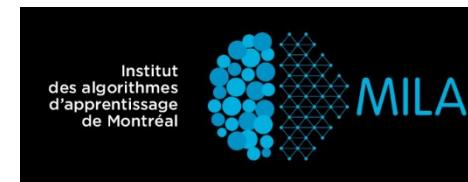


# Convolutional Neural Networks

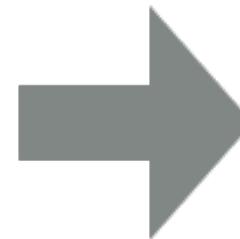
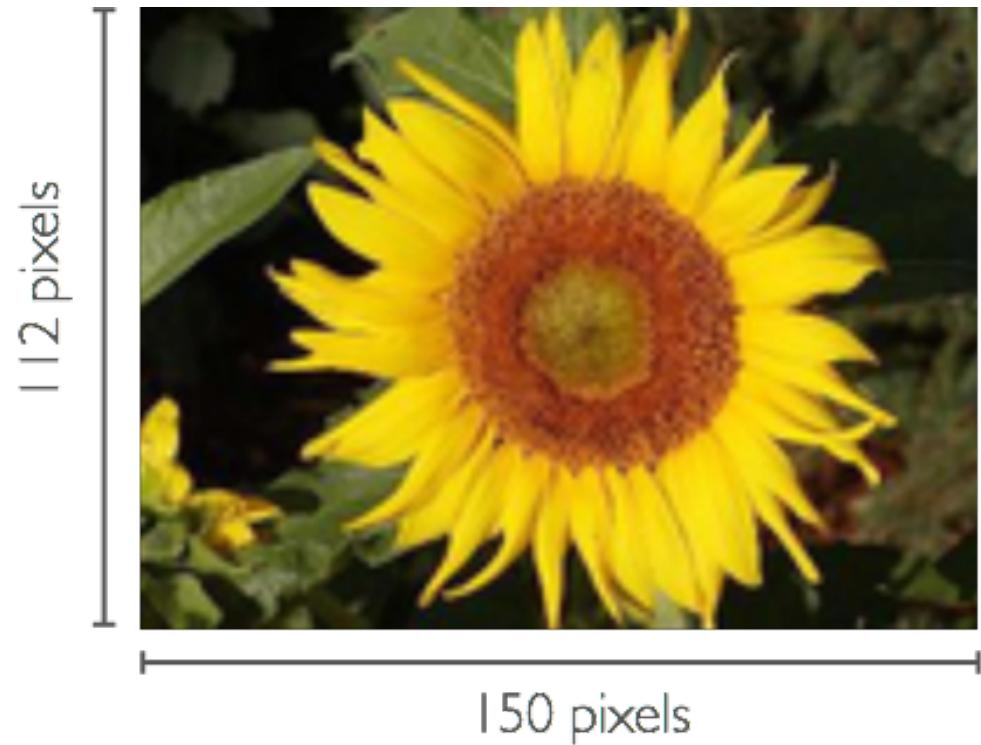
Jian Tang

tangjianpku@gmail.com

HEC MONTRÉAL



# Convolutional Neural Networks for Object Recognition



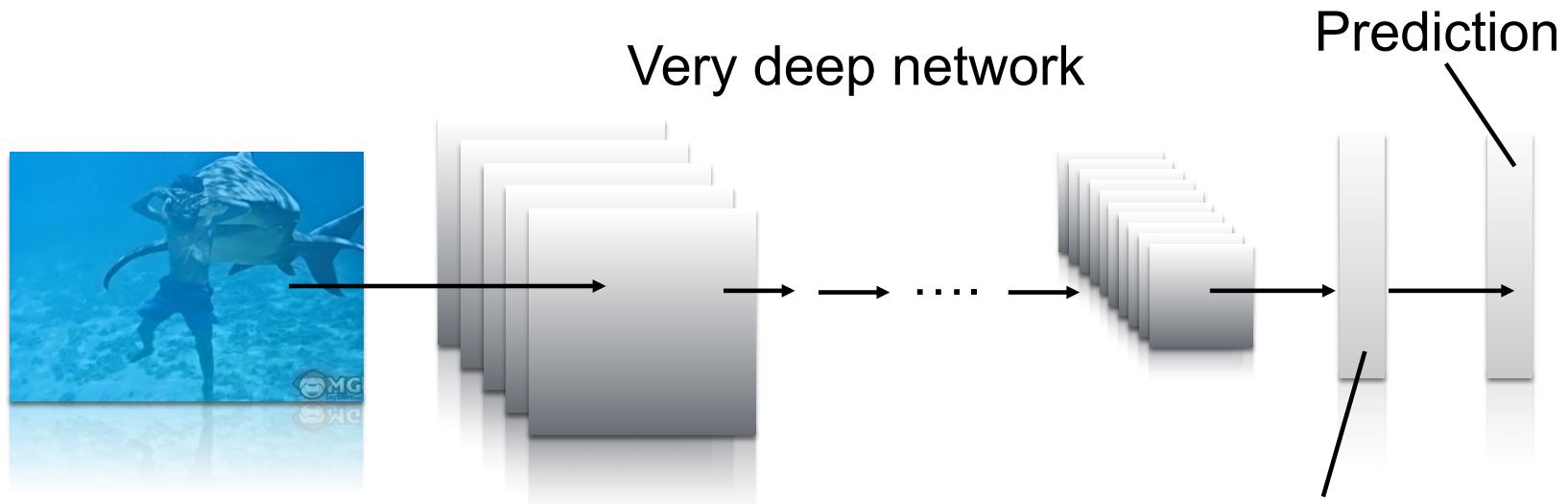
"sun flower"

Object Recognition

# Computer Vision

- Intuitions
  - Deal with very high-dimensional inputs:  $150 \times 150$  pixels = 22500 inputs
  - Can exploit the 2D topology of pixels
  - Can build in invariance to certain variations, e.g., translation, etc.
- Techniques
  - Local connectivity
  - Parameter sharing
  - Convolution
  - Pooling/subsampling hidden units

# Deep Convolutional Neural Networks



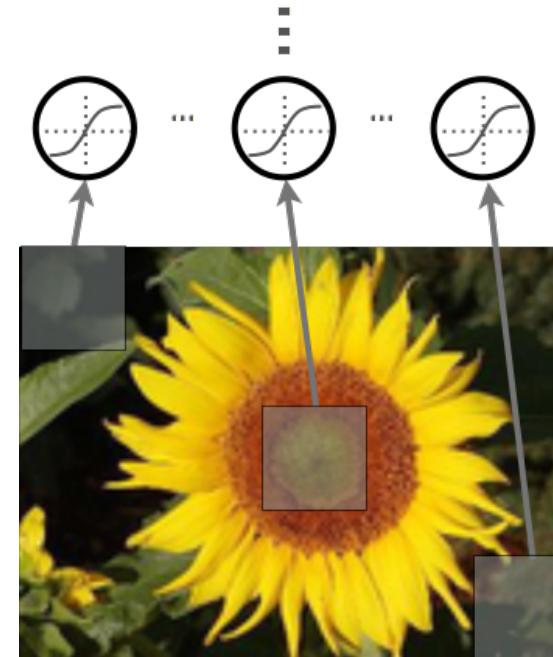
- Convolution
- Pooling
- Normalization
- Densely connected

High-level feature  
space

Prediction

# Local Connectivity

- Use a **local connectivity** of hidden units
  - Each hidden unit is connected only to a sub-region (patch) of the input image
  - It is connected to all channels: 1 if grayscale, 3 (R, G, B) if color image
- Why local connectivity?
  - Fully connected layer has **a lot of parameters** to fit, requires a lot of data
  - Spatial correlation is local

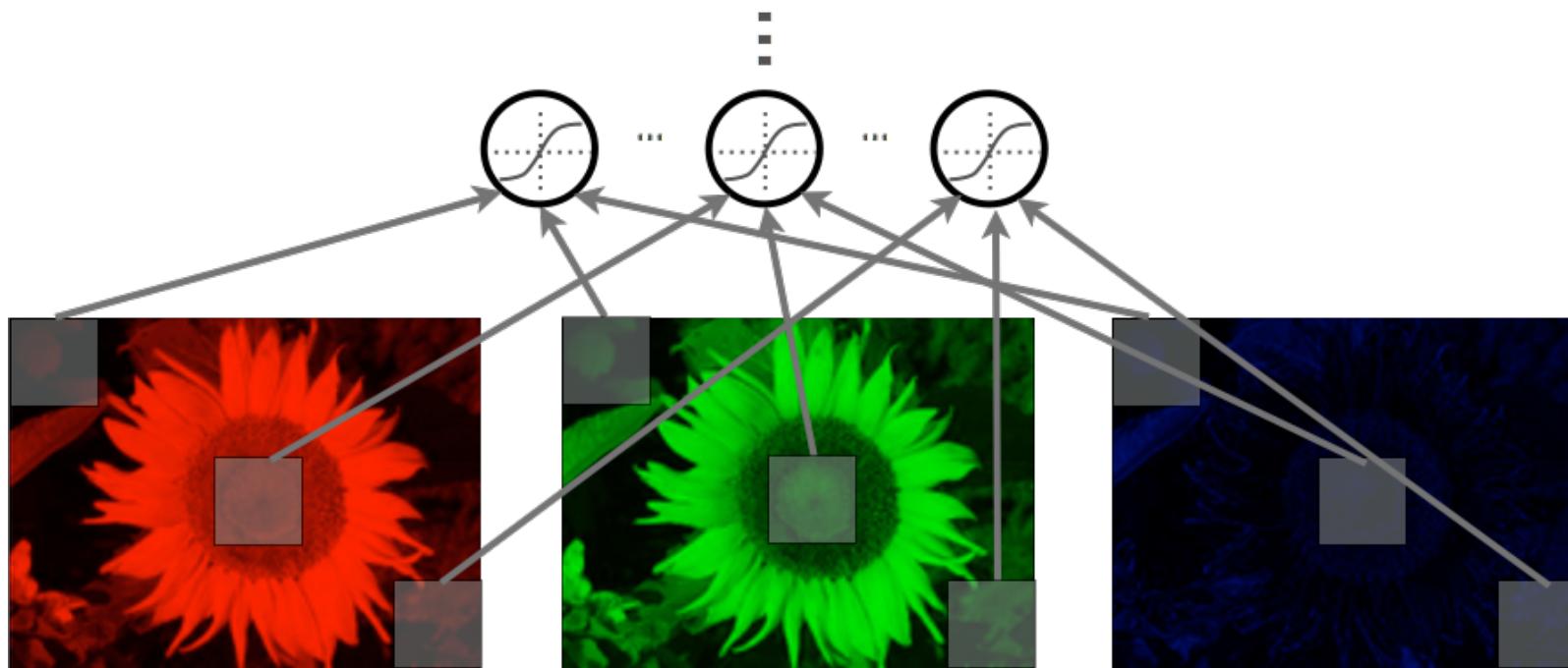


$r$   = receptive field

# Local Connectivity

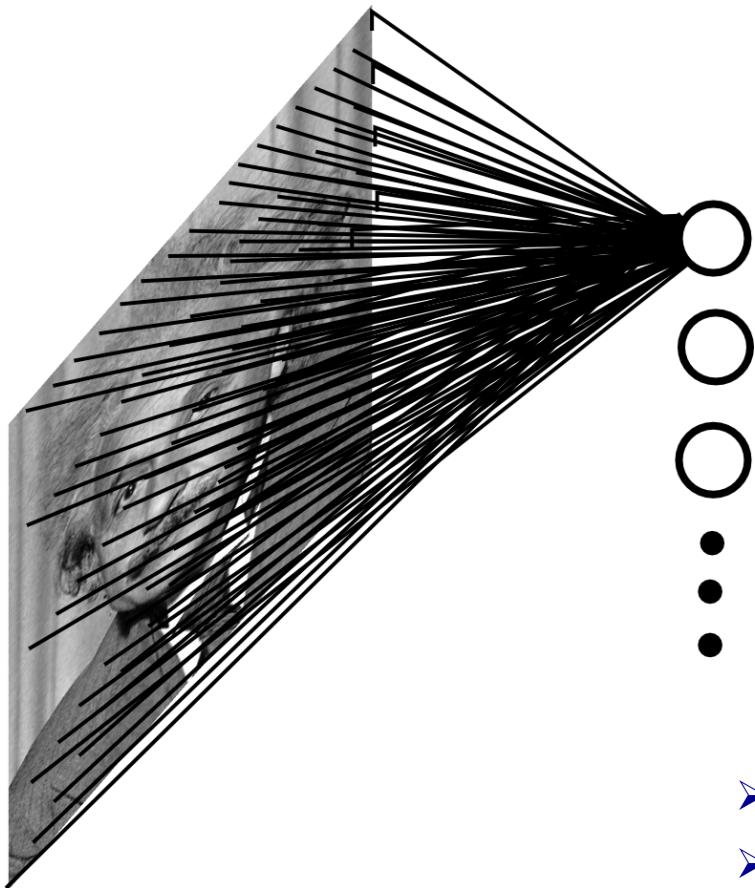
- Units are connected to all channels:

