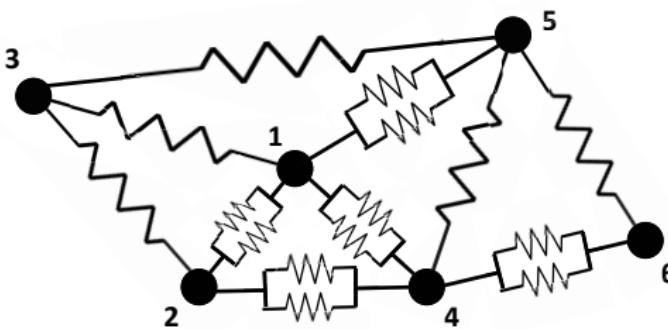


- $R_{ij}(\theta, \xi)$ internalizes the network topology's influence.
 - Intuition: small $R_{ij}(\theta, \xi) \iff$ easy to route vehicles from i to j .
- We design prices based on $\{R_{ij}(\theta, \xi)\}_{i,j}$.

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Our Resistance-Based Pricing

Resistance-Based Pricing

Let $\phi > 0$ be a control parameter. Our resistance-based price for (i, j) is given by

$$p_{ij}^\phi = \underbrace{\frac{1}{2} \left(\frac{1}{\phi} - a_{ij} + c^{\text{full}} \right)}_{\text{parameters of link } (i,j)} + \frac{1}{4\xi_{ij}} \sum_x \sum_y \underbrace{\rho_{ijxy}(\boldsymbol{\theta}, \boldsymbol{\xi})}_{\text{weight function}} \underbrace{\theta_{xy} \left(\frac{1}{\phi} + a_{xy} - c^{\text{full}} \right)}_{\text{parameters of link } (x,y)},$$

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Advertising's Impact on Prices

- If $(x, y) \neq (i, j)$, we have

$$\frac{\partial p_{ij}^\phi}{\partial a_{xy}} = \frac{\theta_{xy}}{4\xi_{ij}} (R_{jx}(\theta, \xi) - R_{ix}(\theta, \xi) - R_{jy}(\theta, \xi) + R_{iy}(\theta, \xi)).$$

- Come back to our example:

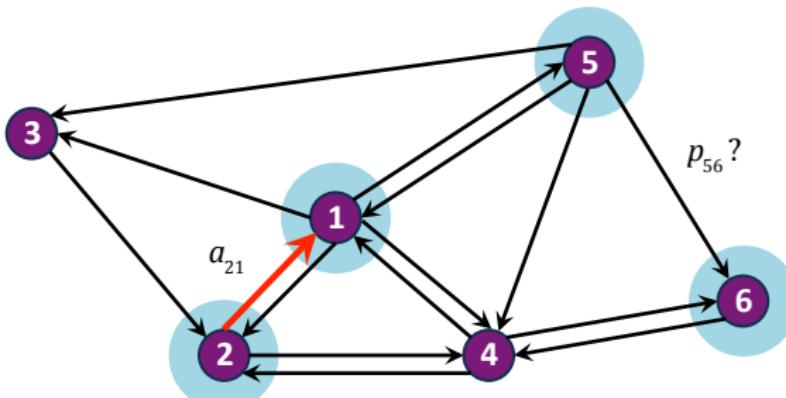
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Sign of $\frac{\partial p_{56}^\phi}{\partial a_{21}}$ depends on $R_{62}(\theta, \xi) - R_{52}(\theta, \xi) - R_{61}(\theta, \xi) + R_{51}(\theta, \xi)$.

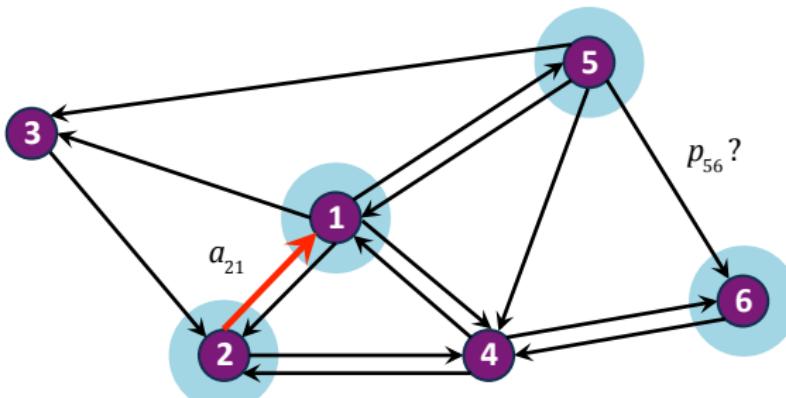
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Performance

- (Theoretical) If demand function D is *linear* and provider cannot route empty vehicles, p_{ij}^ϕ is *optimal*.
- (Experimental) If demand function D is *exponential* and provider can route empty vehicles, p_{ij}^ϕ is *close-to-optimal*.

