Education Examples for Constitutive Material Behavior

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Contents

1 Chapter Summary and Highlights

In the Chapter, the mechanical behaviors of elastoplastic materials is tested. The purpose is to simulate the simplest material points or the simplest solid brick for the sake of education.

Section.2 and Section.3 are testing one material point directly.

Section.4 and Section.5 are testing the material (Gauss Points) through the 8-node solid brick element.

Distinct yield surfaces are tested, including von-Mises and Drucker-Prager yield surfaces with associative and non-associative plastic flow.

Likewise, various hardening rules are simulated, including isotropic hardening, kinematic hardening, Armstrong-Frederick hardening, and multi-yield-surface hardening.

2 Elastic Solid Constitutive Examples

2.1 Linear Elastic Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 100;
bye;
```

Material Response:

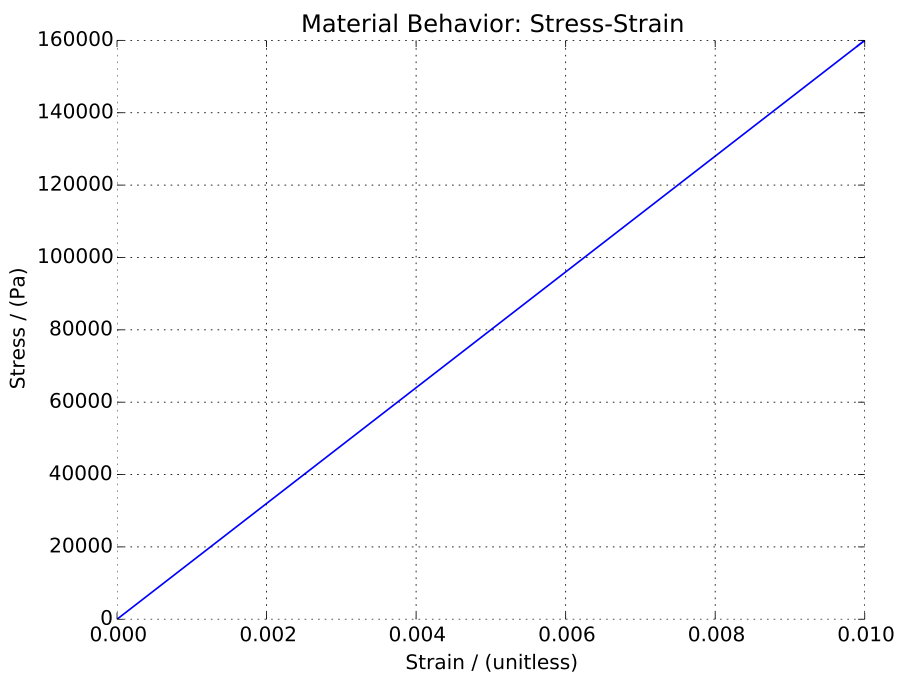

Figure 1: Linear Elastic Pure Shear Monotomic Loading

The fei files for this example are available here.

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

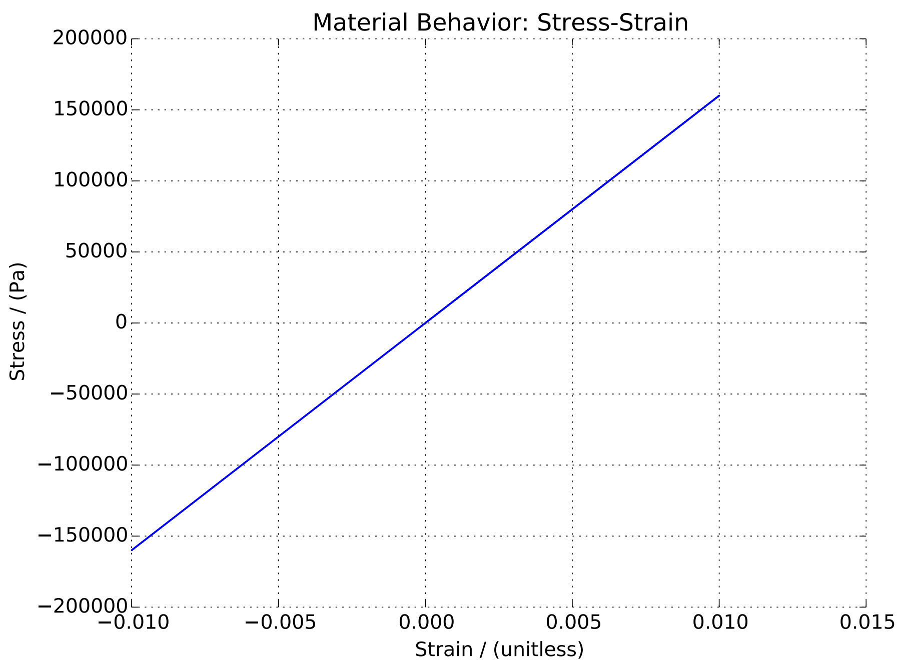

Figure 2: Linear Elastic Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.0 ;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 100;
bye;
```

Material Response:

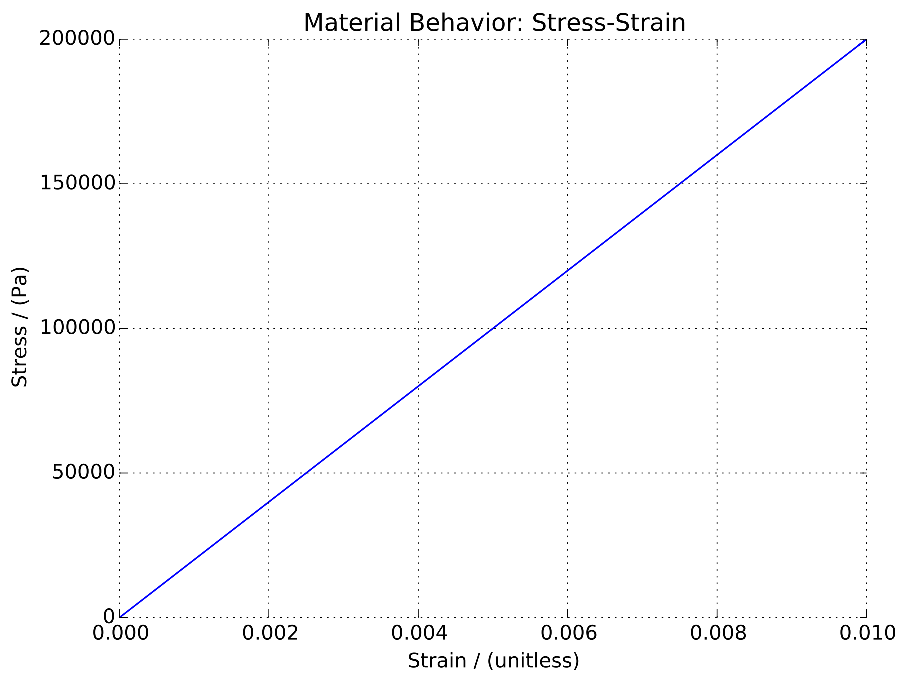

Figure 3: Linear Elastic Uniaxial Monotonic Loading

The fei files for this example are available here.

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type linear_elastic_isotropic_3d
   mass_density = 2E3 * kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio= 0.25 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

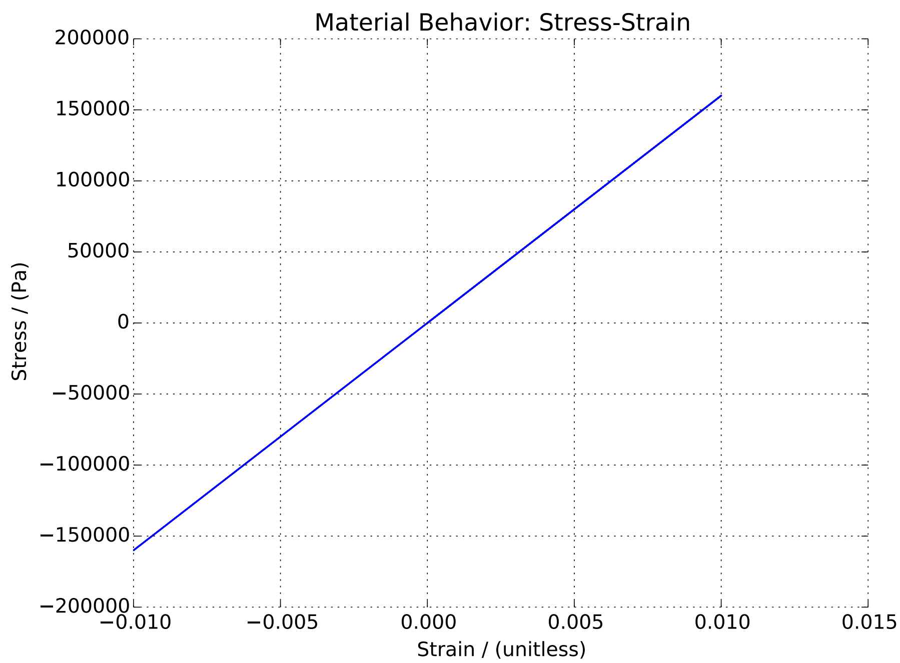

Figure 4: Linear Elastic Uniaxial Cyclic Loading

The fei files for this example are available here.

