Advanced Matrix Structural Analysis Final project report

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1 Program without distributed load

1.1 My flowchart

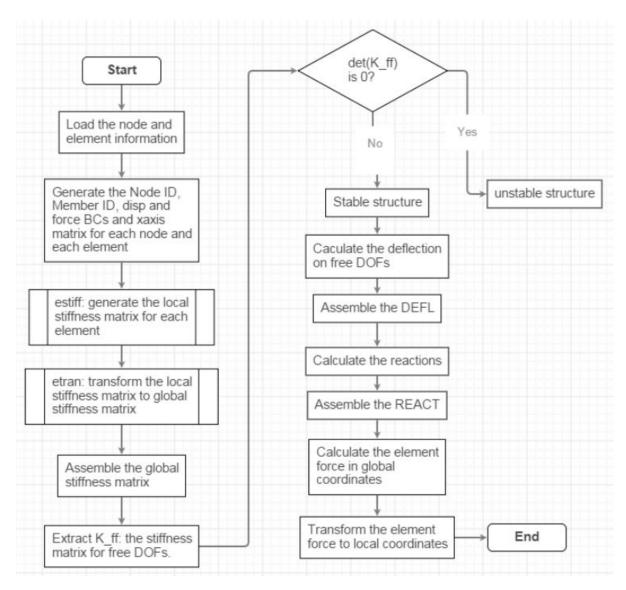


Figure 1: Flowchart

1.2 main program

```
clear
   clc
2
   % load the input information.
   load nnodes.txt;
   load coord.txt;
   load concen.txt;
6
   load fixity.txt;
   load nele.txt;
   load ends.txt;
   load Area.txt;
10
   load Izz.txt;
  load Iyy.txt;
12
   load J.txt;
13
   load E.txt;
   load v.txt;
   load beta_ang.txt;
16
17
   % Generate the node id.
18
   % Each line represents the 6 dofs for one node.
19
   node_id=zeros(nnodes,6);
20
   for i=1:nnodes
21
       for j=1:6
22
           node_id(i,j)=(i-1)*6+j;
23
       end
24
   end
25
26
   % Generate the member id.
27
   % Each line represents the 12 DOFs for one element.
   mem_id=zeros(nele,12);
29
   for i=1:nele
30
       mem_id(i,1:6)=node_id(ends(i,1),1:6);
31
       mem_id(i,7:12)=node_id(ends(i,2),1:6);
32
   end
33
34
   % Generate the displacement boundary conditions.
35
   % Each line in fixity represents the disp BC on 6 DOFs of one node.
36
   % O represents that the DOF is fixed.
   % NaN represnts that the DOF is free.
38
   fixity_tran=fixity';
39
   D=fixity_tran(:);
40
   free_dof=find(isnan(D));
41
   fixed_dof=find(D==0);
42
43
44
   % Generate the force boundary conditions.
   % Each line in concen represents the force BC on 6 DOFs of one node.
46
   \% O represents that the DOF does not has external force.
47
   \% The number represnts that the force values on the DOF.
48
   concen_tran=concen';
  P_total=concen_tran(:);
50
51
   |P_free=P_total(free_dof);
52
```

```
% Generate the xaxis.
    % xaxis will be used in the rotation matrix.
54
    % The rotation matrix is to transform the stiffness matrix
    % from global to local coordinate system.
    for i=1:nele
57
        xaxis(i,:)=(coord(ends(i,2),:)-coord(ends(i,1),:));
58
        Length(i) = norm(xaxis(i,:));
59
        xaxisunit(i,:)=xaxis(i,:)/Length(i) ;
60
    end
61
62
    % Generate the local stiffness matrix for each element.
63
    k_stack_local=zeros(nele,12,12);
    gamma_stack_local=zeros(nele,12,12);
65
    for i=1:nele
        k_stack_local(i,:,:) = stiff(Area(i),Izz(i),Iyy(i),J(i),E(i),v(i),Length(i));
67
        gamma_stack_local(i,:,:)=etran(beta_ang(i),xaxisunit(i,1:3));
    end
69
70
     % Transform the local stiffness matrix to global stiffness matrix.
71
    k_stack_global=zeros(nele,12,12);
72
    for i=1:nele
73
        k_stack_global(i,:,:)=(squeeze(gamma_stack_local(i,:,:))')...
74
                             *squeeze(k_stack_local(i,:,:))...
75
                             *squeeze(gamma_stack_local(i,:,:));
76
    end
77
78
    % Assemble the global stiffness matrix together.
79
    ndof=6*nnodes:
80
    k_total=zeros(ndof,ndof);
81
    for i=1:nele
82
