Q1

4 Points

The inter-arrival time of market shocks is modeled by an exponential distribution with pdf $f(x) = \frac{1}{\theta} e^{-\frac{1}{\theta} x}, x > 0$. Consider a random sample $\{X_1, \cdots, X_n\}$ from this distribution.

Q1.2

1 Point

Compute the standard error of $\bar{X}_n$.

$$E(\bar{X}_n) = \theta$$

$$E(\bar{X}_n^2) = \frac{1}{n} \sum_{i=1}^{n} E(X_i^2)$$

$$\begin{cases} E(X_i^2) = \int_0^\infty x^2 \frac{1}{\theta} e^{-x\frac{1}{\theta}} dx = \frac{1}{\theta} \int_0^\infty x^2 e^{-\frac{1}{\theta} x} \\ = \frac{1}{\theta} \left[e^{-\frac{1}{\theta} x} (-\theta x^2 - 2\theta^2 x - 2\theta^3) \right] \Big|_0^\infty \\ = \frac{1}{\theta} (0 + 2\theta^3) = 2\theta^2 \end{cases}$$

(Continue) $E(\bar{X}_n^2) = \frac{1}{n} \sum_{i=1}^{n} 2\theta^2 = 2\theta^2$

$$Var(\bar{X}_n) = E(\bar{X}_n^2) - E(\bar{X}_n)^2 = \theta^2$$

$$\Rightarrow se(\bar{X}_n) = \theta \quad \#$$

* (the characteristic of exp-dist : mean $= \frac{1}{\lambda}$, std $= \frac{1}{\lambda}$)