4 変数変換

$X = f(Z)$
$Y = g(X)$

$$\frac{f[g^{-1}(y)]}{|g'(g^{-1}(y))|}$$

$$J(x,y) = \frac{\partial(u(x,y), v(x,y))}{\partial(x,y)}$$

$$= \begin{vmatrix} \frac{\partial u(x,y)}{\partial x} & \frac{\partial u(x,y)}{\partial y} \\ \frac{\partial v(x,y)}{\partial x} & \frac{\partial v(x,y)}{\partial y} \end{vmatrix}$$

例1

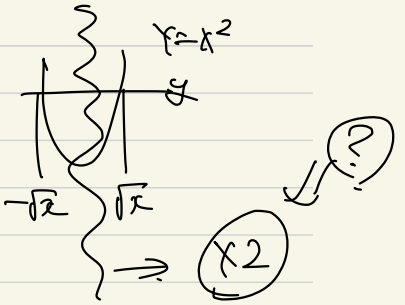
Xの確率密度関数 $\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

$\sigma^2 = 1$ → $\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$ $f(x)$

$\frac{2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \times \frac{1}{2x}$

↓ $y = x^2$

$\frac{1}{\sqrt{2\pi y}} \exp\left(-\frac{y}{2}\right)$



$$\begin{cases} Z = aX + bY \\ W = Y \end{cases} \qquad (X, Y) = \left(\frac{Z}{a} - \frac{bW}{a}, W \right)$$

ヤコビアン

$$J(X,Y) = \begin{vmatrix} a & b \\ 0 & 1 \end{vmatrix} = a$$

例題

問4-1

[1] X のモーメント母関数 $E[e^{\theta X}] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{\sigma^2} + \theta x} dx$

$$-\frac{x^2 + 2\mu x - \mu^2 + \sigma^2\theta x}{\sigma^2}$$

$$= \exp\left(\mu\theta + \frac{\sigma^2\theta^2}{2}\right)$$

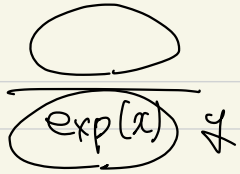
$\theta = 1$ を代入 $E[e^X] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$

$E[e^{2X}] = \exp(2\mu + 2\sigma^2)$

[2] $V[X] = E[Y^2] - E[X]^2$

$\exp(2\mu + \sigma^2)$

$e^{2\mu+\sigma^2} \cdot (\exp(\sigma^2) - 1)$

[3] 

$y = \exp(x)$

$x = \log y$

$\lambda^2 e^{-\lambda(z-w)} \cdot e^{-\lambda w}$

問4.2

$$\begin{cases} Z = X + Y \\ W = Y \end{cases}$$

$$f_Z(z) = \int_{-\infty}^{0} f(z-w) f(w)\, dw + \int_{0}^{z} f(z-w) f(w)\, dw = \lambda^2 z e^{-\lambda z}$$