Manifold Learning and Sparse Representation: Assignment 10

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Exercise 165

The mutual-coherence is 0.9988 (the normalized inner product of the last two columns).

Exercise 167

We first compute $\tilde{A}$ with normalized columns. Then compute Gram matrix $G = \tilde{A}^T\tilde{A}$ of $\tilde{A}$ and then sort every row of G in descending order to obtain G_S. Thus,

$$\mu_1(2) = max_{1\leq j\leq 4} G_S(j,2) + G_S(j,3) = 0.99880.9981 = 1.9969.$$

Exercise 168

Proof. We can take any p columns of A to construct a sub-matrix A_p and obtain the Gram matrix G_p of A_p. Since $\mu_1(p-1) < 1$ deduces to the sum of off-diagonal of elements of each column is strict less than 1 (let one column equal to j and the other Λ), G_p is strictly diagonally dominant with positive diagonal elements (which is 1) and hence positive definite. Therefore, given $\mu_1(p-1) < 1$, any p columns of A are linear independent and $spark(A) > p$. In other words, $spark(A)$ should be large or equal than the smallest p such that $\mu_1(p-1) \leq 1$. □

Exercise 169

The uncertainty and uniqueness properties follow immediately with the lower bound of spark given in the last exercise.

Uncertainty. $\|x_1\|_0 + \|x_1\|_1 \geq \min p|\mu_1(p-1) \geq 1$.

Uniqueness. If a system of linear equations Ax = b has a solution x obeying $\|x\|_0 < \frac{1}{2}\min\{p|\mu_1(p-1) \geq 1\}$, this solution is necessarily the sparsest possible.