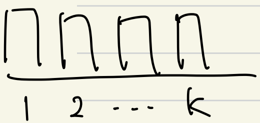
5 離散型分布

○ 一様分布

$$E[X] = \sum_x x p(x) = 1 \cdot \frac{1}{k} + 2 \cdot \frac{1}{k} \cdots + \frac{k}{k} = \frac{1}{k} \cdot \frac{k(k+1)}{2} = \frac{k+1}{2}$$

$$E[X^2] = \frac{1}{k}\left(1+4+9 \cdots + k^2\right) = \frac{1}{k} \cdot \frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(2k+1)}{6}$$



$$V[X] = \frac{2(k+1)(2k+1)}{12} - \frac{3(k+1)^2}{12} = \frac{(k+1)(4k+2-3k-3)}{12} = \frac{k^2-1}{12}$$

○ ベルヌーイ分布 成功確率 p $Bin(1, p)$

$$P(X=x) = p^x (1-p)^{1-x}$$

$$E[X] = \sum_x x\, p(x) = 1 \cdot p + 0 \cdot (1-p) = p$$

$$E[X^2] = \sum_x x^2 p(x) = p$$

$$V[X] = p - p^2 = pq$$

○ 二項分布 $Bin(n, p)$ $Y = X_1 + X_2 + \cdots + X_n$

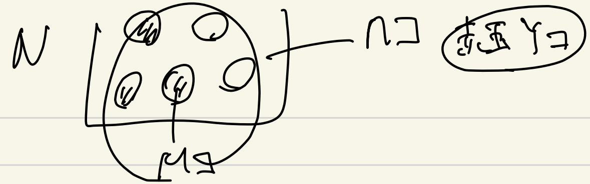
$$E[Y] = E[X_1] + E[X_2] + \cdots + E[X_n] = np$$

$$V[Y] = npq$$

$$E[s^Y] = E[s^{X_1} s^{X_2} \cdots] = E[s^{X_1}] \times E[s^{X_2}] \cdots = (ps+q)^n$$

再生性

○超幾何分布



- ○復元抽出　$Bin\left(n, \frac{M}{N}\right)$
- ○非復元抽出　$HG(N, M, n)$　　$P(Y=y) = \dfrac{{}_M C_y \times {}_{N-M} C_{n-y}}{{}_N C_n}$

○ポアソン分布

$Po(\lambda)$: $P(Y=y) = \dfrac{\lambda^y}{y!} e^{-\lambda}$

$$\begin{cases} E[Y] = \lambda \\ V[Y] = \lambda \end{cases}$$

○幾何分布

$Geo(p)$　　$P(X=x) = (1-p)^x \cdot p$　q

$$G(s) = E[s^X] = \sum_x s^x \cdot p(x) = p + s(1-p)p + s^2(1-p)^2 p \cdots$$

$$= p \cdot \frac{1}{1-sq} = \frac{p}{1-qs}$$

$$G'(s) = E[X \cdot s^{X-1}] = \frac{pq}{(1-qs)^2} \qquad G'(1) = E[X] = \frac{q}{p}$$

$$G''(s) = E[\underbrace{X(X-1)}_{X^2 - X} s^{X-2}] = +\frac{pq \cdot \cancel{(1-qs)} \cdot 2}{(1-qs)^{\cancel{4}\,3}} = \frac{2pq^2}{(1-q)^3 p^3} = \frac{2q^2}{p^2}$$

$$V[X] = \frac{2q^2}{p^2} + \frac{pq}{p^2} - \frac{q^2}{p^2} = \frac{q^2+pq}{p^2} = \frac{q}{p^2}$$

○ <u>負の二項分布</u> $NB(r.p)$

<u>幾何分布</u>を独立にnコ足した分布 → $\begin{cases} E[X] = r \times \frac{q}{p} \\ V[X] = r \times \frac{q^2}{p} \end{cases}$