2.2 Nonlinear Elastic Constitutive Examples

Triaxial Uniform Pressure, Monotonic Loading

The Duncan-Chang nonlinear elastic materials:

$$E = E_0 - \kappa * p \tag{1}$$

where E_0 is the initial elastic modulus, parameter κ is a material constant, and p is the pressure p, where the compresssive pressure is defined as positive in Real ESSI.

Material Parameters:

```
model name "test";
add material # 1 type Duncan_Chang_nonlinear_elastic_isotropic_3d_LT
   mass_density = 2E3 * kg/m^3
   initial_elastic_modulus = 3E5 * Pa
   poisson_ratio= 0.15
   kappa = 60000 ;
simulate constitutive testing strain control triaxial uniform loading use material # 1
   strain_increment_size = 0.000001
   maximum_strain = 0.001
   number_of_increment = 1000;
bye;
```

Material Response:

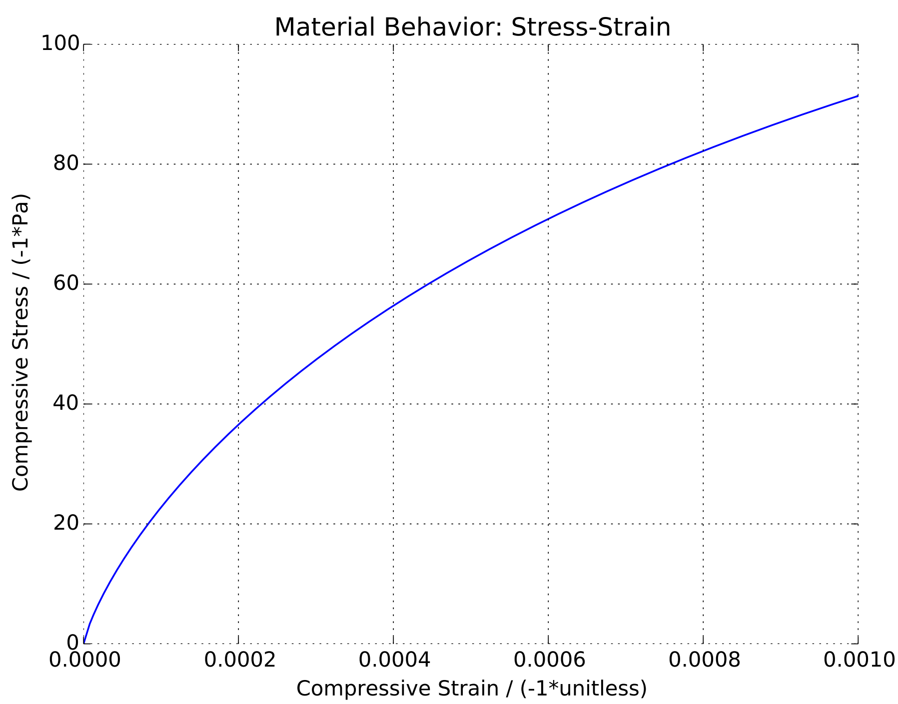

Figure 5: Results of Duncan-Chang Nonlinear Elastic Monotonic Loading

The fei files for this example are available here.

2.3 Nonlinear Elastic Constitutive Examples

Triaxial Uniform Pressure, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type Duncan_Chang_nonlinear_elastic_isotropic_3d_LT
   mass_density = 2E3 * kg/m^3
   initial_elastic_modulus = 3E5 * Pa
   poisson_ratio= 0.15
   kappa = 60000 ;
simulate constitutive testing strain control triaxial uniform loading use material # 1
   strain_increment_size = 0.000001
   maximum_strain = 0.001
   number_of_increment = 2000;
bye;
```

Material Response:

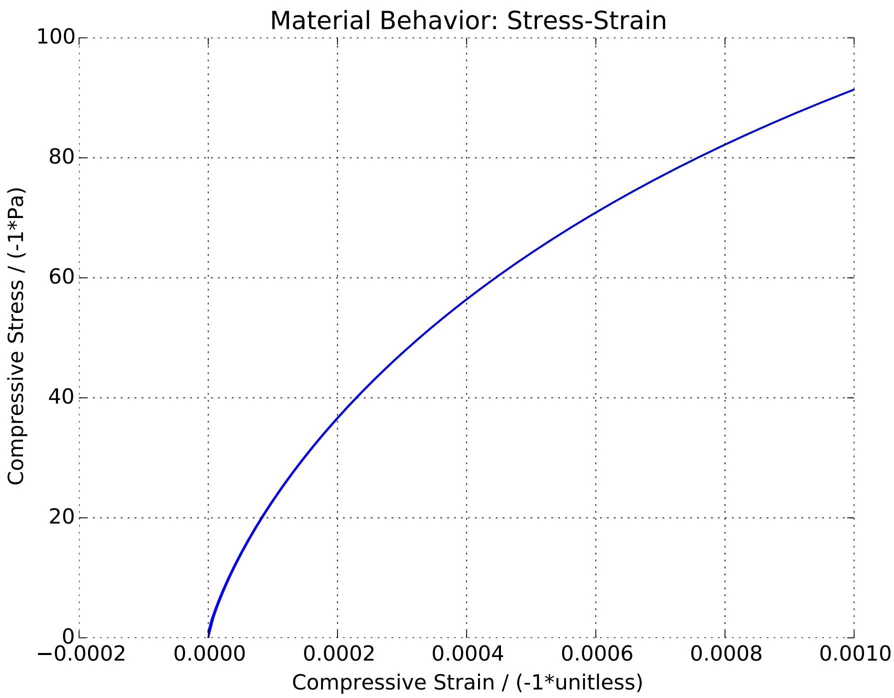

Figure 6: Results of Duncan-Chang Nonlinear Elastic Cyclic Loading

The fei files for this example are available here.

3 Elastic Plastic Solid Constitutive Examples

3.1 Elastic Perfectly Plastic Constitutive Examples

Pure Shear

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0 *Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

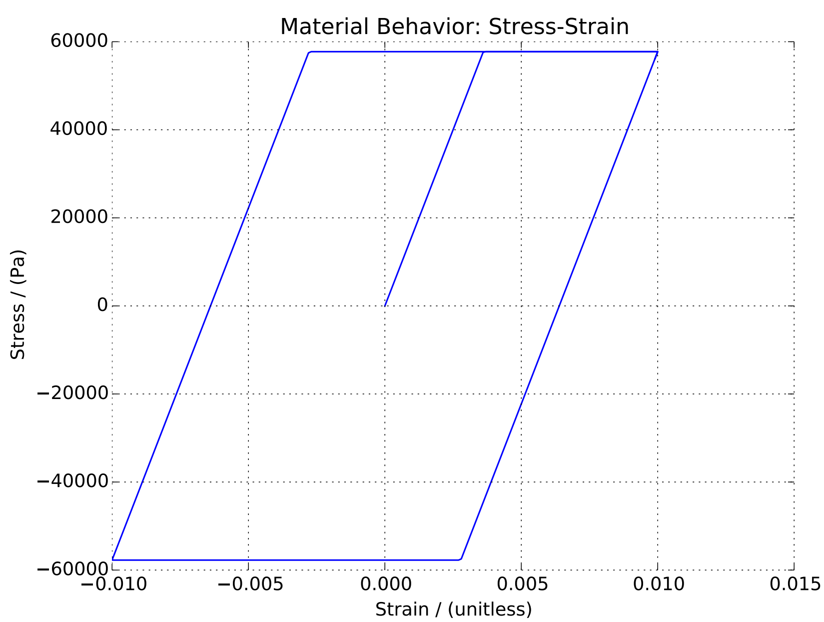

Figure 7: Perfectly Plastic Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0 *Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 500;
bye;
```

