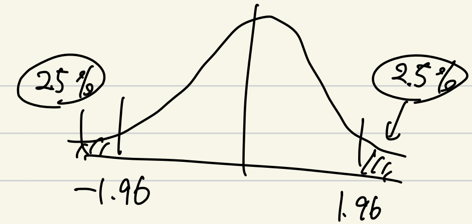
第9章 区間推定



$X \sim N(\mu, \sigma^2)$

推定　既知

$x_1, x_2, \dots, x_n$

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$u = \frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0, 1)$$

(9.2)

$$P\left(\bar{x} - 1.96\sqrt{\frac{\sigma^2}{n}} \leq \mu \leq \bar{x} + 1.96\sqrt{\frac{\sigma^2}{n}}\right) = 0.95$$

☆ 母平均μが $\boxed{\quad}$ の区間に含まれる確率が95%

信頼区間

◎分散の区間推定

$N(\mu,\ \sigma^2)$

未知　　推定

$$\underbrace{X_1,\ X_2,\ \cdots,\ X_n}_{\bar{X}}$$

$$T^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

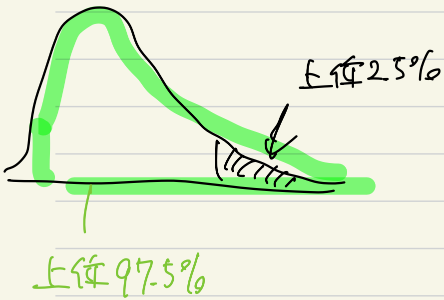
$$\chi^2 = \frac{T^2}{\sigma^2}$$

自由度 $n-1$

$\bar{x}$ 固定だから

$x_1 + x_2 \cdots x_n = n\bar{x}$

自　×　[illegible]



$$P\left(\chi^2_{0.975}(n-1) \le \chi^2 \le \chi^2_{0.025}(n-1)\right) = 0.95$$

$$P\left(\frac{T^2}{\chi^2_{0.975}(n-1)} \le \sigma^2 \le \frac{T^2}{\chi^2_{0.025}(n-1)}\right) = 0.95$$

◎分散の比の区間推定

Ⓐ $\sim N(\mu_1,\ \sigma_1^2)$

Ⓑ $\sim N(\mu_2,\ \sigma_2^2)$

$$F = \frac{V_1 / \sigma_1^2}{V_2 / \sigma_2^2}$$

自由度 $(n_1-1,\ n_2-1)$

$$P\left(\frac{V_1}{V_2} \cdot \frac{1}{F_{0.025}(\ )} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{V_1}{V_2} \frac{1}{F}\right)$$

◎多項分布の信頼区間

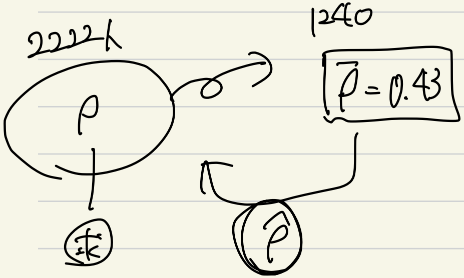
$$P\left(\hat{p}_i - 1.96\sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{n}} \leq p_i \leq \hat{p}_i + 1.96\sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{n}}\right) = 0.95$$

◎多項分布の差の信頼区間

$$\mathrm{Cov}[N_1, N_2] = \underbrace{E[N_1, N_2]}_{n(n-1)p_1p_2} - \underbrace{E[N_1]E[N_2]}_{n^2p_1p_2} = -np_1p_2$$

例題

問9.1



$$\bar{p} - 1.96\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \le \hat{p} \le \bar{p} + 1.96\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\underline{\underline{0.43 \pm \boxed{1.96 \times 0.014}}}$$

$$1.96 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.01$$

$$196 \times \sqrt{\bar{p}(1-\bar{p})} = \sqrt{n}$$

$\underline{\underline{9220}}$

問9.2

(1) 上に同じ

(2) $p_1 = 0.26$

$p_2 = 0.08$

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{p_1(1-p)}{n} + \frac{2p_1p_2}{n}}$$