        k_total(mem_id(i,1:6),mem_id(i,1:6))=...
83
            k_{total}(mem_id(i,1:6),mem_id(i,1:6))+squeeze(k_stack_global(i,1:6,1:6));
84
        k_{total}(mem_{id}(i,1:6),mem_{id}(i,7:12))=...
            k_{total}(mem_id(i,1:6),mem_id(i,7:12))+squeeze(k_stack_global(i,1:6,7:12));
86
        k_{total}(mem_{id}(i,7:12),mem_{id}(i,1:6))=...
87
            k_{total}(mem_{id}(i,7:12),mem_{id}(i,1:6))+squeeze(k_{stack_global}(i,7:12,1:6));
88
        k_{total(mem_id(i,7:12),mem_id(i,7:12))=...}
            k_{total}(mem_{id}(i,7:12),mem_{id}(i,7:12))+squeeze(k_{stack_global}(i,7:12,7:12));
90
    end
91
92
    % Extract the K_ff.
93
    K_ff=k_total(free_dof,free_dof);
    K_sf=k_total(fixed_dof,free_dof);
95
96
97
    % C = rcond(A) returns an estimate for the reciprocal condition of A in 1-norm.
98
    % If A is well conditioned, rcond(A) is near 1.0.
    % If A is badly conditioned, rcond(A) is near 0.
100
    if abs(det(K_ff))<(10^(-15))</pre>
101
        AFLAG=0;
102
        display('Unstable Structure.');
103
        display('Maybe you should fix more DOFs .');
104
    else
105
106
        AFLAG=1;
        display('The structure is stable.');
107
```

```
108
    end
109
    if abs(rcond(K_ff))<(10^(-15))</pre>
110
        BFLAG=0;
111
        display('The matrix is badly conditioned. ');
112
        display('The results may have a large error. ');
113
        display('Some element may have a way bigger stiffness than others.');
114
        display('Maybe you need to check the units of element properties.');
115
    else
116
        BFLAG=1;
117
        display('The matrix is well-conditioned.');
118
119
    end
120
    defl=K_ff\P_free;
121
122
   % Put the deflection to its appropriate location.
123
   % Include the fixed DOF.
124
   defl_vector_total=zeros(ndof,1);
    defl_vector_total(free_dof)=defl;
126
127
    \% Each line in DEFL' represents the 6 DOFs at each node.
128
    DEFL=reshape(defl_vector_total,6,nnodes);
129
    DEFL=DEFL';
130
    display(DEFL)
131
132
    % Generate the reaction.
133
    react=K_sf*defl;
134
135
    react_vector_total=zeros(ndof,1);
136
    react_vector_total(fixed_dof)=react;
137
    REACT=reshape(react_vector_total,6,nnodes);
138
    REACT=REACT';
139
    display(REACT)
140
141
    % Calculate the element force.
142
    % Transform the element from from global to local coordiante system.
143
    ELE_FOR=zeros(nele,12);
144
    for i=1:nele
145
        element_disp_global=zeros(1,12);
146
        element_disp_global(1:6) = DEFL(ends(i,1),1:6);
147
        element_disp_global(7:12) = DEFL(ends(i,2),1:6);
148
        element_disp_global=element_disp_global';
149
        element_force_local=squeeze(k_stack_local(i,:,:))...
150
                           *squeeze(gamma_stack_local(i,:,:))*element_disp_global;
151
        ELE_FOR(i,:)=element_force_local(:);
152
153
    end
    display(ELE_FOR)
154
```

