# IE 522 HW13

# **Q1**

3 Points

Consider a GARCH(2,2) process 
$$a_t=\sigma_t\epsilon_t, \epsilon_t\sim N(0,1), \sigma_t=\sqrt{w+\alpha_1a_{t-1}^2+\alpha_2a_{t-2}^2+\beta_1\sigma_{t-1}^2+\beta_2\sigma_{t-2}^2}$$
 with  $\alpha_1\geq 0, \alpha_2\geq 0, \beta_1\geq 0, \beta_2\geq 0, w>0, \alpha_1+\alpha_2+\beta_1+\beta_2<1.$ 

# Q1.1

1 Point

Enter your answer here

# Q1

3 Points

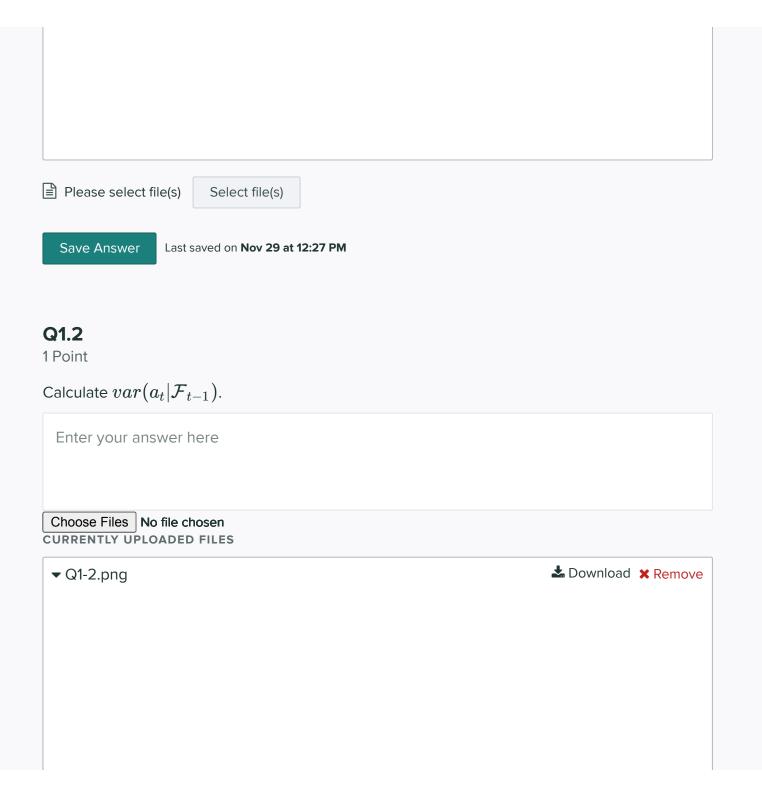
Consider a GARCH(2,2) process 
$$a_t=\sigma_t\epsilon_t, \epsilon_t\sim N(0,1)$$
,  $\sigma_t=\sqrt{w+\alpha_1a_{t-1}^2+\alpha_2a_{t-2}^2+\beta_1\sigma_{t-1}^2+\beta_2\sigma_{t-2}^2}$  with  $\alpha_1\geq 0, \alpha_2\geq 0, \beta_1\geq 0, \beta_2\geq 0,$   $w>0, \alpha_1+\alpha_2+\beta_1+\beta_2<1.$ 

### Q1.1

1 Point

Calculate  $E[a_t|{\cal F}_{t-1}]$  and  $E[a_t]$ .

