Q1

7 Points

Consider a basket call option on two assets. For the first asset,

- the current asset price is \$100,
- it doesn't pay dividend,
- volatility is 50% per year.

For the second asset,

- the current asset price is \$150,
- dividend yield is 3% per year,
- volatility is 40% per year.

The risk free interest rate is 1% per year with continuous compounding. Assume the Black-Scholes-Merton model for both assets. The log returns of the two assets are correlated with coefficient 0.1. The strike price of the basket call is $K = \$250$. The maturity of the option is 1 year.

Q1.1

1 Point

Suppose $S_T = S_{1T} + S_{2T}$ is approximated by a lognormal random variable $e^{\mu+\sigma Z}$. Using the moment matching method, find μ and σ. Keep four digits after the decimal point.

$K = 250$,

$S_{10} = 100$, $S_{20} = 150$ $\quad r = 1\%$

$\sigma_1 = 50\%$, $\sigma_2 = 40\%$ $\quad T = 1$

$q_1 = 0\%$, $q_2 = 3\%$ $\quad \rho = 0.01$

※ We now want μ and σ^2

1st moment) $E(S_T) = E(S_{1T} + S_{2T})$

$$e^{\mu + \frac{1}{2}\sigma^2} = e^{\mu_1 + \frac{1}{2}\sigma_1^2} + e^{\mu_2 + \frac{1}{2}\sigma_2^2} \quad \text{\& } (\ln)$$

$$\mu + \frac{1}{2}\sigma^2 = \ln\left(e^{\mu_1 + \frac{1}{2}\sigma_1^2} + e^{\mu_2 + \frac{1}{2}\sigma_2^2}\right)$$

$$= \ln(a) \quad \text{—①}$$

(2nd moment) $E(S_T^2) = E((S_{1T}+S_{2T})^2)$

$= E(S_{1T}^2) + E(S_{2T}^2) + 2E(S_{1T}S_{2T})$

$\left(\because E(S_{1T}S_{2T}) = e^{M_1+M_2+\frac{1}{2}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)}\right)$

$\Rightarrow e^{2M+2\sigma^2} = e^{2M_1+2\sigma_1^2} + e^{2M_2+2\sigma_2^2} +$

$2e^{M_1+M_2+\frac{1}{2}(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)} = b$

$\Rightarrow 2M + 2\sigma^2 = \ln(b)$ — ②

① ∩ ② :

$M = \ln \frac{a^2}{\sqrt{b}}, \quad \sigma^2 = \ln \frac{b}{a^2}$

(Use R code) $\Rightarrow M = 5.4529, \ \sigma = 0.3337$

$\frac{58}{\sqrt{10020 \cdot x}} < 0.01$

$58 < \sqrt{10020 x}$

$100x > 58$

$x > \frac{58}{100}$

$\sqrt{x} > 58$

$x > 58^2$

$= 3364$