1. Research Objectives

1.1. Problem Statement

A principal challenge in parametric insurance design is the minimization of basis risk. Existing design frameworks, such as that proposed by Steinmann et al. (2023), are fundamentally limited by their reliance on a single, deterministic value for modeled loss for optimization. This approach neglects the inherent stochasticity and uncertainty of catastrophe losses, particularly those stemming from the vulnerability function.

1.2. Proposed Solution

To address this limitation, this research proposes an innovative design paradigm. The central thesis is that the evaluation of modeled loss must be fundamentally transformed. By introducing a **Robust Bayesian Analysis** framework, we will transition the representation of modeled loss from a point estimate to a **complete posterior predictive distribution**. This enables product optimization on a more scientifically rigorous and comprehensive basis.

1.3. Specific Aims

Based on this solution, the specific aims of this research are as follows:

1. **To develop a probabilistic loss model**: To construct an event-level catastrophe loss model capable of generating a full probability distribution, thereby comprehen-

sively quantifying uncertainties from various sources including hazard, exposure, and particularly, the **vulnerability function**.

2. **To establish a novel optimization framework**: To create a new optimization procedure for parametric insurance products that replaces traditional error metrics (e.g., RMSE) with **Proper Scoring Rules**. This will allow for the evaluation of a deterministic payout (a point forecast) against a full probabilistic loss distribution (a distributional forecast).

3. **To design and validate more robust insurance products**: To apply the new framework to design a suite of parametric insurance products and to demonstrate, through systematic methodological comparison, their superior performance in minimizing basis risk compared to products designed using conventional deterministic methods.

4. **To quantify the value of the proposed methodology**: To go beyond technical metrics and elucidate the practical advantages of the framework from the perspectives of risk management and actuarial science, particularly in providing more reliable estimates of expected loss and enabling the scientific quantification of risk loading.

2. Methodology

The research will be executed in six interconnected phases, constituting a complete transition from a deterministic to a probabilistic paradigm.

2.1. Phases I & II: Establishment and Implementation of the Benchmark Framework

This initial phase aims to fully replicate a state-of-the-art deterministic design framework to serve as a rigorous scientific benchmark against which the value added by the proposed probabilistic methodology can be assessed. First, we will apply the CLIMADA framework, utilizing historical and synthetic tropical cyclone tracks from IBTrACS in conjunction with the **Emanuel & Rotunno wind field model**, to establish a foundational catastrophe risk model for North Carolina. Subsequently, high-resolution asset exposure data will be developed using the LitPop methodology, and the Emanuel USA vulnerability function, calibrated for the United States, will be employed. Based on this foundation, we will fully implement the methodology of Steinmann et al. (2023), including the design of **Cat-in-a-Circle parametric indices** and the systematic testing of over 350 combinations of parametric triggers and layered payout functions. The objective of this phase is to **optimize product design by minimizing basis risk**, for which we will use traditional metrics such as RMSE to quantify the discrepancy between modeled losses and payouts, thereby calculating the technical premium for the benchmark product.

2.2. Phase III: Construction of the Hierarchical Bayesian Risk Model

This phase represents the core innovation of this research. Before detailing the specific methods, it is essential to articulate the **rationale for employing Robust Bayesian**

Analysis. The primary objective is to address higher-order **model uncertainty**, which extends beyond mere parameter stochasticity. The conventional model, `Damage = F(hazard, exposure, vulnerability)`, while concise, is predicated on numerous assumptions and choices, each representing a potential source of model misspecification:

- **Uncertainty in the Vulnerability Function**: As acknowledged by Steinmann et al. (2023), the vulnerability function is a **"major source of uncertainty"**. Any single vulnerability curve, such as the Emanuel USA function, is merely a simplification of reality, and reliance upon it introduces significant risk.

- **Uncertainty in the Likelihood Function**: Traditional statistical inference implicitly assumes a specific error distribution for the observed losses (e.g., a normal distribution). If this assumption is invalid, particularly in the presence of extreme loss events, the predictive performance of the model will be compromised.

Robust Bayesian analysis acknowledges the inherent imperfection of any single model and seeks to render the analytical results less sensitive to these "imperfect assumptions." Therefore, this research will address uncertainty at the levels of both the **prior distribution** and the **likelihood function** to ensure the robustness of the final product design.

2.2.1. Hierarchical Bayesian Model Structure for Catastrophe Loss

To systematically capture and quantify the aforementioned uncertainties, we will construct a Hierarchical Bayesian Model. The general structure of this model can be expressed as:

$$L(\text{event}) \sim f(L|H, E, V, \theta, \text{model\_structure}) \tag{1}$$

where L denotes the loss, H, E, V represent hazard, exposure, and vulnerability, respectively, and θ is the set of model parameters. This hierarchical framework allows for the explicit definition and quantification of uncertainty at various levels.

Mathematical Formulation We will model the loss L_{ij} for a single asset at location i (belonging to region $r(i)$) during event j.

