Manifold Learning and Sparse Representation: Assignment 9

ZHANG Yuan, 1601111332

Exercise 155

By definition, we have the following

$$\mu((\mathbf{I},\mathbf{F})) = \max_{i,j} |\frac{1}{\sqrt{N}}\omega^{(i-1)(j-1)}| = \max_{i,j} \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{N}},$$

where $\omega = e^{-2\pi i/N}$ is the primitive N-th root of unity.

Exercise 160

By exhaustive search, we find that any sub-matrices of two columns have full ranks and hence the spark of the given matrix is larger than two. On the other, the sub-matrices of the first three columns has rank two which means those three columns are linear dependent, so that the spark is less or equal than three.

Therefore, the spark is three (3).

Exercise 161

Proof. Suppose $spark(A) > rank(A) + 1$. We denote $m \triangleq spark(A) - 1 > rank(A)$. Since $m < spark(A)$, any m different columns of A CANNOT be linear dependent and hence linear independent, which clearly contradicts the fact that $m > rank(A)$. $\square$