<u>Take-Home Final Exam</u>

Your assignment is to address the questions on the next page. Before beginning, please read the following rules carefully. If you are in doubt about any of them, email your question to *tcj@illinois.edu*.

- Your answers must be uploaded to the Canvas site by **5:00 pm on Sunday, May 7**.
- The questions require you to do some simulations. The simulation can be done in Python, Matlab, Excel, R, or whatever you prefer to use. Submit your answers in one file **without any code**, and upload your code separately.
- In preparing your answers, you **may** consult class notes or any other instructional materials. While it is not necessary to do so, you may also refer to other public sources. Make sure to cite any such sources appropriately. (You do not need to cite the class lecture notes.)
- The previous bullet point also applies to any AI resources. I do not think they will help you. But if any of your submission comes from a chat bot, you <u>must</u> identify it as such. Failure to do so will be treated as plagarism.
- You are **not** allowed to discuss the exam with your classmates, professors, or friends. The document that you submit must contain the following text:

  `I pledge that I have not received assistance from anyone in preparing this exam.`

  Then sign or type your name.
- Be specific in your answers and explain your reasoning in detail. But please do not include include unnecessary material. Do not reproduce (or cut and paste) class lecture notes or the text of the question that you are answering. Do <u>not</u> upload raw output of your simulations.
- Other than clarifications about the rules and logistics, I will not answer questions while the exam is in progress.

Now see the specific instructions on the next page

Betting on Equity Correlations 2020

In the Spring of 2020, it seemed the only news that mattered was news about the coronavirus pandemic. As a result, in equity markets, stocks were highly correlated, as news about individual businesses was much less important than news about the damage that Covid-19 would (or would not) inflict on the economy. This type of pattern is often observed in times of falling markets and high market volatility (for example, in the financial crisis in 2007-2008). However, in 2020 stocks stayed highly correlated even as the stock market rallied and VIX declined from its highs. See the Bloomberg article from May 27.

However, although *realized* correlations remained very high during April and May, *implied* correlations declined to much lower levels. Implied correlations can be imputed by comparing the implied volatilities of the S&P 500 index to the implied volatilities of its component stocks. Visit `https://www.cboe.com/us/indices/implied/` and follow the link to the "White Paper" to see how CBOE does this. Look at the data on `https://www.cboe.com/us/indices/dashboard/cor3m/`, and you can see that on May 27 2020 the inplied correlation using January 2021 options was 60.71. If you plot the historical data. you will find long stretches of time (especially prior to 2016) when the nearest-dated implied correlation was in the 60s or higher.

An interesting fact is that, with traded options, you can "trade correlation". In other words, the CBOE index is not just a mathematical manipulation: if you design the proper combination of swap positions, and hedge appropriately, you can generate a payout profile that is similar to the realized correlation of returns between two assets. Of course, this would be pointless if you thought correlations were constant. So to analyze such trades, we need to look at models with *stochastic correlation.*

- **Question 1** Consider a two-factor stochastic volatility model for a stock:

$$\begin{aligned} dS/S &= \mu^S\ dt + \sigma_t^I\ dW1_t + \beta\ \sigma_t^M\ dW2_t \\ dS^M/S^M &= \mu^M\ dt + \sigma_t^M\ dW2_t \\ d\sigma_t^I &= \kappa^I(\bar{\sigma}^I - \sigma_t^I)dt + b^I\ \sigma_t^I\ dW3_t \\ d\sigma_t^M &= \kappa^M(\bar{\sigma}^M - \sigma_t^M)dt + b^M\ \sigma_t^M\ dW4_t \end{aligned}$$

  where $dW1, dW2, dW3, dW4$ are mutually independent Brownian motions. Here S^M is the market portfolio, β is a constant, and the the volatility of the stock is

$$\sigma_t^S = \sqrt{(\sigma_t^I)^2 + (\beta\ \sigma_t^M)^2}.$$

  The instantaneous correlation of dS and dS^M is then

$$\rho_t = \beta\ \sigma_t^M/\sigma_t^S.$$

  If we have a derivative security, f, whose payoffs are determined by ρ_t what partial differential equation would f's price have to satisfy in a complete market with no arbitrage? (Be sure to define all the notation that you use. Specify any extra assumptions you are making.)

