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Points 15 Published

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Value of St. Petersburg Game 3 pts

If an individual has utility of $U\left(x\right)=\sqrt{x}$, we saw in Note 01 Risk Aversion that this individual would be willing to pay up to 2.914 ducats to obtain the payoff of the St. Petersburg Paradox game. Suppose, instead, that an individual had utility of $U\left(x\right)=\sqrt[4]{x}=x^{\frac{1}{4}}$. What would be the maximum that this individual would be willing to pay to obtain the payoff of the St. Petersburg Paradox game?

State your answer in ducats to the second decimal place, e.g. 2.75 .

nswers 2.31 (with margin: 0.01)

Portfolio Choice with CARA Utility 3 pts

Consider the one-period portfolio choice problem in Note02 Risk Aversion and Portfolio Choice. An individual can choose to invest in a risk-free asset with r_f=0.03 and a risky asset that has a normally distributed rate of return with mean 0.07 and variance 0.02, i.e. $\tilde{r}\sim N(\bar{r},\sigma_r^2)=N(0.07,0.02)$. The individual has negative exponential utility with coefficient of absolute risk aversion equal to *b* = 0.1. If the individual's initial wealth is W_0=$100, what is the proportion of this initial wealth, ω, that the individual will choose to invest in the risky asset? Hint: Use the result that if $\tilde{x}$ ~$N(\mu, \sigma^2)$, then $E[e^{a\tilde{x}}] = e^{a\mu + \frac{1}{2}a^2\sigma^2}$].

Write your answer to 2 decimal places, e.g., 0.55.

nswers 0.2 (with margin: 0.01)

Nominal Interest Rate 3 pts

As in Note 05 Equilibrium Asset Valuation, let $\tilde{m}_{01}$ denote the real stochastic discount factor, which has an expected value of 0.99. Also let CPI_0 be the consumer price index at the beginning of the period and let $C\tilde{P}I_1$ be the consumer price index at the end of the period. $CPI_0/C\tilde{P}I_1$ has an expected value of 0.95 and the covariance between $\tilde{m}_{01}$ and $CPI_0/C\tilde{P}I_1$ is 0.01. A bank invests $1 million in a Treasury bill that is risk-free in nominal (currency or $ terms). At the end of the period, the bank will receive $1×$(1 + r_f^N)$ million where r_f^N is the risk-free nominal interest rate. What is this equilibrium risk-free nominal interest rate, r_f^N?

State your answer as a percentage interest rate to 2 decimal places, e.g., 6.98.

nswers 5.2078 (with margin: 0.01)

Quantitative Easing 3 pts

Following the 2008-2009 Global Financial Crisis (GFC), many central banks implemented a money policy of "Quantitative Easing" (QE) that lowered both short-maturity and long-maturity interest rates. For example, the European Central Bank lowered interest rates to zero and sometimes even below zero. The central banks' goal was to stimulate their economies, but in many countries the recovery from the GFC was much weaker than some policymakers had hoped. In particular, aggregate consumption spending did not grow very quickly. Suppose that power utility

$U(C) = C^\gamma/\gamma$ serves as a good approximation to the utility function of most individuals. Then an explanation consistent with QE failing to stimulate the economy is

nswer

- $\gamma < 0$
- $0 < \gamma < 1$
- $\gamma = 0$
- $\gamma > 1$

Price of a Call Option

3 pts

Consider the complete markets, no arbitrage framework in Note 06 Arbitrage and the Stochastic Discount Factor. An economy has three end-of-period states. State 1 is High Growth, State 2 is Normal Growth, and State 3 is Recession. A futures contract on the stock market pays 5 in State 1, 0 in State 2, and -2 in State 3 and has an initial cost of 0. A stock pays 6 in State 1, 4 in State 2, and 3 in State 3 and has an initial cost of 4. A put option pays 0 in State 1, 0 in State 2, and 2 in State 3 and has an initial cost of 1. In the absence of arbitrage, what would be the initial cost of a call option that pays 2 in State 1 and 0 in both States 2 and 3? Note: It may be helpful to use Excel or some other software to compute the answer.

Write your answer to 2 decimal places, e.g., 0.75.

nswers 0.4 (with margin: 0.01)

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Answer to Value of St. Petersberg Game

Suppose $U_i = U(x_i) = x_i^{\frac{1}{4}}$. Then the expected utility of the St. Petersburg payoff is

$$\begin{aligned}
V &= \sum_{i=1}^{n} p_i U_i = \sum_{i=1}^{\infty} \frac{1}{2^i} 2^{\frac{1}{4}(i-1)} = \sum_{i=1}^{\infty} 2^{-\frac{3}{4}i-\frac{1}{4}} = \sum_{i=1}^{\infty} 2^{\frac{2}{4}} 2^{-\frac{3}{4}(i+1)} \\
&= 2^{\frac{1}{2}} \sum_{i=2}^{\infty} 2^{-\frac{3}{4}i} = 2^{\frac{1}{2}} \left(\sum_{i=0}^{\infty} 2^{-\frac{3}{4}i} - 1 - 2^{-\frac{3}{4}} \right) \\
&= 2^{\frac{1}{2}} \left(\frac{1}{1-2^{-\frac{3}{4}}} - 1 - 2^{-\frac{3}{4}} \right) \\
&= 2^{\frac{1}{2}} \left(\frac{1}{1-2^{-\frac{3}{4}}} - 1 - 2^{-\frac{3}{4}} \right) = 2^{\frac{1}{2}} \left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}-1} - \frac{\left(1+2^{-\frac{3}{4}}\right)\left(2^{\frac{3}{4}}-1\right)}{2^{\frac{3}{4}}-1} \right) \\
&= 2^{\frac{1}{2}} \left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}-1} - \frac{2^{\frac{3}{4}}-2^{-\frac{3}{4}}}{2^{\frac{3}{4}}-1} \right) = 2^{\frac{1}{2}} \left(\frac{2^{-\frac{3}{4}}}{2^{\frac{3}{4}}-1} \right) = \frac{2^{-\frac{1}{4}}}{2^{\frac{3}{4}}-1} = 1.233361
\end{aligned}$$

