Matrix Completion

STAT 542 Final Project
Group 11

Outline

- Dataset Introduction
- Models
- Results Comparison (RMSE)
- Difficulties

Dataset Introduction

Dataset Introduction

Motivation

- Most of the ratings are 3, and could be due to that the students have not tried the restaurants
- The dataset was collected from students, and we want to obtain a similar dataset from students to work on

Description

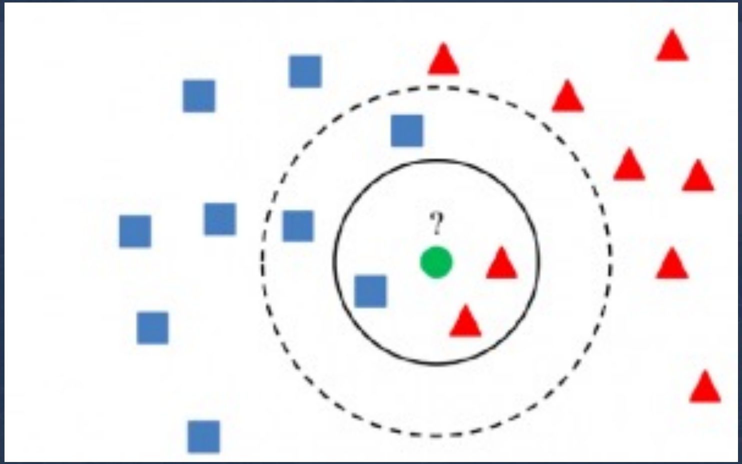
- Created an identical survey with the same 15 restaurants and the same rating scale (1 to 5)
- Collected from 50 students of UIUC
- $(n, p) = (50, 15)$

K Nearest Neighbor

KNN

Methodology

- Compute similarity of the data points
- Define *k* nearest neighbor



Similarity Measures

- Pearson correlation:

$$m_{Pearson}(X,Y) = \frac{1}{n-1}\sum_{l=1}^{n}\left(\frac{x_l-\bar{x}}{s_x}\right)\left(\frac{y_l-\bar{y}}{s_y}\right)$$, Similarity $s = \frac{m+1}{2}$

- Cosine Similarity:

$$m_{cosine}(X,Y) = \frac{X \cdot Y}{\|X\|\|Y\|}$$, Similarity $s = \frac{m+1}{2}$

- Euclidean Distance:

$$d(X,Y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$$, Similarity $s = \frac{1}{1+d}$

User-Based Collaborative Filtering Using KNN

User-Based Collaborative Filtering

Methodology

- Find a neighborhood of similar users:
  - Missing ratings are skipped in the calculation.
  - Compute similarity.
  - Define number k of nearest neighbors (select highest similarity).
- Predict missing ratings by taking the average rating of users in the k nearest neighborhood.

User-Based Collaborative Filtering

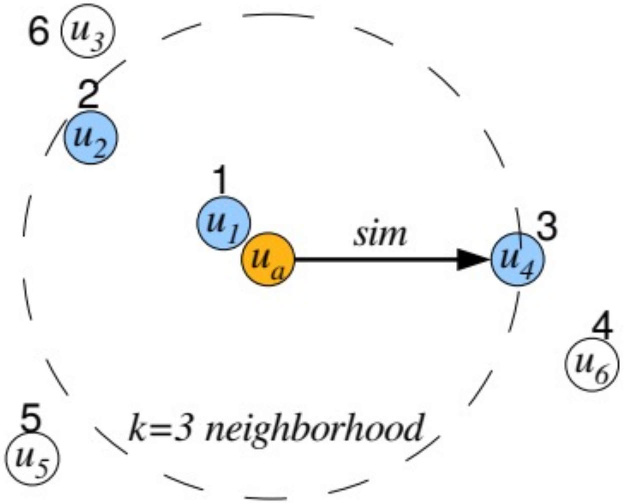
Methodology

R	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
u_1	?	4.0	4.0	2.0	1.0	2.0	?	?
u_2	3.0	?	?	?	5.0	1.0	?	?
u_3	3.0	?	?	3.0	2.0	2.0	?	3.0
u_4	4.0	?	?	2.0	1.0	1.0	2.0	4.0
u_5	1.0	1.0	?	?	?	?	?	1.0
u_6	?	1.0	?	?	1.0	1.0	?	1.0
u_a	?	?	4.0	3.0	?	1.0	?	5.0
$\hat{r}_a$	3.5	4.0			2.3		2.0	

(a)

S_a	u_a
u_1	0.3
u_2	1.0
u_3	0.2
u_4	0.3
u_5	0.1
u_6	0.1

(b)



(c)

https://cran.r-project.org/web/packages/recommenderlab/vignettes/recommenderlab.pdf

User-Based CF using KNN

Results

Similarity measure	Chosen K	RMSE
Pearson	10	0.8609
Pearson	20	0.8394
Cosine	10	0.6780
Cosine	20	0.6484
Euclidean	10	0.6896
Euclidean	20	0.6525

Item-Based Collaborative Filtering Using KNN

Item-Based Collaborative Filtering

Methodology

- Find a neighborhood of similar items:
  - Missing ratings are skipped.
  - Compute similarity.
  - Define number k of nearest neighbors (select highest similarity).
- Predict missing ratings by taking the weighted-average rating of items in the k nearest neighborhood.
  - Weight: similarity
  - Rating: user's rating matched similar items