- 1 channel if grayscale image,
- 3 channels (R, G, B) if color image



# Local Connectivity

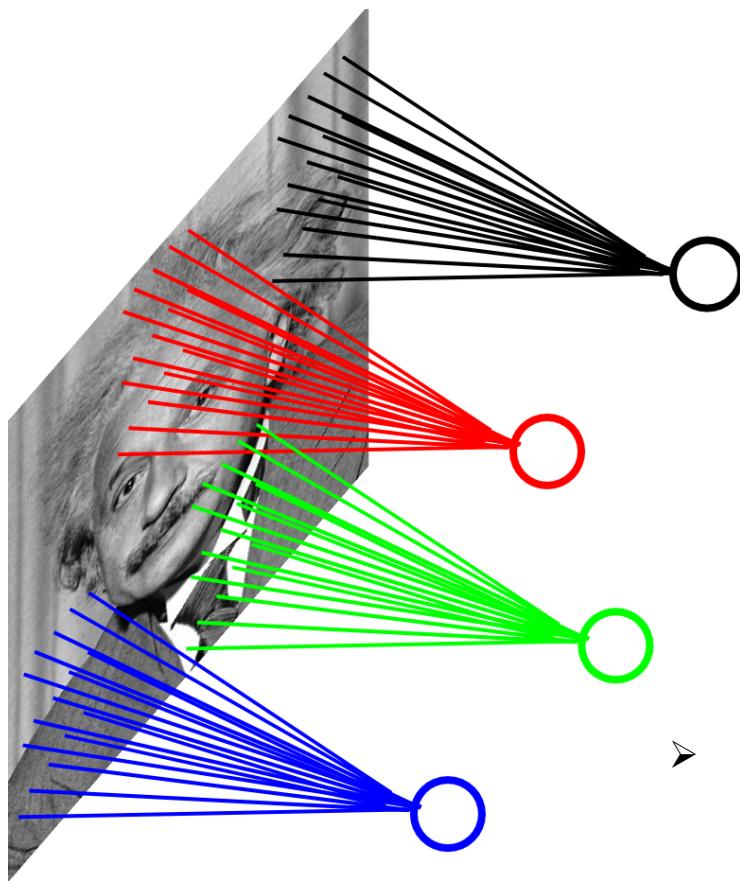
- Example: 200x200 image, 40K hidden units, **~2B parameters!**



- Spatial correlation is local
- Too many parameters, will require a lot of training data!

# Local Connectivity

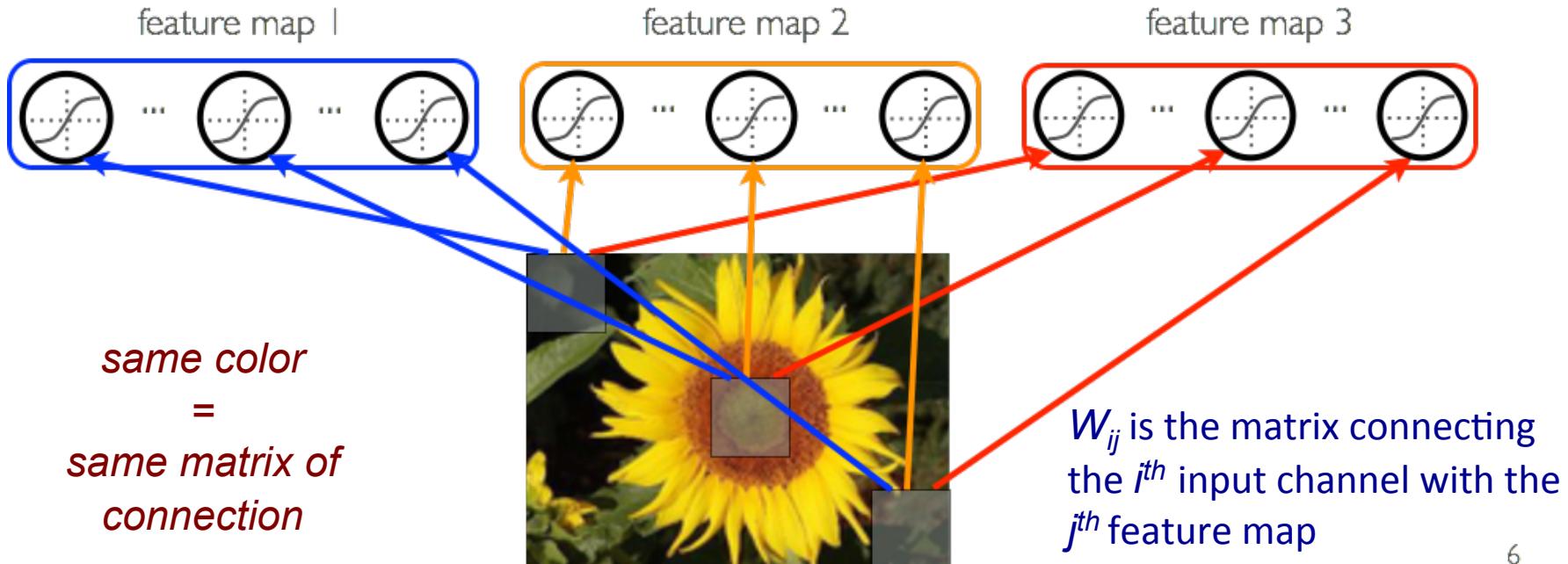
- Example: 200x200 image, 40k hidden units, filter size 10\*10, ~4M parameters



➤ This parameterization is good  
when input **image is registered**

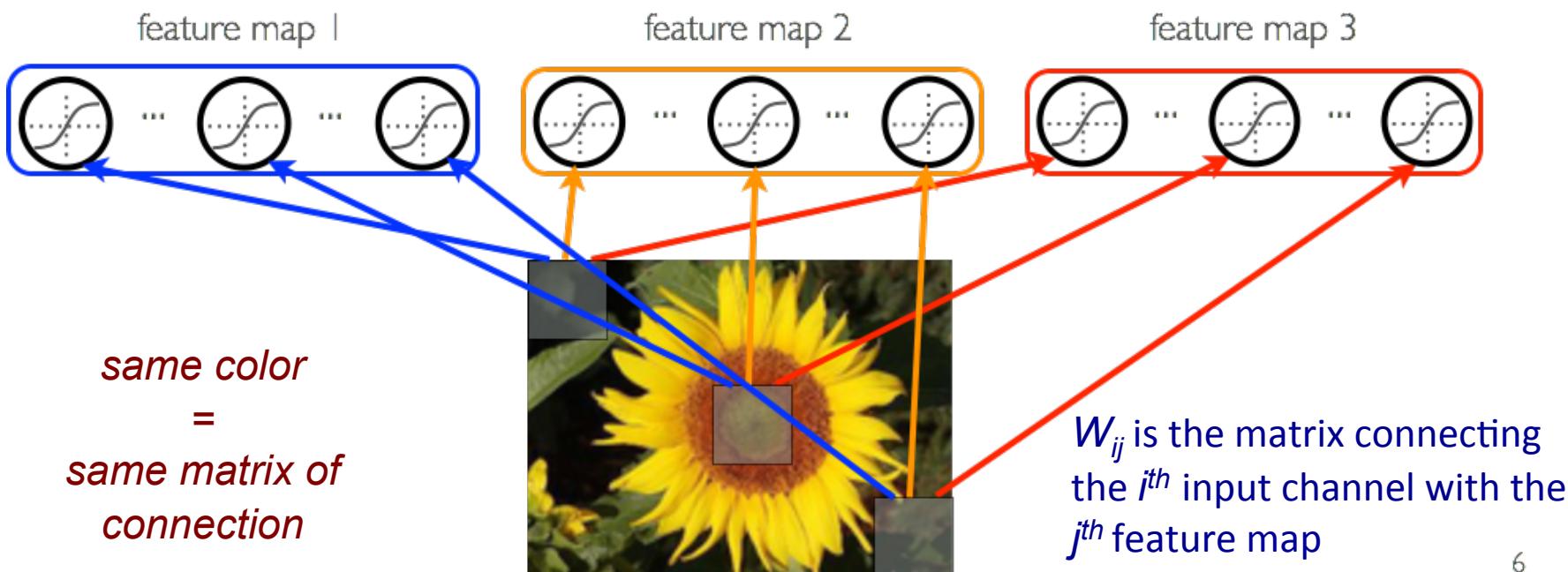
# Parameter Sharing

- Share matrix of parameters across some units
  - Units that are organized into the ‘feature map’ share parameters
  - Hidden units within a feature map cover different positions in the image



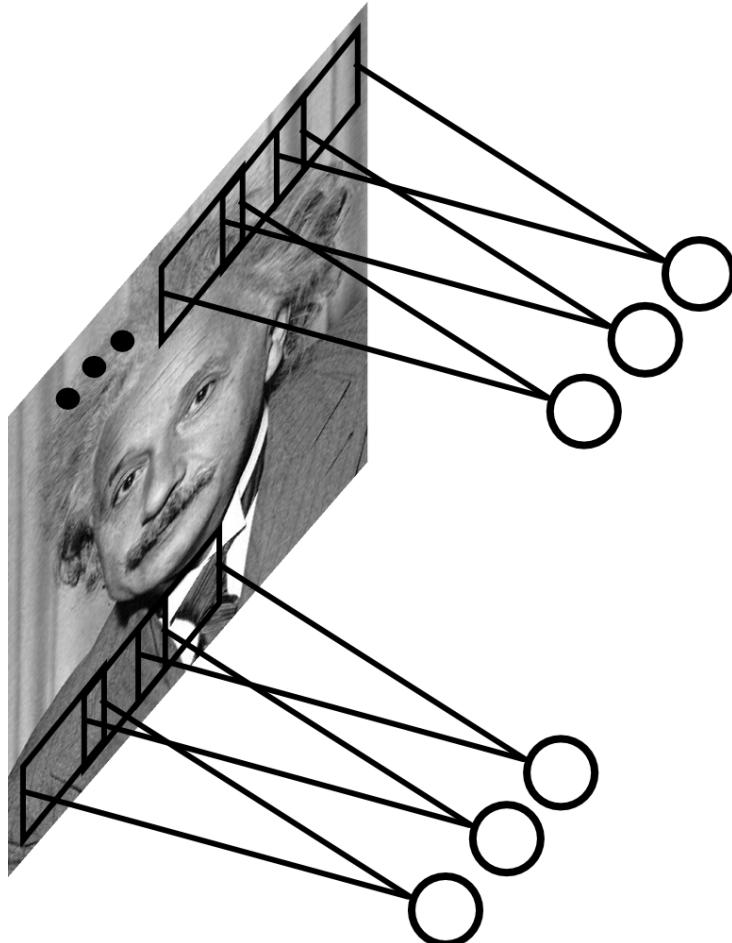
# Parameter Sharing

- Why parameter sharing?
  - Reduces even more the number of parameters
  - Will extract the same features at every position (features are “equivariant”)



# Parameter Sharing

- Share matrix of parameters across certain units



➤ **Convolutions** with certain kernels

# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:

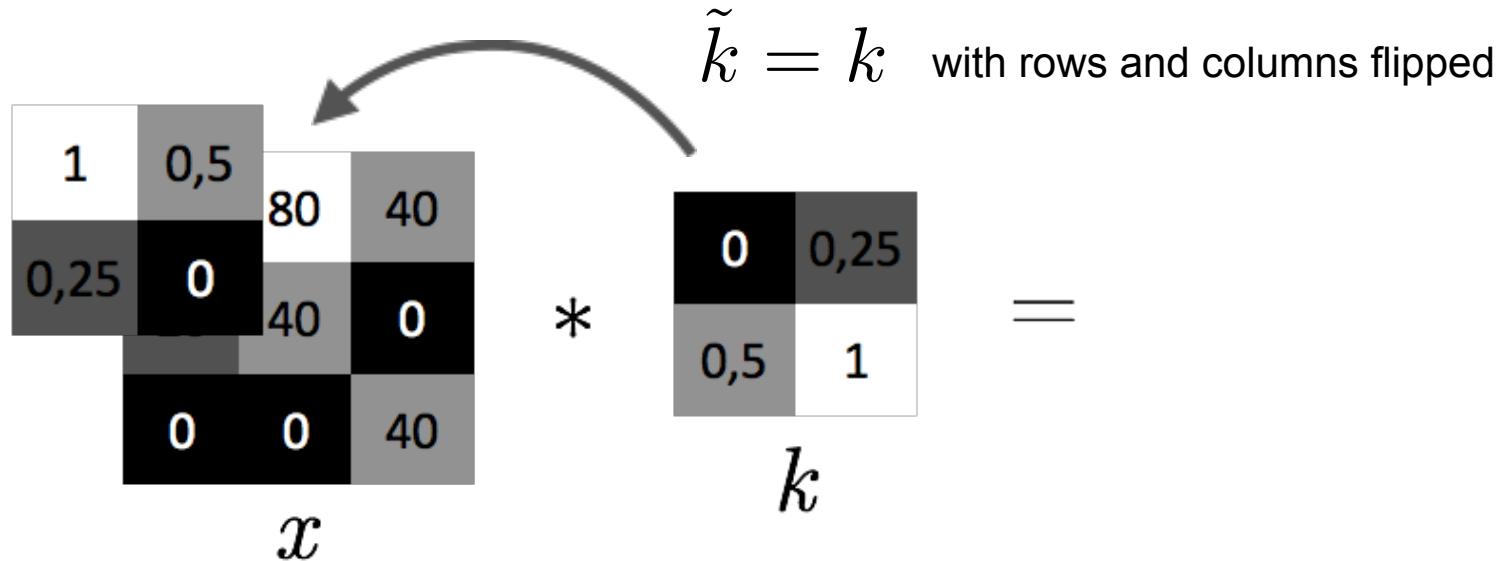
$$\begin{array}{|c|c|c|}\hline 0 & 80 & 40 \\ \hline 20 & 40 & 0 \\ \hline 0 & 0 & 40 \\ \hline \end{array} \quad * \quad \begin{array}{|c|c|}\hline 0 & 0,25 \\ \hline 0,5 & 1 \\ \hline \end{array} \quad = \quad \begin{array}{c} k \end{array}$$

# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:



# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:**  $1 \times 0 + 0.5 \times 80 + 0.25 \times 20 + 0 \times 40 = 45$

The diagram shows the convolution process between two 3x3 matrices. On the left is the input image  $x$ , which has values 1, 0.5, 80, 40, 0.25, 0, 40, 0, and 0 in its respective grid positions. In the center is the kernel  $k$ , with values 0, 0.25, 0.5, and 1. To the right of the multiplication symbol (\*) is an equals sign (=). Below the image  $x$  is the label  $x$ , and below the kernel  $k$  is the label  $k$ . The result of the convolution is a single value 45.

$$\begin{matrix} 1 & 0,5 & 80 \\ 0,25 & 0 & 40 \\ 0 & 0 & 40 \end{matrix} * \begin{matrix} 0 & 0,25 \\ 0,5 & 1 \end{matrix} = 45$$

$x$

$k$

# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:  $1 \times 80 + 0.5 \times 40 + 0.25 \times 40 + 0 \times 0 = 110$

$$\begin{matrix} & 1 & 0,5 & \\ & 0,25 & 0 & 0 \\ & 0 & 0 & 40 \\ \hline x & 0 & 0 & 40 \end{matrix} * \begin{matrix} & 0 & 0,25 \\ & 0,5 & 1 \end{matrix} = \begin{matrix} 45 & 110 \end{matrix}$$

# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:  $1 \times 20 + 0.5 \times 40 + 0.25 \times 0 + 0 \times 0 = 40$

$$\begin{matrix} & \begin{matrix} 0 & 80 & 40 \\ 20 & 40 & 0 \end{matrix} \\ \begin{matrix} 1 & 0,5 \\ 0,5 & 0 \end{matrix} & \begin{matrix} 0 & 0,25 \\ 0,5 & 1 \end{matrix} \\ & \begin{matrix} 45 & 110 \\ 40 & \end{matrix} \end{matrix} = \begin{matrix} x & k \end{matrix}$$

# Discrete Convolution

- The convolution of an image  $x$  with a kernel  $k$  is computed as follows:

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:  $1 \times 40 + 0.5 \times 0 + 0.25 \times 0 + 0 \times 40 = 40$

The diagram shows the convolution process between two 3x3 matrices. The image  $x$  (left) has values: top row [0, 80, 40], second row [20, 40, 0], bottom row [0, 0, 0.25]. The kernel  $k$  (right) has values: top row [0, 0.25], second row [0.5, 1]. The result of the convolution is a 2x2 matrix: [45, 110] in the top row, and [40, 40] in the bottom row.

$$\begin{matrix} 0 & 80 & 40 \\ 20 & 40 & 0 \\ 0 & 0 & 0.25 \end{matrix} \quad * \quad \begin{matrix} 0 & 0.25 \\ 0.5 & 1 \end{matrix} \quad = \quad \begin{matrix} 45 & 110 \\ 40 & 40 \end{matrix}$$

$x$                                      $k$

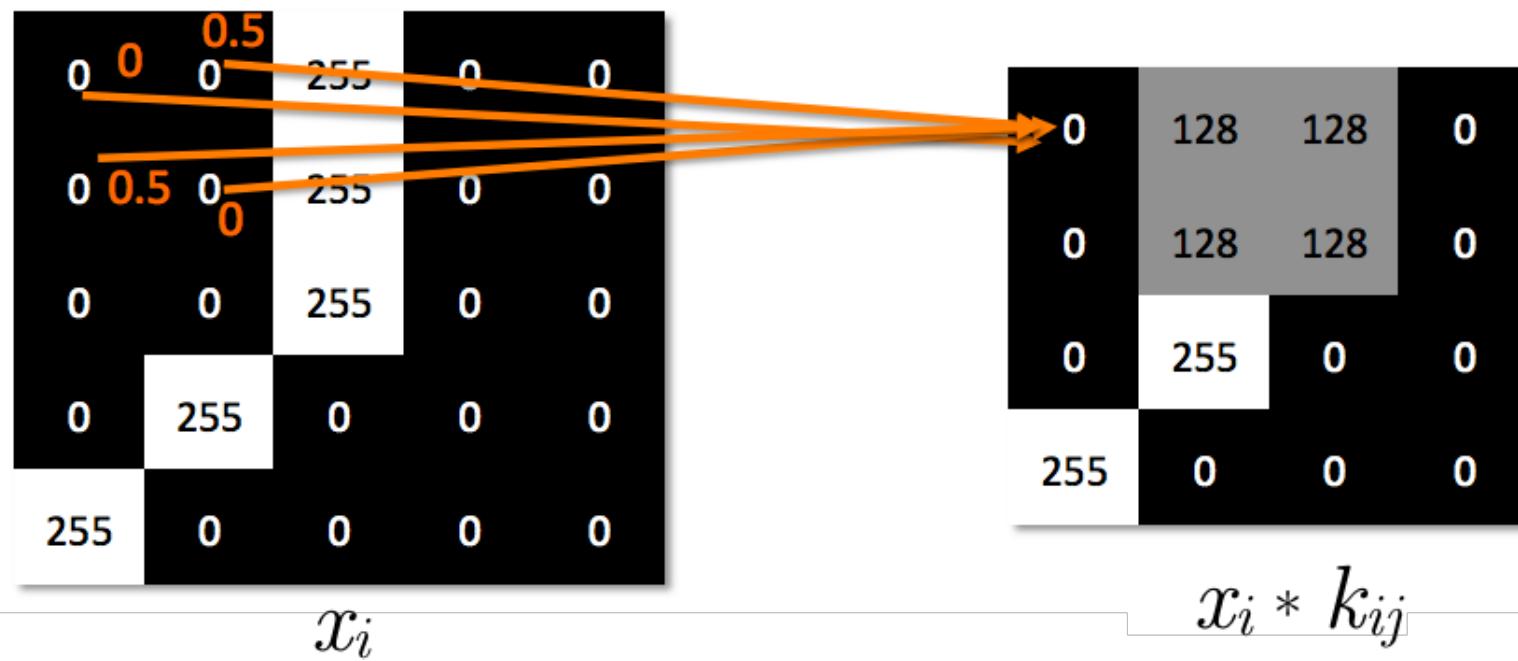
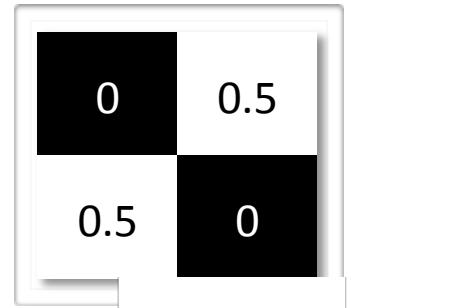
# Discrete Convolution

- Pre-activations from channel  $x_i$  into feature map  $y_j$  can be computed by:
  - getting the convolution kernel where  $k_{ij} = \tilde{W}_{ij}$  from the connection matrix  $W_{ij}$
  - applying the convolution  $x_i * k_{ij}$
- This is equivalent to computing the discrete correlation of  $x_i$  with  $W_{ij}$

# Example

- Illustration:

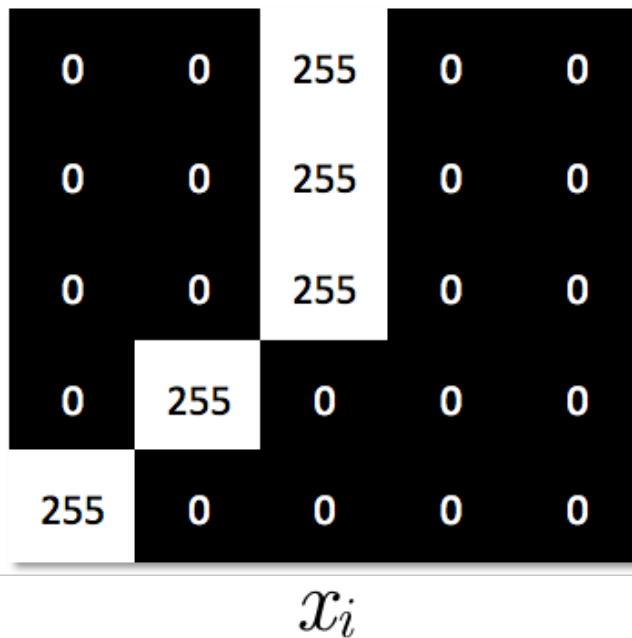
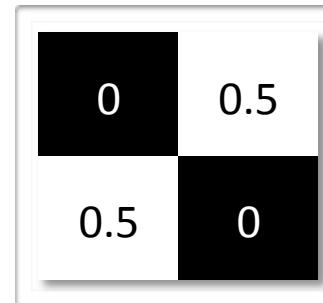
$$(x_i * k_{ij}) \text{ where } k_{ij} = \tilde{W}_{ij}$$



