[4] マルコフ連鎖

$$P_m^{(n)}(x, B) = P(X_{n+m} \in B \mid X_n = x)$$

⑭ マルコフ性

$X \leftarrow$ マルコフ連鎖

mステップ 推移確率

確率行列

$$Q(m) = \begin{pmatrix} P_m(1.1) & \cdots & P_m(1.N) \\ \vdots & & \vdots \\ P_m(N.1) & \cdots & P_m(N.N) \end{pmatrix}$$ 和が1

mステップ
推移確率行列

例1

$$P(i, j) = \begin{array}{ll} q & (j=1) \\ 1-q & (j=2) \end{array} \qquad Q = \begin{pmatrix} q & 1-q \\ q & 1-q \end{pmatrix}$$

问3

$$\theta = \begin{pmatrix} 1-\theta & \theta & 0 \\ 0.1 & 0.9-\theta & 0 \\ 0 & 0.1 & 0.9 \end{pmatrix}$$

$25人$, $5人$ → A→B

49 , $1人$

$10人$

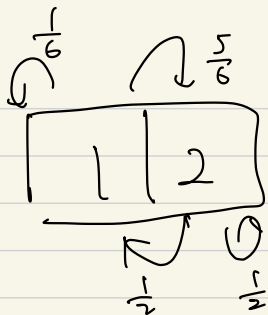$(1-\theta)^{25} \cdot \theta^5 \cdot (0.9-\theta)^{49} \cdot \theta \cdot (0.9)^{10}$

$\log(\theta) = 25 \log(1-\theta) + 6 \log \theta + 49 \log(0.9-\theta)$

$$\frac{\log(\theta)}{d\theta} = \frac{-25}{1-\theta} + \frac{6}{\theta} + \frac{-49}{0.9-\theta}$$

$\Rightarrow \theta = 0.07$

(1)

$\curvearrowright \frac{1}{6}$ $\overset{\frac{5}{6}}{\curvearrowright}$

$\boxed{1 | 2}$

$\overset{\frac{1}{2}}{\curvearrowright}$ $\overset{\frac{1}{2}}{\circlearrowright}$

$Q = \begin{pmatrix} \frac{1}{6}, & \frac{5}{6} \\ \frac{1}{2}, & \frac{1}{2} \end{pmatrix}$

[2]

$\pi = \pi Q$

$\textcircled{$\pi$} \begin{pmatrix} -\frac{5}{6} & \frac{5}{6} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = 0$
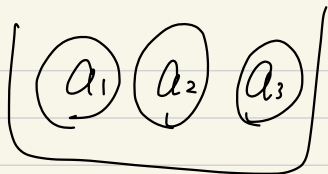
$a + b = 1$

$-\frac{5}{6} a + \frac{1}{2} b = 0$

$\lim_{n \to \infty} \pi_n = \left(\frac{3}{8}, \frac{5}{8} \right)$

$a + b = 1$

$-\frac{5}{3} a + b = 0$

$\frac{8}{3} a = 1$ $a = \frac{3}{8}, b = \frac{5}{8}$

問14.2

$\boxed{(a_1)\ (a_2)\ (a_3)}$

冰稱

| 1 | 2 | 3 |
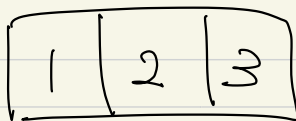
(1) $Q = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{3} \end{pmatrix}$

[2] $\pi_1 = \pi_0 Q = (0.0.1)Q = \left(\frac{1}{9}, \frac{2}{9}, \frac{2}{3}\right)$

$\pi_2 = \pi_0 Q^2 = \left(\frac{1}{9}, \frac{2}{9}, \frac{2}{3}\right)Q = \left(\frac{4}{27}, \frac{8}{27}, \frac{5}{9}\right)$

[3] $\pi = \pi Q$ $\pi Q = \pi$ $\overset{\pi}{\underset{a,b,c}{}}\begin{pmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & -\frac{1}{2} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} & -\frac{1}{3} \end{pmatrix}$

$-\frac{2}{3}a + \frac{1}{6}b + \frac{1}{9}c = 0$

$\frac{1}{3}a - \frac{1}{2}b + \frac{2}{9}c = 0$

$\frac{1}{3}a + \frac{1}{3}b - \frac{1}{3}c = 0$

$a + b + c = 1$

$a + b - c = 0$

$-12a + 3b + 2c = 0$

$2a + 2b = 1$

$-10a + 5b = 0$

$-2a + b = 0$

$3b = 1$

$b = \frac{1}{3}$

$\pi = \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$

$a = \frac{1}{6}, \quad c = \frac{1}{2}$

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$$S = \{0.1.2\}$$

$$\pi_0 = (0. 1. 0)$$

$$P_{ij} = P(X_{n+1} = j-1 \mid X_n = i-1)$$

$$Q = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1-\theta & \theta \\ 1-\theta & \theta & 0 \end{pmatrix}$$

$1 \cdot (1-\theta) \cdot \theta \cdot (1-\theta) \cdot 1 \quad \theta \cdot (1-\theta) \cdot (1-\theta)$

$$\overline{\theta = \frac{1}{3}}$$

[3] $\pi = \pi Q \quad a = \frac{(1-\theta)}{(3-\theta)} = \frac{\frac{2}{3}}{\frac{\theta}{3}} = \frac{1}{4}$