Level 1: Observation Level (Likelihood) This level describes the relationship between the observed loss data and the model's expected loss, capturing measurement error and inherent randomness.

$$L_{ij} \sim \text{LogNormal}(\log(\mu_{ij}), \sigma^2_{\text{obs}}) \tag{2}$$

Here, μ_{ij} is the model's expected mean loss, and σ^2_{obs} is the observational variance. The expected loss is defined by the core catastrophe risk equation:

$$\mu_{ij} = E_i \times V(H_{ij}; \beta_i) \tag{3}$$

where E_i is the exposure value at location i, H_{ij} is the hazard intensity of event j at location i, and $V(\cdot)$ is the vulnerability function, whose behavior is governed by a set of location-specific parameters β_i.

Level 2: Process Level This level models the parameters β_i of the vulnerability function to capture their spatial variability. We assume that the parameters β_i are composed of a regional average, a spatially correlated random effect, and a site-specific random effect:

$$\beta_i = \alpha_{r(i)} + \delta_i + \gamma_i \tag{4}$$

- $\alpha_{r(i)}$: The average vulnerability parameter for region $r(i)$ (fixed effect).

- δ_i: A spatially structured random effect capturing similar vulnerability characteristics in neighboring locations.

- γ_i: An unstructured, site-specific random effect capturing local variability not explained by the spatial structure.

Level 3: Parameter Level This level specifies prior distributions (hyperpriors) for the random effects in the process level.

$$\alpha_r \sim \text{Normal}(0, \sigma_\alpha^2) \tag{5}$$

$$\gamma_i \sim \text{Normal}(0, \sigma_\gamma^2) \tag{6}$$

$$\delta = (\delta_1, \dots, \delta_n) \sim \text{MVN}(0, \Sigma_\delta) \tag{7}$$

where the elements of the covariance matrix Σ_δ are given by $(\Sigma_\delta)_{uv} = \sigma_\delta^2 \exp(-d_{uv}/\rho)$, with d_{uv} being the distance between locations u and v, and ρ being the range parameter of spatial correlation.

Level 4: Hyperparameter Level This level assigns prior distributions to the variance parameters (hyperparameters) from the parameter level.

$$\sigma_{\text{obs}}, \sigma_\alpha, \sigma_\gamma, \sigma_\delta \sim \text{Half-Cauchy}(0, A) \tag{8}$$

$$\rho \sim \text{LogNormal}(m_\rho, s_\rho^2) \tag{9}$$

Through this hierarchical structure, Bayesian inference methods such as MCMC will be used to estimate the posterior distributions of all parameters and hyperparameters, yielding a loss prediction distribution that fully quantifies uncertainty across all levels.

2.2.2. Modeling Strategy

- **Robust Bayesian Framework**: We will employ a robust Bayesian modeling framework constrained by a density ratio class, $\Gamma = \{P : dP/dP_0 \leq \gamma(x)\}$. The core idea is to distrust any single baseline model P_0 and instead define a neighborhood Γ of plausible probability distributions P. By optimizing over this set, we can identify solutions that perform well across the entire class of uncertain models, thereby systematically evaluating and enhancing model robustness.
- **Class of Priors**: To address uncertainty in prior knowledge, we will design and compare a class of prior distributions rather than relying on a single specification. This will include non-informative priors, weakly informative priors, and scenario-based priors (”neutral,” ”optimistic,” and ”pessimistic”).
- **Class of Likelihoods**: To address uncertainty in the data-generating process, we will test a class of likelihood functions to handle outliers common in loss data, primarily comparing the **Normal distribution** with the heavy-tailed **Student’s t-distribution**.

2.2.3. Automatic Distribution Selection, MCMC Diagnostics, and Mixed Predictive Estimation

This section will involve a series of technical steps, including: automatic selection of the best-fitting distribution from six candidates using criteria like AIC/BIC; establishment of a rigorous MCMC diagnostics system including the Gelman-Rubin statistic and effective sample size; and finally, the use of the Mixed Predictive Estimation (MPE) formula, $F_m^{MP}(z) = (1/m)\sum_{i=1}^{m} F_c(z|\theta_i)$, to generate the posterior predictive loss distribution.

2.3. Phase IV: CRPS Evaluation Framework and Climatological Blending

To evaluate product performance within a probabilistic context, we must replace traditional metrics like RMSE with more appropriate **Proper Scoring Rules**. This transition is essential because RMSE is only suitable for comparing two point estimates (a forecast vs. an observation), which is fundamentally incompatible with the core task of this research. Our task is not to model the payout amount, but rather: **first, to probabilistically model the uncertain "loss" to obtain a full distribution; second, to treat the parametric insurance "payout" as a deterministic point forecast; and finally, to evaluate how skillful this point forecast is against our modeled loss distribution.** Proper scoring rules, particularly the **Continuous Ranked Probability Score (CRPS)**, are designed specifically for this purpose. They reward forecasts that are not only accurate but also "honest" about their uncertainty, thus serving as the critical bridge between our probabilistic loss model and the parametric insurance product design.

