10/10 Questions Answered
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IE 522 HW07

Q1
10 Points

The file ZMTSLA.csv contains daily prices of ZM. Compute ZM's daily log returns. Assume that ZM's daily log returns are i.i.d. Assume that one trading day equals $t = 1/252$ year.

Q1.1
1 Point

Fit a Black-Scholes-Merton model to ZM's daily log returns. Report the MLEs for μ, σ and the corresponding value of the log likelihood function.

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BSM_mu = 1.3340069
BSM_sigma = 0.7362867
logliklihoodBSM = 588.0812

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Q1.2

1 Point

Fit a NIG model to ZM's daily log returns. Report the MLEs for $\alpha, \beta, \delta, \mu$ and the corresponding value of the log likelihood function.

(NIGalpha, NIGbeta, NIGdelta, NIGmu, logliklihoodNIG) = (16.4540784598237, 2.34990357077191, 8.50804955093113, 0.106341984286513, 627.465930520146)

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Q1.3

1 Point

Compute the sample mean, variance, skewness and kurtosis of ZM's daily log returns.

(logrt_ZM_mu, logrt_ZM_Var, logrt_ZM_Skew, logrt_ZM_kurt) = (0.00529367809296, 0.00215731473159484, 1.19896749416696, 11.7050223772246)

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Q1.4

1 Point

Fit the NIG model using moment matching. Report the parameters you obtain.

alpha = 15.3997719, beta= 4.0914736, delta = 7.5013727, mu = -0.7332863

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Q1.5

1 Point

What's the value of the log likelihood function when moment matching estimates are used? Do the moment matching estimates maximize the log likelihood function?

625.9847, does no reach the maximum log likelihood function.

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Q1.6

1 Point

$counts in the output of the function optim in R reports the number of times the objective function is called and evaluated. Smaller counts mean faster convergence. In question 1.2, is the convergence faster if you use moment matching estimates as the initial value?

Yes, the outcome is smaller than than that of 1.2.

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```
#Q1-6
#Data Preparation
ZMTSLA = read.csv("/Users/yu-chingliao/Library/CloudStorage/GoogleDrive-josephliao0127@gmail.com/My Drive/Note/UIUC/Fall_2022/Statistical Methods in Finance/Assignment 06/ZMTSLA.csv")
m = nrow(ZMTSLA)
```

```
logrt_ZM = log(ZMTSLA$ZM[2:m]/ZMTSLA$ZM[1:m-1])
t = 1/252

library(moments)

#NIG Model_fitting
initialvalueNIG_star = MMestimates
NIG_star = function(x, theta) {theta[1]*theta[3]*t/pi*besselK(theta[1]*sqrt(theta[3]^2*t^2+(x-theta[4]*t)^2), 1)/(sqrt(theta[3]^2*t^2+(x-theta[4]*t)^2))*exp(theta[3]*sqrt(theta[1]^2-theta[2]^2)*t+theta[2]*(x-theta[4]*t))}
resultNIG_star = optim(initialvalueNIG_star, fn=function(theta){-sum(log(NIG_star(logrt_ZM, theta)))}, method = "L-BFGS-B")
resultNIG_star

```

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Q1.7

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Fit the daily log returns using a Laplace distribution with density $p(x) = \frac{1}{2b} e^{-|x-\mu|/b}, b > 0$. Report the MLEs for μ and b and the corresponding value of the log likelihood function. In this problem, optim won't succeed unless the initial values are appropriately chosen. use moment matching (matching mean and standard deviation) to find good initial values that work.

mu = 0.002637152, b = 0.031774552 maximum log likelihood = 625.2951

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Q1-7.R

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```
#Q1-7
#Data Preparation
ZMTSLA = read.csv("/Users/yu-chingliao/Library/CloudStorage/GoogleDrive-josephliao0127@gmail.com/My Drive/Note/UIUC/Fall_2022/Statistical Methods in Finance/Assignment 06/ZMTSLA.csv")
m = nrow(ZMTSLA)
logrt_ZM = log(ZMTSLA$ZM[2:m]/ZMTSLA$ZM[1:m-1])
t = 1/252

library(moments)

#Moment Matching to generate the initial solution.
lap_mu = mean(logrt_ZM)
lap_b = sqrt((m-1)/(2*m))*sd(logrt_ZM)


#Laplace Model_fitting
initialvalueLaplace = c(lap_mu, lap_b)
Laplace = function(x, theta){exp(-abs(x-theta[1])/theta[2])/(2*theta[2])}
resultLaplace = optim(initialvalueLaplace, fn=function(theta){-sum(log(Laplace(logrt_ZM, theta)))}, method = "BFGS")
Laplace