○ <u>多項分布</u>

式追えない

例題

問5.1

$$\beta = 1-(1-p)^n$$

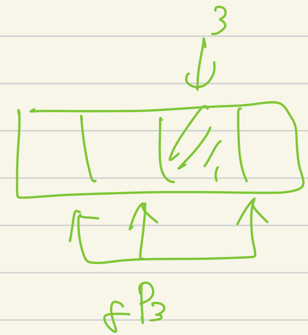
$$(1-p)^n = 0.02$$

$$-np = -3.9$$

$$Y \sim Bin(n, p)$$

$$n = 5000 \times 3.9$$

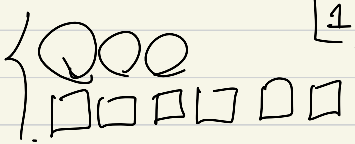
$$= \underline{19500}$$



問5.2

$$P(X=x) = \frac{{}_{40}C_x \cdot {}_{39}C_{25-x}}{{}_{79}C_{25}}$$

問5.3



[1]

$$E[X_i^2] = 0^2 \cdot P(X_i=0) + 1^2 \cdot P(X_i=1) = P(X_i=1)$$

$$= \boxed{\frac{{}_8P_3 \times 3}{{}_9P_4}} \quad ? \quad = \underline{\frac{1}{3}}$$

[2]

$$E[X_i X_j] = 1 \cdot \left(P(X_i = 1) \text{ かつ } P(X_j = 1) \right) + \underbrace{0 \cdots}_{0}$$

$$= \frac{{}_7P_2 \cdot {}_3P_2}{{}_9P_4} = \frac{1}{12}$$

$$\frac{1}{12} - \frac{1}{9} = -\frac{1}{36}$$

$$E[X_i X_j] - E[X_i]E[X_j]$$

[3] $E[X_i] = \frac{1}{3}$ $\qquad V[X_i] = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$

$$V = \frac{1}{16} V[X_1 + X_2 + X_3 + X_4]$$

$$\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$$

$$= \frac{1}{16} \left\{ V[X_1] + V[X_2] + V[X_3] + V[X_4] + \sum_{(12)} \mathrm{cov}[X_i . X_j] \right\} = \frac{5}{144}$$

問5.4

$X \sim Po(1.5)$

$Y \sim Po(3)$

$X \sim Po(\lambda)$

$P(x) = \dfrac{\lambda^x}{x!} e^{-\lambda}$

$E[X] = \lambda$

$V[X] = \lambda$

$Bin\left(5, \dfrac{1}{3}\right)$

$\dfrac{a}{a+b}$

$\boxed{\left(\dfrac{a}{a+b}\right)^x \left(\dfrac{b}{a+b}\right)^{5-x}}$

ア $1.5 + 3 = \underline{4.5}$

イ $X+Y \sim Po(4.5)$

$$P(X = x \mid X+Y = 5) = \frac{P(X=x, Y=5-x)}{P(X+Y=5)} = \frac{\dfrac{(1.5)^x}{x!} \cdot \dfrac{(3)^{5-x}}{(5-x)!}}{\dfrac{(4.5)^5}{5!} \cdot \cancel{e^{-4.5}}}$$

$$= \underbrace{\frac{5!}{x!(5-x)!}}_{{}_5C_x} \cdot \frac{(1.5)^x (3)^{5-x}}{(4.5)^x}$$

$np = 5 \times \dfrac{1}{3} = \underline{1.67}$

問5.5

[1] カード n 種類　k 種類　$k+1$ 種類目　$P_k = \frac{n-k}{n}$　　$X \sim Geo(p)$

$\frac{1}{P_k} = \frac{n}{n-k}$

$n=5$
$k=4$　　$\frac{5}{1} = 5$

$$\sum_{k=0}^{n-1} \frac{n}{n-k} = \sum_{k=0}^{3} \left(\frac{4}{4-0} + \frac{4}{4-1} + \frac{4}{4-2} + \frac{4}{4-3}\right)$$

$$= 4\left(\underbrace{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}_{\frac{12+6+4+3}{12}}\right) = \frac{25}{3}$$

[2] $y = \frac{137}{12}$

$x = \boxed{\frac{25}{3}} + 5 = \frac{40}{3}$