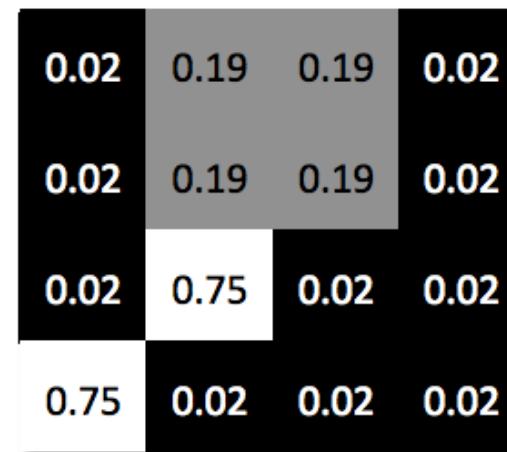
# Example

- With a non-linearity, we get a detector of a feature at any position in the image:

$$(x_i * k_{ij}), \text{ where } W_{ij} = \tilde{W}_{ij}$$

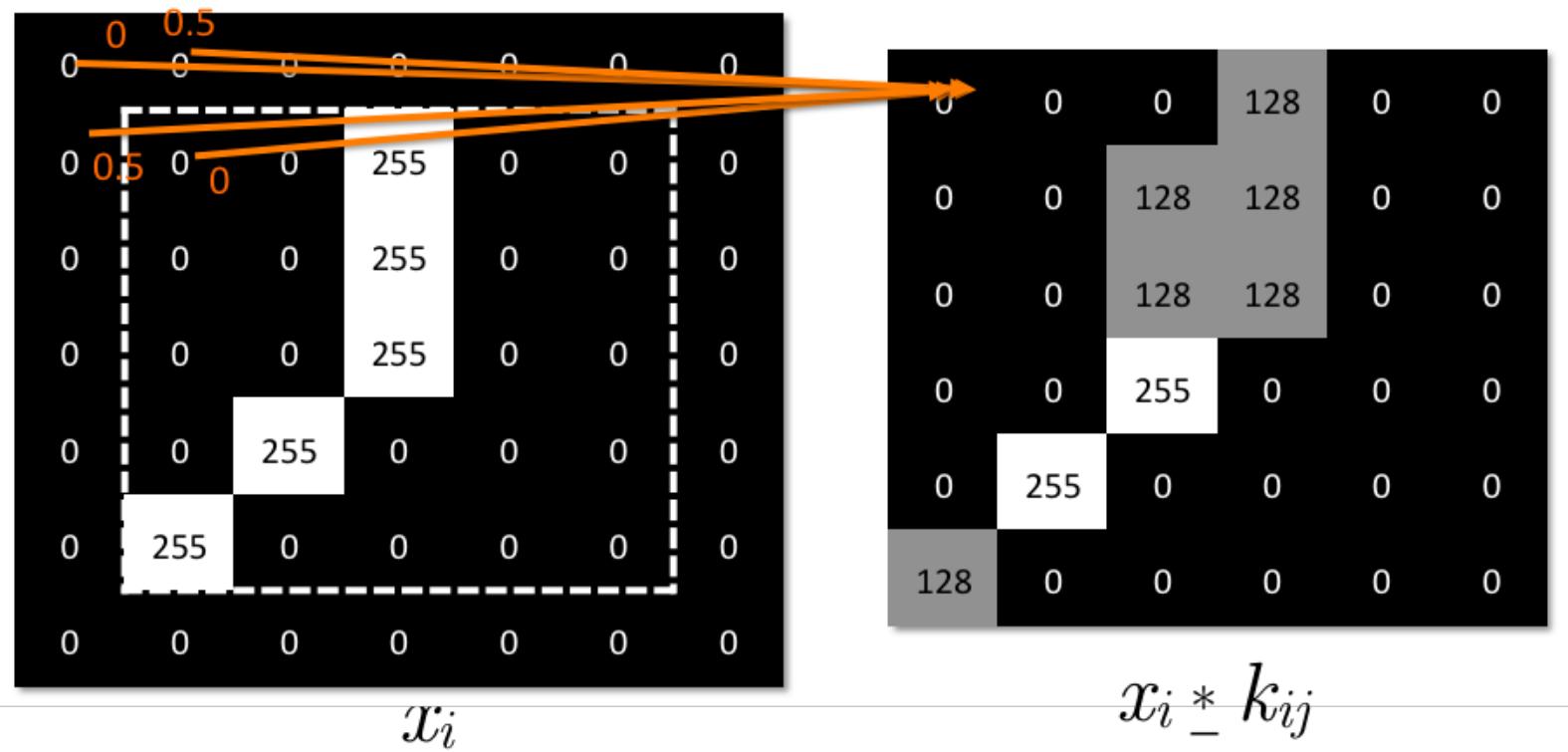


$$\text{sigm}(0.02 x_i * k_{ij} - 4)$$



# Example

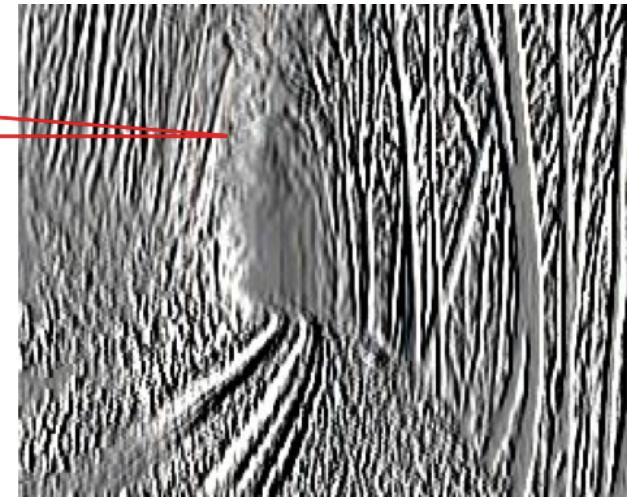
- Can use “zero padding” to allow going over the borders ( \* )



# Example

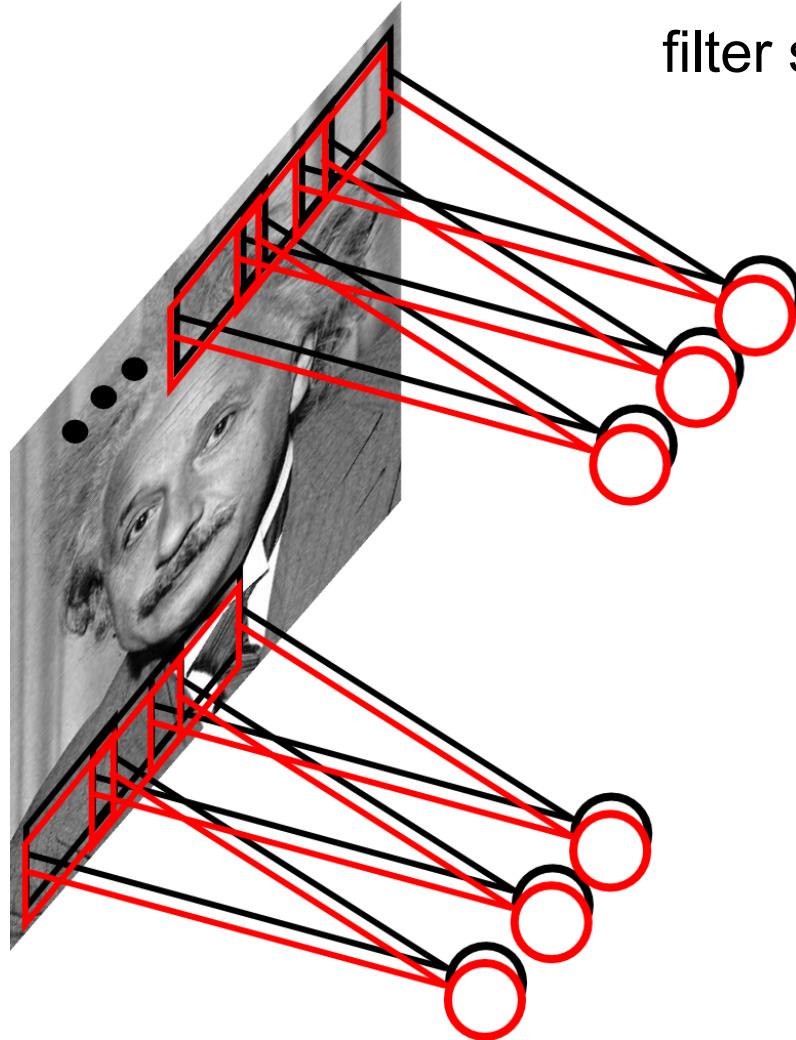


$$* \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} =$$



# Multiple Feature Maps

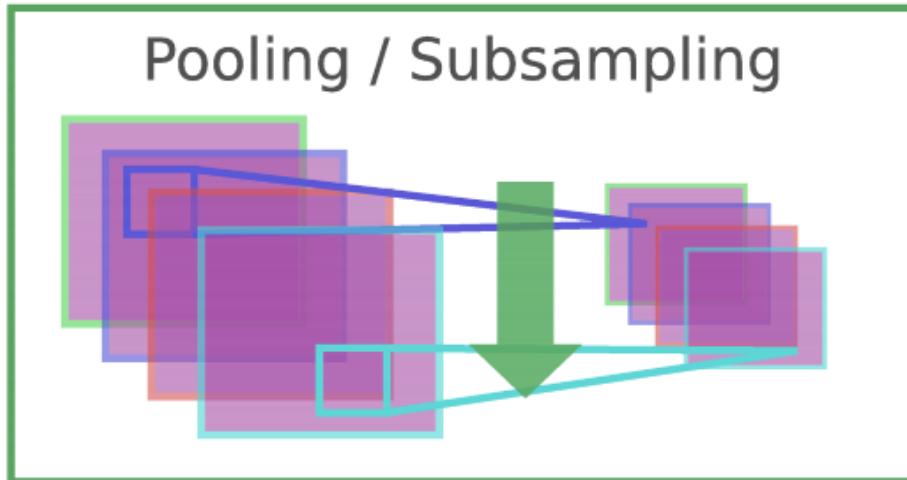
- Example: 200x200 image, 100 filters, filter size 10x10, 10K parameters



# Pooling

- Pool hidden units in same neighborhood
  - Pooling is performed in non-overlapped neighborhoods (subsampling)

$$y_{ijk} = \max_{p,q} x_{i,j+p,k+q}$$



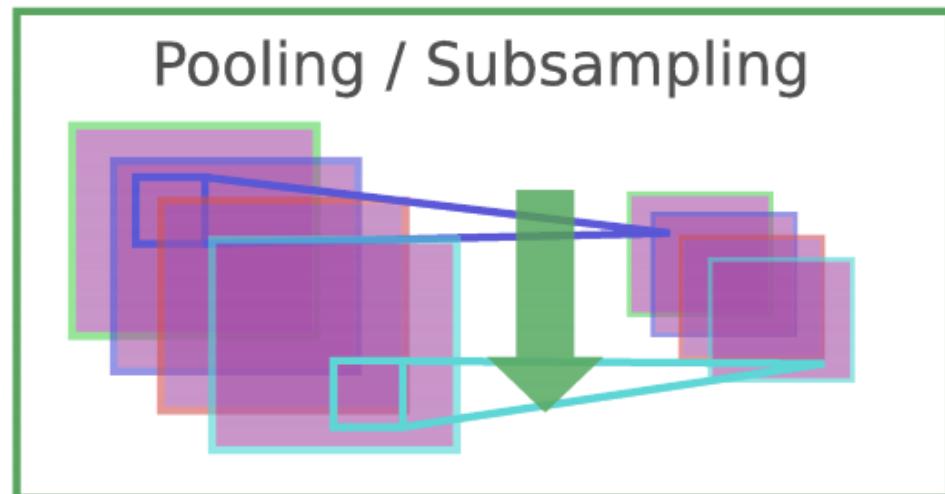
Jarret et al. 2009

- $x_i$  is the  $i^{\text{th}}$  channel of input
- $x_{i,j,k}$  is value of the  $i^{\text{th}}$  feature map at position  $j,k$
- $p$  is vertical index in local neighborhood
- $q$  is horizontal index in local neighborhood
- $y_{ijk}$  is pooled / subsampled layer