1.3 subroutine estiff

This subroutine will generate the local stiffness matrix for each element. The input arguments is listed in the source code comments. The output is the stiffness matrix for this element in local coordinate system.

```
function [stiffmatrix] = estiff(A,Izz,Iyy,J,E,v,Length)
         % A
                -- Area
2
       % Izz
               -- Second moment of area over axis zz
3
       % Iyy
               -- Second moment of area over axis zz
4
       % J
               -- torsion constant
5
       % E
               -- Elastic modulus
6
               -- Poisson's ratio
       % v
       % Length -- Element Length
8
      % write 4 (2 by 2 ) small matrix first and then put them together.
10
      KK1=diag([A/Length, 12*Izz/Length^3, 12*Iyy/Length^3, J/(2*(1+v)*Length), 4*Iyy/Length,
11
          4*Izz/Length]);
      KK1(2,6)=6*Izz/Length^2;
12
      KK1(6,2)=KK1(2,6);
13
      KK1(3,5)=-6*Iyy/Length^2;
14
      KK1(5,3)=KK1(3,5);
15
16
      KK2=diag([-A/Length, -12*Izz/Length^3, -12*Iyy/Length^3, -J/(2*(1+v)*Length), 2*Iyy/Length,
17
          2*Izz/Length]);
      KK2(2,6)=6*Izz/Length^2;
18
      KK2(6,2) = -KK2(2,6);
19
      KK2(3,5)=-6*Iyy/Length^2;
20
      KK2(5,3)=6*Iyy/Length^2;
21
22
      KK3=diag([-A/Length, -12*Izz/Length^3, -12*Iyy/Length^3, -J/(2*(1+v)*Length), 2*Iyy/Length,
23
          2*Izz/Length]);
      KK3(2,6)=-6*Izz/Length^2;
24
      KK3(6,2)=6*Izz/Length^2;
25
26
      KK3(3,5)=6*Iyy/Length^2;
      KK3(5,3)=-6*Iyy/Length^2;
27
      KK4=diag([A/Length, 12*Izz/Length^3, 12*Iyy/Length^3, J/(2*(1+v)*Length), 4*Iyy/Length,
29
          4*Izz/Length]);
      KK4(2,6)=-6*Izz/Length^2;
30
      KK4(6,2)=KK4(2,6);
31
      KK4(3,5)=6*Iyy/Length^2;
32
      KK4(5,3)=KK4(3,5);
33
34
      % Put them together.
35
      stiffmatrix=E*[KK1 KK2;KK3 KK4];
36
37
   end
38
```

1.4 subroutine etran

The beta angle is the rotation angle over the x-axis.

And xaxis is the vector to represent the xaxis in the global coordinate system.

The output is the 12 by 12 transform matrix.

```
function [gamma] = etran(beta_ang,xaxis)
      % Make the transformation matrix by two steps.
2
      % Step 1: take xaxis by matrix StepOne.
3
      % Step 2: rotate xaxis by matrix StepTwo.
4
5
      \% Initialize the matrix StepOne and StepTwo.
6
      StepOne=zeros(3);
      StepTwo=zeros(3);
g
      % StepOne starts!!!
10
      % Deal with the special case when xaxis is parallel to y direction.
11
      if (xaxis(1)==0 & xaxis(3)==0)
12
         StepOne(1,:)=xaxis;
13
         StepOne(2,:)=[-1,0,0];
         StepOne(3,:)=cross(StepOne(1,:),StepOne(2,:));
15
16
      % Deal with the normal case
17
      else
18
         StepOne(1,:)=xaxis;
19
         % Change to unit vector
20
         StepOne(1,:)=StepOne(1,:)/norm(StepOne(1,:));
21
         yaxis=[0 1 0];
22
         StepOne(3,:)=cross(StepOne(1,:),yaxis) ;
23
         StepOne(3,:)=StepOne(3,:)/norm(StepOne(3,:));
24
         StepOne(2,:)=cross(StepOne(3,:),StepOne(1,:));
25
         StepOne(2,:)=StepOne(2,:)/norm(StepOne(2,:));
26
      end
27
      % % StepTwo starts!!!
28
      StepTwo(1,1)=1;
29
      StepTwo(2,2)=cos(beta_ang);
30
      StepTwo(3,3)=cos(beta_ang);
31
      StepTwo(2,3)=sin(beta_ang);
32
      StepTwo(3,2)=-sin(beta_ang);
33
34
      % The final gamma:
35
      gammaPart=StepTwo*StepOne;
36
37
      % To 12 by 12
38
      gamma=zeros(12);
39
      for i=1:4
40
         gamma(3*i-2:3*i,3*i-2:3*i)=gammaPart;
41
      end
42
43
   end
44
```