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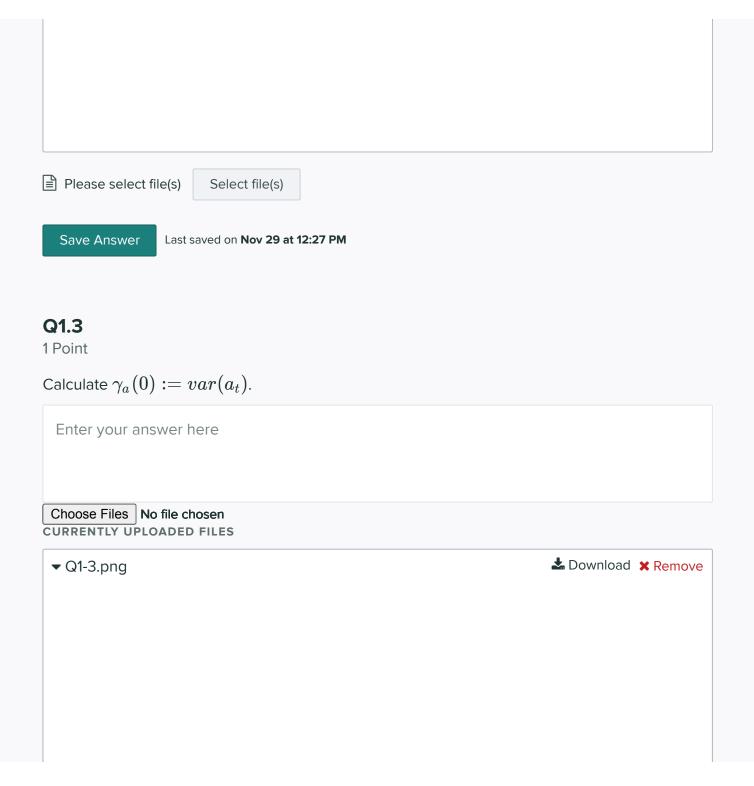


### Q1.2

1 Point

Calculate  $var(a_t|{\cal F}_{t-1})$ .

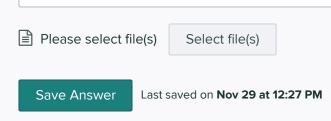
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### Q1.3

1 Point

Calculate  $\gamma_a(0) := var(a_t)$ .



# Q2

7 Points

Use the following code to get S&P 500 daily prices and log returns from <a href="https://finance.yahoo.com">https://finance.yahoo.com</a>. head() and tail() show the first 6 and last 6 observations of the time series. The first return should be on 2000/01/03, and the last return should be on 2022/11/11. In the tests below, the significance level is 5%.

```
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getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12", auto_assign = TRUE)

return=dailyReturn(GSPC, type="log")

head(return)

tail(return)
```

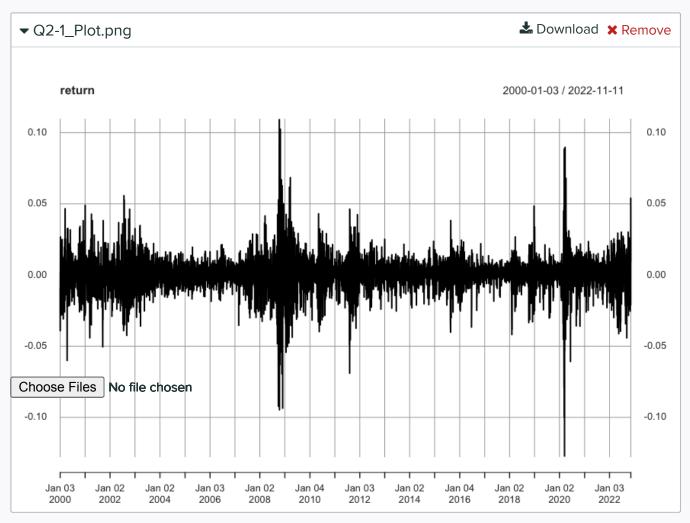
# Q2.1

1 Point

Construct a time series plot for the daily log returns. Does it exhibit volatility clustering?

Yes, there is a volatile on Jan 02 2002 that follows Jan 03 2000, and also Jan 03 2012 after around Jan 03 2009.