A certain payment of $1.2333361^4 = 2.31398$ would give the same expected utility.

Answer to Portfolio Choice with CARA Utility

The individual's problem is to choose the portfolio proportion ω that maximizes expected utility:

$$\begin{aligned}
\max_{\omega} E[U(\tilde{W})] &= \max_{\omega} E\left[U\left(W_0\left[(1+r_f)+\omega(\tilde{r}-r_f)\right]\right)\right] \\
&= \max_{\omega} - E\left[e^{-b(W_0[(1+r_f)+\omega(\tilde{r}-r_f)])}\right] \\
&= \max_{\omega} - e^{-bW_0(1+r_f)} E\left[e^{-b(W_0\omega(\tilde{r}-r_f))}\right] \\
&= \max_{\omega} - e^{-bW_0(1+r_f)} e^{-bW_0\omega(\overline{r}-r_f)+\frac{1}{2}b^2W_0^2\omega^2\sigma_r^2}
\end{aligned}$$

Due to the - sign in the above maximization problem, maximizing this expression with respect to ω is equivalent to minimizing the second exponent. But due to the - sign and factoring out a bW_0, this is equivalent to maximizing

$$\max_{\omega} \omega(\overline{r}-r_f) - \frac{1}{2} bW_0\omega^2\sigma_r^2$$

Differentiating this expression with respect to ω, the first order condition is

$$(\overline{r}-r_f) - bW_0\omega\sigma_r^2 = 0$$

or

$$\omega = \frac{\overline{r}-r_f}{W_0 b\sigma_r^2} = \frac{0.07-0.03}{100 \times 0.1 \times 0.02} = 0.20$$

Answer to Nominal Interest Rate

In the notes we derived the expression

$$\begin{aligned} P_i^N &= E\left[\frac{1}{I_{01}}\frac{\delta U'(C_1)}{U'(C_0)}X_i^N\right] \\ &= E\left[\frac{CPI_0}{CPI_1}m_{01}X_i^N\right] \end{aligned}$$

For the case of investing \$1million in a Treasury bill that is risk-free in nominal terms, this expression implies

$$1 = E\left[\frac{CPI_0}{CPI_1}m_{01}\left(1+r_f^N\right)\right]$$

or dividing both sides by the non-random value $\left(1+r_f^N\right)$:

$$\begin{aligned} \frac{1}{1+r_f^N} &= E\left[\frac{CPI_0}{CPI_1}m_{01}\right] \\ &= E\left[\frac{CPI_0}{CPI_1}\right]E\left[m_{01}\right] + Cov\left(\frac{CPI_0}{CPI_1}, m_{01}\right) \\ &= 0.95 \times 0.99 + 0.01 = 0.9505 \end{aligned}$$

which implies that $1 + r_f^N = 1.052078$ or $r_f^N = 5.2078$ %.

Answer to Quantitative Easing

We showed that for $U(C) = C^\gamma/\gamma$, for $\gamma < 1$, the intertemporal elasticity of substitution is given by

$$\varepsilon \equiv \frac{R_f}{\frac{C_1}{C_0}}\frac{\partial \frac{C_1}{C_0}}{\partial R_f} = \frac{\partial \ln\left(C_1/C_0\right)}{\partial \ln R_f} = \frac{1}{1-\gamma}$$

When $0 < \gamma < 1$, the substitution effect dominates the income effect, implying that consumers would save less (and consume more) when the interest rate declines. Instead, when $\gamma < 1$, the income effect dominates the substitution effect, so that a lower interest rate leads to a consumers to feel poorer, and they consume less and save more. Thus consumption growth falls the most when $\gamma < 0$.

Answer to Price of a Call Option

The payoff matrix and initial prices of the futures contract, stock, and put options are

$$X = \begin{pmatrix} 5 & 6 & 0 \\ 0 & 4 & 0 \\ -2 & 3 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$$

The elementary security prices, p_i, $i = 1, 2, 3$ are given by

$$p_i = P'X^{-1}e_i,\ i = 1, 2, 3$$

where e_i is a 3×1 vector of zeros except for a 1 in the i^{th} row and

$$X^{-1} = \begin{pmatrix} 0.2 & -.3 & 0 \\ 0 & 0.25 & 0 \\ 0.2 & -0.675 & 0.5 \end{pmatrix}$$

This calculation results in $p_1 = 0.2$, $p_2 = 0.325$, and $p_3 = 0.5$. Since p_1 is the price of a security that pays 1 only in state 1, the initial price of the call option that pays 2 only in state 1 is $2 \times p_1 = 0.40$.