- **Question 2**

  Now imagine that you can do a trade with a counterparty in two securities ("vol swaps") that pay off based on the realized squared returns of each of the two assets. Specifically, suppose these returns (denoted r_t^M and r_t^S) are computed at every time interval Δ, from $t = 0$ to T. In your first trade, at T you receive $\sqrt{\frac{1}{T}\sum_t (r_t^M)^2}$ in exchange for a fee agreed in advance, which we will denote $\tilde{\sigma^M}$. In the second trade, you will receive a (different) fee, $\tilde{\sigma^S}$, and pay $\sqrt{\frac{1}{T}\sum_t (r_t^S)^2}$. We can view the payoffs and fees as percentages, and do the trade for any notional amount that we want.

  We want to analyze the profits (under the physical measure) of this two-sided "vol swap". Proceed as follows:

  1. Simulate the above system 200 times over a time interval of length $T = 1/4$ or three months, assuming there are 21 trading days per month (252 per year) and the time interval is $\Delta = 1$ hour, or $1/(24 * 252)$. Use parameters $\mu^S = \mu^M = 0.0$, $\kappa^I = \kappa^M = 2.0$, $b^I = b^M = 0.5$, $\beta = 1.0$, and $\bar{\sigma}^I = 0.25$, $\bar{\sigma}^M = 0.15$. Also assume the starting values are $\sigma_0^I = \bar{\sigma}^I$ and $\sigma_0^M = \bar{\sigma}^M$.
  2. For each simulation at each step, save the returns and the true volatilities. From these paths compute
     (a) The total profit of the vol swap, assuming the inital prices were $\tilde{\sigma}^M = \bar{\sigma}^M$ and $\tilde{\sigma}^S = \sqrt{(\bar{\sigma}^I)^2 + (\beta\ \bar{\sigma}^M)^2}$.
     (b) The pathwise average of the true correlation.
     (c) The realized sample correlation of the returns dS^M/S^M and dS/S.
  3. Show scatter plots of the swap profit against (i) the average true correlation, and (ii) the realized return correlation. Also plot the two correlation series against each other.

  What is the breakeven correlation for the trade? If you do the trade for a notional amount of $10 million, how much do you make or lose if correlation is 0.10 higher/lower than the breakeven value?

- **Question 3** Now plot the time series realizations for some of your simulated histories. Show sample paths for ρ_t, σ_t^S, and σ_t^M. Also plot the realized returns, i.e. the cumulative sums of r_t^M and r_t^S for some paths.

  How well does the model capture the empirical facts alluded to in the Bloomberg article? Specifically, (i) is correlation *directional* in this model? That is, does your trade tend to make more money in down markets than up markets? And (ii) do we tend to observe higher correlation when market volatility is higher? Explain what assumptions of the model specification lead to each finding.

- **Question 4**

  Consider the sector volatilities in the spreadsheet on Canvas. Suppose that on 5/26/2020 you could do 3-month vol-swaps of the type analyzed in Question 2 at the rates listed in the "implied" column for the SPY ETF and for the component sector ETFs. You have shown that going long one unit of the index vol swap and going short vol swaps on one of the sector ETFs will result in a profit/loss that is positively related to the realized (and true) correlation. One way to reduce the noise in this relation is to sell separate vol swaps on the components of the index, with weights that add up to one.

  Suppose that, to bet on continued high correlation, you did that trade on May 26, 2020, and equally weighted the sectors. Using your findings from Question 2, and the column showing the realized vols over the subsequent 60 trading days to 8/26/20, estimate how much money you would have made/lost if the notional value of the market vol swap was $10 million.

  What do your results imply about the observations in the Bloomberg column from May 26?