Q1

4 Points

The inter-arrival time of market shocks is modeled by an exponential distribution with pdf $f(x) = \frac{1}{\theta} e^{-\frac{1}{\theta}x}, x > 0$. Consider a random sample $\{X_1, \cdots, X_n\}$ from this distribution.

Q1.1

1 Point

The sample mean $\bar{X}_n$ is used for estimating θ. Compute the bias of $\bar{X}_n$.

$$\bar{X}_n = E(X_i) = \int_{x=0}^{x=\infty} x f(x)\, dx$$

$$= \int_0^\infty x \frac{1}{\theta} e^{-\frac{1}{\theta}x}\, dx = \frac{1}{\theta}\int_0^\infty x e^{-\frac{1}{\theta}x}$$

$$= \frac{1}{\theta}\left[\theta^2 e^{-\frac{1}{\theta}x}\left(-\frac{1}{\theta}x - 1\right)\right]\Bigg|_0^\infty$$

$$= \frac{1}{\theta}(0 + \theta^2) = \theta = \bar{X}_n = E(X_i)$$

$$E(\bar{X}_n) = E\left(\frac{1}{n}\sum_{i=1}^n X_i\right) = \frac{1}{n}\sum_{i=1}^n [\underbrace{E(X_i)}_{=\theta}]$$

$$= \frac{1}{n}\sum_{i=1}^n \theta = \theta \qquad (\theta = E(\bar{X}_n) = E(X_i))$$

(Sample mean) (population mean)

* Since, if Sample mean $(\bar{X}_n)$ is unbiased,

$E(\bar{X}_n) - \theta = 0$

and Since, $E(\bar{X}_n) = \theta$

$\Rightarrow \bar{X}_n$ is unbiased $\Rightarrow$ bias$(\bar{X}_n) = 0$ #