Material Response:

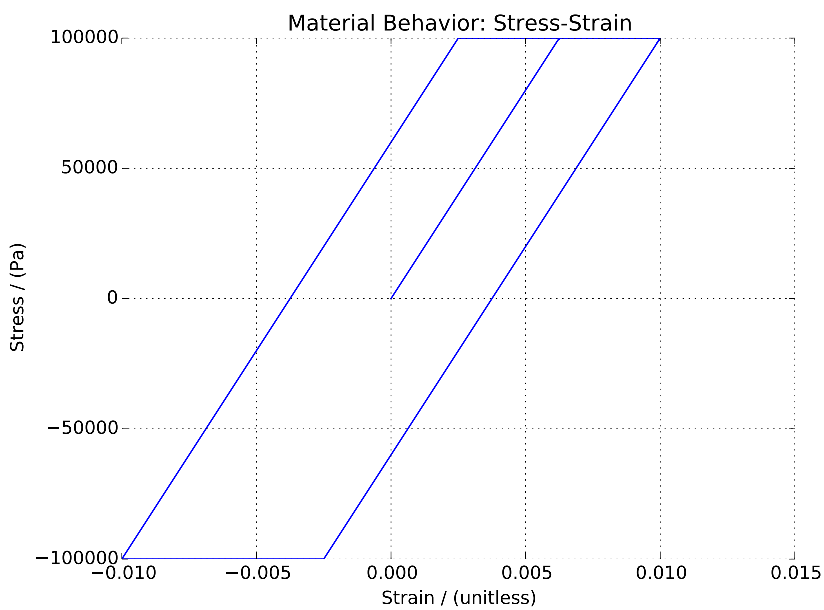

Figure 8: Perfectly Plastic Uniaxial Cyclic Loading

The fei files for this example are available here.

3.2 Elastic Plastic, Isotropic Hardening, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
  mass_density = 2E3*kg/m^3
  elastic_modulus = 2E7 * Pa
  poisson_ratio=0.25
  von_mises_radius = 1E5*Pa
  kinematic_hardening_rate = 0.0*Pa
  isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
  yield_function_relative_tolerance = 1E-2
  stress_relative_tolerance = 1E-3
  maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
  confinement_strain = 0.001
  strain_increment_size = 0.0001
  maximum_strain = 0.01
  number_of_increment = 99;
bye;
```

Material Response:

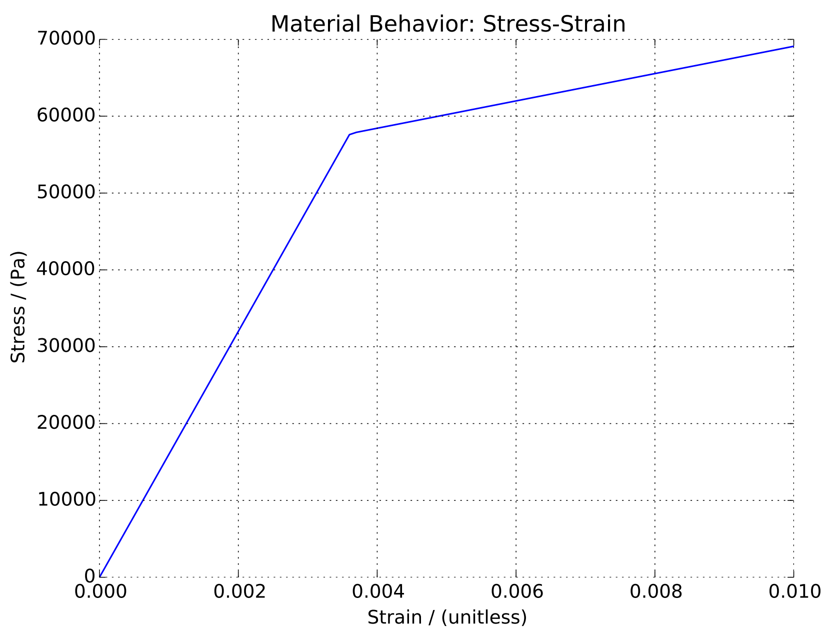

Figure 9: Isotropic Hardening Pure Shear Monotonic Loading

The fei files for this example are available here.

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

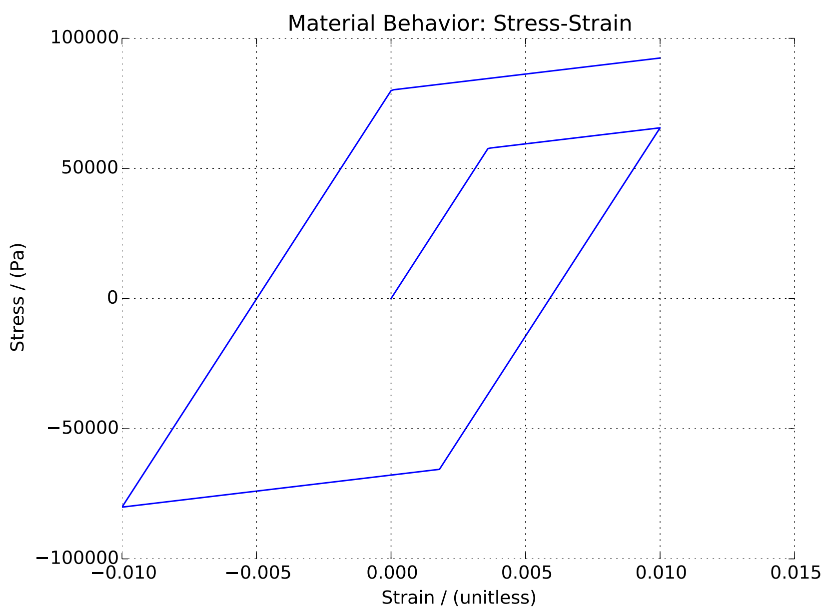

Figure 10: Isotropic Hardening Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

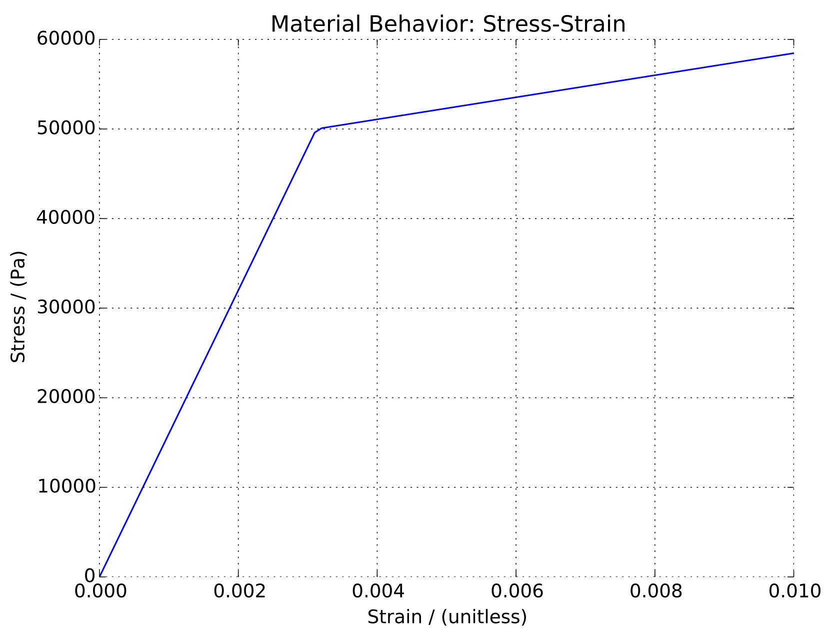

Figure 11: Isotropic Hardening Uniaxial Monotonic Loading

The fei files for this example are available here.

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 0.0*Pa
   isotropic_hardening_rate = 2E6 *Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

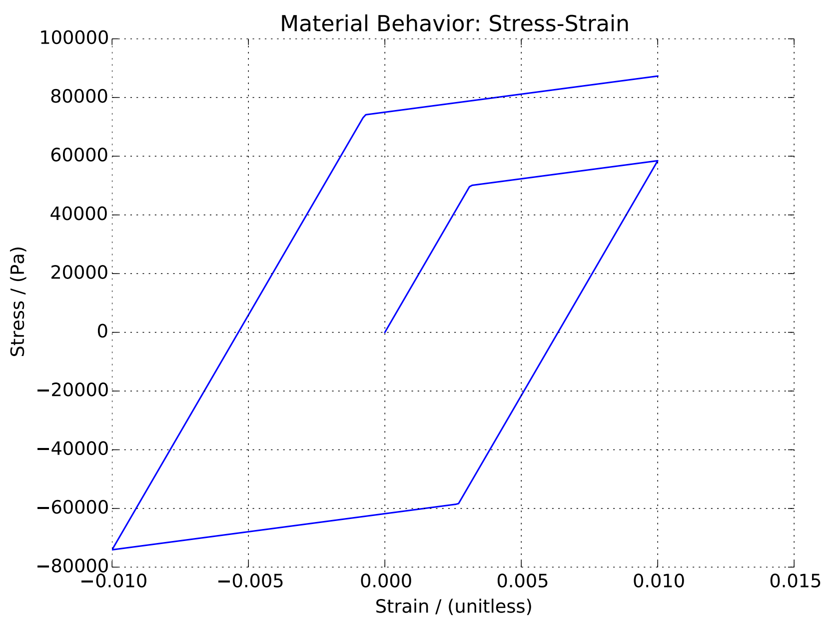

Figure 12: Isotropic Hardening Uniaxial Cyclic Loading

The fei files for this example are available here.

3.3 Elastic Plastic, Kinematic Hardening, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

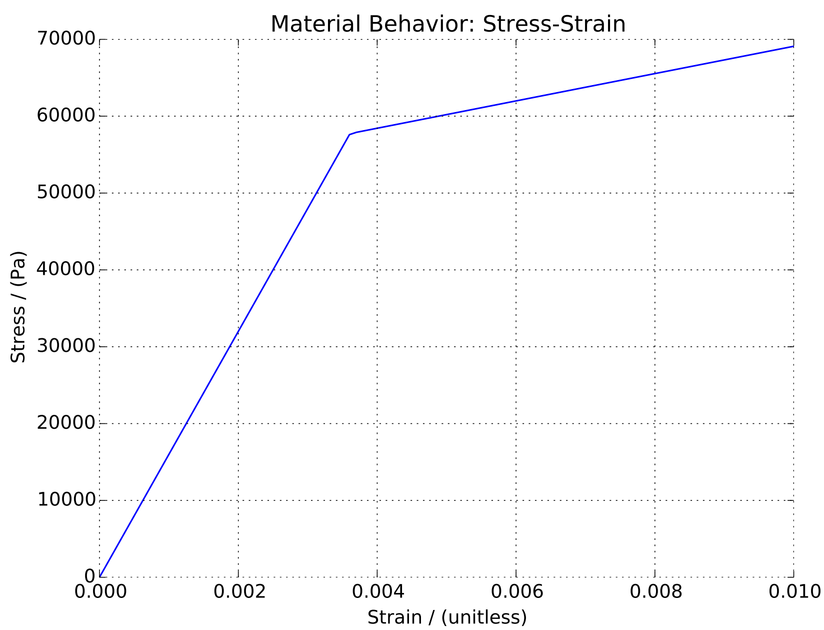

Figure 13: Kinematic Hardening Monotonic Cyclic Loading

The fei files for this example are available here.

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 1E5*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

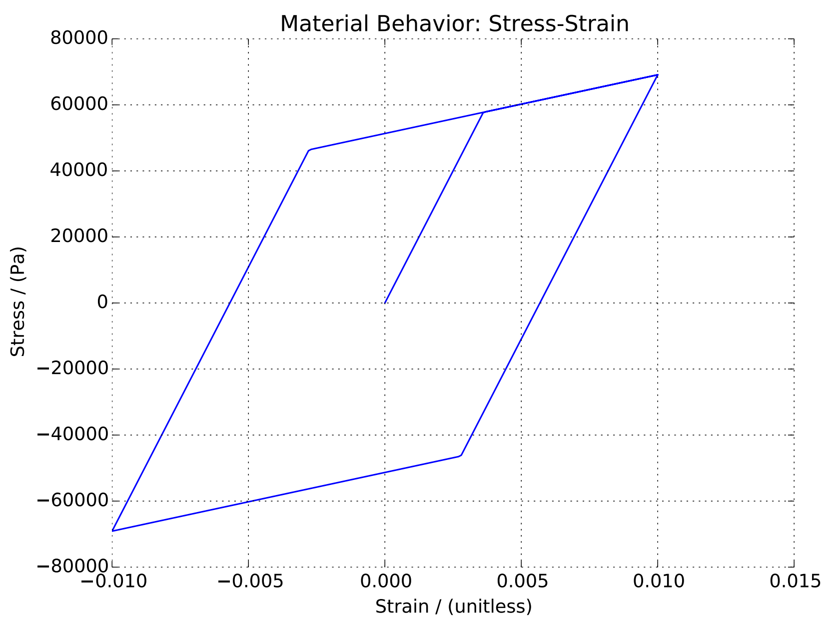

Figure 14: Kinematic Hardening Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 99;
bye;
```