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#### 

- 1 #Q2-1
- 2 #Data Preparation
- 3 library(quantmod)

```
getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12",
    auto_assign = TRUE)

return=dailyReturn(GSPC,type="log")
head(return)
tail(return)
alpha = 0.05

#Construct a time series plot for the daily log returns.
plot(return)

plot(return)
```

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### Q2.2

1 Point

We want to test whether the daily log returns are random (independent) or autocorrelated. Does the runs test show evidence that the daily log returns are non-random? The Ljung-Box test is another commonly used test, where  $H_0$  is that data are independent and hence not Choose Files No. file chosen autocorrelated,  $H_1$  is that data are not independent and there exists autocorrelation. In R, use Box.test(return,lag=10,type="Ljung-Box") to conduct the Ljung-Box test for lags up to 10. Does it show evidence that the daily log returns are auto-correlated?

```
Runs Test

data: x

statistic = 4.6408, runs = 3054, n1 = 2877, n2 = 2877, n = 5754,
p-value = 3.47e-06

alternative hypothesis: nonrandomness
```

.....

We conclude that return is not random since the p value is small enough for us to reject H0: Randomness.

Box-Ljung test

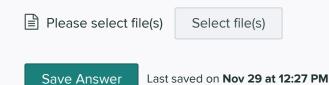
data: return

X-squared = 96.921, df = 10, p-value = 2.22e-16

------

We reject the H0: independent since the p-value is small. Hence, we conclude that return is autocorrelated.

```
▲ Download ★ Remove
▼ Q2-2.R
    #02-2
   #Data Preparation
    library(quantmod)
    getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12",
     auto_assign = TRUE)
    return=dailyReturn(GSPC, type="log")
Choose Files (No file) chosen
    tail(return)
    alpha = 0.05
 9
10 #Run test
11 library(randtests)
12 x <- as.vector(return[,"daily.returns"])</pre>
    runs.test(x,alternative = "two.sided", threshold = median(x), pvalue =
13
     "normal", plot = FALSE)
14
15
    #Ljung-Box test
   Box.test(return, lag=10, type="Ljung-Box")
16
```



### Q2.3

1 Point

The Augmented Dickey-Fuller Test is used to test whether a time series is stationary.  $H_0$  is that there exists a unit root (AR component with coefficient 1) and it's not stationary,  $H_1$  is that there is no unit root and it's stationary. This could be done in R using 'adf.test' function in

library(tseries) . Is there evidence that there is no unit root in the daily log returns?

Augmented Dickey-Fuller Test

data: return

Dickey-Fuller = -18.101, Lag order = 17, p-value = 0.01

alternative hypothesis: stationary

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We conclude that return is stationary since p value is small enough to be rejected.

```
5    return=dailyReturn(GSPC,type="log")
6    head(return)
7    tail(return)
8    alpha = 0.05
9
10    #Augmented Dickey-Fuller Test
11    library(tseries)
12    adf.test(return)
13
```

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### Q2.4

1 Point

Using arima(), find the best ARMA(p,q) model for the daily log returns with  $0 \le p \le 3, 0 \le q \le 3$ . Denote the daily log return on day t by  $Y_t$ . Write down the equation of the best model, including the distribution for the white noise (don't use auto.arima since it might pick a model

Choose Files set to zero). What's the AIC of the best model?

```
[1] "Find the best ARMA Model:"
[1] "p = 0, q = 0, AIC = -34066.4839765114"
[1] "p = 0, q = 1, AIC = -34129.3491511865"
[1] "p = 0, q = 2, AIC = -34127.4834424758"
[1] "p = 0, q = 3, AIC = -34125.6811487401"
[1] "p = 1, q = 0, AIC = -34127.9372075773"
[1] "p = 1, q = 1, AIC = -34127.4617479751"
[1] "p = 1, q = 2, AIC = -34126.6751340284"
[1] "p = 1, q = 3, AIC = -34123.7865976053"
```

```
[1] "p = 2 , q = 0 , AIC = -34127.6617590531"

[1] "p = 2 , q = 1 , AIC = -34125.5744935221"

[1] "p = 2 , q = 2 , AIC = -34123.8282991566"
```

[1] "
$$p = 2$$
,  $q = 3$ , AIC = -34121.7914499761"

$$[1]$$
 "p = 3, q = 0, AIC = -34126.112121182"

