

# HW3

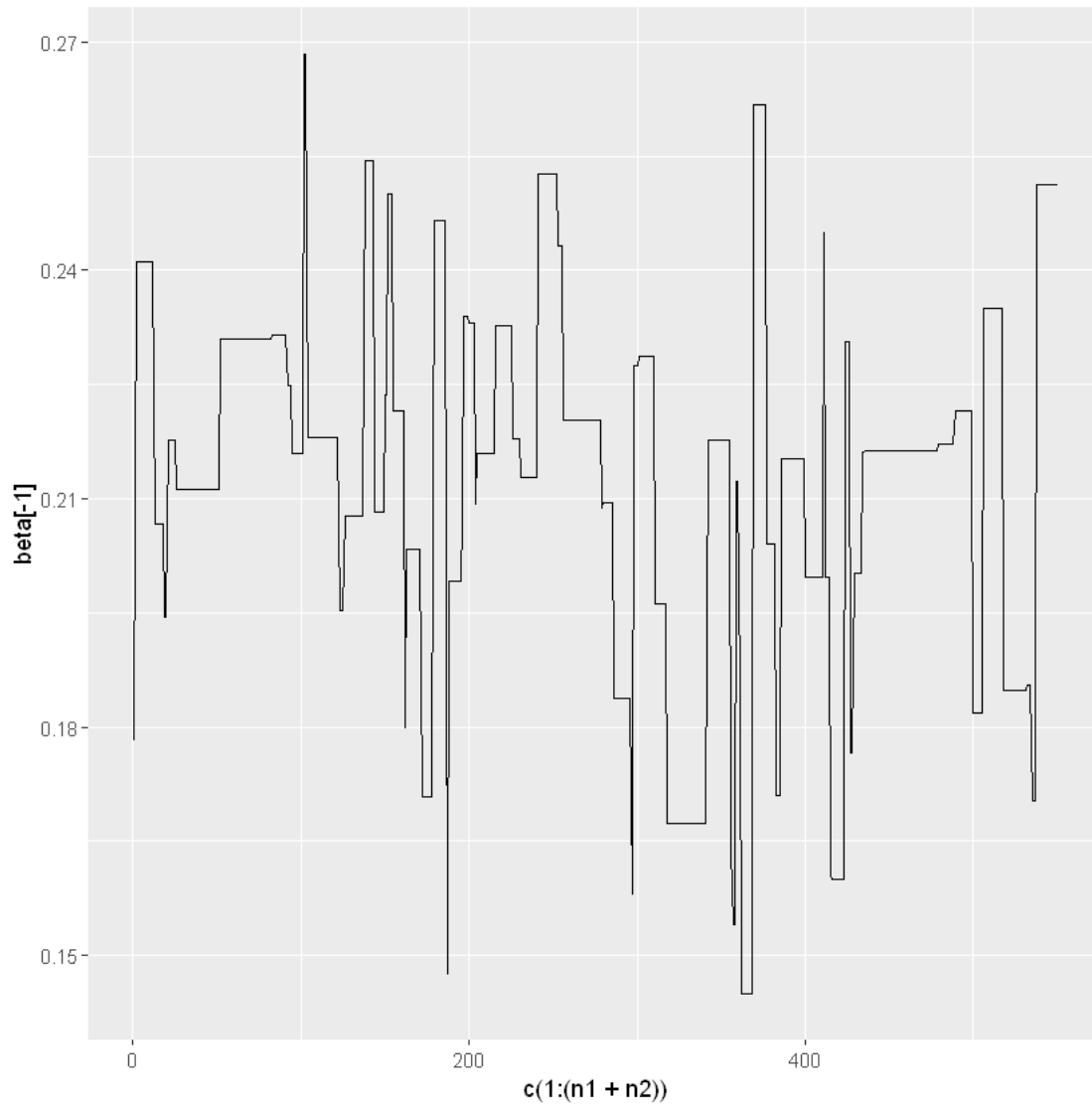
Congyuan Duan

December 30, 2021

```
[2]: library(ggplot2)
```

## 1 3.21

```
[5]: postbeta <- function(beta, x1, x2, x3, x4, x5) {  
  new <- (1 - beta) ^ x2 * (1 - 2 * beta) ^ x3 * beta ^ (x4 + x5) * (beta > 0)  
  ↪0) * (beta < 0.5)  
  return (new)  
}  
MH1 <- function(beta0, x1, x2, x3, x4, x5, n1, n2) { #n1 burn-in and n2 samples  
  beta <- c(beta0, rep(0, (n1+n2)))  
  for (i in 1:(n1+n2)) {  
    y <- runif(1, 0, 0.5)  
    u <- runif(1, 0, 1)  
    r <- min(1, postbeta(y, x1, x2, x3, x4, x5)/postbeta(beta[i], x1, x2, ↪  
    ↪x3, x4, x5))  
    if (u <= r) {beta[i+1] <- y}  
    else {beta[i+1] <- beta[i]}  
  }  
  return (beta)  
}  
n1 <- 50  
n2 <- 500  
beta <- MH1(0.1, 82, 72, 45, 34, 17, n1, n2)  
p <- ggplot() + geom_line(aes(x = c(1:(n1+n2)), y = beta[-1]))  
p
```



```
[6]: mean(beta[(n1+1):n2])
```

0.212722430411571

When the initial value of  $\beta$  is 0, the posterior mean is 0.213.

## 2 3.25

```
[4]: postx <- function(n, y) {
  return (rbinom(1, n, y))
}
posty <- function(x, alpha, beta, n) {
  return (rbeta(1, x+alpha, n-x+beta))
}
```

```

}
Gibbs1 <- function(alpha, beta, n, x0, y0, m1, m2) { #m is the sample size
  X <- c(x0, rep(0, (m1+m2)))
  Y <- c(y0, rep(0, (m1+m2)))
  for (i in 1:(m1+m2)) {
    X[i+1] <- postx(n, Y[i])
    Y[i+1] <- posty(X[i+1], alpha, beta, n)
  }
  return(list(X = X, Y = Y))
}
alpha <- 0.5
beta <- 0.5
n <- 20
x0 <- 1
y0 <- 0.5
m1 <- 5000
m2 <- 10000
Y <- Gibbs1(alpha, beta, n, x0, y0, m1, m2)$Y[(1+m1):(m1+m2+1)]
x1 <- seq(0, 1, length.out = m2)
p <- ggplot() + geom_density(aes(x=Y)) + geom_line(aes(x = x1, y = dbeta(x1,
↪alpha, beta)), color = "red") +
  geom_histogram(aes(x=Y, y = ..density..), binwidth=0.05, alpha = 0.25)
↪+ ylim(0,2.5)
p

```