# Pooling

- Pool hidden units in same neighborhood
  - an alternative to “max” pooling is “average” pooling

$$y_{ijk} = \frac{1}{m^2} \sum_{p,q} x_{i,j+p,k+q}$$

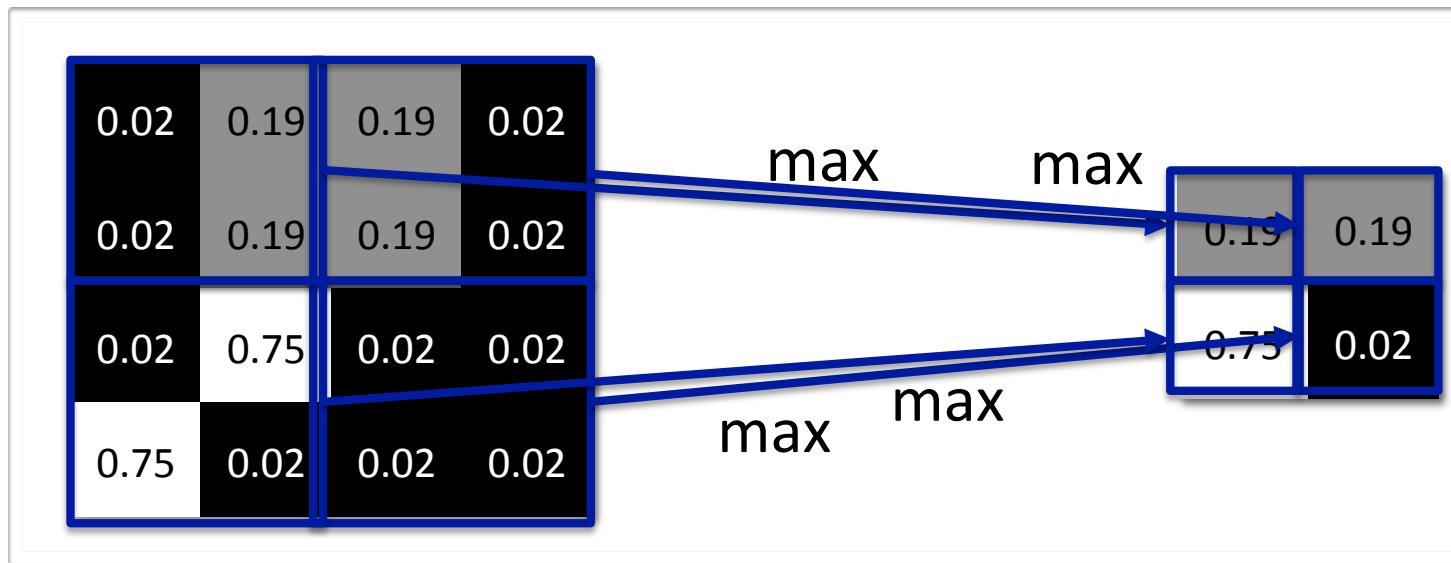


Jarret et al. 2009

- $x_i$  is the  $i^{\text{th}}$  channel of input
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- $p$  is vertical index in local neighborhood
- $q$  is horizontal index in local neighborhood
- $y_{ijk}$  is pooled / subsampled layer
- $m$  is the neighborhood height/width

# Example: Pooling

- Illustration of pooling/subsampling operation

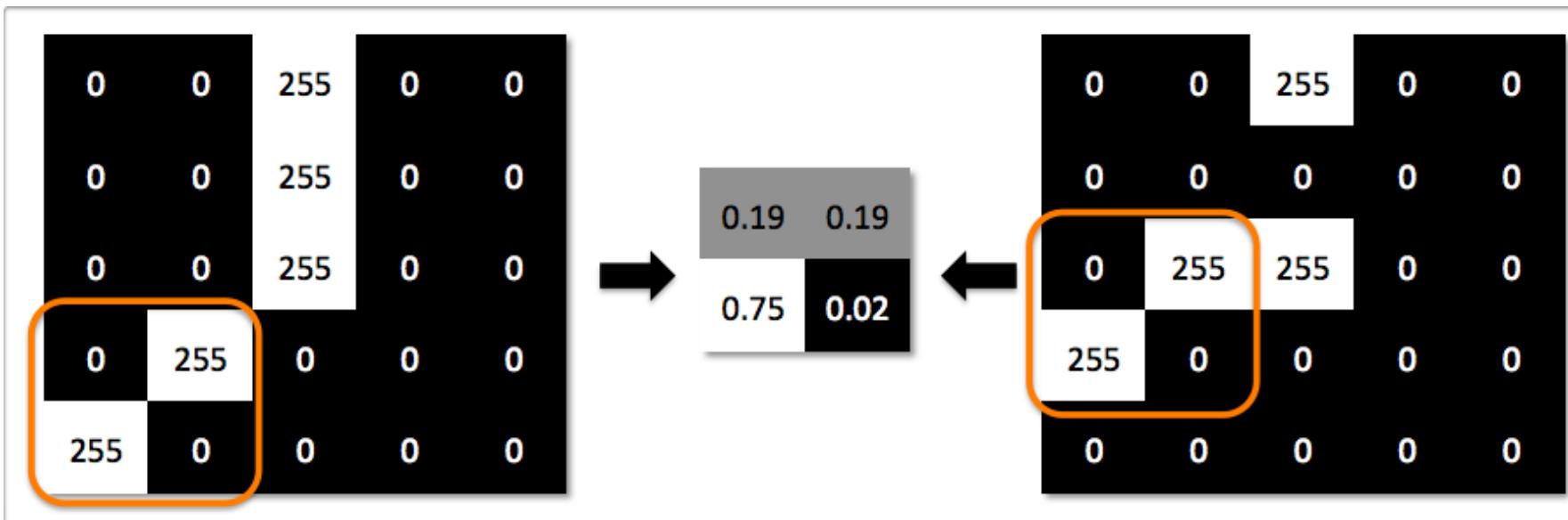


- Why pooling?

- Introduces invariance to local translations
- Reduces the number of hidden units in hidden layer

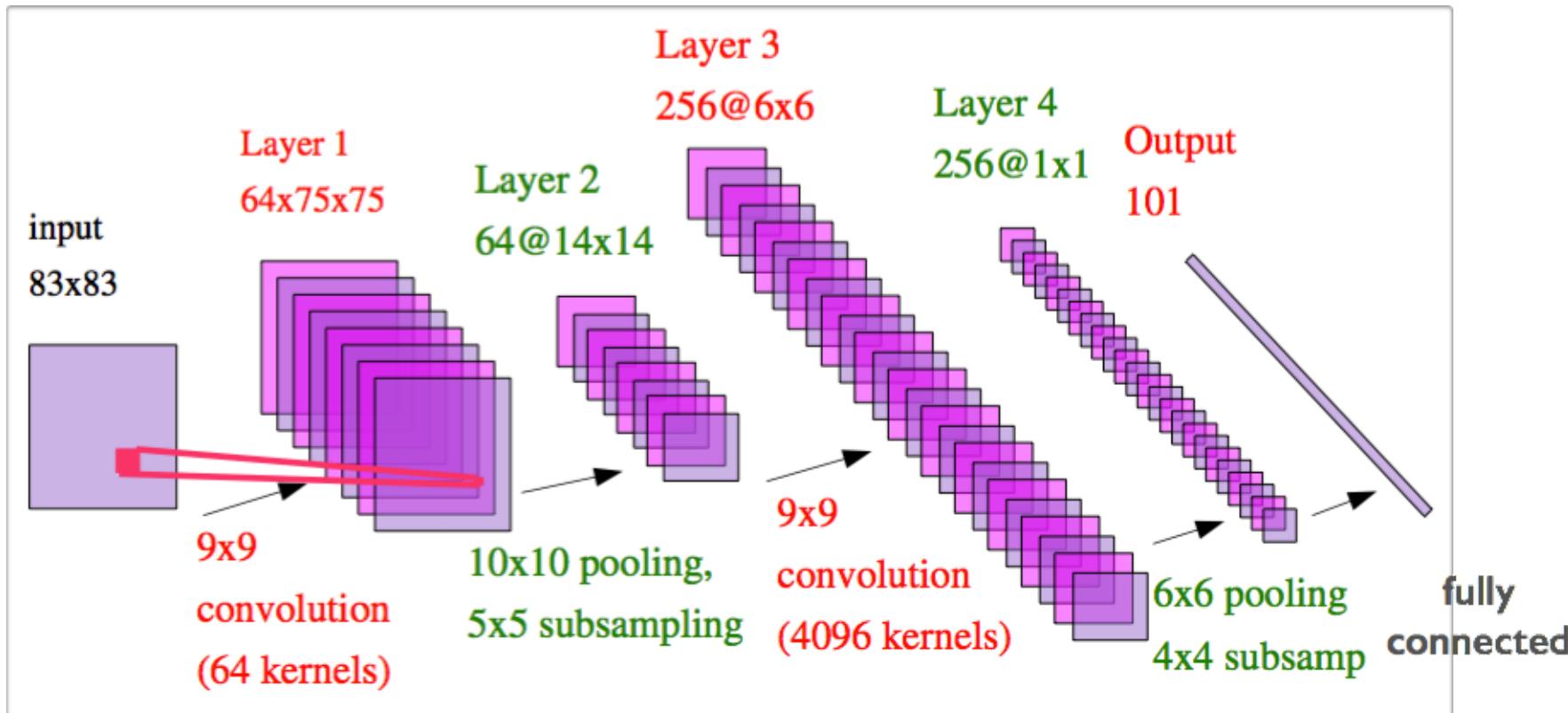
# Translation Invariance

- Illustration of local translation invariance
  - both images result in the same feature map after pooling/subsampling



# Convolutional Neural Network

- Convolutional neural network alternates between the convolutional and pooling layers