[1] "
$$p = 3$$
,  $q = 1$ , AIC = -34124.0002462696"

[1] "p = 3, q = 2, AIC = 
$$-34122.0164945384$$
"

[1] "p = 3, 
$$q = 3$$
, AIC = -34123.7734978826"

- [1] "The best fit is the one with the minimal AIC value:"
- [1] "The best ARMA model is ARMA( 0, 1) Model, AIC = -34129.3491511865"

#### Call:

arima(x = x, order = c(smallest\_p, 0, smallest\_q))

### Coefficients:

ma1 intercept

Choose Files interfied has en 0001553: log likelihood = 17067.67, aic = -34129.35

[1] "-----

$$Y_t = 0.0002 + \epsilon_t + -0.1070\epsilon_{t-1}$$

\_\_\_\_\_

$$AIC = -34129.35$$

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**▼** Q2-4.R

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- 1 #Q2-4
- 2 #Data Preparation

```
library(quantmod)
   getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12",
    auto assign = TRUE)
   return=dailyReturn(GSPC, type="log")
    head(return)
 7
    tail(return)
 8
    alpha = 0.05
9
10 #find the best arima
    find the best ARMA <- function(x){</pre>
11
12
      smallest p = 0
      smallest_q = 0
13
14
      smallest AIC = 0
15
      print("Find the best ARMA Model:")
16
17
      for (p in 0:3){
18
      for (q in 0:3){
19
         fit = arima(x, order = c(p, 0, q))
20
         print(paste("p =", p, ", ", "q =", q, ", ", "AIC =", AIC(fit)))
21
      if (smallest AIC > AIC(fit)){
22
          smallest AIC = AIC(fit)
23
          smallest p = p
24
           smallest q = q
25
          }
Choose Files | No file chosen
27 }
28
    ----")
29
      print("The best fit is the one with the minimal AIC value:")
      print(paste("The best ARMA model is ARMA(", smallest p, ",", smallest q,")
30
    Model, AIC = ", smallest AIC))
      fit4 = arima(x, order = c(smallest p,0,smallest q))
31
32
      print(fit4)
33
     print("-----
    ----")
34 }
35 find the best ARMA(return)
```

```
36
37
38
39
40
41
```

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### **Q2.5**

1 Point

The <code>garchFit</code> function in <code>library(fGarch)</code> only reports scaled AIC, that is, AIC/length of the series. If <code>result=garchFit(...)</code> is the fitting result, AIC can be obtained with <code>result@fit\$ics[1]\*nrow(return)</code>. Using AIC as the criteria, find the best model among ARMA(i,j)/GARCH(p,q), where  $0 \le i \le 2, 0 \le j \le 2, 1 \le p \le 2, 0 \le q \le 2$  (54 models in total). Write down the equation of the best model. Compare the AIC of the best model in 2.5 and the one in 2.4.

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[1] "Find the best GARCH Model:"

[1] "i, j, p, 
$$q = 0$$
, 0, 1, 2, AIC = -36985.4333670941"

```
[1] "i, j, p, q = 0 , 1 , 1 , 2 , AIC = -37003.0005888905"
```

[1] "i, j, p, 
$$q = 0$$
, 1, 2, 0, AIC = -35915.5292905145"

[1] "i, j, p, 
$$q = 0$$
, 2, 1, 0, AIC = -35113.1765883622"

[1] "i, i, p, 
$$q = 0$$
, 2, 1, 2, AIC = -37011.4357192459"

[1] "i, j, p, 
$$q = 0$$
, 2, 2, 0, AIC = -35929.4764288811"

[1] "i, j, p, 
$$q = 0$$
, 2, 2, 2, AIC = -37023.8277883297"

[1] "i, j, p, 
$$q = 1, 0, 1, 0$$
, AIC = -35091.9346436501"

[1] "i, j, p, 
$$q = 1, 0, 1, 2$$
, AIC = -37001.9634564251"