```

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Moment Matching Estimators for Laplace Dist. .png

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$$\hat{\mu} = \bar{X}_n, \hat{b} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}_n^2} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2} = \sqrt{\frac{n-1}{2n}}S_n,$$

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Q1.8

1 Point

Derive and compute the theoretical maximum likelihood estimates for the Laplace distribution. Are they close to what you get in problem 1.7? Is the value of the log likelihood the same (to 4 digits after the decimal point)? It's okey for you to search online for this question.

T_lap_mu = median(logrt_ZM)
T_lap_b = mean(abs(logrt_ZM - T_lap_mu))

theoretical mean = 0.002542841, theoretical b = 0.031758523, it is somewhat close to 1.7.
log-likelihood is 625.2951, which is same as 1.7.

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```
#Q1-8
#Data Preparation
ZMTSLA = read.csv("/Users/yu-chingliao/Library/CloudStorage/GoogleDrive-josephliao0127@gmail.com/My Drive/Note/UIUC/Fall_2022/Statistical Methods in Finance/Assignment 06/ZMTSLA.csv")
m = nrow(ZMTSLA)
logrt_ZM = log(ZMTSLA$ZM[2:m]/ZMTSLA$ZM[1:m-1])
t = 1/252

library(moments)

```

```
T_lap_mu = median(logrt_ZM)
T_lap_b = mean(abs(logrt_ZM - T_lap_mu))
theoretical = c(T_lap_mu, T_lap_b)
theoretical

loglikelihoodLaplace = sum(log(Laplace(logrt_ZM, theoretical)))
loglikelihoodLaplace

```

▾ Theoretical Estimators for Laplace Dist..png Download ✖ Remove

Given n independent and identically distributed samples $x_1, x_2, \ldots, x_n$, the maximum likelihood (MLE) estimator of μ is the sample median,[4]

$$\hat{\mu} = \mathrm{med}(x).$$

The MLE estimator of b is the mean absolute deviation from the median,[citation needed]

$$\hat{b} = \frac{1}{N}\sum_{i=1}^{n}|x_i - \hat{\mu}|.$$

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Q1.9

1 Point

Compute AIC for all three cases. In terms of AIC, which model is fitting the data the best?

(AIC_BSM, AIC_NIG, AIC_Laplce) =
(-1172.16237122154 -1246.93186104029 -1246.5901445823)
the NIG proviide the best estimation.

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Q1-9.R

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```
#Q1-9
#Data Preparation
ZMTSLA = read.csv("/Users/yu-chingliao/Library/CloudStorage/GoogleDrive-josephliao0127@gmail.com/My Drive/Note/UIUC/Fall_2022/Statistical Methods in Finance/Assignment 06/ZMTSLA.csv")
m = nrow(ZMTSLA)
logrt_ZM = log(ZMTSLA$ZM[2:m]/ZMTSLA$ZM[1:m-1])
t = 1/252

library(moments)

AIC_BSM = -2*(-resultBSM$value-2)
AIC_NIG = -2*(-resultNIG$value-4)
AIC_Laplce= -2*(-resultLaplace$value-2)

print(paste(AIC_BSM, AIC_NIG, AIC_Laplce))


```

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Q1.10

1 Point

Construct the histogram (density) of ZM's daily log returns, and add your BSM, NIG, Laplace fits to the histogram. Graphically, which model provides the worst fit?

BSM provides the worst fit.

