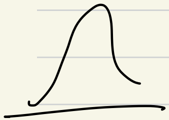
$\boxed{8}$ 統計的推定の基礎



θ

$x_1, x_2 \cdots, x_n$

$\Downarrow$

$\hat{\theta} = h(x_1, x_2 \cdots x_n)$

点推定　　統計量

尤度関数

$$L(\theta) = \prod_{i=1}^{n} f(x_i : \theta)$$

$$l(\theta) = \log L(\theta) = \sum_{i=1}^{n} \log f(x_i : \theta)$$

例2　対数尤度　$\bar{x}$, s

$$l(\mu, u) = \log \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$= \log\left(\left(\frac{1}{\sqrt{2\pi u}}\right)^n - \frac{1}{2u}\sum_{i=1}^{n}(x_i-\mu)^2\right)$$

$$= -\frac{n}{2}\log(2\pi u) - \frac{1}{2u}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\sum_{i=1}^{n}(x_i-\mu)^2$$

$$= \sum_{i=1}^{n}\left((x_i-\bar{x}) + (\bar{x}-\mu)\right)^2$$

$$= \sum_{i=1}^{n}(x_i-\bar{x})^2 + \cancel{n2(\bar{x}-\mu)\left(\sum_{i=1}^{n} x_i - n\bar{x}\right)} + n(\bar{x}-\mu)^2$$

$$\bar{x} = \mu$$

$$l(\mu, v) = -\frac{n}{2}\log(2\pi v) - \frac{1}{2v}\sum_{i=1}^{n}(x_i-\mu)^2$$

標本分散

$$S = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

$$= -\frac{n}{2}\log(2\pi v) - \frac{nS}{2v}$$

vで微分

$$l(\mu, v)' = \frac{-n\,\cancel{2\pi} v}{2\cdot\cancel{2\pi} v^2} + \frac{nS}{2}\cdot\frac{1}{v^2}$$

$$-nv = -nS \qquad v = S$$

$\boxed{例}$ モーメント法

$$\theta_j = g_j(\mu_1 \cdots \mu_k) \qquad j = 1, \cdots, m$$

例43 平均 $\mu_1 = \dfrac{\alpha}{\lambda}$

分散 $\mu_2 = \dfrac{\alpha}{\lambda^2}$

$$\begin{cases} \alpha = \dfrac{\mu_1^2}{\mu_2} \\ \lambda = \dfrac{\mu_1}{\mu_2} \end{cases}$$

$$\hat{\alpha} = \frac{\bar{x}^2}{S}$$

$$\hat{\lambda} = \frac{\bar{x}}{S}$$

$\boxed{証}$

$E_\theta[\hat{\theta}] = \theta$ ： 不偏推定量

$b_\theta(\hat{\theta}) = E_\theta[\hat{\theta}] - \theta$

$$E[(\hat{\theta}-\theta)^2] = (\underbrace{E[\hat{\theta}] - \theta}_{b_\theta(\hat{\theta})})^2 + V_\theta[\hat{\theta}]$$

バイアス　　バリアンス

一様最小分散不偏推定量 ← クラーメル・ラオの不等式

$V_\theta[\hat{\theta}] \geq J_n(\theta)^{-1}$

満たしたら．有効推定量．

フィッシャー情報量の逆数

フィッシャー情報量

$$J_n(\theta) = E_\theta\left[\left(\frac{\partial}{\partial\theta}\log f(x_1 \dots x_n ; \theta)\right)^2\right]$$

(4) 十分統計量

$$P(X=x \mid T(X)=t, \theta) = P(X=x \mid T(X)=t)$$

θ の十分統計量

$$f(x;\theta) = h(x) \times \boxed{g(T(x), \theta)}$$

十分統計量のみを含む

∴ フィッシャー・ネイマンの分解定理

$$\hat{\theta}_{jack} = n\hat{\theta} - (n-1)\hat{\theta}_{(\cdot)}$$

ジャックナイフ推定量

リサンプリング法

例題

問8.1

①, ②, ④

問8.2 ポアソン分布 $f(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

[1] 尤度関数 $L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$$\log L(\lambda) = n \log(e^{-\lambda}) + \sum_{i=1}^{n} \log \lambda^{x_i} + \sum_{i=1}^{n} \log(x_i!)$$

$$= \sum_{i=1}^{n} x_i \log \lambda - n\lambda + \sum_{i=1}^{n} \log(x_i!)$$

① $\frac{n^2 \cdot \bar{x}}{\lambda} - n = 0$ $\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i$

[2]

$$E\left[\sum_{i=1}^{n} X_i\right] = n\lambda$$

$$J_n(\lambda) = -E_\lambda\left[\underbrace{\frac{d^2}{d\lambda^2}\log L(\lambda)}\right] = \underbrace{\frac{n}{\lambda}}$$

$$\left(\frac{\sum_{i=1}^{n} x_i}{\lambda} - n\right)' = -\frac{\sum_{i=1}^{n} x_i}{\lambda^2}$$

[3]

$$J_n(\lambda)^{-1} = \frac{\lambda}{n}$$

$$V\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}V[X_i] = \underbrace{\frac{\lambda}{n}}$$

=

問ふ3

ばりむずい