6 連続型分布と標本分布

◎正規分布. $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ $N(\mu, \sigma^2)$

$\therefore N(0.1)$

↓

標準正規分布

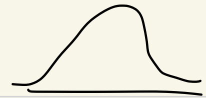
◎指数分布. $f(x) = \lambda e^{-\lambda x}$ $: Exp(\lambda)$ 無記憶性.

◎ガンマ分布. $f(x) = \frac{1}{\Gamma(a)\, b^a} x^{a-1} \cdot e^{-\frac{x}{b}}$

◎ベータ分布 $f(x) = \frac{1}{B(a.b)} x^{a-1} \cdot (1-x)^{b-1}$

◎多変量正規分布. $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$

$$f(x) = \frac{1}{(2\pi)^{\frac{p}{2}} \cdot (\det\Sigma)^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

$Z \sim N(0.1)$ 確率密度関数  $\phi(z)$

← ファイ

累積分布関数  $\Phi(z)$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)\,dt$$

$X \sim N(\mu . \sigma^2)$ のとき $E[X] = \mu$ $V[X] = \sigma^2$

$$M(t) = E[e^{tX}] = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$$

$$P(X \leq x) = P(Z \leq z) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

再現性あり

モーメント母関数 $M(t) = E[e^{tx}]$

$M'(0) = E[x]$

$M''(0) = E[x^2]$

◎カイ二乗分布 $\chi^2(n)$

$Y = Z_1^2 + Z_2^2 + \cdots + Z_n^2$ $Z_i \sim N(0.1)$

$N(\mu.\sigma^2)$

$X_1, X_2, \ldots, X_n$ → $\bar{X}$, S^2

$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0.1)$$

$$\frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2$$

$$\sim \chi^2(n-1)$$

◎t分布

$Z \sim N(0.1)$

$Y \sim \chi^2(n)$

$$T = \frac{Z}{\sqrt{\frac{Y}{n}}}$$ ← 自由度nのt分布.

☆ 分散が未知の正規分布の平均の検定

$$t = \frac{\bar{X}}{\frac{s}{\sqrt{n}}}$$ 自由度n-1

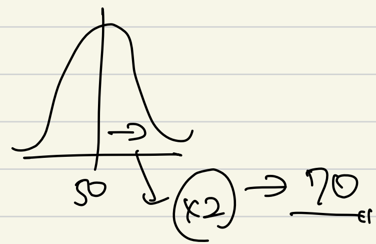
⑪ F分布 $Y_1 \sim \chi^2(n_1)$ $X = \dfrac{Y_1/n_1}{Y_2/n_2}$

$Y_2 \sim \chi^2(n_2)$ 自由度 (n_1, n_2) の F分布.

例題

問6.1

[1] $\dfrac{85-65}{10} = 2$



$\dfrac{60-65}{10} = -0.5$

45点

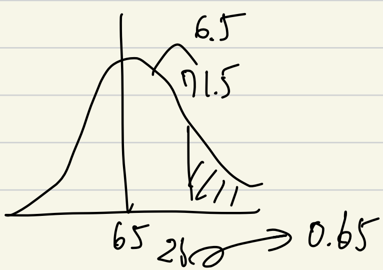
[2]

$P(X \leqq 2) - P(X \leqq -0.5)$

$= (1 - P(X \geqq 2)) - P(X \geqq 0.5) = 0.6687$

668人 ← ×1000.