1.5 The verification problem

1.5.1 My node and element definition for the verification problem

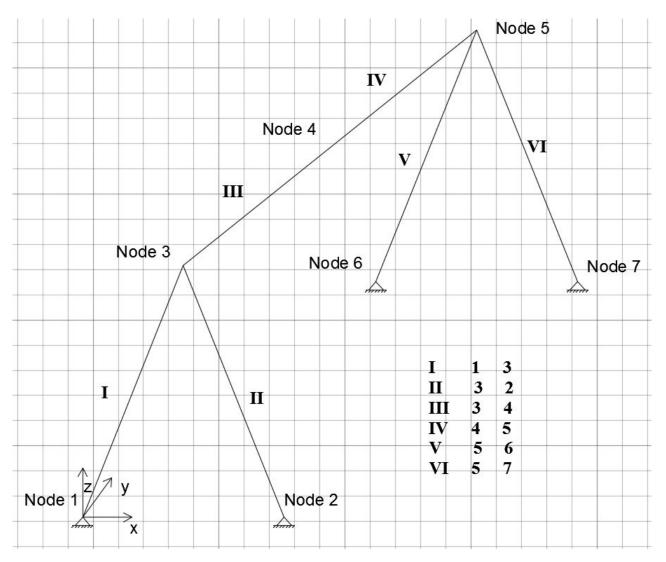


Figure 2: The node and element definition

1.5.2 The full results for the verification problem

```
The structure is stable.
   The matrix is well-conditioned.
2
3
   DEFL =
4
5
            0
                      0
                                0
                                     0.0008
                                               0.0000
                                                        -0.0013
6
            0
                      0
                                0
                                     0.0008
                                              -0.0000
                                                         0.0013
7
      -0.0000
                 0.0026
                          -0.0123
                                    -0.0026
                                              -0.0000
                                                        -0.0000
8
                          -4.4649
       0.0000
                -0.0000
                                    -0.0000
                                              -0.0000
                                                         0.0000
9
      -0.0000
                -0.0026
                          -0.0123
                                     0.0026
                                              -0.0000
                                                         0.0000
10
                                    -0.0008
            0
                      0
                                0
                                               0.0000
                                                         0.0013
11
                                                        -0.0013
            0
                      0
                                0
                                    -0.0008
                                              -0.0000
12
13
14
   REACT =
15
16
                 0.2505
                                          0
                                                    0
                                                              0
17
       0.4498
                           1.1250
      -0.4498
                 0.2505
                           1.1250
                                          0
                                                    0
                                                              0
18
                                          0
                                                    0
                                                              0
            0
                      0
                                0
19
            0
                      0
                                0
                                          0
                                                    0
                                                              0
20
            0
                      0
                                0
                                          0
                                                    0
                                                              0
21
       0.4498
                -0.2505
                           1.1250
                                          0
                                                    0
                                                              0
22
                                          0
                                                              0
      -0.4498
                -0.2505
                           1.1250
                                                    0
23
24
25
   ELE_FOR =
26
27
       1.21
              0.25
                     0.00
                            0.00
                                      0.00
                                               0.00 -1.21 -0.25 -0.00 -0.00
                                                                                    -0.48 674.55
28
                                      0.48 -674.55 -1.21
       1.21 -0.25 -0.00 -0.00
                                                             0.25
                                                                    0.00
                                                                           0.00
                                                                                    -0.00
                                                                                             0.00
29
       0.50
              0.00
                     2.25 -0.00 -1252.61
                                               0.00 -0.50 -0.00
                                                                   -2.25
                                                                           0.00
                                                                                -2122.39
                                                                                             0.00
30
       0.50 -0.00 -2.25 -0.00
                                   2122.39
                                              -0.00 -0.50
                                                             0.00
                                                                    2.25
                                                                           0.00
                                                                                  1252.61
                                                                                            -0.00
31
       1.21
              0.25
                     0.00
                               0
                                     -0.48
                                             674.55 -1.21 -0.25
                                                                   -0.00
                                                                              0
                                                                                     0.00
                                                                                            -0.00
32
              0.25 -0.00 -0.00
                                      0.48
                                             674.55 -1.21 -0.25
                                                                                    -0.00
                                                                                            -0.00
       1.21
                                                                    0.00
                                                                           0.00
33
```

1.6 The question in the verification problem

1.6.1 Deflections at points c and d

Deflections at point c in global coordinate system (unit: mm for displacement and unitless for θ):

u	v	w	θ_x	θ_y	θ_z
-0.0000	0.0026	-0.0123	-0.0026	-0.0000	-0.0000

Deflections at point d in global coordinate system (unit: mm for displacement and unitless for θ):

u	v	w	θ_x	θ_y	θ_z
0.0000	-0.0000	-4.4649	-0.0000	-0.0000	0.0000

1.6.2 Reactions at points a and b

Reactions at points a in global coordinate system (unit: kN for force and $N \cdot m$ for moment):

F_x	F_y	F_z	M_x	M_y	M_z
0.4498	0.2505	1.1250	0	0	0

Reactions at points b in global coordinate system (unit: kN for force and $N \cdot m$ for moment):

F_x	F_y	F_z	M_x	M_y	M_z
-0.4498	0.2505	1.1250	0	0	0

1.6.3 Axial forces in members a-c, c-b, and c-d

Axial forces in members a-c is 1.21 kN.