Item-Based Collaborative Filtering

Methodology

S	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	$\hat{r}_a$
i_1	-	0.1	0	**0.3**	**0.2**	**0.4**	0	0.1	-
i_2	0.1	-	**0.8**	**0.9**	0	**0.2**	0.1	0	0.0
i_3	0	**0.8**	-	0	**0.4**	0.1	0.3	**0.5**	4.6
i_4	**0.3**	**0.9**	0	-	0	0.1	0	**0.2**	3.2
i_5	**0.2**	0	**0.4**	0	-	0.1	**0.2**	0.1	-
i_6	**0.4**	**0.2**	0.1	**0.3**	0.1	-	0	0.1	2.0
i_7	0	**0.1**	**0.3**	0	**0.2**	0	-	0	4.0
i_8	**0.1**	0	**0.5**	**0.2**	0.1	0.1	0	-	-

k=3

u_a	2	?	?	?	4	?	?	5

https://cran.r-project.org/web/packages/recommenderlab/vignettes/recommenderlab.pdf

Item-Based CF using KNN

Results

Similarity measure	Optimal K	RMSE
Pearson	7	0.6829
Cosine	8	0.6338
Euclidean	11	0.6337

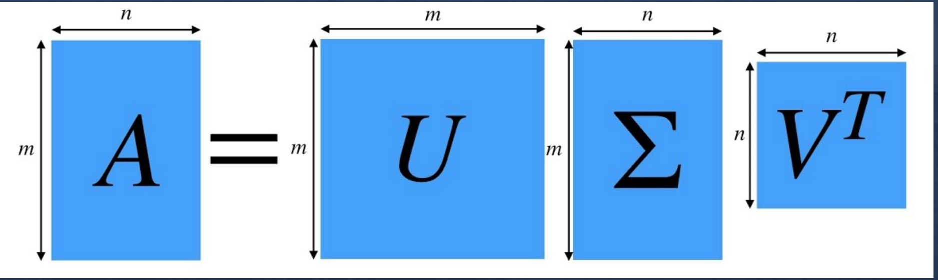
Singular Value Decomposition (SVD)

SVD

Methodology

- Singular Value Decomposition (SVD) of a matrix A is a factorization into three matrices U, Σ, and V, with U and V being orthogonal matrices and Σ being a diagonal matrix with singular value entries.

$$A = U\Sigma V^T$$



Iterative SVD

Steps:

1. Initial guess for NaN values in the matrix A
2. Apply SVD to A
3. Apply Low-Rank Matrix Approximation
4. Replace the known value in A to get matrix A'
5. Repeat the process until the difference between A and A' is less than a pre-determined threshold

*Note that the results highly depend on the initial matrix.

SVD

Results

How NaN values were initialized	RMSE
3	0.5080
Row mean	0.4435
Column mean	0.4070

What if we used other initial matrices to implement SVD?

KNN+SVD

Steps:

1. Use the results from KNN as matrix A
2. Apply SVD to A
3. Apply Low-Rank Matrix Approximation
4. Replace the known value in A to get matrix A’
5. Repeat the process until the difference between A and A’ is less than a pre-determined threshold

KNN+SVD

Results

KNN	RMSE
User-Based	0.4350
Item Based	0.4016

Results Comparison

Results Comparison

	Item-Based KNN	Used-Based KNN	SVD	KNN+SVD
RMSE	0.6337	0.6484	0.4070	0.4016

Difficulties

Difficulties

Problem 1- Dataset

- We observed that most of the values in the provided dataset contains 3, making google reviews or other ratings online unreliable

Solution:

- We decided to collect our own dataset, in ways that maximizes its similarity with Feedback.csv

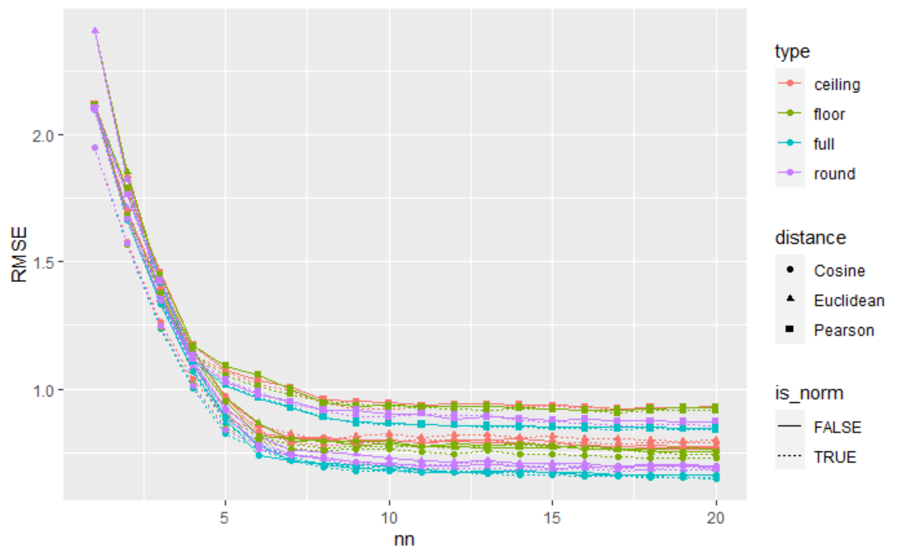
Difficulties

Problem 2- User-Based CF

- Although increasing k still results in a decrease in RMSE, the decrease is only marginal after reaching k=10

Solution:

- Decided to choose k=20 as it covers around 50% of the data



Difficulties

Problem 3- User-Based and Item-Based CF

- User rating bias: some users tend to use higher ratings while some tend to use lower ratings

Solution:

- Center the rows of user-item rating by doing normalization

$$h(r_{jl}) = r_{jl} - \bar{r_j}$$

Thank you