2.3.1. Multi-dimensional Skill Score System and Multi-Objective Optimization

This research will employ a multi-dimensional scoring system, incorporating both deterministic and probabilistic metrics, to comprehensively evaluate the performance of different product designs.

Table 1: Multi-dimensional Skill Score System

Score	**Category**	**Orientation**	**Purpose**
RMSE	Deterministic	$\downarrow$ (Lower is better)	Basic prediction error
CRPSS	Probabilistic	$\uparrow$ (Higher is better)	Skill of the distributional forecast
EDI	Extreme Events	$\uparrow$ (Higher is better)	Dependence in extreme events
TSS	Categorical	$\uparrow$ (Higher is better)	Accuracy of binary classification
Brier	Probabilistic	$\downarrow$ (Lower is better)	Calibration of probability forecasts

The final optimization will be achieved through a weighted objective function:

$$\text{Optimal Product} = \arg\min\{w_1 \times (1 - \text{CRPSS}) + w_2 \times (1 - \text{EDI}) + w_3 \times (1 - \text{TSS})\} \quad (10)$$

with default weights set to $w_1 = 0.4, w_2 = 0.3, w_3 = 0.3$.

2.4. Phases V & VI: Comprehensive Comparative Analysis and Unified Framework Refactoring

Finally, a comprehensive performance comparison will be conducted. Using statistical tools such as t-tests, Wilcoxon rank-sum tests, and Bootstrap confidence intervals, we will systematically compare the traditional Steinmann product with the robust Bayesian products in terms of skill scores, basis risk characteristics, and robustness. The objective of this final phase is not only to demonstrate the superiority of the new method but also to **refactor the entire analytical pipeline**, transitioning the codebase and underlying logic from a deterministic to a fully probabilistic paradigm, resulting in a unified, efficient, and scientifically sound framework.

3. Anticipated Outcomes

This research is expected to yield significant outcomes with both academic and practical value:

1. **A More Scientific Risk Assessment Framework**: This study will deliver a complete tropical cyclone risk assessment system for North Carolina. Unlike traditional methods, the core output of this framework will not be a single loss value but a **complete probability distribution of loss** for each catastrophe event.

2. **A Probabilistic Characterization of Basis Risk**: This research will fundamentally change how basis risk is quantified. By transitioning from a single "loss - payout" value to a **full probability distribution**, it will offer profound insights for both the theory and practice of parametric insurance, manifested in several ways:

    - **More Comprehensive and Realistic Risk Description**: The framework acknowledges and quantifies the inherent uncertainty in loss estimation, moving from the question "What is the basis risk?" to "What is the **likely range** of the basis risk, and what is its distribution?"

    - **More Granular Characterization of Basis Risk**: The full probability distribution enables a nuanced analysis of critical questions, such as the probability of underpayment, the probability of overpayment, and the magnitude of **tail risk**.

- **Design of More Robust Insurance Products**: By calibrating against the entire loss distribution rather than a single mean value, the resulting products will exhibit greater **robustness** to unexpected and extreme loss scenarios.

3. **A Data-Driven Framework for Premium Ratemaking**: The probabilistic loss distributions generated by this framework serve as a direct and scientifically grounded input for premium calculation, fundamentally improving the reliability of the pure premium and the precision of the risk loading.

   - **Calculation of a More Reliable Expected Loss**: The blind spot of traditional methods is that averaging point estimates of loss ignores the uncertainty within each estimate. By averaging thousands of full loss distributions, our framework produces an expected loss value that is inherently more robust and reliable, as it has already integrated model uncertainty.

   - **Scientific Quantification of Risk Loading**: The primary challenge in traditional ratemaking is scientifically pricing "uncertainty." Our framework's output of a basis risk probability distribution directly solves this problem. Since a premium must cover not only the expected loss but also a risk load for uncertainty, the shape of this distribution provides a data-driven basis for this calculation:

     – **Precise Capital Requirements**: The loss distribution allows for the direct calculation of risk measures such as **Value at Risk (VaR)** or **Expected Shortfall (ES)**, aligning directly with regulatory frameworks like Solvency II.

     – **Data-Driven Pricing**: The shape of the basis risk distribution (its variance, skewness, kurtosis) directly informs the nature of the risk. A wider, more skewed distribution implies a higher probability of extreme outcomes, thus justifying a higher risk load.

Illustrative Example of Outcomes Using the historical event of TC Idai as a case study, the final analysis will clearly present:

- **Loss Distribution Generation**: A visualization of the posterior predictive loss distribution for TC Idai generated via the robust Bayesian analysis (e.g., a chart showing losses ranging from $70M to $95M, with the highest probability density around $80M).
- **Product Evaluation and Comparison**: A comparison of two parametric product designs (e.g., a traditional one and one from this study) with their respective payout amounts (e.g., $40M and $60M).
- **CRPS Score Presentation**: The calculation of the CRPS scores for both payouts against the loss distribution, explaining why the lower score represents a superior product.
- **Basis Risk Visualization**: A graphical representation of the basis risk distributions for both products, intuitively displaying their respective risks of underpayment and overpayment.