Axial forces in members c-b is 1.21 kN.

Axial forces in members c-d is 0.50 kN.

2 Program with distributed load

2.1 main program

The main program is here. The subroutines are the same.

```
clear
   clc
2
   % load the input information.
   load nnodes.txt;
   load coord.txt;
5
   load concen.txt;
   load fixity.txt;
   load nele.txt;
   load ends.txt;
   load Area.txt;
   load Izz.txt;
11
   load Iyy.txt;
12
   load J.txt;
13
   load E.txt;
14
   load v.txt;
15
   load beta_ang.txt;
16
17
18
   % Generate the node id.
19
   % Each line represents the 6 dofs for one node.
20
   node_id=zeros(nnodes,6);
21
   for i=1:nnodes
22
       for j=1:6
23
           node_id(i,j)=(i-1)*6+j;
24
25
       end
   end
26
27
   % Generate the member id.
28
   % Each line represents the 12 DOFs for one element.
   mem_id=zeros(nele,12);
30
   for i=1:nele
31
       mem_id(i,1:6)=node_id(ends(i,1),1:6);
32
       mem_id(i,7:12)=node_id(ends(i,2),1:6);
33
34
   end
35
   \% Generate the displacement boundary conditions.
   % Each line in fixity represents the disp BC on 6 DOFs of one node.
37
   % O represents that the DOF is fixed.
   % NaN represnts that the DOF is free.
39
   fixity_tran=fixity';
40
   D=fixity_tran(:);
41
42
   free_dof=find(isnan(D));
43
   fixed_dof=find(D==0);
45
46
47
   % Generate the force boundary conditions.
   % Each line in concen represents the force BC on 6 DOFs of one node.
```

```
% O represents that the DOF does not has external force.
    % The number represnts that the force values on the DOF.
51
    concen_tran=concen';
    P_total=concen_tran(:);
53
    P_free=P_total(free_dof);
54
55
    % Generate the xaxis.
56
    % xaxis will be used in the rotation matrix.
57
    % The rotation matrix is to transform the stiffness matrix
58
    % from global to local coordinate system.
59
    for i=1:nele
60
        xaxis(i,:)=(coord(ends(i,2),:)-coord(ends(i,1),:));
61
        Length(i) = norm(xaxis(i,:));
62
        xaxisunit(i,:)=xaxis(i,:)/Length(i) ;
63
    end
64
65
66
    % Generate the local stiffness matrix for each element.
    k_stack_local=zeros(nele,12,12);
68
    gamma_stack_local=zeros(nele,12,12);
69
    for i=1:nele
70
        k_stack_local(i,:,:)=estiff(Area(i),Izz(i),Iyy(i),J(i),E(i),v(i),Length(i));
71
        gamma_stack_local(i,:,:)=etran(beta_ang(i),xaxisunit(i,1:3));
72
    end
73
74
     % Transform the local stiffness matrix to global stiffness matrix.
75
    k_stack_global=zeros(nele,12,12);
76
    for i=1:nele
77
        k_stack_global(i,:,:)=(squeeze(gamma_stack_local(i,:,:))')...
78
                             *squeeze(k_stack_local(i,:,:))...
79
                             *squeeze(gamma_stack_local(i,:,:));
80
    end
81
82
83
    % Assemble the global stiffness matrix together.
    ndof=6*nnodes;
85
    k_total=zeros(ndof,ndof);
86
    for i=1:nele
87
        k_{total}(mem_{id}(i,1:6),mem_{id}(i,1:6))=...
88
            k_{total}(mem_id(i,1:6),mem_id(i,1:6))+squeeze(k_stack_global(i,1:6,1:6));
89
        k_{total}(mem_{id}(i,1:6),mem_{id}(i,7:12))=...
90
            k_{total}(mem_{id}(i,1:6),mem_{id}(i,7:12))+squeeze(k_{stack_global}(i,1:6,7:12));
91
        k_{total}(mem_{id}(i,7:12),mem_{id}(i,1:6))=...
92
            k_{total}(mem_id(i,7:12),mem_id(i,1:6))+squeeze(k_stack_global(i,7:12,1:6));
93
        k_{total}(mem_{id}(i,7:12),mem_{id}(i,7:12))=...
94
            k_{total}(mem_id(i,7:12),mem_id(i,7:12))+squeeze(k_stack_global(i,7:12,7:12));
95
    end
96
97
98
    % Extract the K_ff.
    K_ff=k_total(free_dof,free_dof);
100
    K_sf=k_total(fixed_dof,free_dof);
101
102
103
   \% C = rcond(A) returns an estimate for the reciprocal condition of A in 1-norm.
104
```

```
% If A is well conditioned, rcond(A) is near 1.0.
