Q1

4 Points

The inter-arrival time of market shocks is modeled by an exponential distribution with pdf $f(x) = \frac{1}{\theta}e^{-\frac{1}{\theta}x}, x > 0$. Consider a random sample $\{X_1, \cdots, X_n\}$ from this distribution.

Q1.3

1 Point

Compute the Cramér-Rao lower bound for unbiased estimators of θ. Will you be able to find an unbiased estimator for θ that is more precise than the sample mean $\bar{X}_n$?

$E(\bar{X}_n) = \theta$

$$I_n(\theta) = n E\left[\left(\frac{d}{d\theta} \log_e \frac{1}{\theta} e^{-\frac{1}{\theta}x}\right)^2\right]$$

$$= n E\left[\left(\frac{d}{d\theta}\left(\log \frac{1}{\theta} + -\frac{1}{\theta}x\right)\right)^2\right]$$

$$= n E\left[\left(\frac{d}{d\theta}\left(-\ln \theta - \frac{1}{\theta}x\right)\right)^2\right]$$

$$= n E\left[\left(\frac{1}{\theta^2}x - \frac{1}{\theta}\right)^2\right]$$

$$= \theta^{-4} n E[(X-\theta)^2] = \theta^{-4} n \, Var(X) = \theta^{-2} n$$

$$\frac{1}{I_n(\theta)} = \frac{1}{\theta^{-2} n} = \frac{1}{n}\theta^2 \Rightarrow$$

$Var(\bar{X}_n)$ attain the lower bound $\rightarrow$ best est.

(Cannot find the better)