Mathematical Solutions and Python Simulations

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System Explanations

Methods:

1) Old: Exp(Black) = 3%;
2) New Reset Threshold: If the 32 consecutively draws are none black, the 33th is black
3) New Fixed Threshold: For every 33 draws, the 33th draw is black regardless the outcomes of previous draws.

Closed Form Solutions

1) Old Method

Bionomial Distribution, so $Exp = np$, and $SD = \sqrt(np(1-p))$.

```
Exp = 0.03
SD = sqrt(0.03*0.97)
SD
```

```
## [1] 0.1705872
```

2) New Reset Threshold

Consider the expected draws needed to get the next black is $\sum_1^{32} 0.97^i * 0.03 * i + 0.97 * 1 * 33$.

```
i = seq(1,32)
draws_32 = (0.97^i)*0.03*i
draws_33 = 0.97^32*1*33
```

Deeded draws are

```
sum(draws_32,draws_33)
```

```
## [1] 20.87325
```

Thus, the expectation is

```
1/sum(draws_32,draws_33)
```

```
## [1] 0.04790822
```

3) New Fixed Threshold

Suppose there are 33 draws. The first 32 draws have $Exp(black) = 0.03$, and the 33th draw's Exp is 1. The $Var = E(x^2) - E(x)$, where $E(x^2) = 32 * p * (1 - p + 1)$.

```r
Exp = ( 32*0.03 + 1 )/33
SD = sqrt( (32*0.03*(0.97+1)-Exp^2)/33 )
```

Thus, the results are

```r
Exp
```

```
## [1] 0.05939394
```

```r
SD
```

```
## [1] 0.2391698
```

Python Simulation

For 10^7 Draws, the result is

```python
import os
os.system('python draw_system.py')
```

```
## number of draws: 1e+07
## method_old run time: 1.0547 seconds
## method_reset run time: 1.1338 seconds
## method_fix run time: 1.0357 seconds
##        Expectation        SD
## old       0.029880  0.170257
## reset     0.047537  0.212785
## fixed     0.059485  0.236531
```