    % If A is badly conditioned, rcond(A) is near 0.
106
    if abs(det(K_ff))<(10^(-15))</pre>
107
        AFLAG=0;
108
        display('Unstable Structure.');
109
        display('Maybe you should fix more DOFs .');
110
    else
111
        AFLAG=1;
112
        display('The structure is stable.');
113
    end
114
115
    if abs(rcond(K_ff))<(10^(-15))
116
        BFLAG=0;
117
        display('The matrix is badly conditioned. ');
118
        display('The results may have a large error. ');
119
        display('Some element may have a way bigger stiffness than others.');
120
        display('Maybe you need to check the units of element properties.');
121
    else
122
        BFLAG=1;
123
        display('The matrix is well-conditioned.');
124
    end
125
126
127
    % Input the distributed load.
128
    load distributedLoad.txt;
129
    Fequiva_stack_local=zeros(nele,12);
130
    for i=1:nele
131
132
        qx=distributedLoad(i,1);
        qy=distributedLoad(i,2);
133
        qz=distributedLoad(i,3);
134
        Fequiva_stack_local(i,:)=[-qx*Length(i)/2; -qy*Length(i)/2; -qz*Length(i)/2; ...
135
                                           qz*Length(i)^2/12; -qy*Length(i)^2/12; ...
                               0;
136
                                                                -qz*Length(i)/2; ...
                    -qx*Length(i)/2;
                                          -qy*Length(i)/2;
137
                                       -qz*Length(i)^2/12;
                               0;
                                                            qy*Length(i)^2/12];
138
    end
139
140
    % Transform the equivalent node force from local to global coordinate system.
141
    Fequiva_stack_global=zeros(nele,12);
142
    for i=1:nele
143
        Fequiva_stack_global(i,:)=squeeze(gamma_stack_local(i,:,:))'*Fequiva_stack_local(i,:)';
144
145
    % Assembly the equivalent node force
146
    Fequiva_global=zeros(ndof,1);
147
    for i=1:nele
148
        Fequiva_global(mem_id(i,1:12),1)=Fequiva_global(mem_id(i,1:12),1)+Fequiva_stack_global(i,:)';
149
150
    end
151
    Fequiva_free=Fequiva_global(free_dof);
152
    Fequiva_fixed=Fequiva_global(fixed_dof);
153
154
155
156
    defl=K_ff\(P_free+Fequiva_free);
157
158
159
```

```
% Put the deflection to its appropriate location.
   % Include the fixed DOF.
161
    defl_vector_total=zeros(ndof,1);
162
    defl_vector_total(free_dof)=defl;
163
164
    % Each line in DEFL' represents the 6 DOFs at each node.
165
    DEFL=reshape(defl_vector_total,6,nnodes);
166
    DEFL=DEFL';
167
    display(DEFL)
168
169
170
171
    % Generate the reaction.
   react=K_sf*defl;
172
    react_vector_total=zeros(ndof,1);
174
    react_vector_total(fixed_dof)=react;
   REACT=reshape(react_vector_total,6,nnodes);
176
177
    REACT=REACT';
    display(REACT)
178
179
180
    % Calculate the element force.
181
    % Transform the element from from global to local coordiante system.
182
    ELE_FOR=zeros(nele,12);
183
    for i=1:nele
184
        element_disp_global=zeros(1,12);
185
        element_disp_global(1:6) = DEFL(ends(i,1),1:6);
186
        element_disp_global(7:12)=DEFL(ends(i,2),1:6);
187
        element_disp_global=element_disp_global';
        element_force_local=squeeze(k_stack_local(i,:,:))...
189
                           *squeeze(gamma_stack_local(i,:,:))*element_disp_global;
190
        for j=1:12
191
           % When we have distributed load,
192
           % we should add the equivalent node force to element force.
193
           ELE_FOR(i,j)=element_force_local(j)+Fequiva_stack_local(i,j);
194
        end
195
    end
196
    display(ELE_FOR)
197
```

To verify my program for distributed load, two problems are tested.

