Education Examples for Constitutive Material Behavior

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9. February, 2017

Contents

1 Chapter Summary and Highlights

In the Chapter, the mechanical behaviors of elastoplastic materials is tested. The purpose is to simulate the simplest material points or the simplest solid brick for the sake of education.

Section.2 and Section.3 are testing one material point directly.

Section.4 and Section.5 are testing the material (Gauss Points) through the 8-node solid brick element.

Distinct yield surfaces are tested, including von-Mises and Drucker-Prager yield surfaces with associative and non-associative plastic flow.

Likewise, various hardening rules are simulated, including isotropic hardening, kinematic hardening, Armstrong-Frederick hardening, and multi-yield-surface hardening.

2 Elastic Solid Constitutive Examples

2.1 Linear Elastic Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 100;
bye;
```

Material Response:

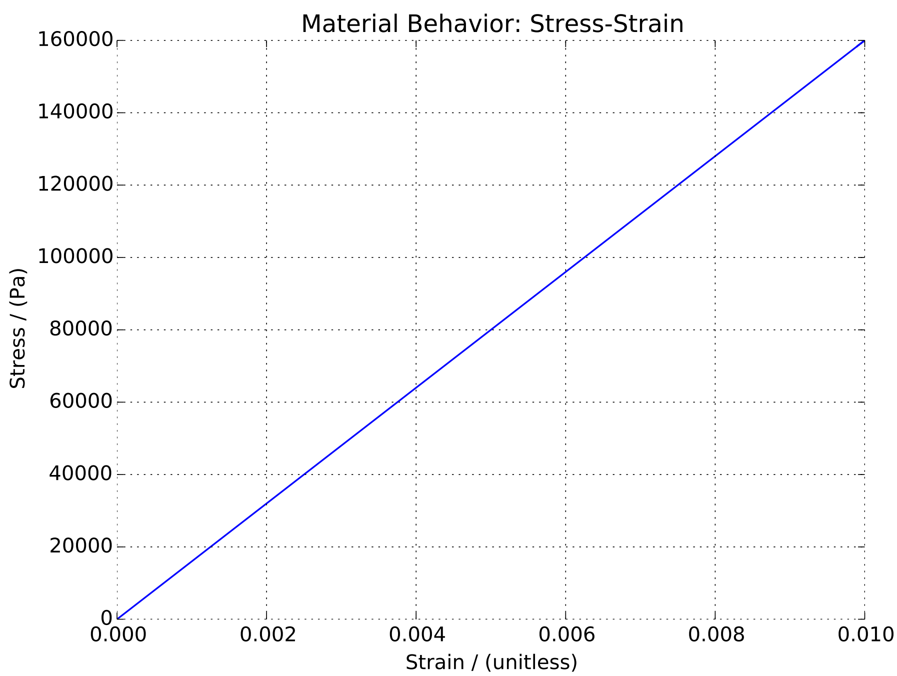

Figure 1: Linear Elastic Pure Shear Monotomic Loading

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

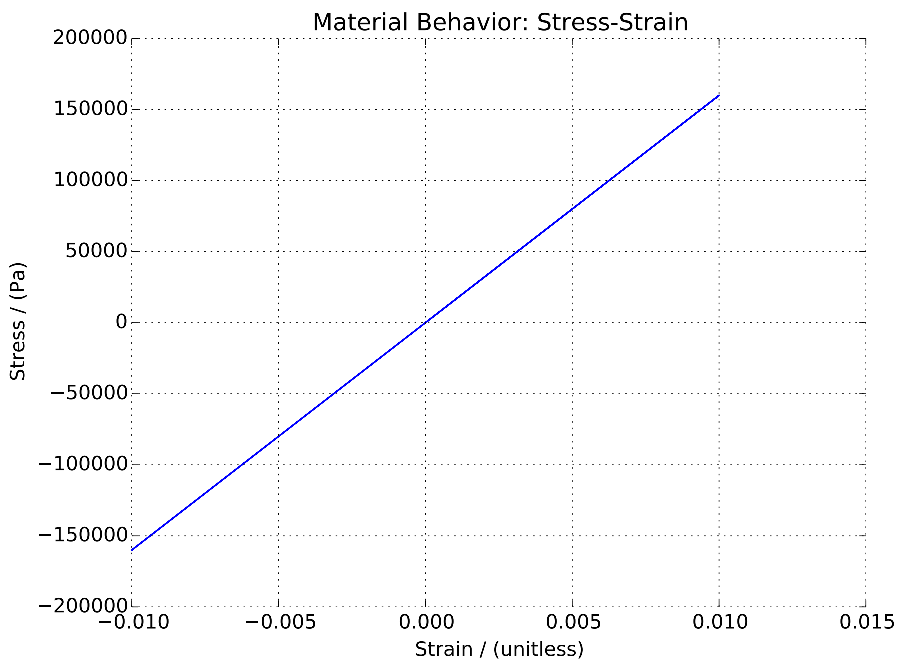

Figure 2: Linear Elastic Pure Shear Cyclic Loading

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.0 ;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 100;
bye;
```

Material Response:

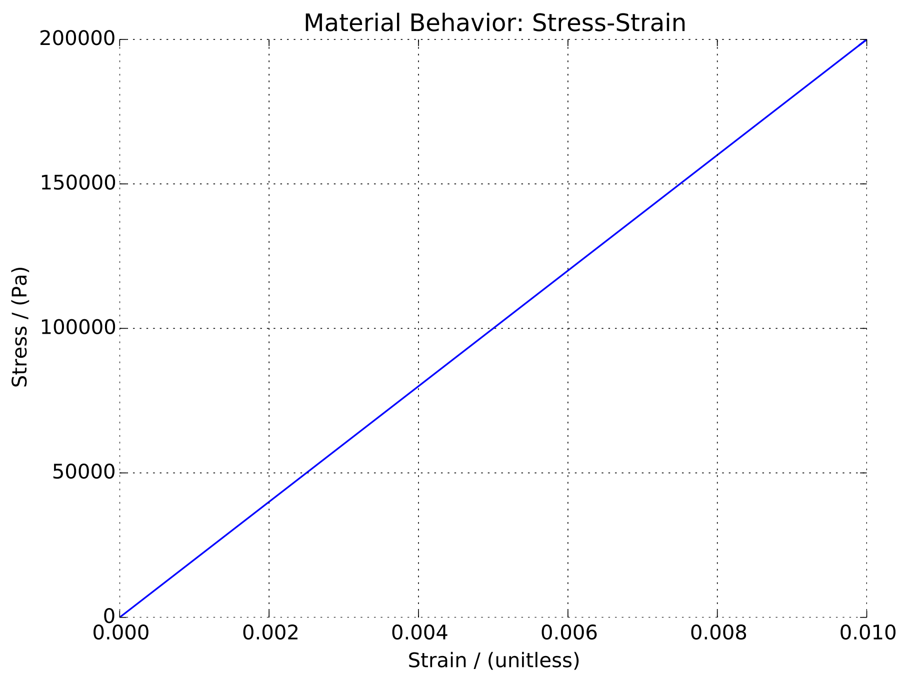

Figure 3: Linear Elastic Uniaxial Monotonic Loading

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

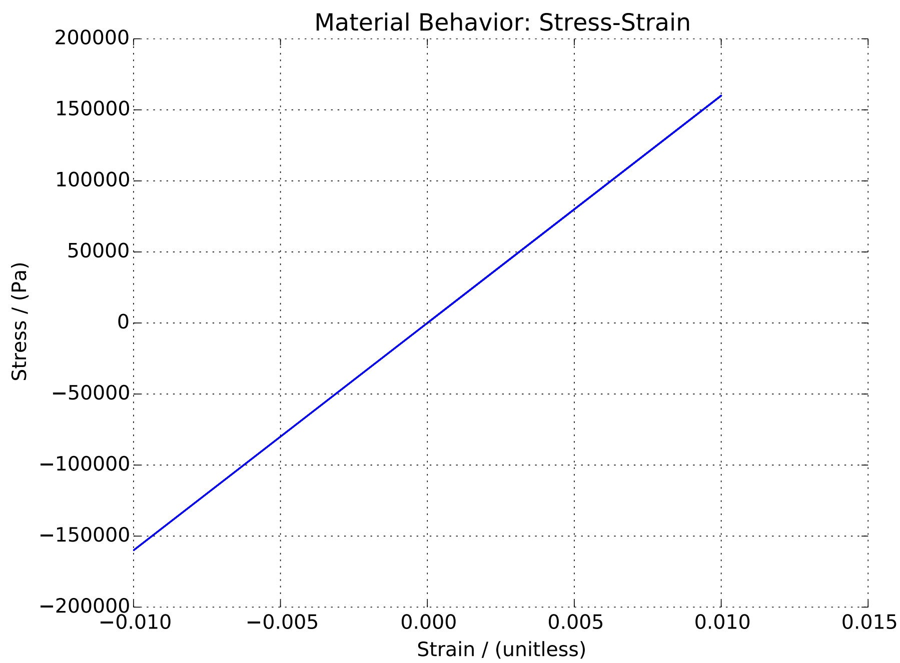

Figure 4: Linear Elastic Uniaxial Cyclic Loading

3 Elastic Plastic Solid Constitutive Examples

3.1 Elastic Perfectly Plastic Constitutive Examples

Pure Shear

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0 *Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

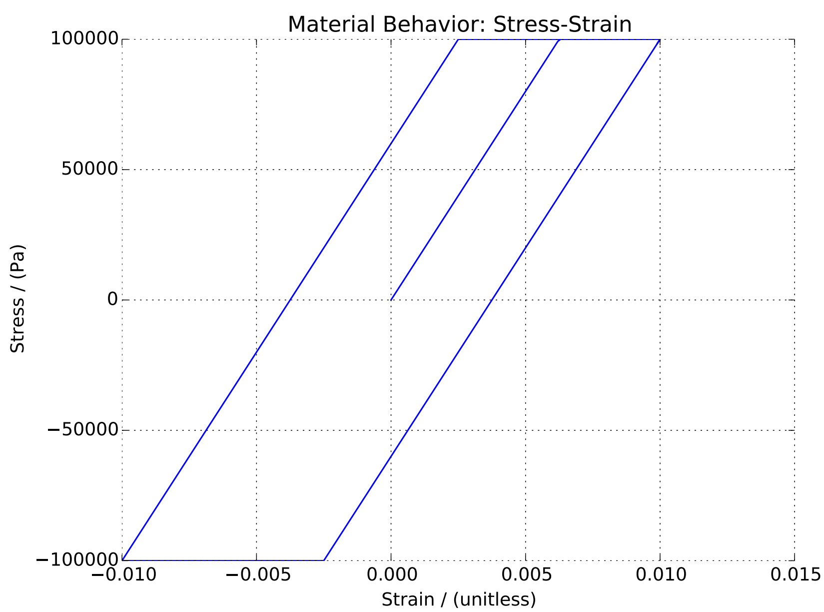

Figure 5: Perfectly Plastic Pure Shear Cyclic Loading

Uniaxial Strain

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0 *Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