From Yann LeCun's slides

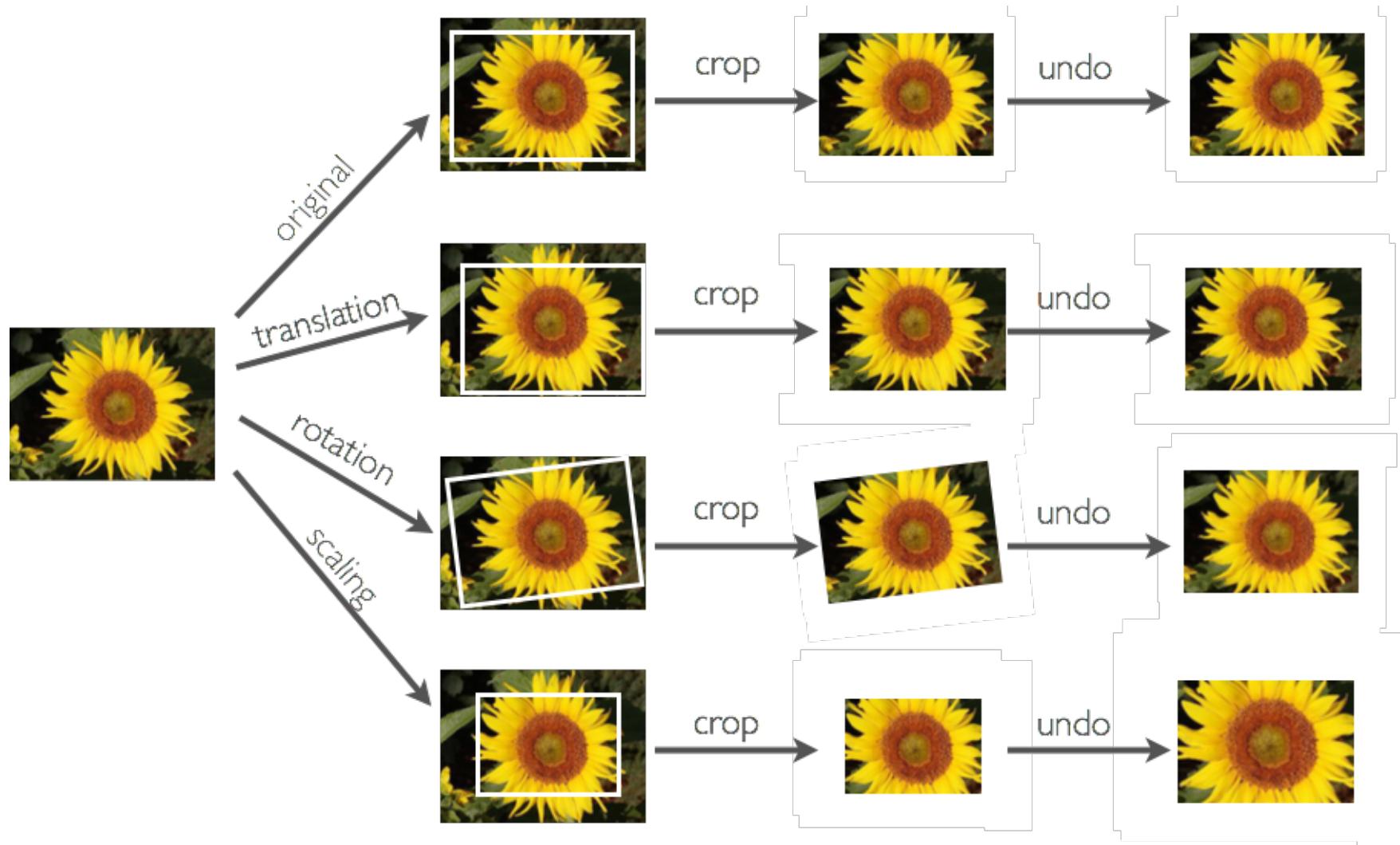
# Convolutional Neural Network

- For **classification**: Output layer is a regular, fully connected layer with softmax non-linearity
  - Output provides an estimate of the conditional probability of each class
- The network is trained by **stochastic gradient descent**
  - Backpropagation is used similarly as in a fully connected network
  - We have seen how to pass gradients through element-wise activation function
  - We also need to pass gradients through the convolution operation and the pooling operation

# Invariance by Dataset Expansion

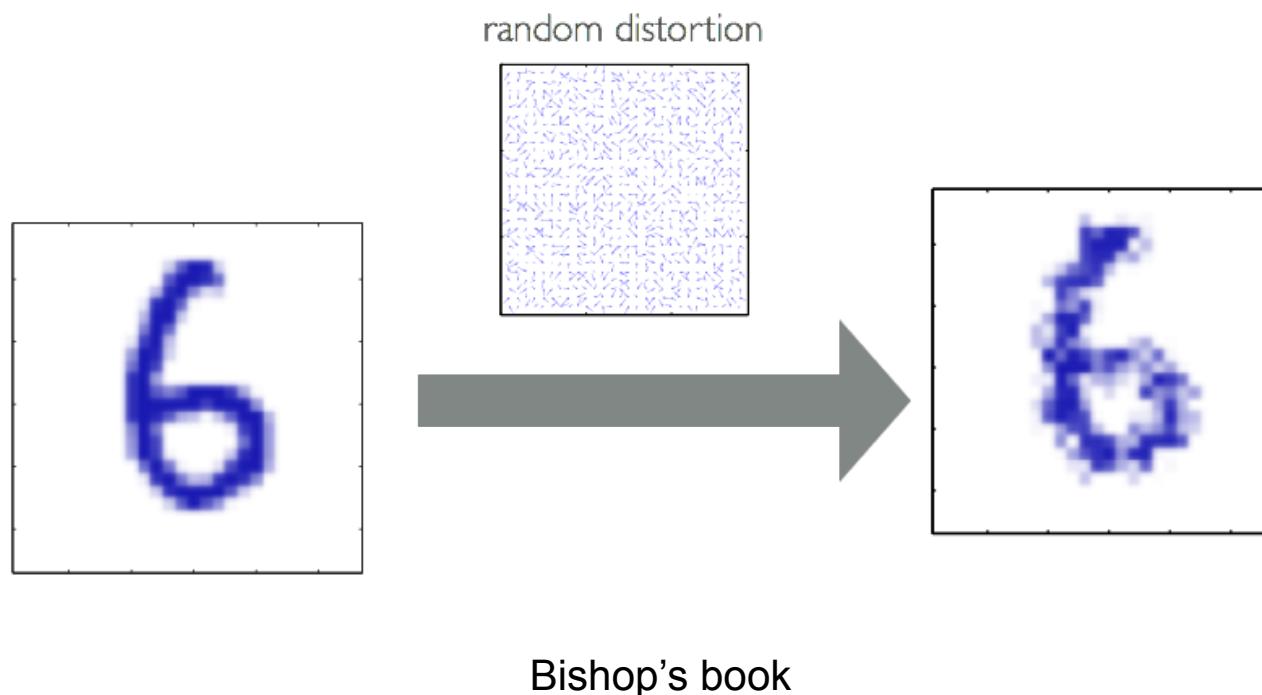
- Invariances built-in in convolutional network:
  - small translations: due to convolution and max pooling
  - small illumination changes: due to local contrast normalization
- It is not invariant to other important variations such as rotations and scale changes
- However, it's easy to artificially generate data with such transformations
  - could use such data as additional training data
  - neural network can potentially learn to be invariant to such transformations

# Generating Additional Examples



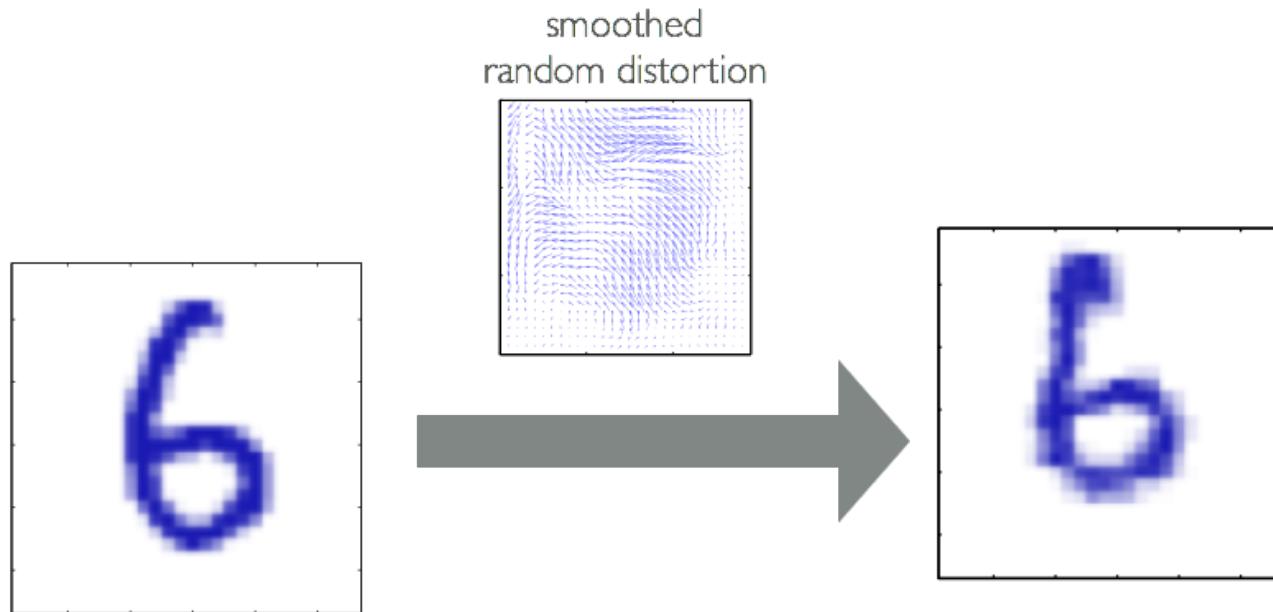
# Elastic Distortions

- Can add “elastic” deformations (useful in character recognition)
- We can do this by applying a “distortion field” to the image
  - a distortion field specifies where to displace each pixel value



# Elastic Distortions

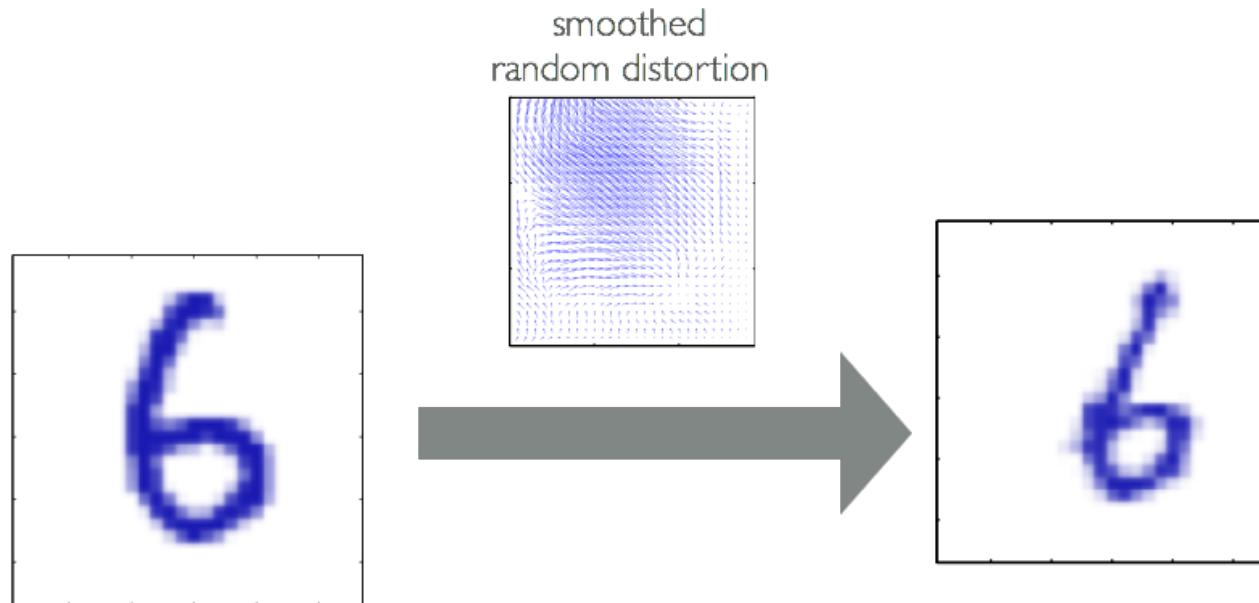
- Can add “elastic” deformations (useful in character recognition)
- We can do this by applying a “distortion field” to the image
  - a distortion field specifies where to displace each pixel value



