Q1

1 Point

The pdf of a probability distribution is given by $p(x) = ac^a/x^{a+1}, x \geq c$. Given a random sample $\{x_1, \cdots, x_n\}$, write down the log likelihood function. What c maximizes the log likelihood? What's the mle for a?

$$p(x) = \frac{ac^a}{x^{a+1}}$$

$$L(x) = \prod_{i=1}^{n} \frac{ac^a}{x_i^{a+1}} = \frac{ac^a}{x_1^{a+1}} \times \frac{ac^a}{x_2^{a+1}} \times \cdots \times \frac{ac^a}{x_n^{a+1}}$$

$$\log(L(x)) = \log \frac{ac^a}{x_1^{a+1}} + \log \frac{ac^a}{x_2^{a+1}} + \cdots + \log \frac{ac^a}{x_n^{a+1}}$$

$$= \sum \log \frac{a \times c^a}{x_i^{a+1}} = \sum \log \frac{a}{x_i} + \sum \log \frac{c^a}{x_i^a}$$

$$= \sum^n \log a - \sum^n \log x_i + \sum \log c^a - \sum \log x_i^a$$

$$= n \log a - (a+1) \sum \log x_i + an \log c$$

(log likelihood func)

$$\frac{d \log(L(x))}{dc} = \frac{an}{c} > \quad \text{as } c \to \infty, \ \frac{d \log(L(x))}{dc} = \frac{an}{c} \to 0$$

but $x \geq c \Rightarrow c = x_{min}$ #

$$\frac{d \log(L(x))}{da} = \frac{n}{a} - \sum \log x_i + n \log c \overset{\text{let}}{=} 0$$

$$\Rightarrow \frac{n}{a} = \sum \log x_i - n \log c$$

$$\hat{a} = n \left(\sum \log x_i - n \log c \right)^{-1}$$ #

mle of a.