Performance Evaluation (Theoretical)

Theorem

When the following three conditions hold:

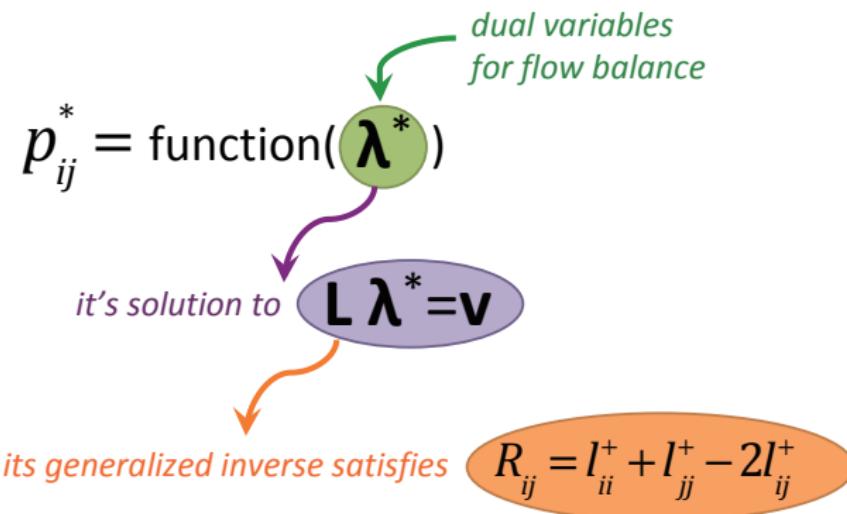
- (i) $D(p_{ij}) = \max \{1 - \psi p_{ij}, 0\}$ for $\psi > 0$ (linear demand function),
- (ii) $c^{\text{empty}} \rightarrow \infty$,
- (iii) The auxiliary constraint $p_{ij} \leq \frac{1}{\psi}$ is not binding,

the provider can **achieve the maximum profit** by choosing p_{ij}^ψ ,

$$q_{ij}^{\text{full}} = \theta_{ij} D\left(p_{ij}^\psi\right), \text{ and } q_{ij}^{\text{empty}} = 0 \text{ for all } i, j.$$

Performance Evaluation (Theoretical)

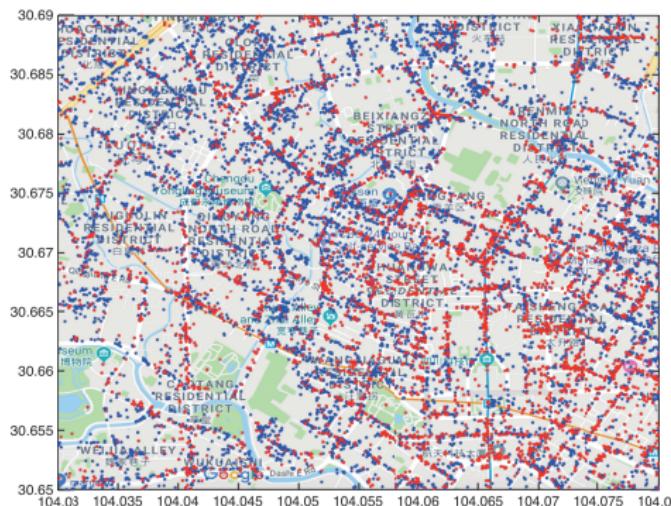
Key Idea of Proof: When conditions are satisfied,



Performance Evaluation (Experimental)

We consider non-linear $D(p_{ij})$ and finite c^{empty} in experiments.

- Real-world dataset (DiDi Chuxing GAIA Open Data Initiative)
 - Information of DiDi rides during November, 2016 in Chengdu.¹



Pick-Up (Blue) and Drop-Off (Red) Dots During 7-9 am On Weekdays.

- We divide an area into 15 locations and derive θ and ξ .

¹DiDi Chuxing GAIA Open Data Initiative (<https://gaia.didichuxing.com>).

Performance Evaluation (Experimental)

- Other experiment settings

- $D(p_{ij}) = e^{-\gamma p_{ij}}$ (verified by real data in [Fang *et al.* 2017]).
- $c^{\text{full}} = c^{\text{empty}} = 0.4$.
- a_{ij} follows an exponential distribution with mean 0.15.

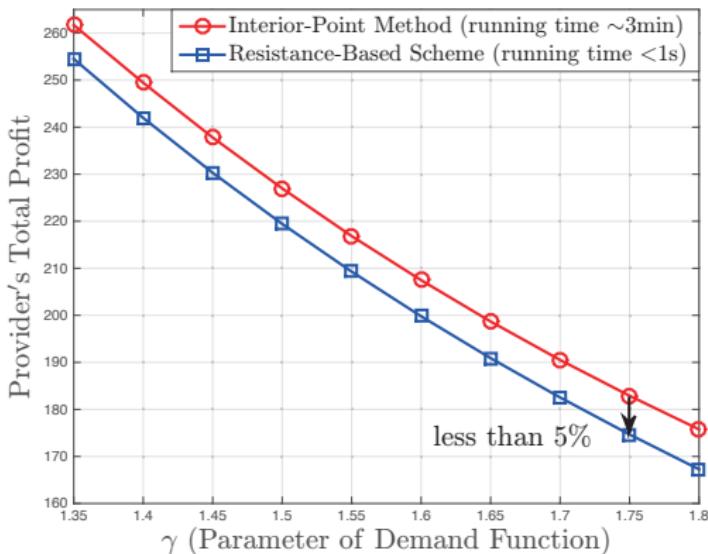
- Evaluated schemes

- Our resistance-based scheme (complexity: polynomial in number of locations):

- Choose p_{ij}^ϕ for all i, j (where $\phi = \frac{\gamma}{2}$);
- Choose $q_{ij}^{\text{full}} = \theta_{ij} D(p_{ij}^\phi)$ for all i, j ;
- Choose q_{ij}^{empty} by solving an LP problem.

- Interior-point method

Performance Evaluation (Experimental)



Our resistance-based scheme achieves at least 95% of the profit achieved by the interior-point method.

Conclusion

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- Use **effective resistances** (capturing network topology's influence) to design prices.
- Provide a simple and effective approach to measure location-based advertising's impact on network pricing.

- Other results in our work

- Investigate the advertising's impact on users' payoffs.
- Study the provider's optimal advertiser selection strategy.

- Future directions

- Consider driver-side design in ride-sharing systems.
- Consider time-variant traffic demand.

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THANK YOU