Bishop's book

# Elastic Distortions

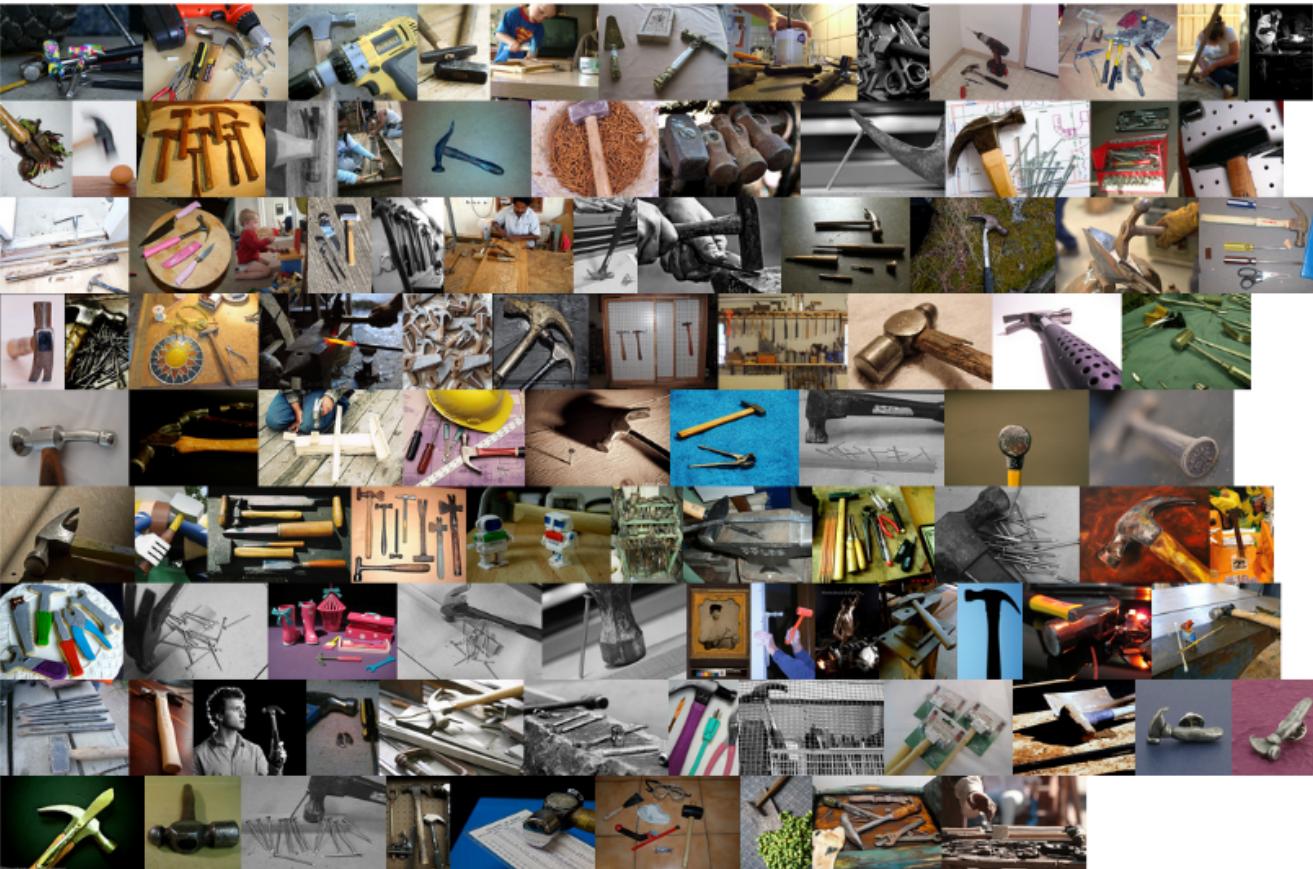
- Can add “elastic” deformations (useful in character recognition)
- We can do this by applying a “distortion field” to the image
  - a distortion field specifies where to displace each pixel value



# ImageNet Dataset

- 1.2 million images, 1000 classes

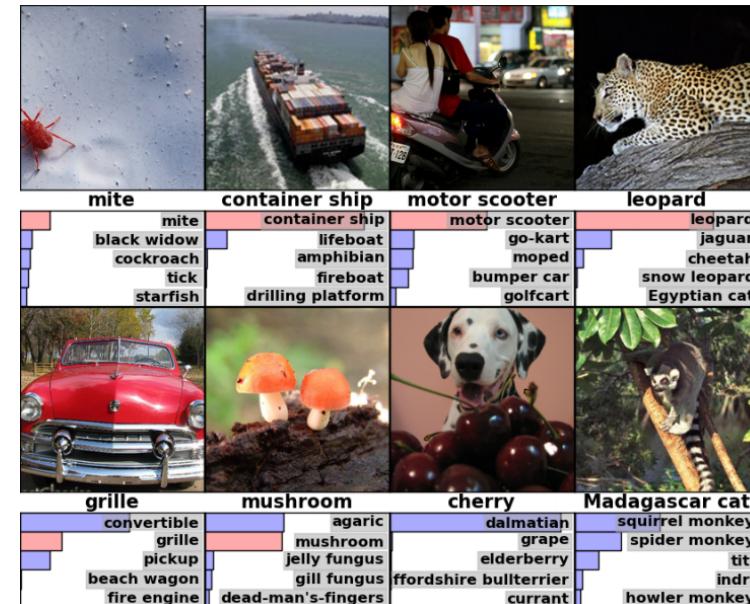
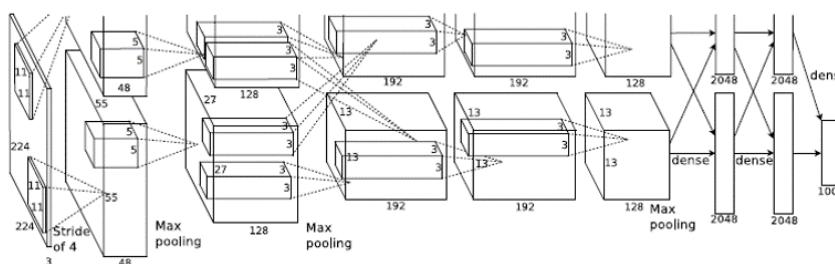
Examples of Hammer



# Important Breakthroughs

- Deep Convolutional Nets for Vision (Supervised)

Krizhevsky, A., Sutskever, I. and Hinton, G. E., ImageNet Classification with Deep Convolutional Neural Networks, NIPS, 2012.



IMAGENET

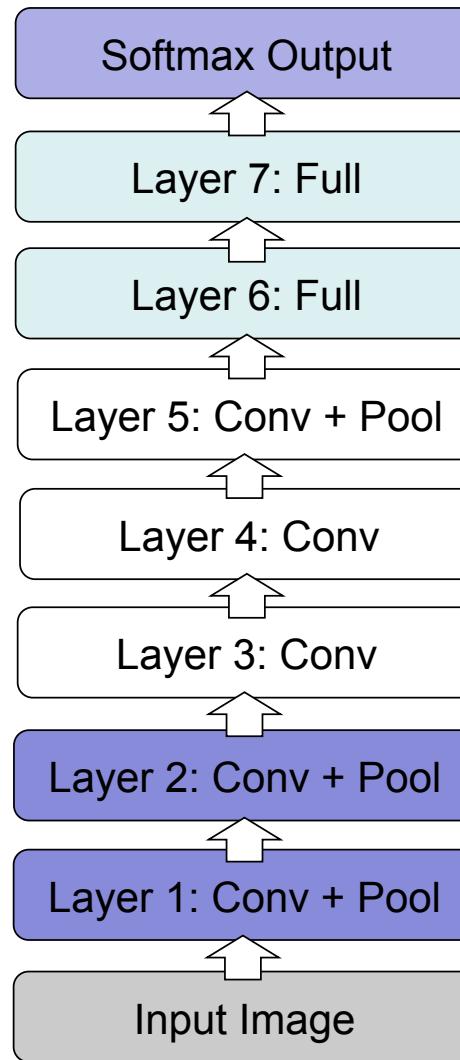
1.2 million training images  
1000 classes

# How to Select the Right Architecture?

- From manual tuning features => manual tuning architectures
- Many hyper-parameters
  - Number of layers (depth), number of feature maps (width)
- Cross validation
- Grid search (need lots of GPUs)
- Smarter Strategies
  - Random search
  - Bayesian optimization
  - Reinforcement Learning (Zoph et al. 2016)

# AlexNet

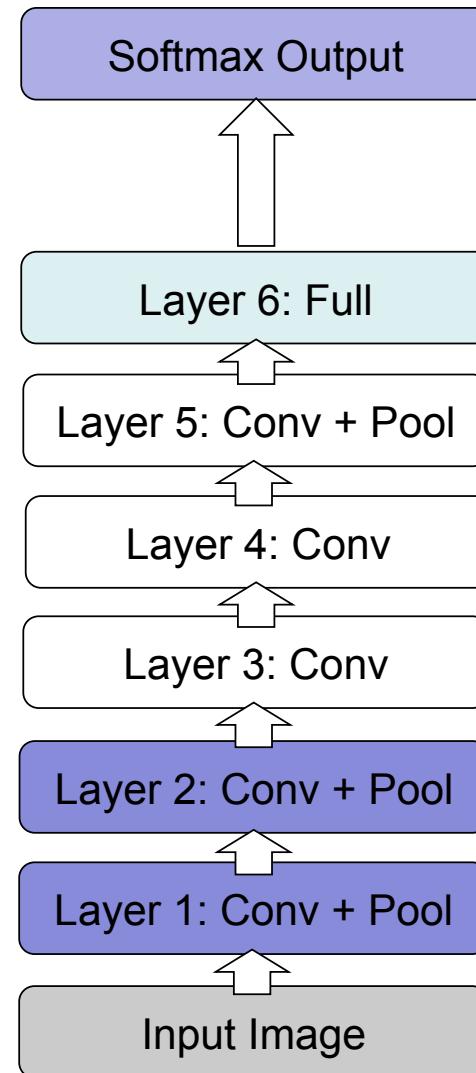
- 8 layers total
- Trained on Imagenet dataset [Deng et al. CVPR'09]
- 18.2% top-5 error



[From Rob Fergus' CIFAR 2016 tutorial]

# AlexNet

- Remove top fully connected layer 7
- Drop ~**16 million** parameters
- Only 1.1% drop in performance!



[From Rob Fergus' CIFAR 2016 tutorial]

# AlexNet

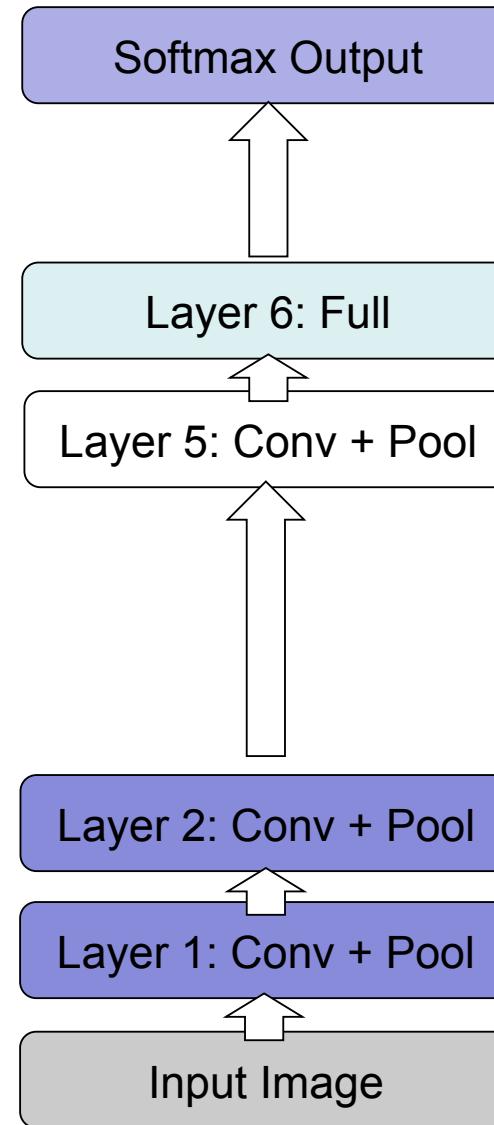
- Let us remove upper feature extractor layers and fully connected:

- Layers 3,4, 6 and 7

- Drop ~50 million parameters

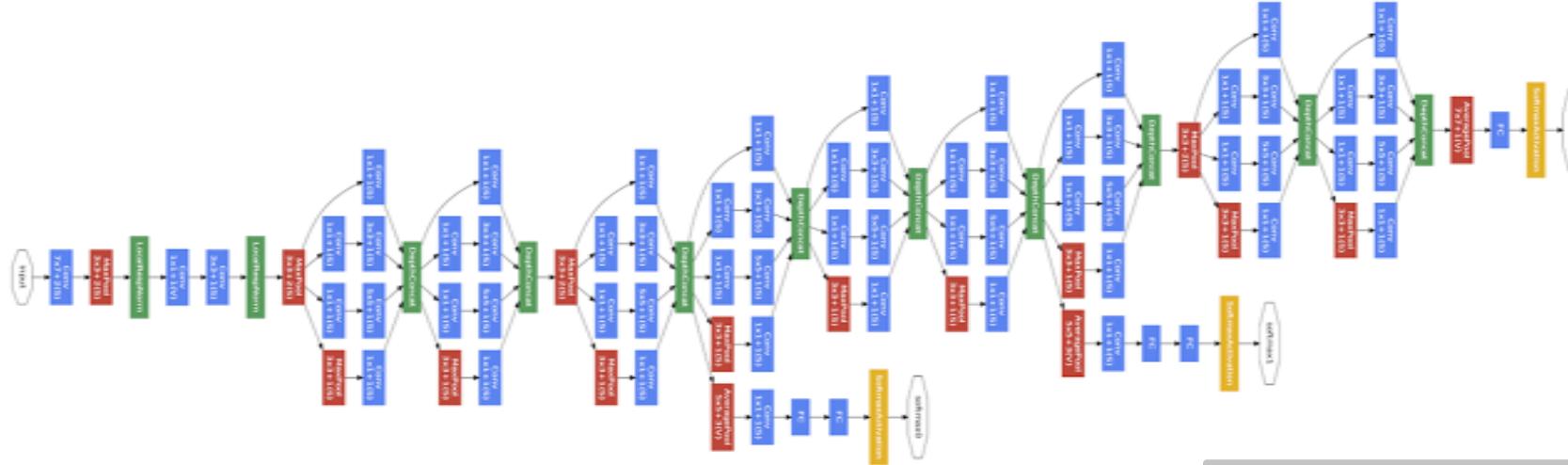
- **33.5 drop in performance!**

- Depth of the network is the key.