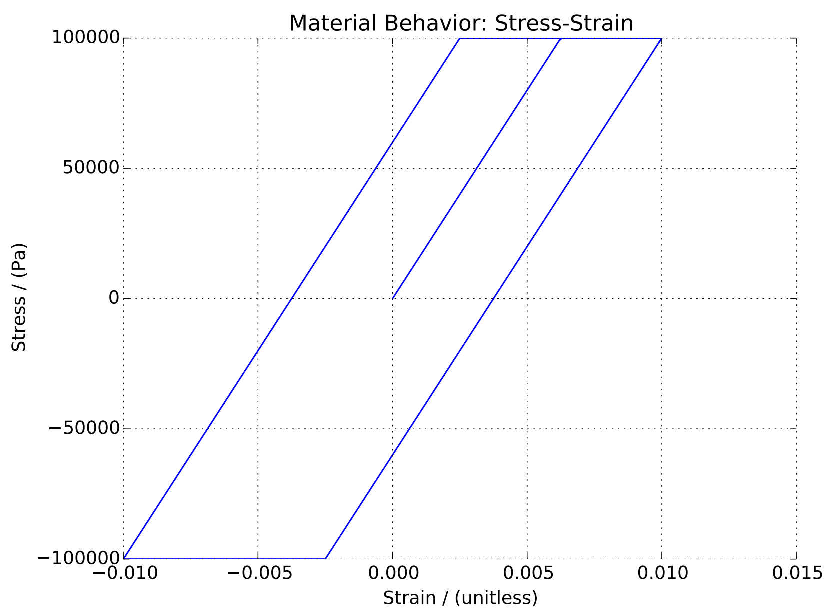

Figure 6: Perfectly Plastic Uniaxial Cyclic Loading

3.2 Elastic Plastic, Isotropic Hardening, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

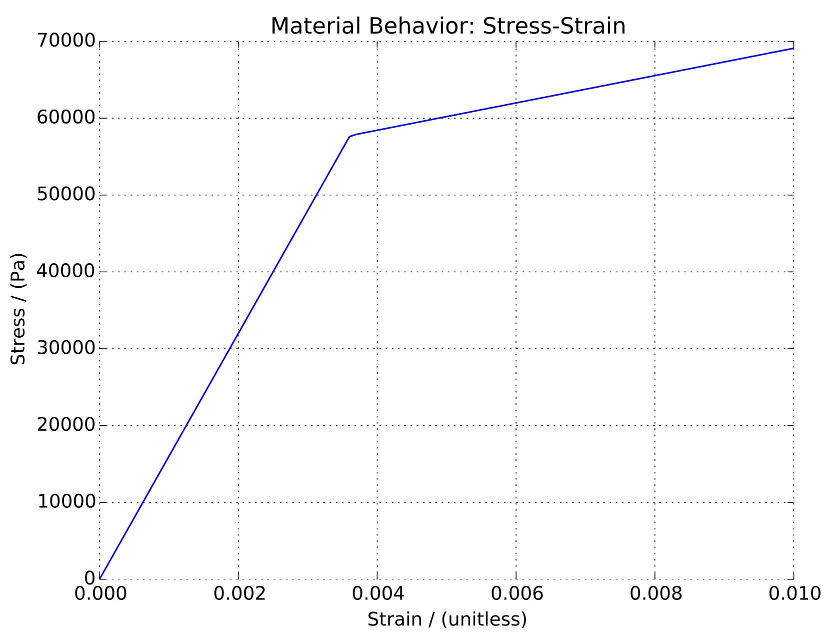

Figure 7: Isotropic Hardening Pure Shear Monotonic Loading

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

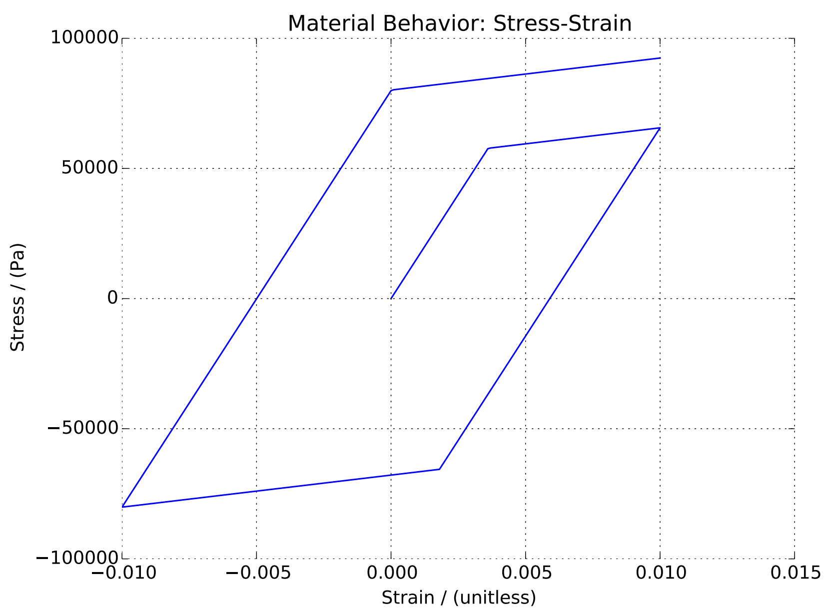

Figure 8: Isotropic Hardening Pure Shear Cyclic Loading

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

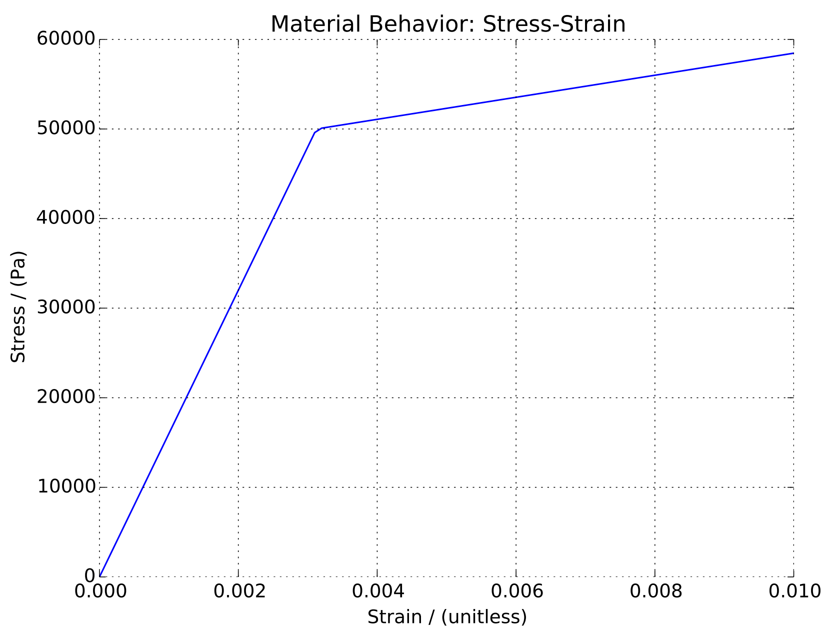

Figure 9: Isotropic Hardening Uniaxial Monotonic Loading

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

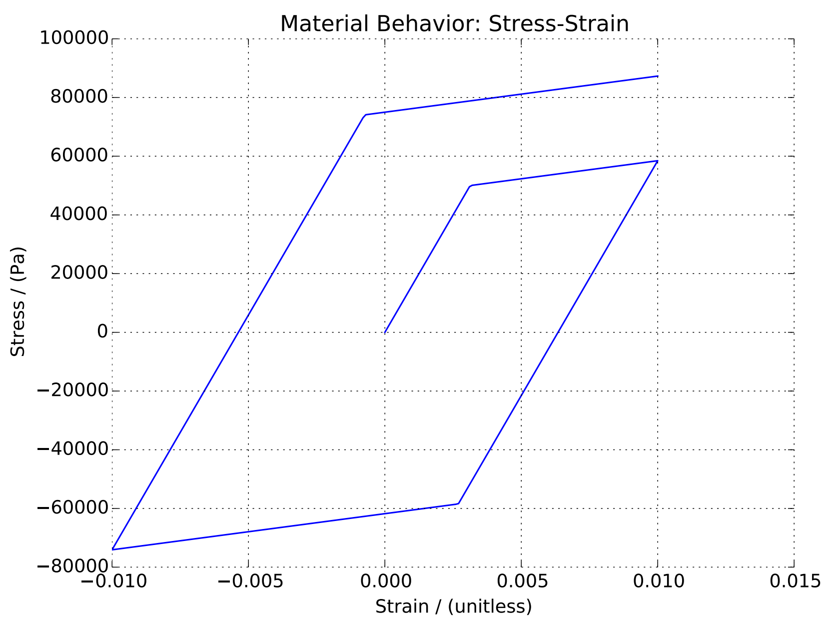

Figure 10: Isotropic Hardening Uniaxial Cyclic Loading

3.3 Elastic Plastic, Kinematic Hardening, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

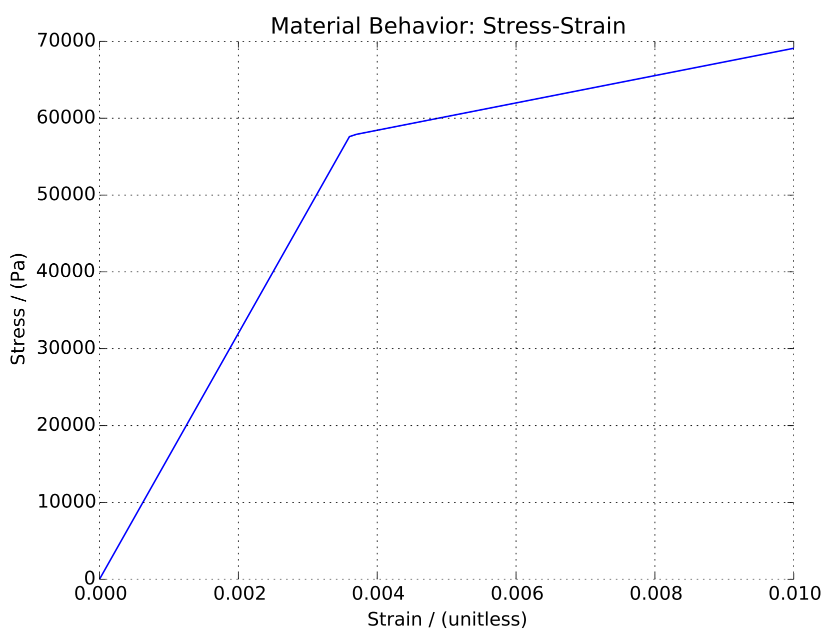

Figure 11: Kinematic Hardening Monotonic Cyclic Loading

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

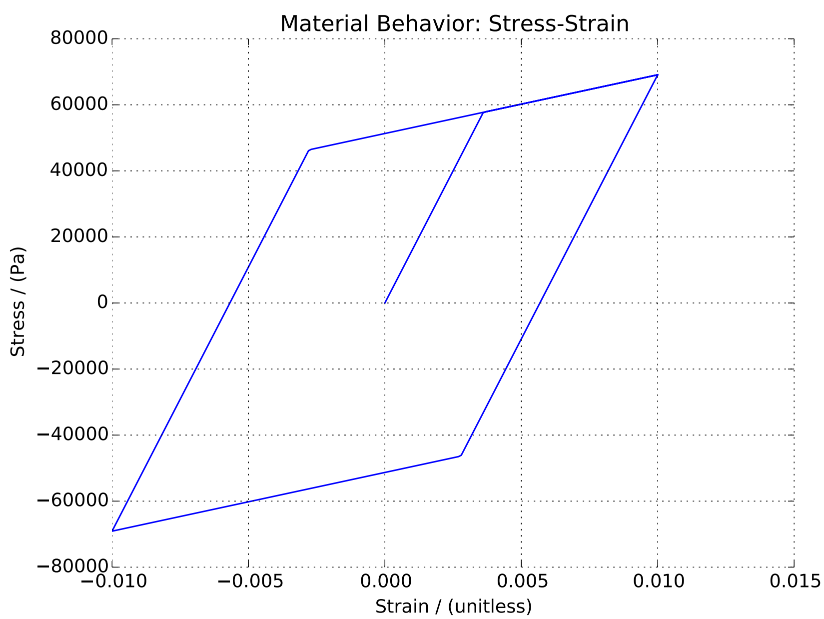

Figure 12: Kinematic Hardening Pure Shear Cyclic Loading

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

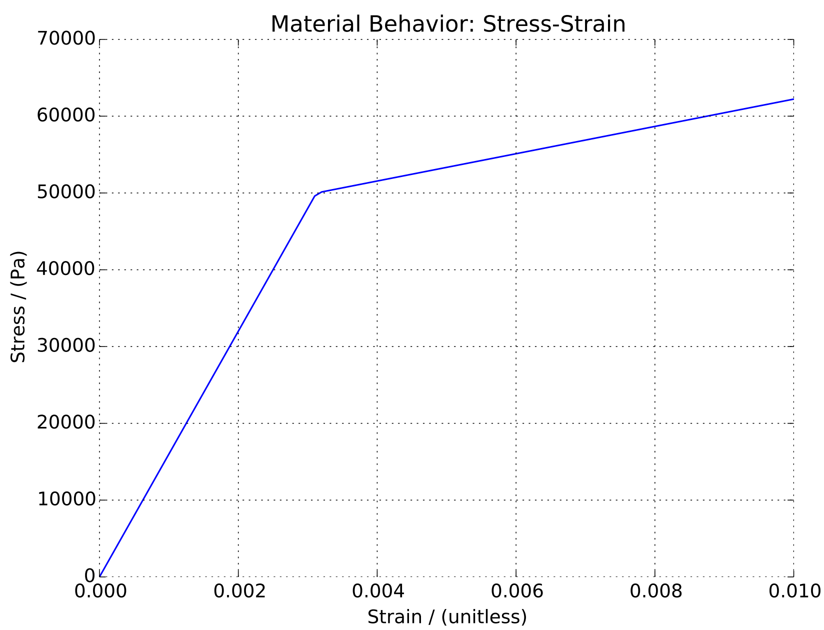

Figure 13: Kinematic Hardening Uniaxial Monotonic Loading

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

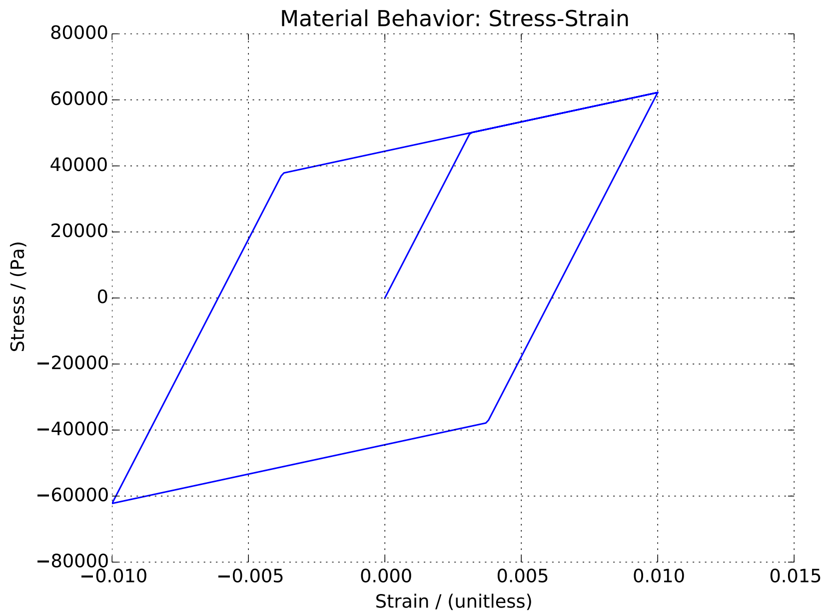

Figure 14: Kinematic Hardening Uniaxial Cyclic Loading

