

IE 522 HW08

Q1

1 Point

Suppose you want to predict the price of an asset using the following model:

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \epsilon,$$

where X_2 is the value of a factor, X_3 is the value of another factor and X_4 is the average value of the two factors. What assumption is violated for this linear regression model? Why?

It violates the "No multicollinearity assumption", since X_4 is the linear combination of X_2 and X_3 .

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Q2

9 Points

Consider the simple linear regression model $Y = \beta_1 + \beta_2 X + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$.

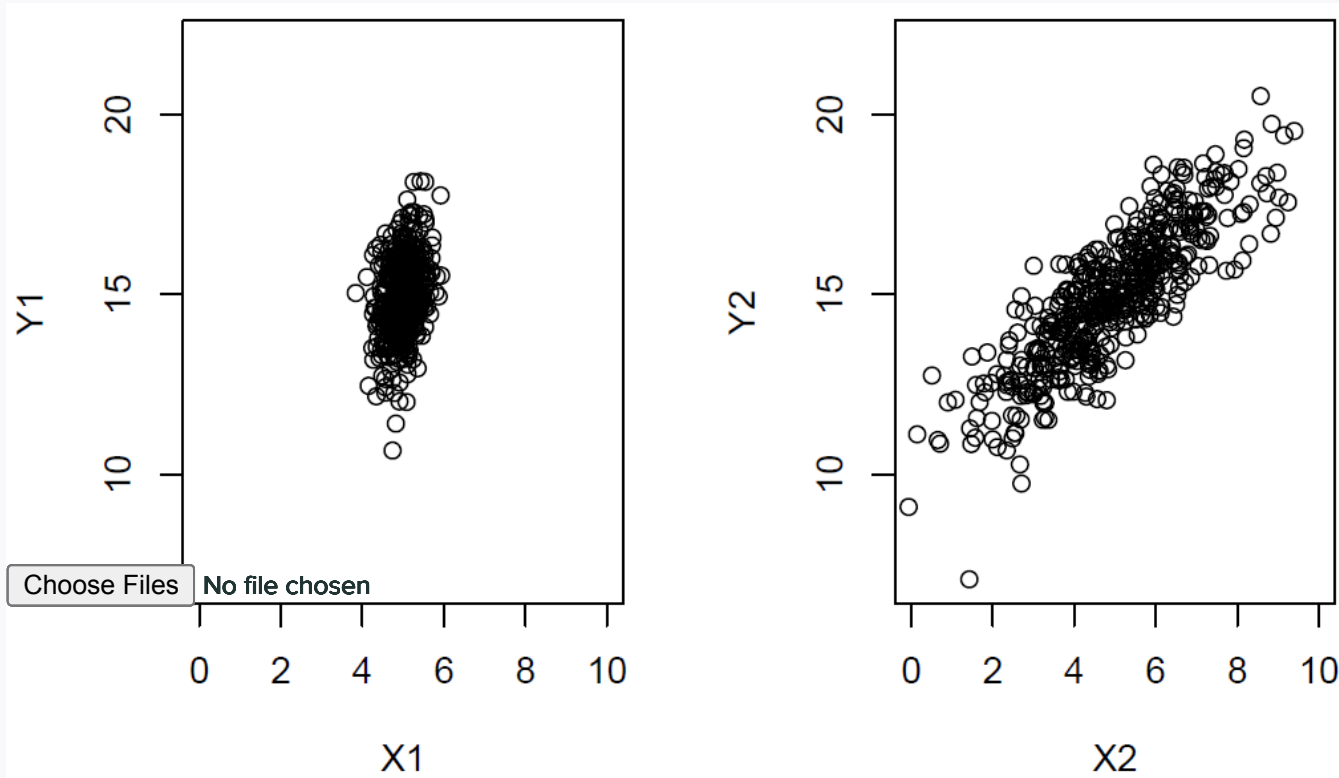
There are two sets of data of the same size $n = 500$ from this model: $\{(X_{1i}, Y_{1i}), 1 \leq i \leq$

$n\}$ and $\{(X_{2i}, Y_{2i}), 1 \leq i \leq n\}$.

Q2.1

1 Point

According to the following scatter plots, which data set will give a more precise estimate for β_2 ? Why?




Data set 2 (the one on the right hand side) has more precise estimation.

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$$se(\hat{\beta}_x | x) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} = \frac{\hat{\sigma}}{\sqrt{S_{xx}}}$$

⇒ as X_i 's more spreading out, $\hat{\beta}_x$ has better estimation with β_x .

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Q2.2

1 Point

The sample mean and sample variance of $\{X_{1i}, 1 \leq i \leq n\}$ are 4.99 and 0.32, respectively. The sample mean and sample variance of $\{X_{2i}, 1 \leq i \leq n\}$ are 5.04 and 8.62, respectively. Which data set will give a more precise estimate for β_1 ? Why?