[1] "i, j, p, 
$$q = 1, 0, 2, 2$$
, AIC = -37013.8081397373"

[1] "i, j, p, 
$$q = 1, 1, 2, 0$$
, AIC = -35928.8258286894"

# Choose Files No file 2hose AIC = -37021.7789607776"

```
[1] "i, j, p, q = 2, 0, 1, 2, AIC = -37010.9507238099"
[1] "i, j, p, q = 2, 0, 2, 0, AIC = -35928.7349856275"
[1] "i, j, p, q = 2, 0, 2, 1, AIC = -37021.2053472111"
[1] "i, j, p, q = 2, 0, 2, 2, AIC = -37023.2959905097"
[1] "i, j, p, q = 2, 1, 1, 0, AIC = -35111.6040234149"
[1] "i, j, p, q = 2, 1, 1, 1, AIC = -37011.3521014812"
[1] "i, j, p, q = 2, 1, 1, 2, AIC = -37009.7578516936"
[1] "i, j, p, q = 2, 1, 2, 0, AIC = -35927.9366690353"
[1] "i, j, p, q = 2, 1, 2, 1, AIC = -37020.2536261821"
[1] "i, j, p, q = 2, 1, 2, 2, AIC = -37022.4701777692"
[1] "i, j, p, q = 2, 2, 1, 0, AIC = -35110.0248585506"
[1] "i, j, p, q = 2, 2, 1, 1, AIC = -37013.2780992923"
[1] "i, j, p, q = 2, 2, 1, 2, AIC = -37011.6373348586"
[1] "i, j, p, q = 2, 2, 2, 0, AIC = -35932.1497493543"
[1] "i, j, p, q = 2, 2, 2, 1, AIC = -37021.273683065"
[1] "i, j, p, q = 2, 2, 2, 2, AIC = -37023.5221536438"
[1] "------"
[1] "The best fit is the one with the minimal AIC value:"
[1] "The best GARCH model is ARMA(1,1)| GARCH(2,2) Model, AIC =
-37023.9449796672"
Choose Files No file chosen
Title:
 GARCH Modelling
Call:
garchFit(formula = Formulax(smallest_i, smallest_j, smallest_p,
  smallest_q), data = return, cond.dist = "norm", trace = FALSE)
Mean and Variance Equation:
data ~ arma(1, 1) + garch(2, 2)
```

```
<environment: 0x7ff3f9fa7110>
 [data = return]
Conditional Distribution:
 norm
Coefficient(s):
                                         alpha1 alpha2
     mu
              ar1
                      ma1
                              omega
                                                              beta1
1.7250e-04 7.2105e-01 -7.7157e-01 4.1956e-06 8.7558e-02 1.3365e-01 1.7393e-01
   beta2
 5.7718e-01
Std. Errors:
 based on Hessian
Error Analysis:
     Estimate Std. Error t value Pr(>ltl)
mu 1.725e-04 6.940e-05 2.486 0.0129 *
ar1 7.211e-01 1.085e-01 6.648 2.98e-11 ***
ma1 -7.716e-01 9.998e-02 -7.717 1.20e-14 ***
Choose Files 100 File Chose 11e-07 7.491 6.84e-14 ***
alpha1 8.756e-02 1.373e-02 6.379 1.79e-10 ***
alpha2 1.336e-01 1.681e-02 7.948 2.00e-15 ***
beta1 1.739e-01 1.401e-01 1.241 0.2144
beta2 5.772e-01 1.262e-01 4.574 4.78e-06 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
18519.97 normalized: 3.218626
```

```
▲ Download ★ Remove
▼ Q2-5.R
   #02-5
   #Data Preparation
 3 library(quantmod)
    getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12",
    auto_assign = TRUE)
    return=dailyReturn(GSPC, type="log")
Choose Files No file chosen
 8
    alpha = 0.05
 9
10
    #GARCH fit
11
    library(fGarch)
12
    find_the_best_GARCH <- function(x){</pre>
13
      smallest p = 0
14
      smallest q = 0
15
      smallest i = 0
      smallest j = 0
16
17
      smallest AIC = 0
18
       print("Find the best GARCH Model:")
```

```
19
      for (i in 0:2){
20
       for (j in 0:2){
21
         for (p in 1:2) {
22
            for (q in 0:2){
              Formula<-function(i, j, p, q)</pre>
23
     as.formula(paste('~arma(',i,',',j,')+garch(',p,',',q,')'))
24
               fit = garchFit(formula = Formula(i, j, p, q), data = x, cond.dist =
     "norm", trace = FALSE)
25
              AIC = fit@fit$ics[1]*nrow(return)
              print(paste("i, j, p, q =", i,",", j,",", p,",", q, ", AIC =", AIC))
26
27
               if (smallest_AIC > AIC){
28
                smallest\_AIC = AIC
29
                 smallest p = p
30
                smallest q = q
31
                 smallest i = i
32
                 smallest j = j
33
              }
34
             }
35
           }
36
37
      }
38
39
     ----")
Choose Files No file chosen fit is the one with the minimal AIC value:")
      print(paste("The best GARCH model is ARMA(", smallest i, ",", smallest j,")
     GARCH(", smallest p, ", ", smallest q, ") Model, AIC = ", smallest AIC))
     Formula5<-function(i, j, p, q)</pre>
42
     as.formula(paste('~arma(',i,',',j,')+garch(',p,',',q,')'))
      fit5 = garchFit(formula = Formula5(smallest_i, smallest_j, smallest_p,
43
     smallest q), data = return, cond.dist = "norm", trace = FALSE)
44
      print(fit5)
45
      print("-----
     ----")
46 }
47
    find the best GARCH(return)
48
```