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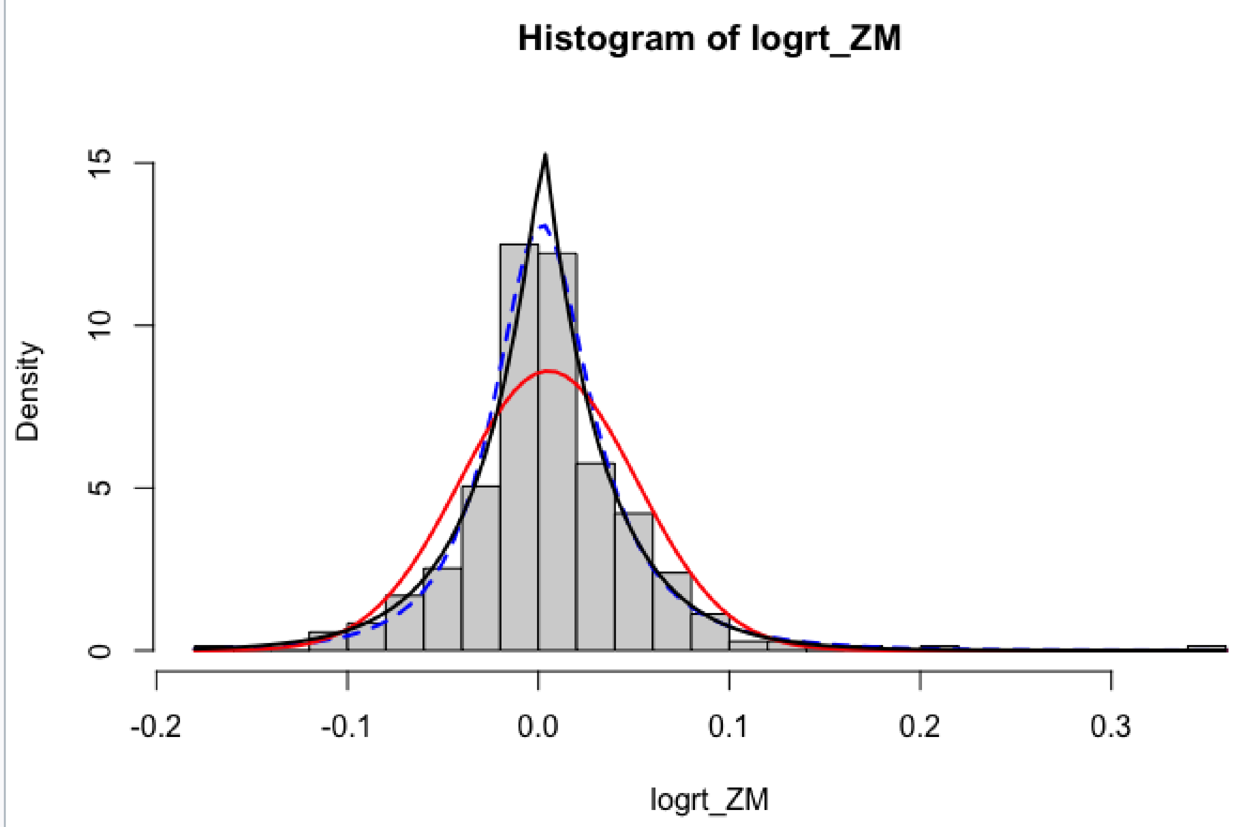

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```
#Q1-10
#Data Preparation
ZMTSLA = read.csv("/Users/yu-chingliao/Library/CloudStorage/GoogleDrive-josephliao0127@gmail.com/My Drive/Note/UIUC/Fall_2022/Statistical Methods in Finance/Assignment 06/ZMTSLA.csv")
m = nrow(ZMTSLA)
logrt_ZM = log(ZMTSLA$ZM[2:m]/ZMTSLA$ZM[1:m-1])
t = 1/252

```

```r
library(moments)

hist(logrt_ZM, breaks = 30, prob = TRUE, ylim = c(0, 16))
thetaNIG = resultNIG$par
thetaBSM = resultBSM$par
thetaLaplace = resultLaplace$par

curve(NIG(x, thetaNIG), add = TRUE, col = "blue", lwd = 2, lty = 2)
curve(BSM(x, thetaBSM), add = TRUE, col = "red", lwd = 2, lty = 1)
curve(Laplace(x, thetaLaplace), add = TRUE, col = "black", lwd = 2, lty = 1)


```

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