We are not able to define which data set has better estimation for beta 1.

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$$se(\hat{\beta}_1 | X) = \hat{\sigma} \sqrt{\frac{1}{n} + \frac{\bar{X}_b^2}{S_{XX}}}$$

$$\text{Data set 1 : } \hat{\sigma}_1 \sqrt{\frac{1}{500} + \frac{(9.99)^2}{0.32 \times 499}} = 0.39741 \hat{\sigma}_1$$

$$\text{Data set 2 : } \hat{\sigma}_2 \sqrt{\frac{1}{500} + \frac{(5.04)^2}{8.62 \times 499}} = 0.08 \hat{\sigma}_2$$

Since we don't know $\hat{\sigma}_1$ and $\hat{\sigma}_2$,

it's not possible for us to assure which
can provide us better estimation.

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Q2.3

1 Point

In the following, $\{(X_{2i}, Y_{2i}), 1 \leq i \leq n\}$ is used to fit the simple linear regression model. The sample variance of $\{Y_{2i}, 1 \leq i \leq n\}$ is 9.24. What's the total sum of squares?

SST = 4610.76.

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2-3

$$SST = \sum (Y_i - \bar{Y})^2$$

$$\frac{1}{n-1} \sum (Y_i - \bar{Y})^2 = 9.24$$

$$\Rightarrow \sum (Y_i - \bar{Y})^2 = 4610.76$$

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Q2.4

1 Point

The residual standard error is 1.008. What's the corresponding error sum of squares?

SSE = 505.999872

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$$\hat{\sigma} = 1.008, \text{ SSE} = ?$$

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = 1.016064$$

$$\Rightarrow \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = 505.999872$$

$$\Rightarrow \text{SSE} = 505.999872$$

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Q2.5

1 Point

What's the regression sum of squares?

SSR = 4104.760128.

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$$SSR = \sum (\hat{Y}_i - \bar{Y}_n)^2$$

$$SST = \sum (Y_i - \bar{Y}_n)^2 = 4610.76$$

$$\left(\sum_{i=1}^n \frac{1}{n} (Y_i - \bar{Y}_n)^2 = 9.24 \right) \uparrow$$

$$\left(\sum (Y_i - \bar{Y}_n)^2 = 4610.76 \right)$$


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$$SSE = 505.999872$$

$$SST = SSE + SSR$$

$$4610.76 = 505.999872 + SSR$$

$$SSR = 4104.760128$$

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Q2.6



1 Point

What's R^2 ? What's the sample correlation coefficient between $\{X_{2i}, 1 \leq i \leq n\}$ and $\{Y_{2i}, 1 \leq i \leq n\}$?

$$R^2 = \rho^2 = 0.8902567316$$

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$$R = \frac{SSR}{SST} = \frac{4104.760128}{4610.76} = 0.8902567316$$
$$\rho^2 = R^2$$

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Q2.7

1 Point



The sample mean of $\{Y_{2i}, 1 \leq i \leq n\}$ is 15.03. What are $\hat{\beta}_1$ and $\hat{\beta}_2$?

beta 1 hat = 10.10653877

beta 2 hat = 0.9768772279

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$$\bar{Y}_n = 15.03, \quad \bar{X}_n = 5.04$$

$$\frac{1}{n-1} \sum (Y_i - \bar{Y}_n)^2 = 9.24 \rightarrow \sum (Y_i - \bar{Y}_n)^2 = 4610.26$$

$$\frac{1}{n-1} \sum (X_i - \bar{X}_n)^2 = 8.62 \rightarrow \sum (X_i - \bar{X}_n)^2 = 4301.38$$

↓

$$S_{XX} = 4301.38$$

$$R^2 = \frac{S_{XY}^2}{S_{XX} S_{YY}} \Rightarrow S_{XY} = \sqrt{R^2 S_{XX} S_{YY}}$$

$$= (0.89056736 \times 4301.38 \times 4610.26)^{1/2}$$


$$\Rightarrow S_{XY} = 4201.92017$$

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$$\frac{4201.92017}{4301.38} = S_{XY} = 0.9768772279$$

$$\hat{\beta}_1 = \bar{Y}_n - \hat{\beta}_0 \bar{X}_n = 15.03 - \hat{\beta}_0 \times 5.04$$

$$= 10.10653877$$

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Q2.8



1 Point

What's the average of the fitted values $\{\hat{Y}_{2i}, 1 \leq i \leq n\}$?

15.03

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$$\frac{1}{n} \sum \hat{Y}_i = \bar{\hat{Y}} = 15.03$$

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Q2.9



1 Point

What's the average of the residuals?

0


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$E(\hat{e}_i) \Rightarrow$ (zero-mean assumption)

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