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# **Q2.6**

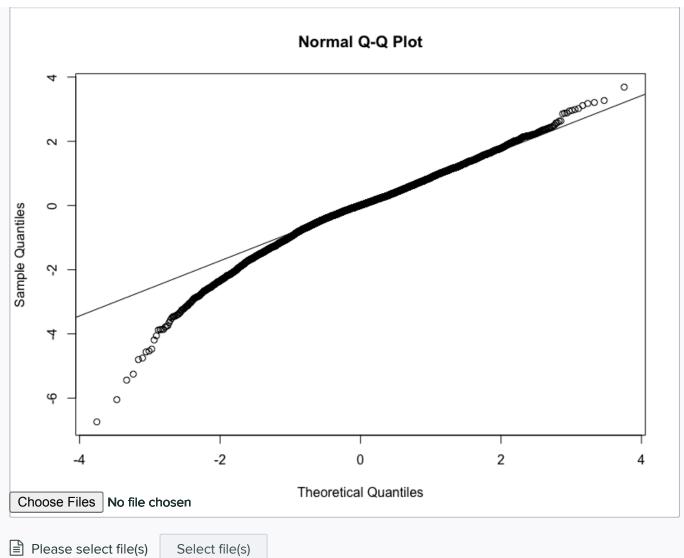
1 Point

For the best model chosen in 2.5, construct a normal probability plot for the residuals. According to the probability plot, is normal distribution good enough for  $\epsilon_t$ ?

