第9章 区間推定

$$X \sim N(\mu, \sigma^2)$$

既知

推定

$$x_1, x_2 \cdots x_n$$

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$U = \frac{\bar{x} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$$

25% 25%

-1.96 1.96

(9.2)

$$P\left(\bar{X} - 1.96\sqrt{\frac{\sigma^2}{n}} \leqq \mu \leqq \bar{X} + 1.96\sqrt{\frac{\sigma^2}{n}}\right) = 0.95$$
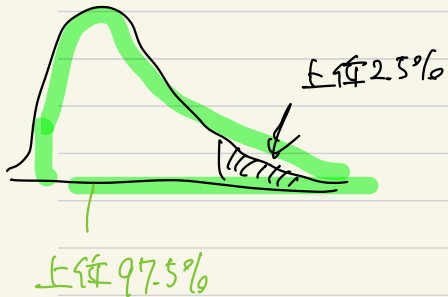
☆ 母平均 μ が ☐ の区間に含まれるかくりつが 95%

信頼区間

① 分散の 区間推定.

$N(\underline{\underline{\mu}}, \underline{\underline{\sigma^2}})$
未知（推定）

$X_1, X_2, \cdots, X_n$
$\overline{X}$

$T^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2$

$\chi^2 = \dfrac{T^2}{\sigma^2}$ 　自由度 $n-1$

$\overline{X}$ 固定だから
$x_1 + x_2 \cdots = n\overline{X}$
回 × コテイ

上位 2.5%

上位 97.5%

$P\left(\chi^2_{0.975}(n-1) \leq \chi^2 \leq \chi^2_{0.025}(n-1) \right) = 0.95$

$P\left(\dfrac{T^2}{\chi^2_{0.975}(n-1)} \leq \sigma^2 \leq \dfrac{T^2}{\chi^2_{0.025}(n-1)} \right) = 0.95$

② 分散の 比の 区間推定.

① $\sim N(\mu_1, \sigma_1^2)$

② $\sim N(\mu_2, \sigma_2^2)$

$F = \dfrac{V_1 / \sigma_1^2}{V_2 / \sigma_2^2}$

$P\left(\dfrac{V_1}{V_2} \cdot \dfrac{1}{F_{0.025}()} \leq \dfrac{\sigma_1^2}{\sigma_2^2} \leq \dfrac{V_1}{V_2} F \right)$

自由度 (n_1-1, n_2-1)

◎ 多項分布の 信頼区間

$$P \left(\hat{P_i} - 1.96 \sqrt{\frac{\hat{P_i}(1-\hat{P_i})}{n}} \leq P_i \leq \hat{P_i} + 1.96 \sqrt{\frac{\hat{P_i}(1-\hat{P_i})}{n}} \right) = 0.95$$
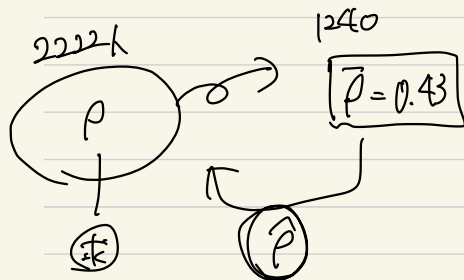
◎ 多項分布 の差の 信頼区間

$$Cov[N_1, N_2] = \underbrace{E[N_1, N_2]}_{n(n-1)P_1 P_2} - \underbrace{E[N_1] E[N_2]}_{n^2 P_1 P_2} = -n P_1 P_2$$

$$E[\hat{P_1} - \hat{P_2}] = \frac{n P_1}{n} - \frac{n P_2}{n} = P_1 - P_2$$

$$V[\hat{P_1} - \hat{P_2}] = \frac{P_1(1-P_1)}{n} + \frac{P_2(1-P_2)}{n} + \frac{2 P_1 P_2}{n}$$

例題

例9.1

2222人 → 1240

ρ

求

$\hat{\rho}$

$\bar{\rho} = 0.43$

$\bar{\rho} - 1.96 \sqrt{\dfrac{\bar{\rho}(1-\bar{\rho})}{n}} \le \hat{\rho} \le \bar{\rho} + 1.96 \sqrt{\dfrac{\bar{\rho}(1-\bar{\rho})}{n}}$

$0.43 \pm \boxed{1.96 \times 0.014}$

$1.96 \times \sqrt{\dfrac{\bar{\rho}(1-\bar{\rho})}{n}} = 0.01$

$196 \times \sqrt{\bar{\rho}(1-\bar{\rho})} = \sqrt{n}$ $\underline{9220}$

例9.2

(1) 上に同じ

(2) $\rho_1 = 0.26$
    $P_2 = 0.08$

$(\hat{P_1} - \hat{P_2}) \pm 1.96 \sqrt{\dfrac{\hat{P_1}(1-\hat{P_1})}{n} + \dfrac{P_1(1-P_1)}{n} + \dfrac{2P_1P_2}{n}}$