2.2 Problem 1 to verify the main program with distributed load

Problem 1 is very simple. It is a one-element cantilever. We can solve this problem by hand.

As shown in the figure below, E=1, Izz=Iyy=1, J=2, A=6, L=2 and the distributed load is q=1. Since we just want to verify the program, the parameters here are unitless to be simple.

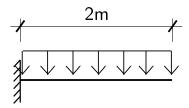


Figure 3: Problem 1 to verify the distributed load

According to the mechanics of materials, we know the vertical displacement at the free end is

$$\triangle = \frac{qL^4}{8EI} = 2\tag{1}$$

The rotation angle is

$$\theta = \frac{qL^3}{6EI} = -1.33\tag{2}$$

Then, I run my code with the distributed load. I get the same results below. The code and input for this example were emailed to professor in the folder "final Project Distributed Load for Cantilever". Just run "main" in the folder will give this result.

```
The structure is stable.
2
    The matrix is well-conditioned.
3
    DEFL =
4
5
             0
                         0
                                    0
                                                                     0
6
                              2.0000
                                                   -1.3333
7
8
9
    REACT =
10
11
             0
                             -1.0000
                                                    1.6667
                         0
                                               0
                                                                     0
12
             0
                         0
                                               0
                                    0
                                                          0
                                                                     0
13
14
15
    ELE_FOR =
16
17
             0
                             -0.0000
                         0
                                               0
                                                    1.3333
                                                                     0
18
             0
                         0
                              2.0000
                                               0
                                                    0.6667
                                                                     0
19
```

2.3 Problem 2 to verify the main program with distributed load

Problem 2 is the HW#3 Problem#1.

We already got the results by hand (with two elements) that the rotation at Point B is 0.0012.

I run my final project code with five elements and I got the same results below.

The 4th line in "DEFL" represents the displacement of the 4th node, which is Point B in the problem. We can see the rotation θ_y is 0.0012, the same result. The code and input for this example were emailed to professor in the folder "final Project Distributed Load for HW3Prob1". Just run "main" in the folder will give this result.

```
The structure is stable.
1
   The matrix is well-conditioned.
2
3
   DEFL =
4
5
             0
                        0
                                  0
                                             0
                                                        0
                                                                   0
6
             0
                        0
                             0.0238
                                             0
                                                  -0.0014
                                                                   0
7
             0
                        0
                             0.0347
                                             0
                                                   0.0013
                                                                   0
8
             0
                        0
                                  0
                                             0
                                                   0.0012
                                                                   0
9
                        0
                                   0
                                             0
                                                                   0
10
11
12
   REACT =
13
14
             0
                        0
                            -2.9297
                                                  71.6146
                                             0
                                                                   0
15
             0
                        0
                                             0
                                                                   0
                                   0
                                                        0
16
             0
                        0
                                   0
                                             0
                                                        0
                                                                   0
17
             0
                        0
                                             0
                            -2.5000
                                                        0
                                                                   0
18
             0
                        0
                                             0
                                                  14.3229
                             0.4297
                                                                   0
19
20
21
    ELE_FOR =
22
23
             0
                                                    0
                                                        0
                                                             2.9297
                 0
                      -2.9297
                                0
                                     71.6146
                                               0
                                                                       0
                                                                             1.6276
24
             0
                 0
                       2.0703
                                0
                                    -43.2943
                                               0
                                                    0
                                                        0
                                                             2.9297
                                                                       0
                                                                           64.7786
                                                                                      0
25
             0
                 0
                                                    0
                                                                          -28.6458
                                                                                      0
                       2.0703
                                0
                                   -23.1120
                                               0
                                                        0
                                                            -2.0703
                                                                       0
26
                 0
             0
                      -0.4297
                                     28.6458
                                                    0
                                                        0
                                                             0.4297
                                                                           14.3229
27
```