No, it does not fit to the normal line.

```
▼ Q2-6.R
    #Q2-6
   #Data Preparation
 2
    library(quantmod)
Choose Files No file chosen = "^GSPC", from = "2000-01-01", to = "2022-11-12",
    auto_assign = TRUE)
    return=dailyReturn(GSPC, type="log")
    head(return)
 6
 7
    tail(return)
    alpha = 0.05
 8
 9
    #Get GARCH model
10
   smallest p = 0
11
12 smallest_q = 0
13 smallest i = 0
    smallest j = 0
14
    smallest AIC = 0
```

```
for (i in 0:2){
16
17
      for (j in 0:2){
18
        for (p in 1:2) {
19
           for (q in 0:2){
20
             Formula<-function(i, j, p, q)</pre>
     as.formula(paste('~arma(',i,',',j,')+garch(',p,',',q,')'))
21
             fit = garchFit(formula = Formula(i, j, p, q), data = return, cond.dist
     = "norm", trace = FALSE)
22
             AIC = fit@fit$ics[1]*nrow(return)
23
             if (smallest_AIC > AIC){
24
               smallest\_AIC = AIC
25
               smallest_p = p
26
               smallest q = q
27
               smallest i = i
               smallest j = j
28
29
             }
30
           }
31
32
33
34
35 Formula6<-function(i, j, p, q)
     as.formula(paste('~arma(',i,',',j,')+garch(',p,',',q,')'))
36 | fit6 = garchFit(formula = Formula6(smallest_i, smallest_j, smallest_p,
Choose Files No file chosen = return, cond.dist = "norm", trace = FALSE)
    print(fit6)
37
38
39 #Get residual
40 res6 = residuals(fit6, standardize = T)
41
    qqnorm(res6)
42
    qqline(res6)
43
```



Save Answer Last saved on Nov 29 at 12:27 PM Replace the normal distribution by sged in the model chosen in 2.5 (cond.dist="sged"). According to AIC, is sged better than the normal distribution for  $\epsilon_t$ ? Use qqplot(res,rsged(10000,nu=shape parameter,xi=skew parameter)) to construct a probability plot for residuals. Here shape and skew parameters are the numeric estimates you obtain in garchFit. According to the probability plot, is the sged distribution a better candidate for  $\epsilon_t$ ?

```
AIC = -37383.07
```

\_\_\_\_\_\_

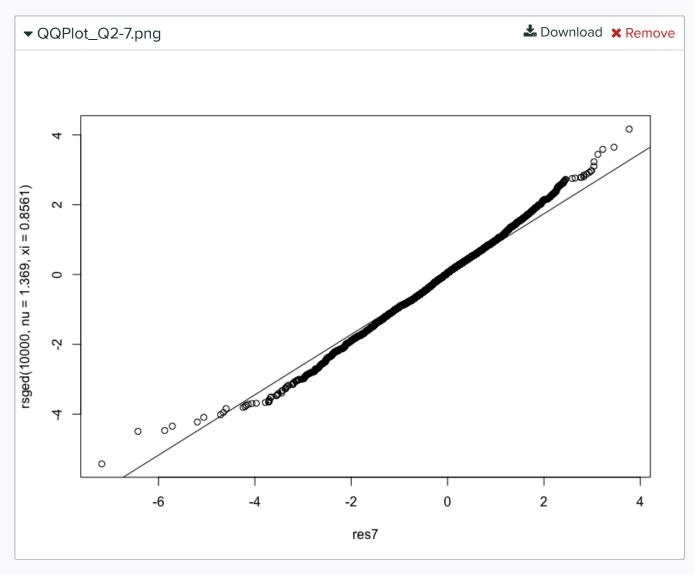
Yes, it is better normal since it has smaller AIC value.

\_\_\_\_\_\_

Yes, it fits the normal line better.

```
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▼ Q2-7.R
    #02-7
 1
 2 #Data Preparation
 3
    library(quantmod)
    getSymbols(Symbols = "^GSPC", from = "2000-01-01", to = "2022-11-12",
     auto_assign = TRUE)
Choose Files | No filey chosem (GSPC, type="log")
    head(return)
    tail(return)
 8
    alpha = 0.05
 9
10 #Get sged model
11 Formula7<-function(i, j, p, q)
    as.formula(paste('~arma(',i,',',j,')+garch(',p,',',q,')'))
12 | fit7 = garchFit(formula = Formula7(smallest i, smallest j, smallest p,
     smallest q), data = return, cond.dist = "sged", trace = FALSE)
13 AIC7 = fit7@fit$ics[1]*nrow(return)
14 print(AIC7)
```

```
15
16 #QQPlot
17 res7 = residuals(fit7, standardize = T)
18 qqplot(res7,rsged(10000,nu=1.369,xi=0.8561))
19 qqline(res7)
20
```



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Save All Answers

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