Brownian Motion

6. (a) Compute $\mathbb{E}[W_1^2 W_s^2]$ for $0 \leq s \leq 1$.
   (b) Compute
   $$\mathbb{E}\left[W_1^2 \int_{s=0}^{1} W_s^2 ds\right].$$

7. Compute
   (a) $\mathbb{E}[(W_{10} - W_7)^2]$.
   (b) $\mathbb{E}[(W_5 - W_1)^2]$.
   (c) $\mathbb{E}[(W_{10} - W_7)(W_5 - W_1)]$.
   (d) $\mathbb{E}\left[(3\{W_{10} - W_7\} + 4\{W_5 - W_1\})^2\right]$.

8. This is a key calculation for *exponential martingales* (useful in *numéraire* calculations). Compute
   (a) $\mathbb{E}[W_{10} - W_6]$.
   (b) $\mathbb{E}[(W_{10} - W_6)^2]$.
   (c) $\mathbb{E}\left[\exp\left[3(W_{10} - W_6)\right]\right]$.
   (d)
   $$\mathbb{E}\left[\exp\left[3(W_{10} - W_6) - 18\right]\right]$$