3.4 Elastic Plastic, Multiple Yield Surface, von-Mises, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type vonMisesMultipleYieldSurface
   mass_density = 0.0*kg/m^3
   elastic_modulus = 35175 * Pa
   poisson_ratio = 0.15
   total_number_of_yield_surface = 15
   radiuses_of_yield_surface_file = "radiuses_of_yield_surface.txt"
   hardening_parameters_of_yield_surface_file = "hardening_parameters_of_yield_surface.txt" ;
incr_size = 0.0000001 ;
max_strain= 0.001 ;
num_of_increm = max_strain/incr_size -1 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.0
   strain_increment_size = incr_size
   maximum_strain = max_strain
   number_of_increment = num_of_increm;
bye;
```

Material Response:

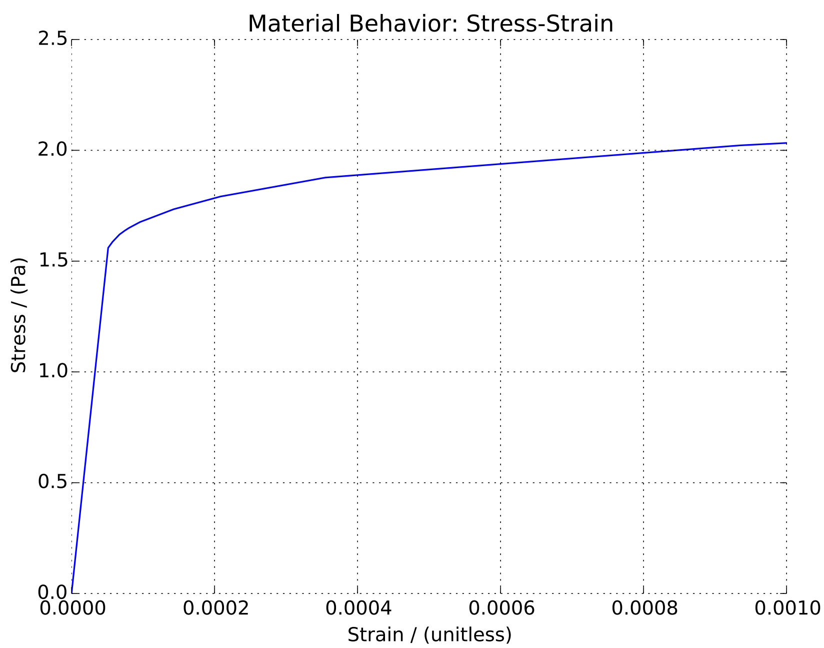

Figure 15: Multiple Yield Surface Pure Shear Monotonic Loading

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type vonMisesMultipleYieldSurface
   mass_density = 0.0*kg/m^3
   elastic_modulus = 35175 * Pa
   poisson_ratio = 0.15
   total_number_of_yield_surface = 15
   radiuses_of_yield_surface =
   "2.7 2.74 2.8 2.82 2.85 2.9 3.0 3.1 3.25 3.5 4.2 5.3 6.5 7.5 10.0 12.65"
   hardening_parameters_of_yield_surface =
   "5500 4000 2700 2400 1890 1300 915 600 254 167 79 65.4 23 2.2 1.2" ;
incr_size = 0.0000001 ;
max_strain= 0.001 ;
num_of_increm = 5* max_strain/incr_size ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.0
   strain_increment_size = incr_size
   maximum_strain = max_strain
   number_of_increment = num_of_increm;
bye;
```

