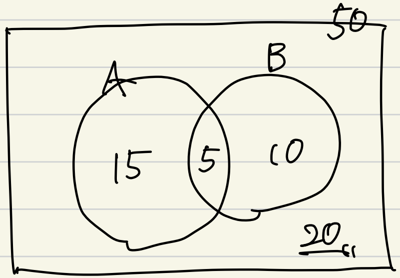
1 事象と確率

例1



独立性 $P(B|A) = P(B)$ とも書ける.

ベイズの定理

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(B|A^c)P(A^c)}$$

$P(A|B)$: 事後確率

$P(A)$: 事前確率

A=原因
B=結果

確率変数

$$\underbrace{p(x)}_{\text{確率関数}} = P(X = x)$$

$$\mu = E[X] = \sum_x x\,p(x)$$

$$E[g(X)] = \sum_x g(x)\,p(x)$$

$$\sigma^2 = V[X] = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$$

$$= \underbrace{\sum_x x^2 p(x)}_{E[X^2]} - \underbrace{2\mu \sum_x x\,p(x)}_{-2\mu^2} + \underbrace{\mu^2 \boxed{\sum_x p(x)}}_{\mu^2} = E[X^2] - \mu^2$$

$\swarrow 1$

確率密度関数 : $f(x) = \lim_{\varepsilon \to +0} \frac{P(x < X \leq x+\varepsilon)}{\varepsilon}$

$E[X] = \int_{-\infty}^{\infty} x f(x) \, dx$, $V[X] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \, dx$

$\boxed{\text{例題}}$

問1.1

[1] $0.6 \times 0.4 + 0.4 \times 0.5 = \underline{0.44}$ //

[2] $\frac{0.4 \times 0.5}{0.44} = \frac{20}{44} = \underline{\frac{5}{11}}$ //

問1.2

条件から $\begin{cases} P(X=1)=P(X=2)=\frac{1}{6} \\ P(X=3)=\frac{2}{3} \end{cases}$

[1]

$$E[X]=\sum_{x} x\cdot P(x)=1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{2}{3}=\frac{1+2+12}{6}=\frac{15}{6}=\underline{\frac{5}{2}}_{\,//}$$

$$V[X]=E[X^2]-\mu^2=1\times\frac{1}{6}+4\times\frac{1}{6}+9\times\frac{4}{6}-\frac{25}{4}=\frac{1+4+36}{6}-\frac{25}{4}$$

$$=\frac{82-75}{12}=\underline{\frac{7}{12}}_{\,//}$$

[2] $(1,3)$, $(2,3)$, $(3,3)$
$(3,1)$, $(3,2)$

$$4\times\left(\frac{1}{6}\times\frac{2}{3}\right)+\frac{4}{9}=\underline{\frac{8}{9}}_{\,//}$$

問1.3　$P(A)=\frac{1}{100}$ ← 病気にかかっている確率

検査1. $\begin{cases} P(陽|A)=0.99 \\ P(陽|{}^cA)=0.02 \end{cases}$

⇓

検査2. $\begin{cases} P(陽_2|陽_1, A)=0.9 \\ P(陽_2|陽_1, {}^cA)=0.1 \end{cases}$

[1]

$$P(A|陽_1)=\frac{P(A)P(陽_1|A)}{P(陽_1)}$$

$$=\frac{0.01\times0.99}{0.01\times0.99+0.99\times0.02}$$

$$=\frac{1}{3}$$

$P(A^c|陽_1)$

$$=\frac{2}{3}$$

$$P(A|陽_2, 陽_1)=\frac{P(陽_2|陽_1, A)P(A|陽_1)}{P(陽_2|陽_1)}=\frac{0.9\times\frac{1}{3}}{P(陽_2|陽_1, A)P(A|陽_1)+P(陽_2|陽_1, {}^cA)P(A^c|陽_1)}$$

$$=\frac{0.9\times\frac{1}{3}}{0.9\times\frac{1}{3}+0.1\times\frac{2}{3}}=\frac{0.9}{1.1}=\frac{9}{11}$$

$$P(A,B,C)=P(A,B|C)P(C)$$

$$=P(A|B,C)P(B|C)P(C)$$

$$=P(B|A,C)P(A|C)P(C)$$

$$P(B|AC)=\frac{P(A|B,C)P(B|C)}{P(A|C)}$$