Q2

3 Points

Consider a sample $\{x_1, \cdots, x_n\}$ from an exponential distribution with pdf $p(x|\lambda) = \lambda e^{-\lambda x}, x > 0$. For a random variable X with this distribution, $\mathbb{E}[X] = \frac{1}{\lambda}, var(X) = \frac{1}{\lambda^2}$.

Q2.1

1 Point

Derive the maximum likelihood estimate $\hat{\lambda}$ for λ and compute its value for the data given in canvas (exp.csv). What's the maximum likelihood estimate for the mean of the distribution $\theta = 1/\lambda$?

$$L(\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x} = \lambda e^{-\lambda x_1} \times \lambda e^{-\lambda x_2} \times \cdots \times \lambda e^{-\lambda x_n}$$

$$= \lambda^n e^{-\lambda(x_1 + x_2 + \cdots + x_n)} = \lambda^n e^{-\lambda \sum x_i}$$

$$\log(L(\lambda)) = \log \lambda^n - \lambda \sum x_i$$

$$= n \log \lambda - \lambda \sum x_i$$

$$\frac{d \log(L(\lambda))}{\lambda} = \frac{n}{\lambda} - n\bar{x} \Rightarrow \frac{n}{\hat{\lambda}} - n\bar{x} = 0$$

$$n\bar{x} = \frac{n}{\hat{\lambda}}, \quad \hat{\lambda} = \frac{1}{\bar{x}} = \frac{1}{E(x)}$$

$$\theta = \frac{1}{\lambda} \Rightarrow \hat{\theta} = \frac{1}{\hat{\lambda}}$$