[3]



$\dfrac{\theta - 65}{10} = 0.65$

$\theta = 71.5$

$6.5 \times 2 = 13$

[4] $\begin{cases} \cdot X \sim N(65, 10^2) \\ \cdot Z \sim N(0, 1) = \dfrac{X-65}{10} \end{cases}$

求 $E[X \mid X \geqq 65] = E[10Z + 65 \mid Z \geqq 0]$

$= 65 + 10\, E[Z \mid Z \geqq 0]$

条件付き期待値

$f_{Z|Z\geqq 0}(z) = \dfrac{f(z)}{P(Z \geqq 0)} = 2 \cdot f(z) = 2 \cdot \dfrac{1}{\sqrt{2\pi}} \exp\left(-\dfrac{z^2}{2}\right)$

よって $E[Z \mid Z \geqq 0] = \displaystyle\int_0^\infty z \cdot \dfrac{2}{\sqrt{2\pi}} \exp\left(-\dfrac{z^2}{2}\right) dz$

$= -\dfrac{2}{\sqrt{2\pi}} \left[\exp\left(-\dfrac{z^2}{2}\right)\right]_0^\infty = \sqrt{\dfrac{2}{\pi}} \fallingdotseq 0.79$

求 $= 65 + 7.9 = 72.9 \fallingdotseq \underline{73}$

問6.2

$L \sim N(305, 80^2)$

$R \sim N(250, 90^2)$

$T \sim N(555, 150^2)$

[1]

$$\text{相関係数}\ \rho = \frac{\mathrm{cov}[R.L]}{\sqrt{V[R]}\cdot\sqrt{V[L]}}$$

$$= \frac{40\cancel{00}}{8\cancel{0}\cdot 9\cancel{0}} \therefore \frac{5}{9} = 0.55..$$

$$V[T] = V[R] + V[L] + 2\,\mathrm{cov}[R.L]$$

$$225 = \underbrace{81 + 64}_{145} + 2\,\underbrace{\mathrm{cov}[R.L]}_{40}$$

$\times 100$

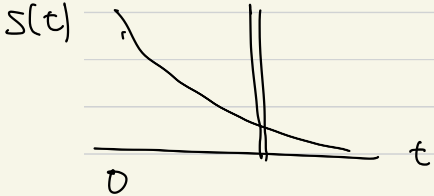
[2]

$$E[\overset{R}{Y} \mid \overset{L}{X} = x] = E[Y] + \rho[X\,Y]\cdot\sqrt{\frac{V[Y]}{V[X]}}\,(x - E[x])$$

公式?!

$$\underbrace{250 + 0.55\times\underbrace{\sqrt{\frac{81}{64}}}_{\frac{9}{8}}\,(\underbrace{335-305}_{30})}_{18.56} = 268.56$$

問6.3



[1] $F(t) = P(T<t) = 1 - S(t)$

$= 1 - \exp(-\lambda t)$

$f(t) = \lambda \exp(-\lambda t) \quad (t \geqq 0)$ //

[2] $E[T] = \int_0^\infty t \cdot \lambda \cdot e^{-\lambda t} \, dt = \left[t \cdot (-e^{-\lambda t}) \right]_0^\infty + \int_0^\infty e^{-\lambda t} \, dt$

$= \left[-\frac{1}{\lambda} e^{-\lambda t} \right]_0^\infty = \frac{1}{\lambda}$ //

[3] $\lambda = \frac{1}{3}$

$S(t) = \exp(-\lambda t) = \frac{1}{4}$

$t = \frac{\log 4}{\lambda}$ //

$\therefore t = 1.4 \times 3 = 4.2$

?

問6.4

300人 文 $\sim N(65, 4^2)$

200人 理 $\sim N(85, 3^2)$

[1] $\frac{67-65}{4} = 0.5$ $\quad 50+0.5\times10 = 55$

$\frac{82-85}{3} = -1$ $\quad 50-1\times10 = \underline{40}$

[2] $P(X \geq 60) = P\left(Z \geq \frac{60-65}{4}\right) = 1 - \underbrace{P(Z \leq -1.25)}_{}^{0.10} = 0.9$ $\quad 300$

$\boxed{270人}$

同様 $\underline{P\left(Z \geq \frac{60-85}{4}\right) = 1 - P\left(Z \leq \frac{25}{4}\right)}$ $\quad 6.$ $\rightarrow 200$

$\frac{470}{500} = \underline{94\%}$

$P(X \leq x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$