Information

9. Consider the event space $\Omega \stackrel{\text{def}}{=} [0,1)$ with probability measure dx. Consider the partition

$$\mathcal{P} \stackrel{\text{def}}{=} \{[0, {}^1\!/_3), [{}^1\!/_3, {}^2\!/_3), [{}^2\!/_3, 1)\}.$$

Assume that $f : \Omega \to \mathbb{R}$ is such that

$$\mathbb{E}\left[f\middle|\mathcal{P}\right](x) = 5\mathbf{1}_{[0,1/3)}(x) - 2\mathbf{1}_{[1/3,2/3)}(x) + 6\mathbf{1}_{[2/3,1)}(x)$$

Define

$$g(x) = 4\mathbf{1}_{[0,1/3)}(x) - 2\mathbf{1}_{[1/3,2/3)}(x).$$

and compute $\mathbb{E}[fg]$.

10. Define

$$\Omega \stackrel{\text{def}}{=} [0,1)^2$$
$$A \stackrel{\text{def}}{=} [0, {}^2\!/_3) \times ({}^1\!/_2, 1)$$
$$B \stackrel{\text{def}}{=} [{}^1\!/_3, 1) \times [0, 1).$$

Enumerate and sketch all sets in $\sigma(\{A, B\})$.

11. Define $\Omega \stackrel{\text{def}}{=} [0,1)$ and

$$f(x) \stackrel{\text{def}}{=} x^3. \qquad x \in [0,1)$$

Compute the conditional expectation of f given the partition

$$P \stackrel{\text{def}}{=} \{[0, {}^1\!/_2], ({}^1\!/_2, {}^3\!/_4], ({}^3\!/_4, 1)\}.$$

12. Let X and Y be independent Gaussian random variables; let X have mean 1 and variance 4, and let Y have mean -1 and variance 2. Define

$$S_0 \stackrel{\text{def}}{=} 3, \qquad S_1 \stackrel{\text{def}}{=} S_0 \exp[X], \qquad \text{and} \qquad S_2 \stackrel{\text{def}}{=} S_1 \exp[Y].$$

Compute

(a) $\mathbb{E}[S_1]$

(b) $\mathbb{E}[S_2]$

(c) $\mathbb{E}[S_2|S_1]$.