6 連続型分布と標本分布

① 正規分布

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$N(\mu, \sigma^2)$

② 指数分布 $f(x) = \lambda e^{-\lambda x} = Exp(\lambda)$ 無記憶性.

③ ガンマ分布 $f(x) = \dfrac{1}{\Gamma(a) b^a} x^{a-1} \cdot e^{-\frac{x}{b}}$

④ ベータ分布 $f(x) = \dfrac{1}{B(a,b)} x^{a-1} \cdot (1-x)^{b-1}$

⑤ 多変量正規分布 $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$

$$f(x) = \frac{1}{(2\pi)^{\frac{p}{2}} \cdot (\det\Sigma)^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

◎カイ二乗分布 $\chi^2(n)$

$Z_i \sim N(0.1)$

$Y = Z_1^2 + Z_2^2 + \cdots + Z_n^2$

$N(\mu.\sigma^2)$

X_1

X_2

$\vdots$

X_n

$\bar{X}, S^2$

$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0.1)$$

$$\frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2$$

$$\sim \chi^2(n-1)$$

◎t分布

$Z \sim N(0.1)$

$Y \sim \chi^2(n)$

$$T = \frac{Z}{\sqrt{\frac{Y}{n}}}$$ ← 自由度nのt分布.

☆ 分散が未知の正規分布の平均の検定

$$t = \frac{\bar{X}}{\frac{s}{\sqrt{n}}}$$

自由度n-1

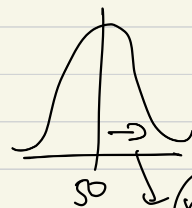
⑪ F分布 $Y_1 \sim \chi^2(n_1)$ $X = \dfrac{Y_1/n_1}{Y_2/n_2}$

$Y_2 \sim \chi^2(n_2)$ 自由度 (n_1, n_2) の F分布.

例題

問6.1

[1] $\dfrac{85-65}{10} = 2$

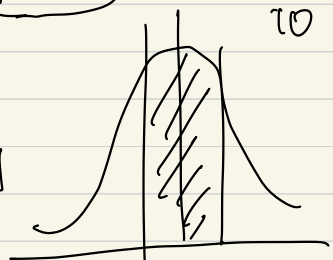


$\times 2 \to 70$点

$\dfrac{60-65}{10} = -0.5$

45点

[2]

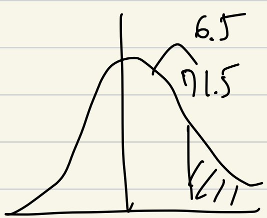


$P(x \leq 2) - P(x \leq -0.5)$

$= (1 - P(x \geq 2)) - P(x \geq 0.5) = \underbrace{0.6687}$

668人] $\times 1000$.

[3]



$\dfrac{\theta - 65}{10} = 0.65$

$\theta = 71.5$

$6.5 \times 2 = 13$

[4] $\begin{cases} \cdot X \sim N(65, 10^2) \\ \cdot Z \sim N(0, 1) = \dfrac{X-65}{10} \end{cases}$

求 $E[X \mid X \geqq 65] = E[10Z + 65 \mid Z \geqq 0]$

$= 65 + 10\, E[Z \mid Z \geqq 0]$

条件付き期待値

$$f_{Z \mid Z \geqq 0}(z) = \frac{f(z)}{P(Z \geqq 0)} = 2 \cdot f(z) = 2 \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

よって $E[Z \mid Z \geqq 0] = \displaystyle\int_0^\infty z \cdot \frac{2}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz$

$$= -\frac{2}{\sqrt{2\pi}} \left[\exp\left(-\frac{z^2}{2}\right)\right]_0^\infty = \sqrt{\frac{2}{\pi}} \fallingdotseq 0.79$$

求 $= 65 + 7.9 = 72.9 \fallingdotseq \underline{73}$

問6.2

L $\sim N(305, 80^2)$

R $\sim N(250, 90^2)$

T $\sim N(555, 150^2)$

[1] 相関係数 $\rho = \dfrac{cov[R.L]}{\sqrt{V[R]}\cdot\sqrt{V[L]}}$

$= \dfrac{40\cancel{00}}{8\cancel{0}\cdot 9\cancel{0}} = \dfrac{5}{9} = \underline{0.55\ldots}$

$V[T] = V[R] + V[L] + 2\,cov[R.L]$

$225 = \underbrace{81 + 64}_{145} + 2\,\underbrace{cov[R.L]}_{40} \quad \times 100$

[2]

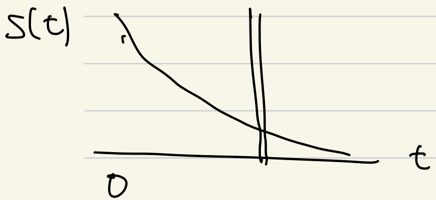
$$\underset{R}{E[Y} \mid \underset{L}{X} = x] = E[Y] + \rho[X\,Y] \cdot \sqrt{\frac{V[Y]}{V[X]}}\,(x - E[x])$$

公式?!

$250 + 0.55 \times \underbrace{\sqrt{\frac{81}{64}}}_{\frac{9}{8}}\,(\underbrace{335-305}_{30}) = \underline{268.56}$

18.56

問6.3



[1] $F(t) = P(T < t) = 1 - S(t)$

$= 1 - \exp(-\lambda t)$

$f(t) = \lambda \exp(-\lambda t) \quad (t \geqq 0)$

[2] $E[T] = \int_0^\infty t \cdot \lambda \cdot e^{-\lambda t} \, dt = \left[t \cdot (-e^{-\lambda t}) \right]_0^\infty + \int_0^\infty e^{-\lambda t} \, dt$

$= \left[-\frac{1}{\lambda} e^{-\lambda t} \right]_0^\infty = \frac{1}{\lambda}$

[3] $\lambda = \frac{1}{3}$

$S(t) = \exp(-\lambda t) = \frac{1}{4}$

$t = \frac{\log 4}{\lambda}$

$\therefore t = 1.4 \times 3 = 4.2$

?

問6.4

[1]

300人 文 $\sim N(65,\ 4^2)$

200人 理 $\sim N(85,\ 3^2)$

$\frac{67-65}{4} = 0.5$ $\quad 50+0.5\times 10 = 55$

$\frac{82-85}{3} = -1$ $\quad 50-1\times 10 = \underline{40}$ //

[2] $P(X \geq 60) = P\left(Z \geq \frac{60-65}{4}\right) = 1 - \underline{P(Z \leq -1.25)}^{0.10} = 0.9$ 300 → 270人

同様 $\underline{P\left(Z \geq \frac{60-85}{4}\right) = 1 - P\left(Z \leq \frac{25}{4}\right)}$ 6. → 200

$\frac{470}{500} = \underline{94\%}$ //

$P(X \leq x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$