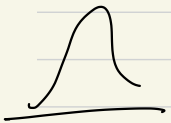
8 統計的推定の基礎



$x_1, x_2 \cdots, x_n$

$\Downarrow$

$\hat{\theta} = h(x_1, x_2 \cdots x_n)$

点推定　　統計量

尤度関数

$$L(\theta) = \prod_{i=1}^{n} f(x_i ; \theta)$$

$$\ell(\theta) = \log L(\theta) = \sum_{i=1}^{n} \log f(x_i ; \theta)$$

例2　対数尤度　　$\bar{x}$, s

$$\ell(\mu, \sigma) = \log \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$= \log \left(\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n - \frac{1}{2\sigma} \sum_{i=1}^{n} (x_i - \mu)^2 \right.$$

$$= -\frac{n}{2} \log(2\pi\sigma) - \frac{1}{2\sigma} \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\sum_{i=1}^{n} (x_i - \mu)^2$$

$$= \sum_{i=1}^{n} (x_i - \bar{x} + \bar{x} - \mu)^2$$

$$= \sum_{i=1}^{n} (x_i - \bar{x})^2 + n2(\bar{x} - \mu)\left(\sum_{i=1}^{n} x_i - n\bar{x}\right) + n(\bar{x} - \mu)^2$$

$\bar{x} = \mu$

$$\ell(\mu,\sigma) = -\frac{n}{2}\log(2\pi\sigma) - \frac{1}{2\sigma}\sum_{i=1}^{n}(x_i-\mu)^2$$

標本分散 $S = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$$= -\frac{n}{2}\log(2\pi\sigma) - \frac{nS}{2\sigma}$$

σ で微分 $\ell(\mu,\sigma)' = \frac{-n\,2\pi\sigma}{2\cdot 2\pi\sigma^2} + \frac{nS}{2}\cdot\frac{1}{\sigma^2}$

$$-n\sigma = -nS \qquad \underline{\sigma = S}.$$

(例43) 平均 $\mu_1 = \frac{\alpha}{\lambda}$

分散 $\mu_2 = \frac{\alpha}{\lambda^2}$

$$\begin{cases} \alpha = \dfrac{\mu_1^2}{\mu_2} & \hat{\alpha} = \dfrac{\bar{x}^2}{S} \\ \lambda = \dfrac{\mu_1}{\mu_2} & \hat{\lambda} = \dfrac{\bar{x}}{S} \end{cases}$$

図

$E_\theta[\hat{\theta}] = \theta$: 不偏推定量

$b_\theta(\theta) = E_\theta[\hat{\theta}] - \theta$

$E[(\hat{\theta} - \theta)^2] = \underbrace{(E[\hat{\theta}] - \theta)^2}_{b_\theta(\theta)} + V_\theta[\hat{\theta}]$

バイアス　　バリアンス

$f(x;\theta) = h(x) \times \boxed{g(T(x), \theta)}$

十分統計量のみを含む

$\hat{\theta}_{jack} = n\hat{\theta} - (n-1)\hat{\theta}_{(\cdot)}$

ジャックナイフ推定量

リサンプリング法

例題

問8.1

① , ② , ④

問8.2 ポアソン分布 $f(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

[1] 尤度関数 $L(\lambda) = \prod_{i=1}^{n} \dfrac{e^{-\lambda} \cdot \lambda^{x_i}}{x_i!}$

$$\log L(\lambda) = n \log\left(e^{-\lambda}\right) + \sum_{i=1}^{n} \log \lambda^{x_i} + \sum_{i=1}^{n} \log(x_i!)$$

$$= \sum_{i=1}^{n} x_i \log \lambda - n\lambda + \sum_{i=1}^{n} \log(x_i!)$$

① $'$ $\dfrac{n^2 \bar{x}}{\lambda} - n = 0$ $\quad \lambda = \dfrac{1}{n} \sum_{i=1}^{n} x_i$

$$E\left[\sum_{i=1}^{n} X_i\right] = n\lambda$$

[2]

$$J_n(\lambda) = -E_\lambda\left[\frac{d^2}{d\lambda^2}\log L(\lambda)\right] = \frac{n}{\lambda}$$

$$\left(\frac{\sum_{i=1}^{n} x_i}{\lambda} - n\right)' = -\frac{\sum_{i=1}^{n} x_i}{\lambda^2}$$

[3]

$$J_n(\lambda)^{-1} = \frac{\lambda}{n}$$

$$V\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n} V[X_1] = \frac{\lambda}{n} \quad (=)$$

問ふ3

ばりむずい