Material Response:

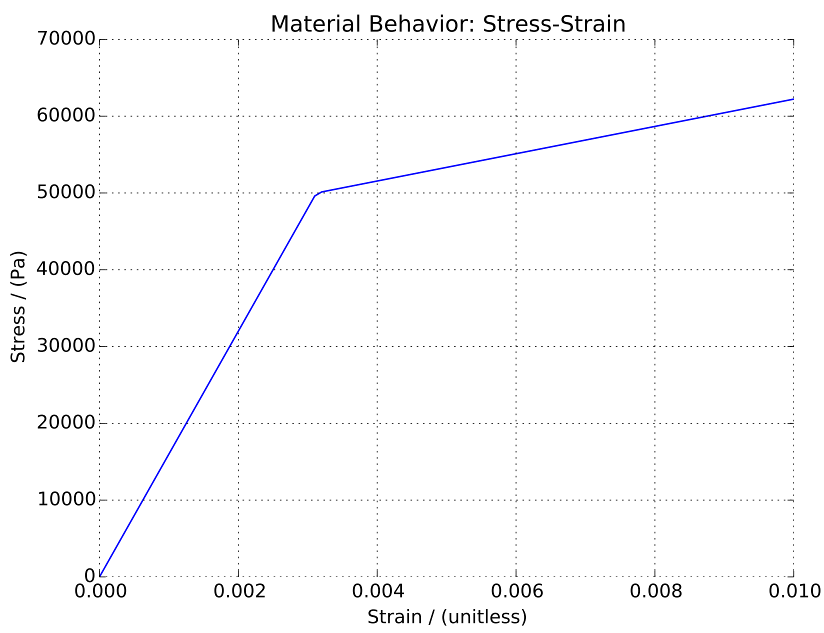

Figure 15: Kinematic Hardening Uniaxial Monotonic Loading

The fei files for this example are available here.

Uniaxial Strain, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type VonMises
   mass_density = 2E3*kg/m^3
   elastic_modulus = 2E7 * Pa
   poisson_ratio=0.25
   von_mises_radius = 5E4*Pa
   kinematic_hardening_rate = 2E6*Pa
   isotropic_hardening_rate = 0.0*Pa ;
define NDMaterial constitutive integration algorithm Backward_Euler
   yield_function_relative_tolerance = 1E-2
   stress_relative_tolerance = 1E-3
   maximum_iterations = 30;
simulate constitutive testing strain control uniaxial loading use material # 1
   confinement_strain = 0.001
   strain_increment_size = 0.0001
   maximum_strain = 0.01
   number_of_increment = 499;
bye;
```

Material Response:

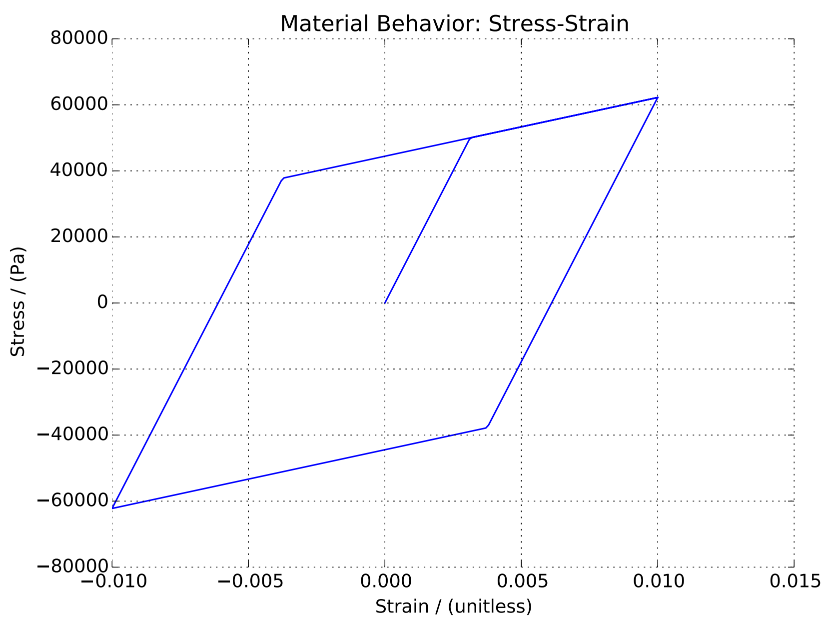

Figure 16: Kinematic Hardening Uniaxial Cyclic Loading

The fei files for this example are available here.

3.4 Elastic Plastic, Multiple Yield Surface, von-Mises, Constitutive Examples

Pure Shear, Monotonic Loading

Material Parameters:

```
model name "test";
add material # 1 type vonMisesMultipleYieldSurface
   mass_density = 0.0*kg/m^3
   elastic_modulus = 35175 * Pa
   poisson_ratio = 0.15
   total_number_of_yield_surface = 15
   radiuses_of_yield_surface =
   "2.7 2.74 2.8 2.82 2.85 2.9 3.0 3.1 3.25 3.5 4.2 5.3 6.5 7.5 10.0 12.65"
   hardening_parameters_of_yield_surface =
   "5500 4000 2700 2400 1890 1300 915 600 254 167 79 65.4 23 2.2 1.2" ;
incr_size = 0.0000001 ;
max_strain= 0.001 ;
num_of_increm = max_strain/incr_size -1 ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.0
   strain_increment_size = incr_size
   maximum_strain = max_strain
   number_of_increment = num_of_increm;
bye;
```

Material Response:

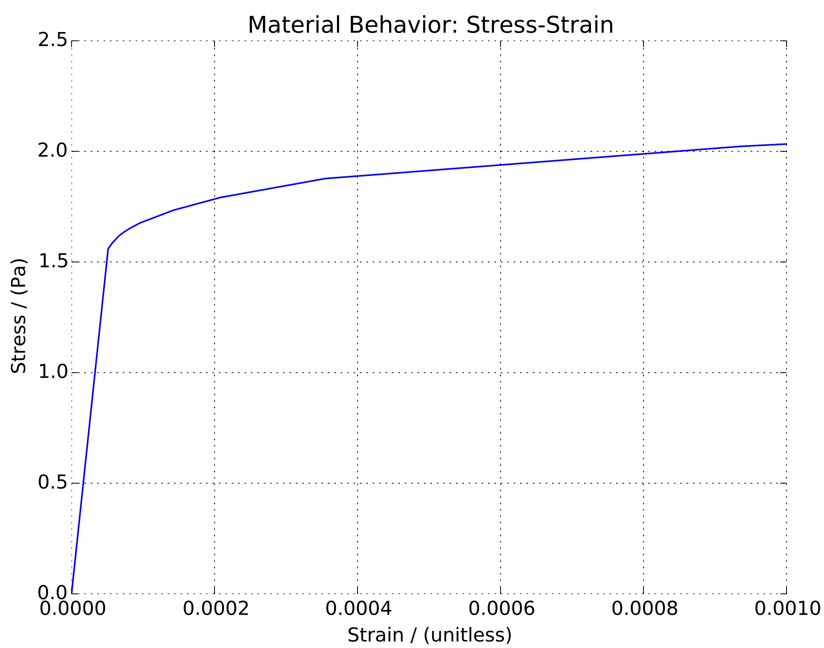

Figure 17: Multiple Yield Surface Pure Shear Monotonic Loading

The fei files for this example are available here.

Pure Shear, Cyclic Loading

Material Parameters:

```
model name "test";
add material # 1 type vonMisesMultipleYieldSurface
   mass_density = 0.0*kg/m^3
   elastic_modulus = 35175 * Pa
   poisson_ratio = 0.15
   total_number_of_yield_surface = 15
   radiuses_of_yield_surface =
   "2.7 2.74 2.8 2.82 2.85 2.9 3.0 3.1 3.25 3.5 4.2 5.3 6.5 7.5 10.0 12.65"
   hardening_parameters_of_yield_surface =
   "5500 4000 2700 2400 1890 1300 915 600 254 167 79 65.4 23 2.2 1.2" ;
incr_size = 0.0000001 ;
max_strain= 0.001 ;
num_of_increm = 5* max_strain/incr_size ;
simulate constitutive testing strain control pure shear use material # 1
   confinement_strain = 0.0
   strain_increment_size = incr_size
   maximum_strain = max_strain
   number_of_increment = num_of_increm;
bye;
```

Material Response:

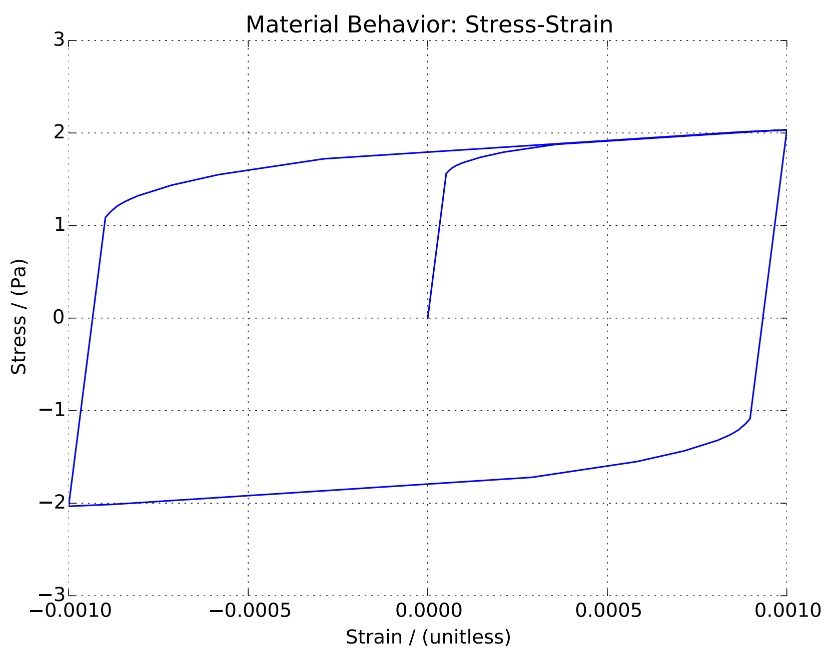

Figure 18: Multiple Yield Surface Pure Shear Cyclic Loading

The fei files for this example are available here.

4 Elastic Single Solid Finite Finite Element Examples

4.1 Linear Elastic, Solid Examples

Pure Shear, Monotonic Loading

Model Description:

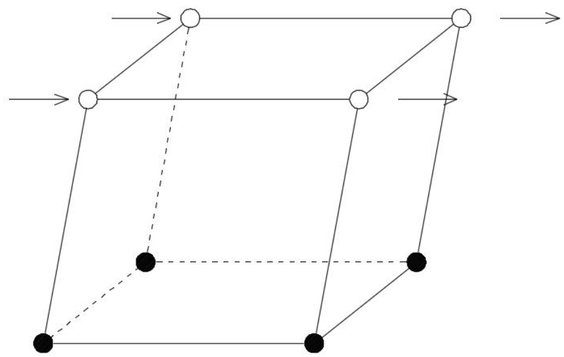

Figure 19: Diagram Linear Elastic Solid Pure Shear Monotonic Loading

Material Response at Gauss Point:

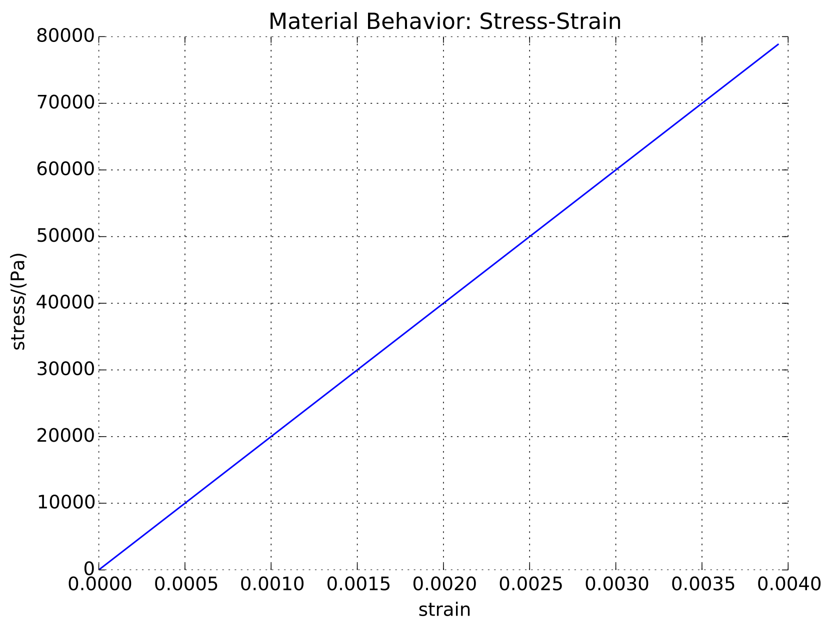

Figure 20: Results of Linear Elastic Solid Pure Shear Monotonic Loading

The fei files for this example are available here.

Pure Shear, Cyclic Loading

Model Description:

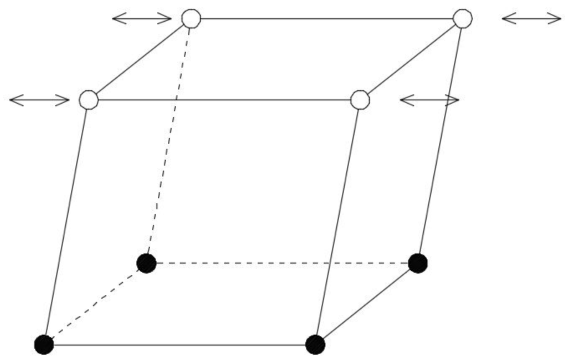

Figure 21: Diagram Linear Elastic Solid Pure Shear Cyclic Loading

Material Response at Gauss Point:

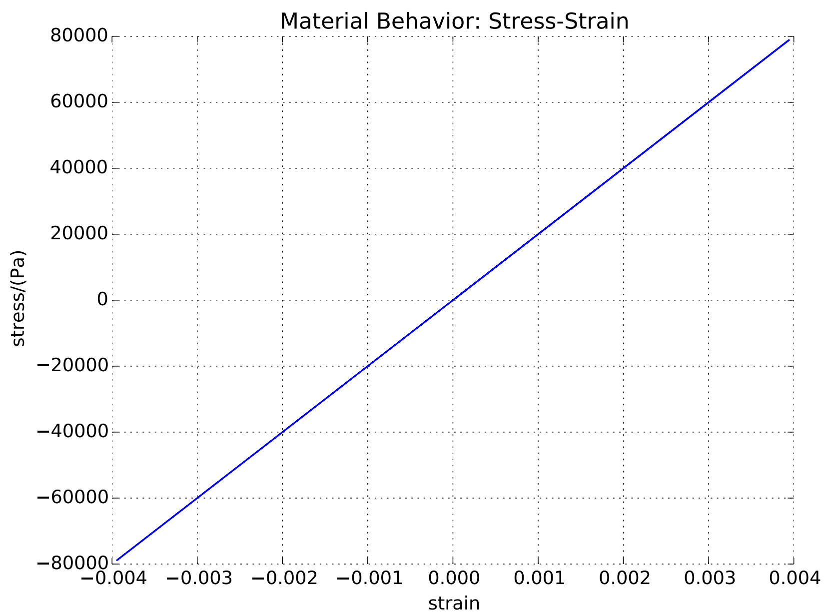

Figure 22: Results of Linear Elastic Solid Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain, Monotonic Loading

