

## Problem 1

1.

By definition, 2-D Gaussian kernel is the cross product of 2 1-D Gaussian kernels. If we denote 1-D Gaussian kernel as  $H$ , then 2-D Gaussian kernel is  $H \times H'$ .

Hence for image  $I$ ,  $I * (H \times H') = I * H * H' = I * H' * H$

2.

The Sobel kernel is spatially separable.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \times [1 \ 2 \ 1]$$

3.

For an  $a \times b$  image and  $c \times d$  kernel, for each pixel there are  $cd$  multiplications. In total there are  $abcd$  multiplications.

For an  $a \times b$  image and  $c \times 1$  kernel, for each pixel there are  $cd$  multiplications. In total there are  $abc$  multiplications. Convolute the resulting  $a \times b$  image with  $1 \times d$  kernel, there will be  $abd$  multiplications.

Since it's likely that  $abcd > ab(c+d)$ , separable convolution is preferred.