Material Response:

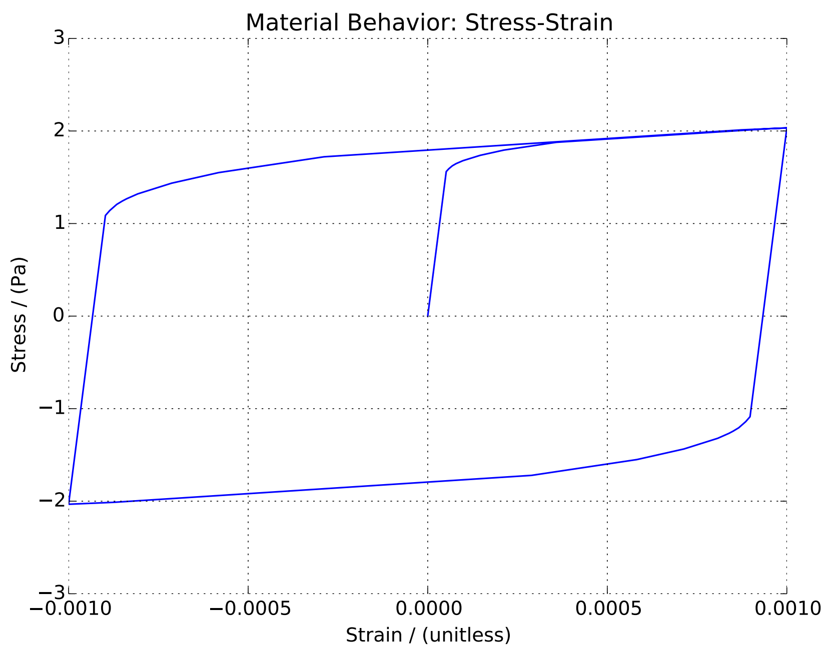

Figure 16: Multiple Yield Surface Pure Shear Cyclic Loading

4 Elastic Single Solid Finite Finite Element Examples

4.1 Linear Elastic, Solid Examples

Pure Shear, Monotonic Loading

Model Description:

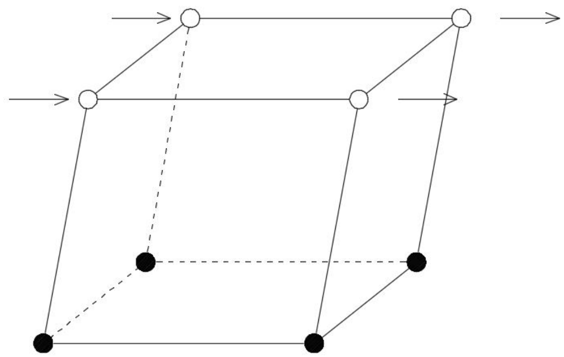

Figure 17: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

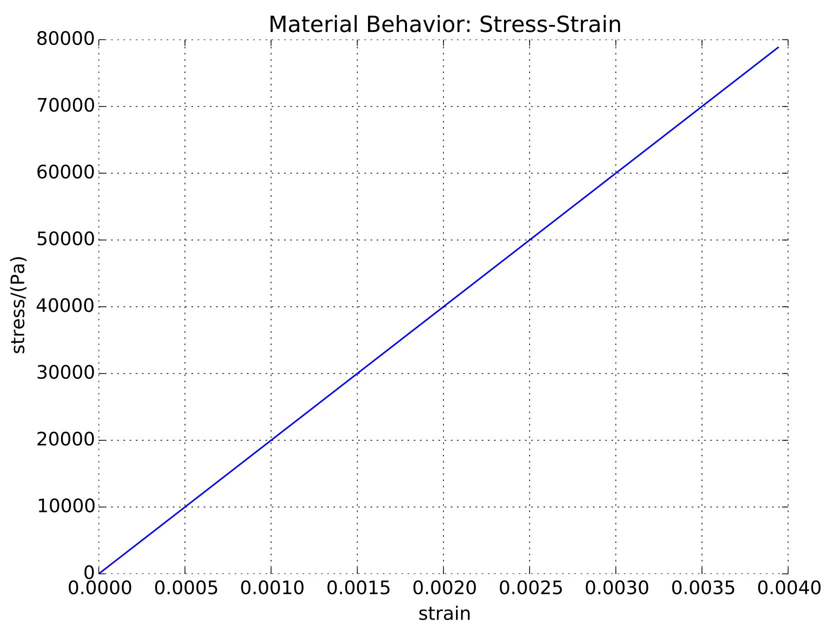

Figure 18: Linear Elastic Pure Shear Cyclic Loading

Pure Shear, Cyclic Loading

Model Description:

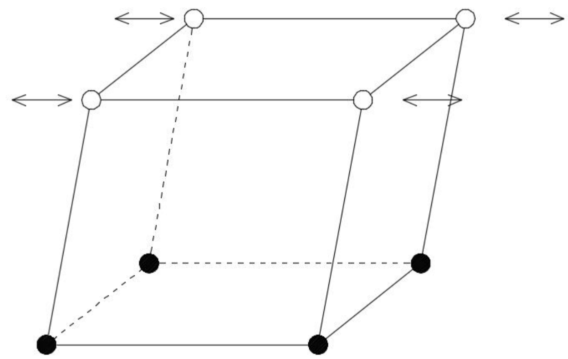

Figure 19: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

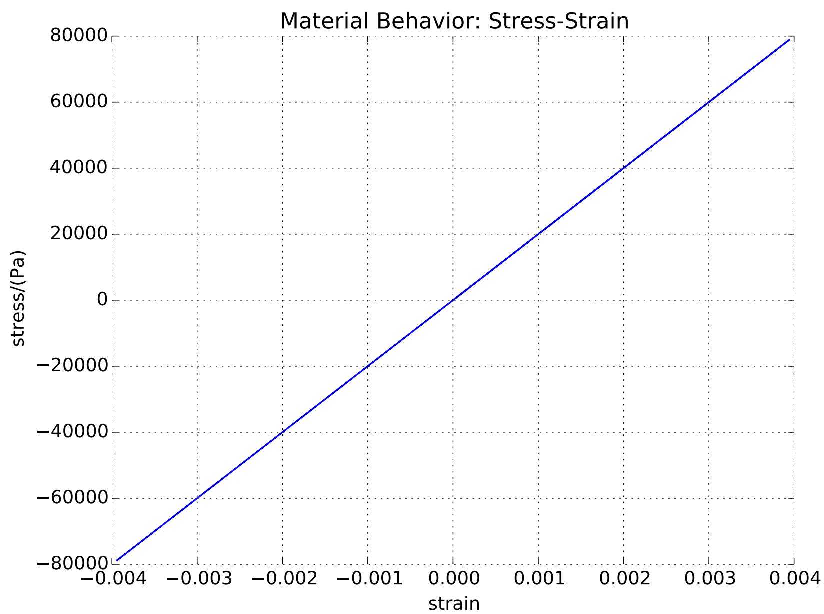

Figure 20: Linear Elastic Pure Shear Cyclic Loading

Uniaxial Strain, Monotonic Loading

Model Description:

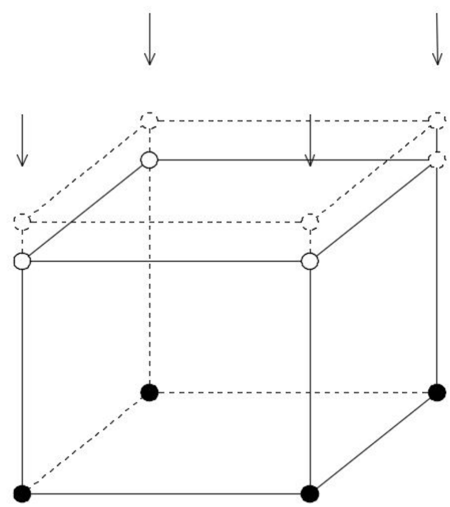

Figure 21: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

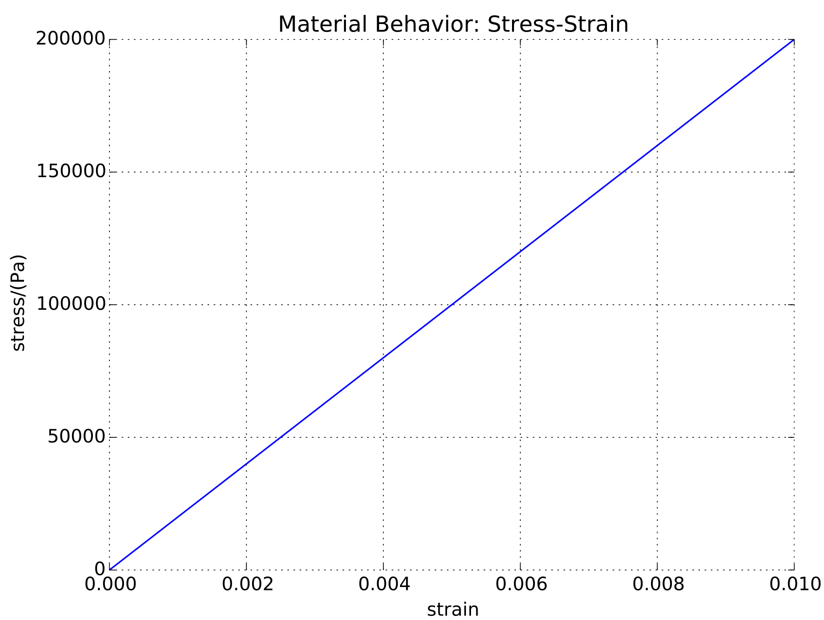

Figure 22: Linear Elastic Pure Shear Cyclic Loading

Uniaxial Strain, Cyclic Loading

Model Description:

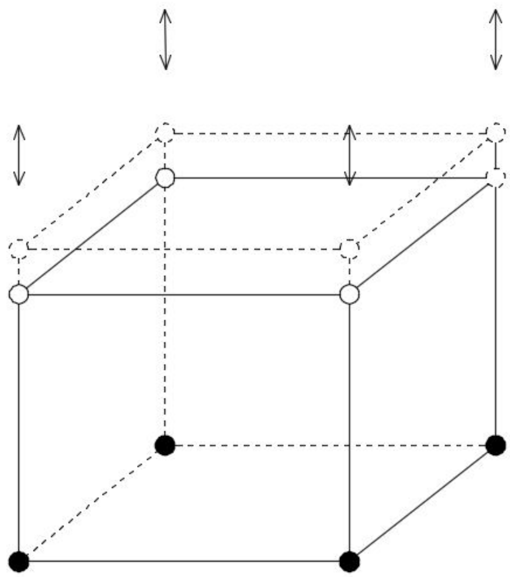

Figure 23: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

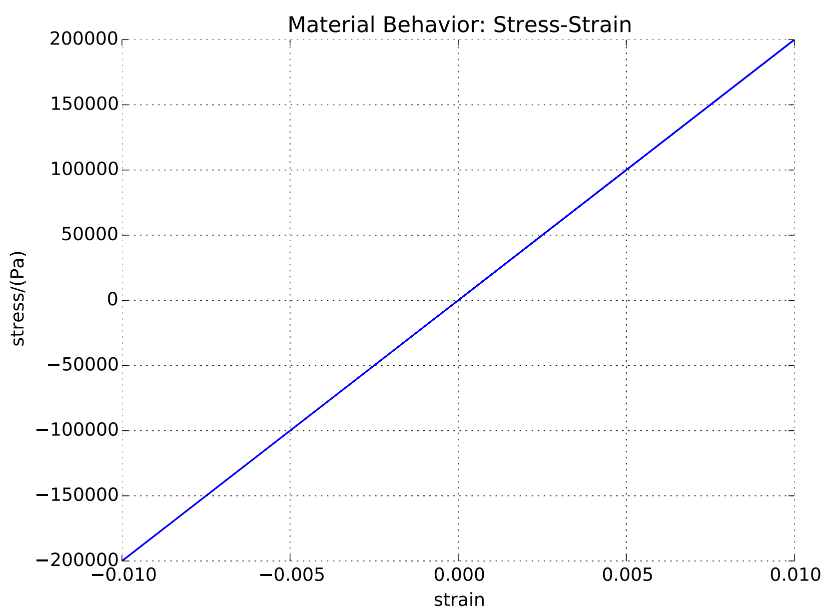

Figure 24: Linear Elastic Pure Shear Cyclic Loading

5 Elastic-Plastic Single Solid Finite Element Examples

5.1 Elastic Perfectly Plastic, Cyclic Loading, Pure Shear Solid Examples

Model Description:

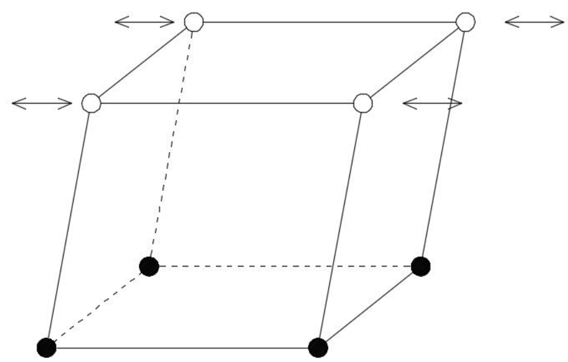

Figure 25: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

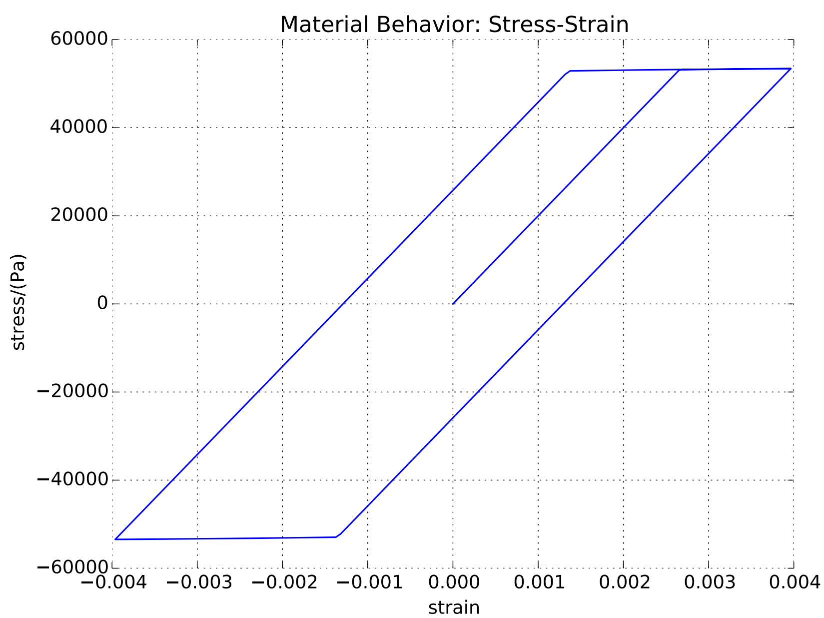

Figure 26: Linear Elastic Pure Shear Cyclic Loading

5.1.1 von Mises Yield Function, Isotropic Hardening

Model Description:

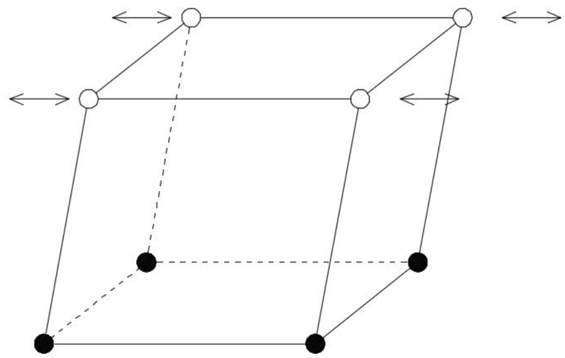

Figure 27: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

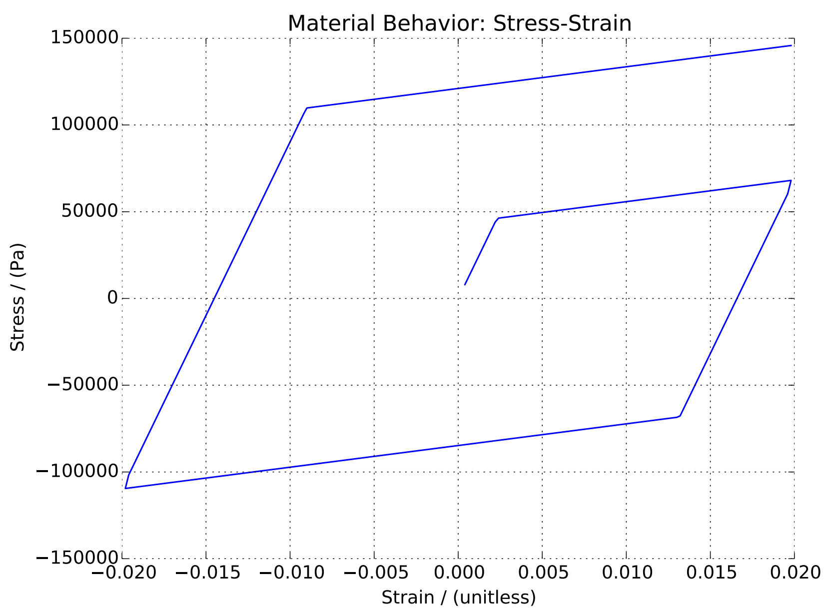

Figure 28: Linear Elastic Pure Shear Cyclic Loading

5.1.2 von Mises Yield Function, Kinematic Hardening

Model Description:

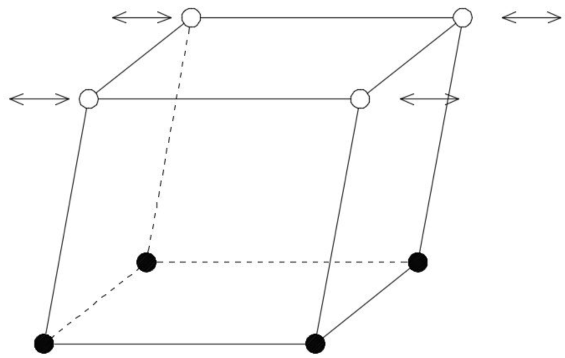

Figure 29: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

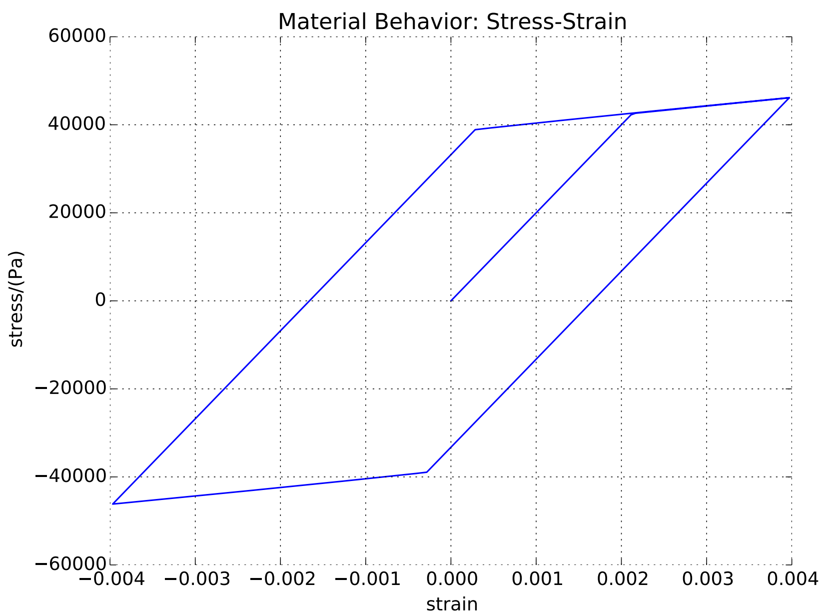

Figure 30: Linear Elastic Pure Shear Cyclic Loading

5.1.3 Drucker Prager Yield Function, von Mises Plastic Potential Function, Perfectly Plastic Hardening Rule

Model Description:

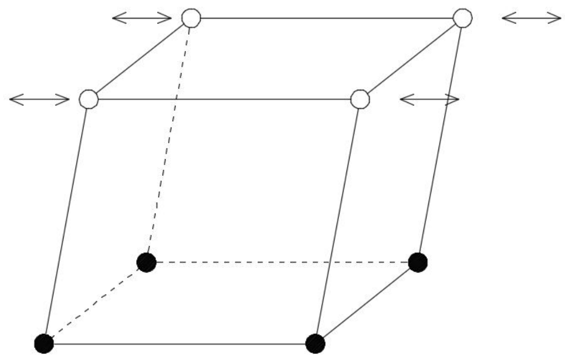

Figure 31: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

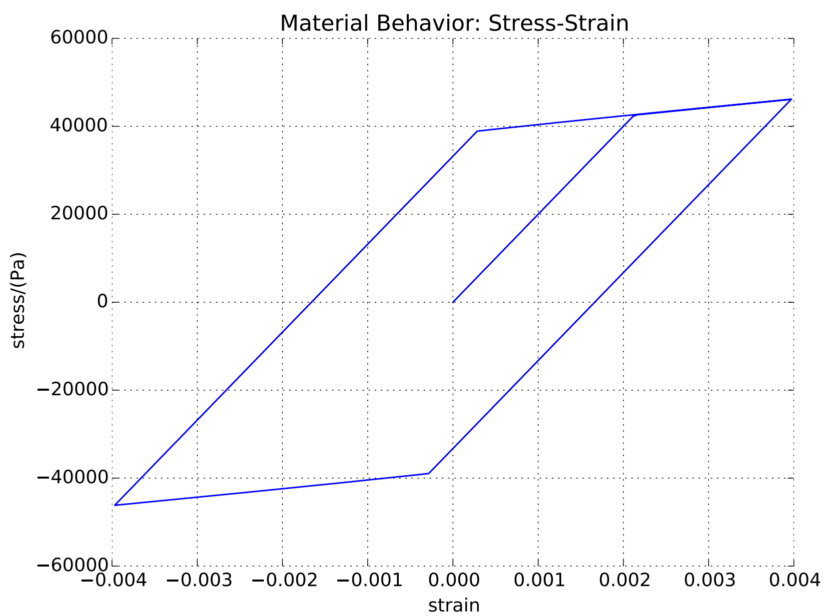

Figure 32: Linear Elastic Pure Shear Cyclic Loading

5.1.4 Drucker Prager Yield Function, Drucker Prager Plastic Potential Function, Perfectly Plastic Hardening Rule

Model Description:

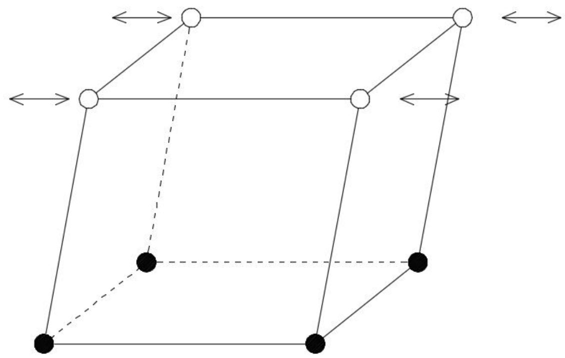

Figure 33: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

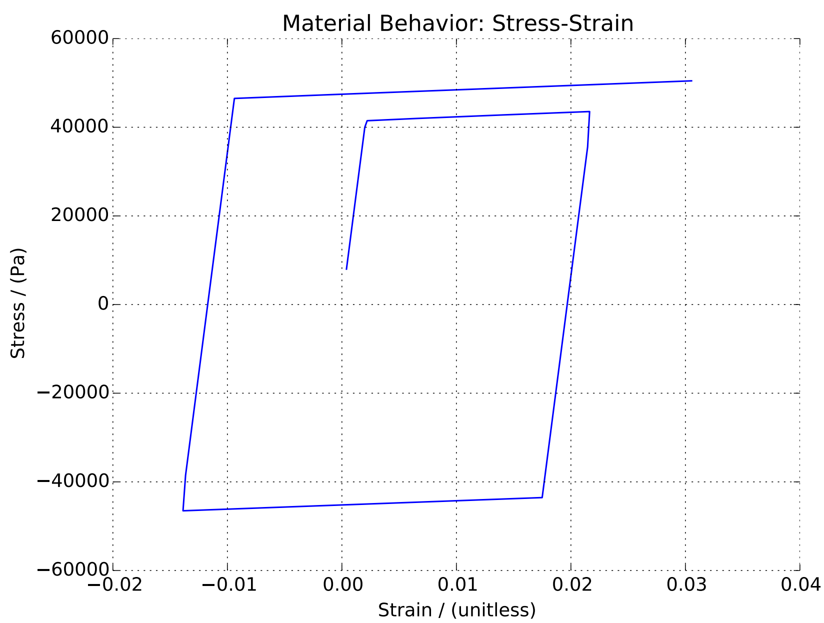

Figure 34: Linear Elastic Pure Shear Cyclic Loading

5.2 Elastic Plastic, Cam Clay Model, Various Stress Paths (To Be Added)

5.3 Elastic Plastic, SaniSand Models, Pure Shear Solid Examples (To Be Added)

6 Stiffness Reduction and Damping Curves Modeling

6.1 Pisano Material Model (To Be Added)

6.2 Drucker Prager with Armstrong Frederick Nonlinear Kinematic Hardening Material Model

Model Description:

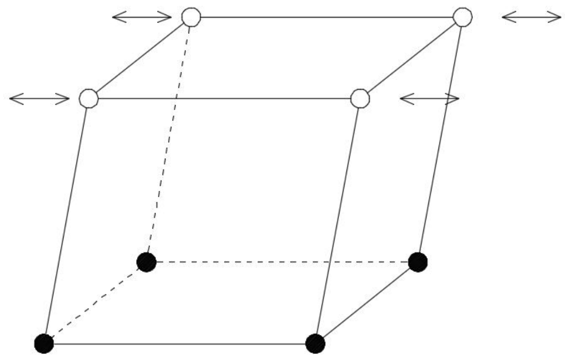

Figure 35: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

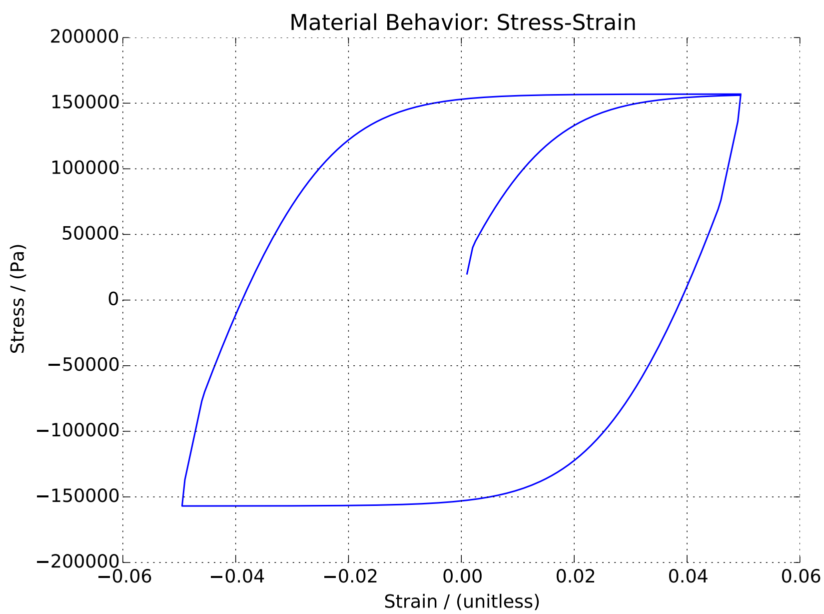

Figure 36: Linear Elastic Pure Shear Cyclic Loading