Model Description:

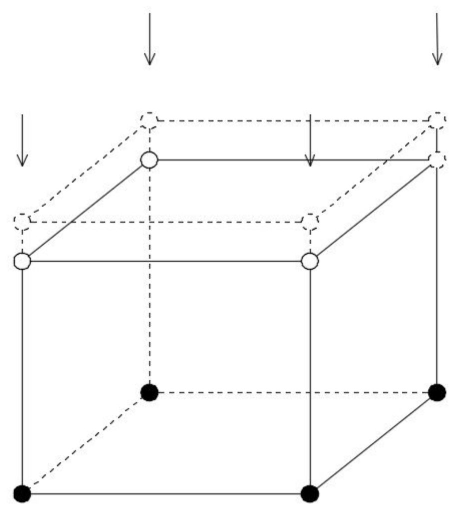

Figure 23: Diagram Linear Elastic Uniaxial Strain Solid Monotonic Loading

Material Response at Gauss Point:

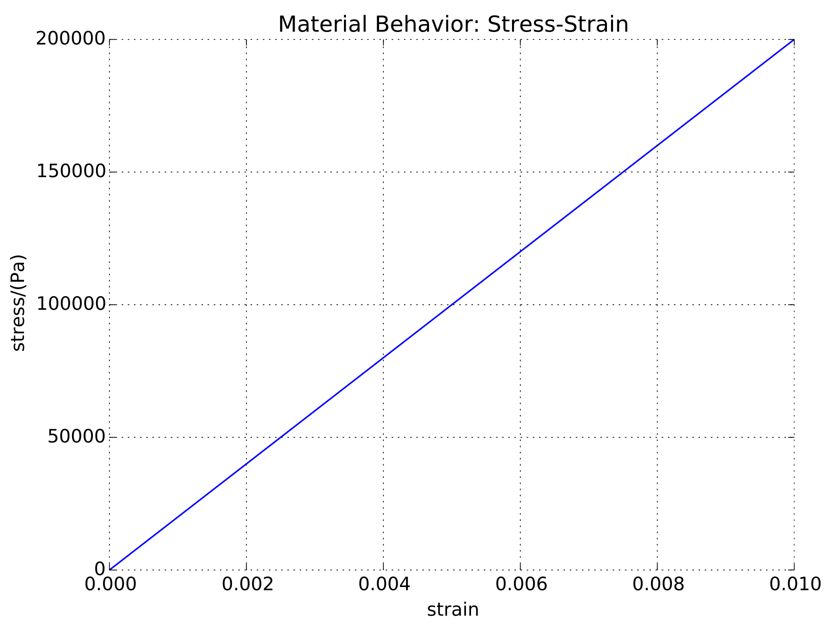

Figure 24: Results of Linear Elastic Pure Shear Cyclic Loading

The fei files for this example are available here.

Uniaxial Strain, Cyclic Loading

Model Description:

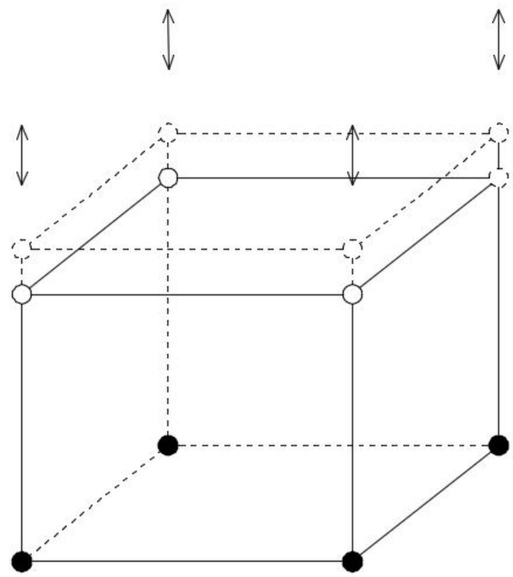

Figure 25: Linear Elastic Uniaxial Strain Cyclic Loading

Material Response at Gauss Point:

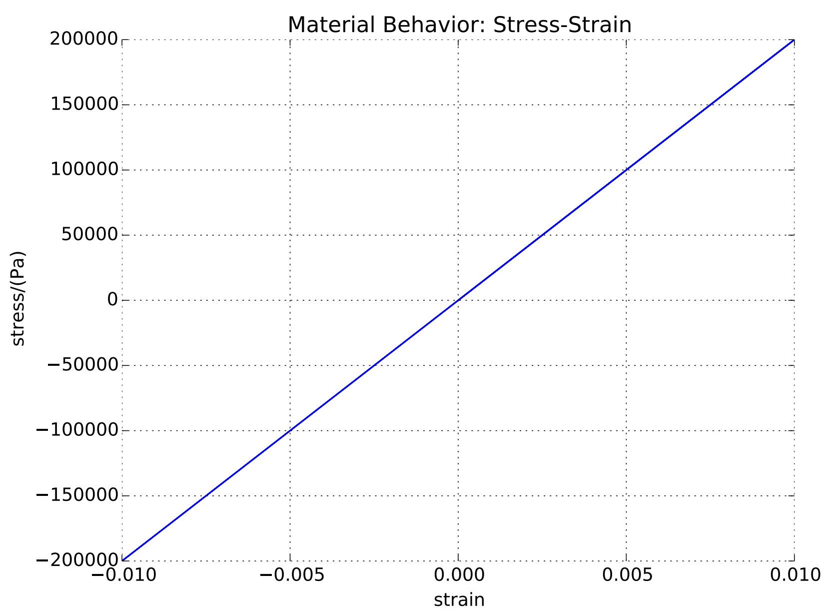

Figure 26: Results of Linear Elastic Pure Shear Cyclic Loading

The fei files for this example are available here.

5 Elastic-Plastic Single Solid Finite Element Examples

5.1 Elastic Perfectly Plastic, Cyclic Loading, Pure Shear Solid Examples

Model Description:

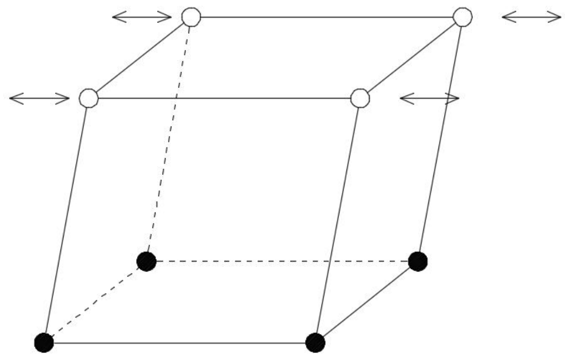

Figure 27: Perfectly Plastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

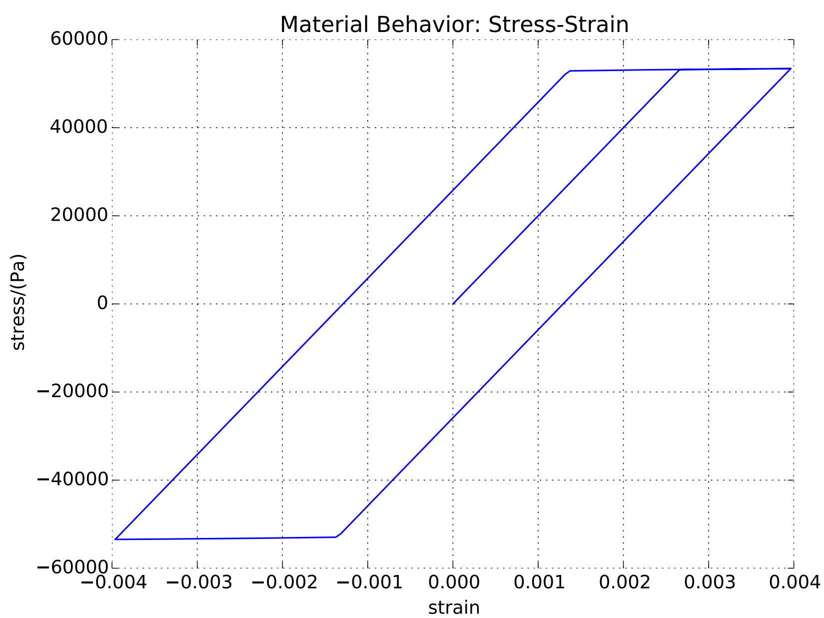

Figure 28: Results of Linear Elastic Pure Shear Cyclic Loading

The fei files for this example are available here.

5.1.1 von Mises Yield Function, Isotropic Hardening

Model Description:

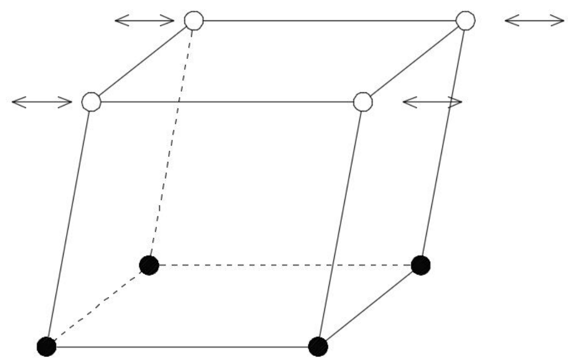

Figure 29: Linear Elastic Pure Shear Cyclic Loading

Material Response at Gauss Point:

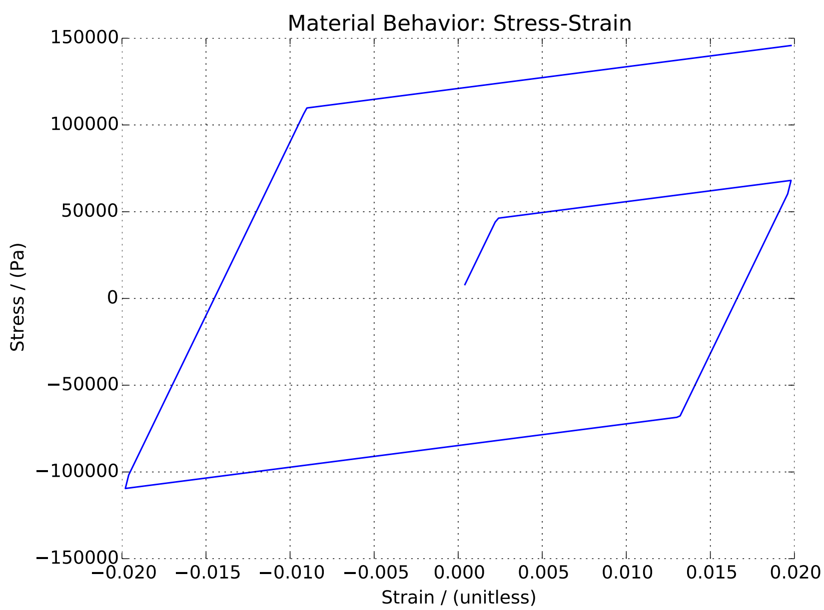

Figure 30: Linear Elastic Pure Shear Cyclic Loading

The fei files for this example are available here.

5.1.2 von Mises Yield Function, Kinematic Hardening

Model Description:

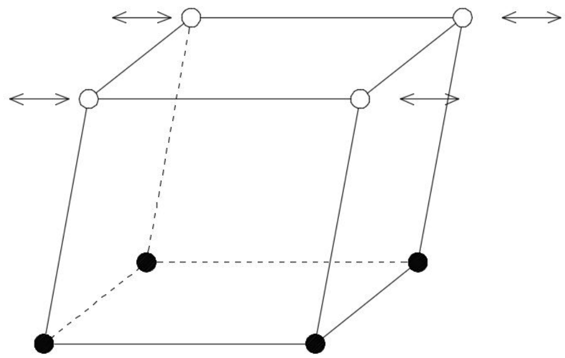

Figure 31: von Mises Pure Shear Cyclic Loading

Material Response at Gauss Point:

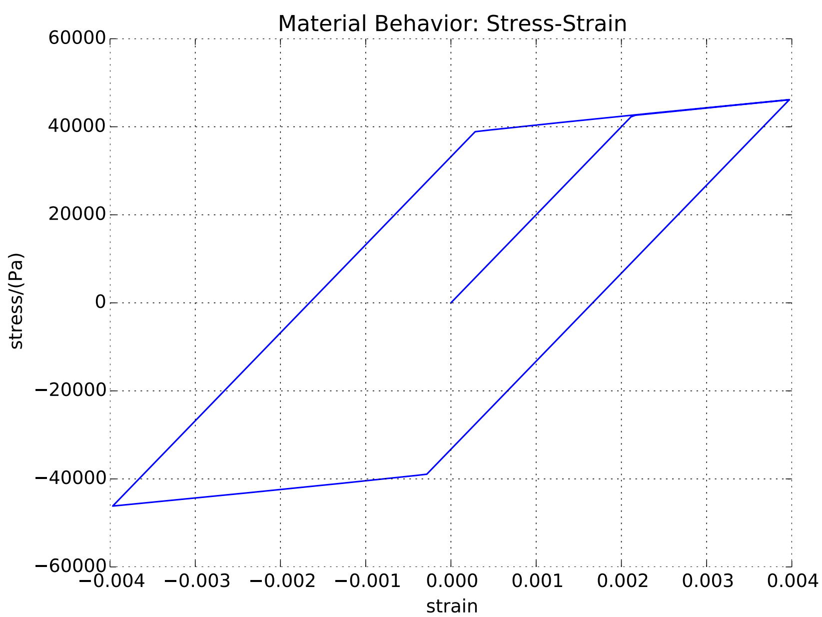

Figure 32: Results of von Mises Pure Shear Cyclic Loading

The fei files for this example are available here.

5.1.3 Drucker Prager Yield Function, von Mises Plastic Potential Function, Perfectly Plastic Hardening Rule

Model Description:

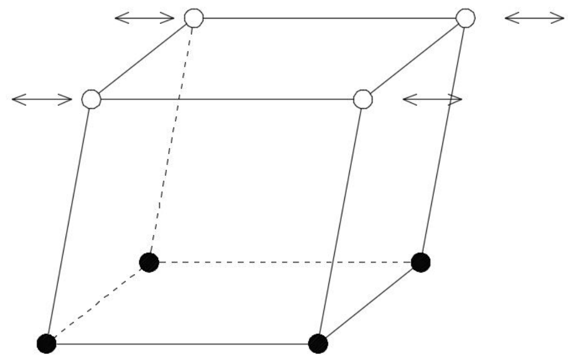

Figure 33: Diagram of Non-associative Drucker Prager Pure Shear Cyclic Loading

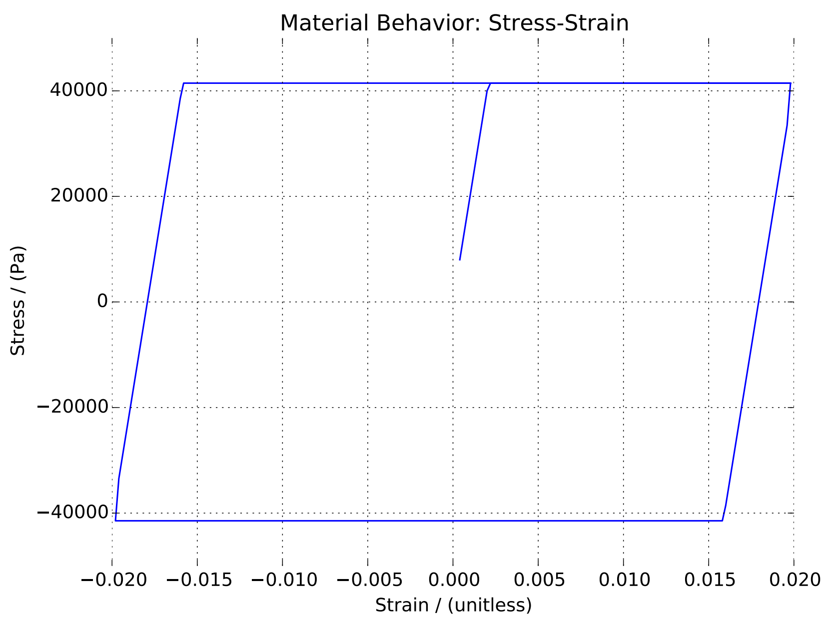

Figure 34: Results of Drucker Prager Pure Shear Cyclic Loading

The fei files for this example are available here.

5.1.4 Drucker Prager Yield Function, Drucker Prager Plastic Potential Function, Perfectly Plastic Hardening Rule

Model Description:

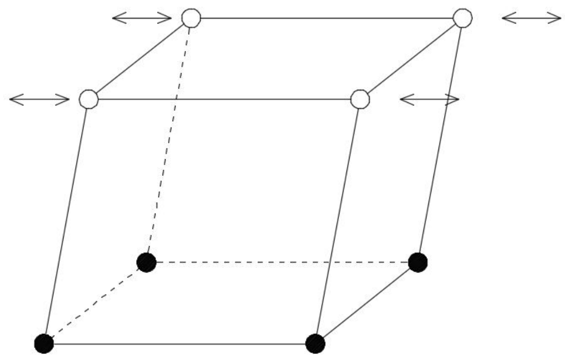

Figure 35: Associative Drucker Prager Pure Shear Cyclic Loading

Material Response at Gauss Point:

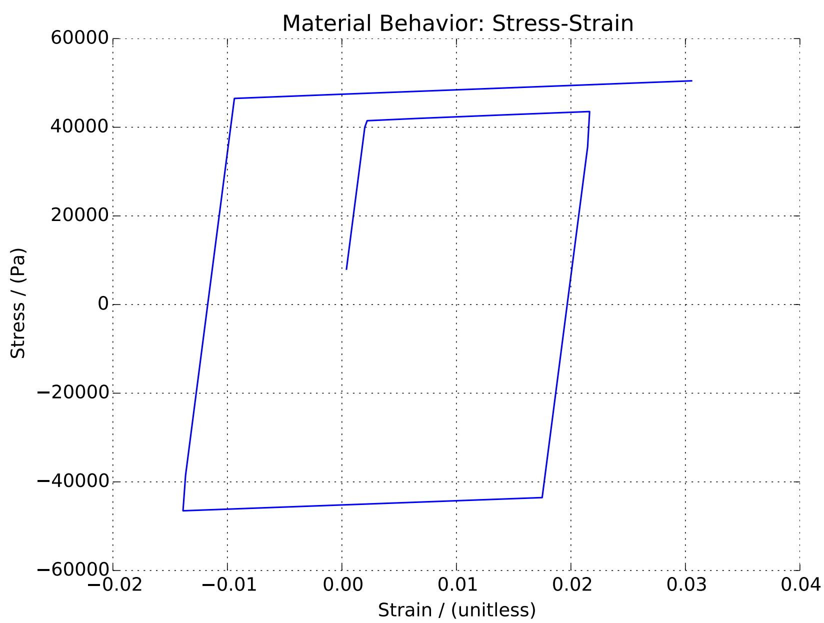

Figure 36: Results of Associative Drucker Prager Pure Shear Cyclic Loading

The fei files for this example are available here.

5.2 Elastic Plastic, Cam Clay Model, Various Stress Paths (To Be Added)

5.3 Elastic Plastic, SaniSand Models, Pure Shear Solid Examples (To Be Added)

6 Stiffness Reduction and Damping Curves Modeling

6.1 Pisano Material Model (To Be Added)

6.2 Drucker Prager with Armstrong Frederick Nonlinear Kinematic Hardening Material Model

Model Description:

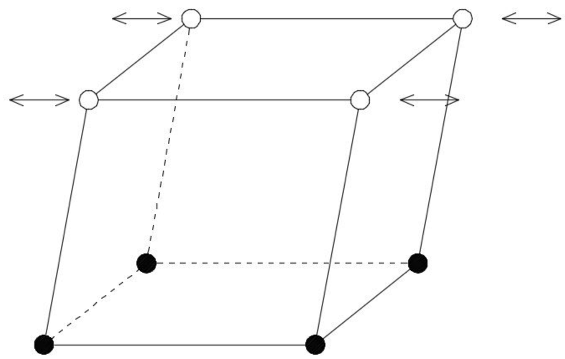

Figure 37: Diagram of Drucker-Prager Armstrong-Frederick Pure Shear Cyclic Loading

Material Response at Gauss Point:

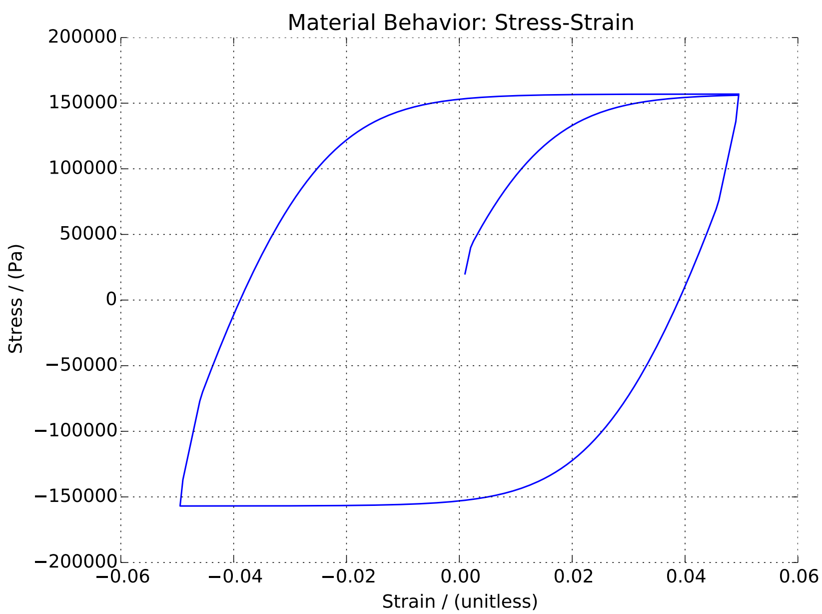

Figure 38: Result of Drucker-Prager Armstrong-Frederick Pure Shear Cyclic Loading

The fei files for this example are available here.