[From Rob Fergus' CIFAR 2016 tutorial]

# GoogLeNet



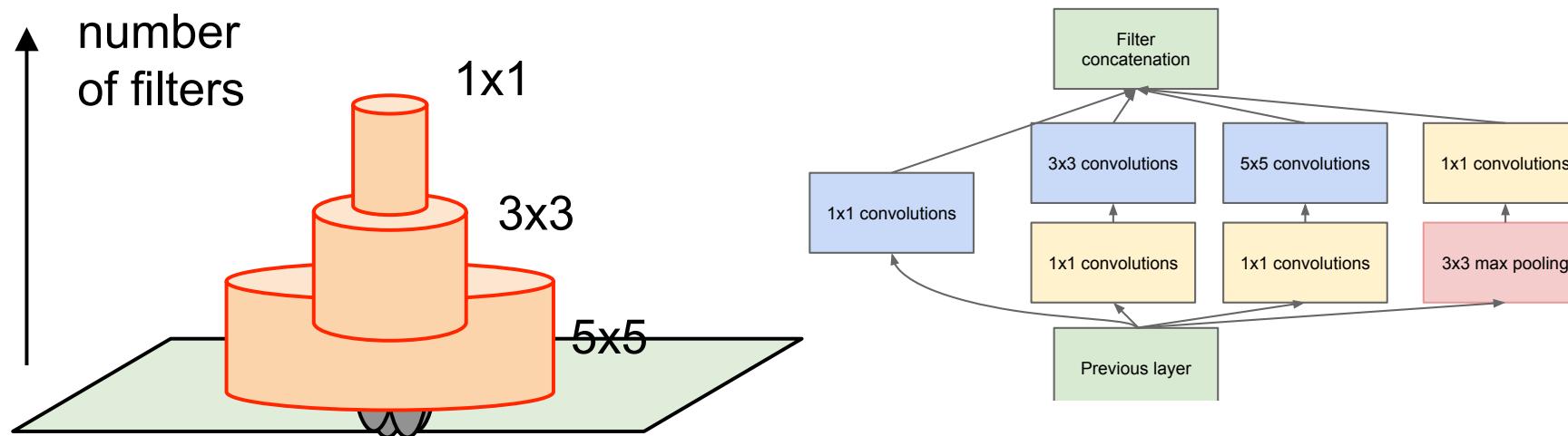
- 24 layer model that uses so-called inception module.

Convolution  
Pooling  
Softmax  
Other

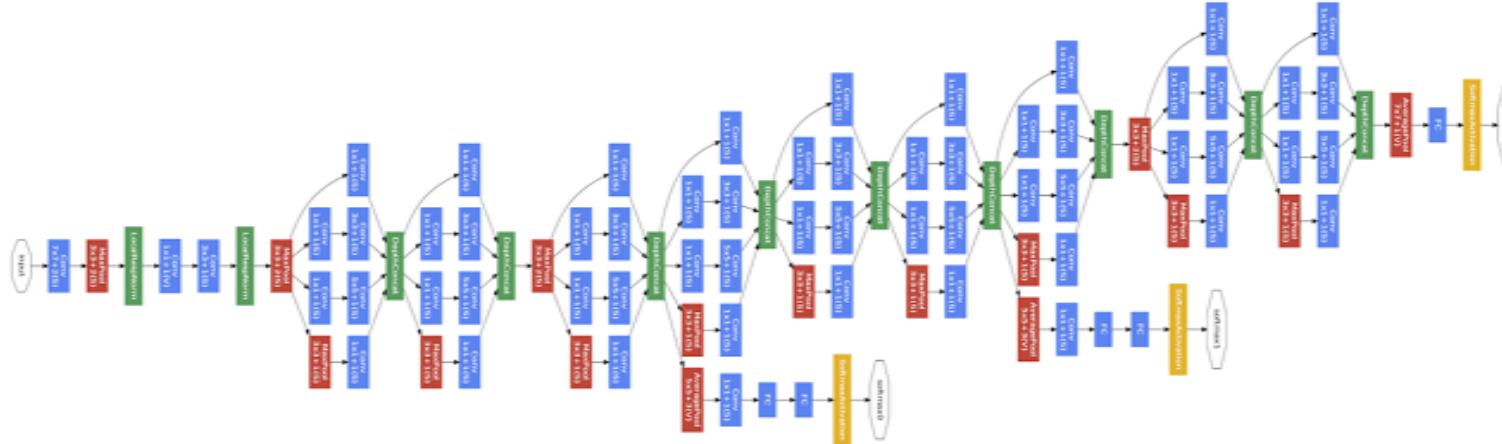
# GoogLeNet

- GoogLeNet inception module:

- Multiple filter scales at each layer
- Dimensionality reduction to keep computational requirements down



# GoogLeNet

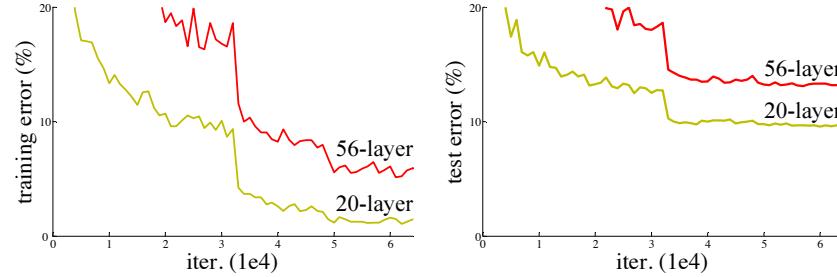


- Width of inception modules ranges from 256 filters (in early modules) to 1024 in top inception modules.
- Can remove fully connected layers on top completely
- Number of parameters is reduced to 5 million
- 6.7% top-5 validation error on Imagnet

[Going Deep with Convolutions, Szegedy et al., arXiv:1409.4842, 2014]

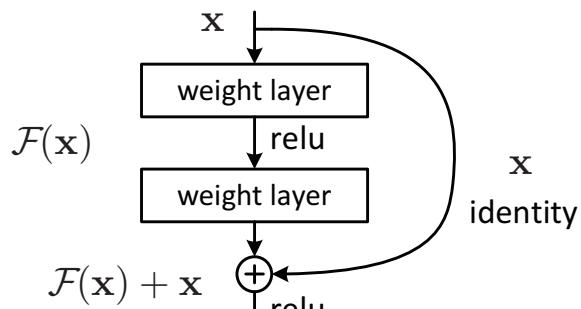
# Residual Networks

Really, really deep convnets do not train well,  
E.g. CIFAR10:



Key idea: introduce “pass through” into each layer

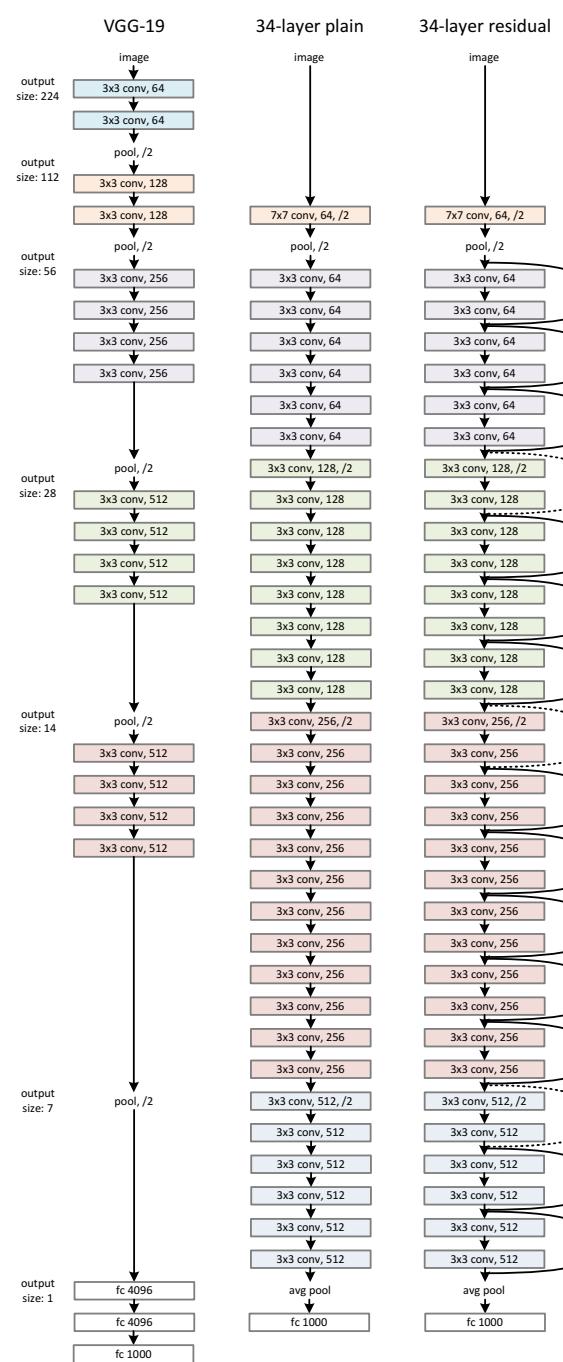
Thus only residual now  
needs to be learned



method	top-1 err.	top-5 err.
VGG [41] (ILSVRC’14)	-	8.43 <sup>†</sup>
GoogLeNet [44] (ILSVRC’14)	-	7.89
VGG [41] (v5)	24.4	7.1
PReLU-net [13]	21.59	5.71
BN-inception [16]	21.99	5.81
ResNet-34 B	21.84	5.71
ResNet-34 C	21.53	5.60
ResNet-50	20.74	5.25
ResNet-101	19.87	4.60
ResNet-152	<b>19.38</b>	<b>4.49</b>

Table 4. Error rates (%) of single-model results on the ImageNet validation set (except <sup>†</sup> reported on the test set).

With ensembling, 3.57% top-5  
test error on ImageNet



# Selecting the Architecture

- Task dependent
- Cross-validation
- [Convolution → pooling]\* + fully connected layer
- The more data: the more layers and the more kernels
  - Look at the **number of parameters** at each layer
  - Look at the **number of flops** at each layer
- Computational resources

# Optimization Tricks

- SGD with momentum, batch-normalization, and dropout usually works very well
- Pick learning rate by running on a subset of the data
  - Start with large learning rate & divide by 2 until loss does not diverge
  - Decay learning rate by a factor of ~100 or more by the end of training
- Use ReLU nonlinearity
- Initialize parameters so that each feature across layers has similar variance. Avoid units in saturation.

# Improve Generalization

- Weight sharing (greatly reduce the number of parameters)
- Data augmentation (e.g., jittering, noise injection, etc.)
- Dropout
- Weight decay (L2, L1)
- Sparsity in the hidden units
- Multi-task (unsupervised learning)

# References

- Chapter 9, deep learning book