

1 Thermodynamics

If two objects are in thermal equilibrium with a third system, then they are in equilibrium with each other.

1.1 Thermal Expansion

For linear expansion, the change in length is:

$$\Delta L = \alpha L_0 \Delta T$$

Where α is the coefficient of linear expansion. For area expansion, use approximately 2α . For volume expansion, use approximately 3α .

1.2 Kinetic Theory of Gases

1.2.1 Ideal Gas Law

An ideal gas' molecules are treated as non-interacting point particles. For an ideal gas of N particles at pressure P , volume V and temperature T :

$$PV = NK_B T$$

For a non-ideal gas, the Van der Waals correction to the ideal gas law is:

$$\left(P + a \left(\frac{n}{V}\right)^2\right)(V - bn) = nRT$$

Where a and b are constants.

1.2.2 Internal Energy

Different gases at the same temperature have the same average kinetic energy. Thus we define temperature of a substance to be its average kinetic energy. For a monatomic ideal gas:

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$$

For a gas molecule with r atoms, its total kinetic energy, center of mass kinetic energy and internal vibrational/rotational energy are given by:

$$E_{\text{Total}} = \frac{3r}{2}kT$$

$$E_{\text{COM}} = \frac{3}{2}kT$$

$$E_{\text{Internal}} = \frac{3(r-1)}{2}kT$$

The equipartition theorem states that each degree of freedom a molecule has contributes an extra $\frac{1}{2}kT$ of kinetic energy.

1.2.3 Maxwell Distribution

For an ideal gas, the distribution of its velocities is:

$$f(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}}$$

From this distribution, we can get the average speed of a particle:

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m}}$$

The most probable velocity is the maximum point of the distribution:

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}}$$

For any two particles, their average relative speed is:

$$\langle v_{\text{rel}} \rangle = \sqrt{2}\langle v \rangle = \sqrt{\frac{16kT}{\pi m}}$$

From this, we can get the mean free path of a particle, the average distance a particle travels before hitting another particle:

$$l_m = \frac{1}{4\pi\sqrt{2}r^2n}$$

Where n is the number density of the particle and r is its radius.

1.2.4 Diffusion

For a substance undergoing diffusion due to a concentration gradient $\frac{dc}{dx}$, the diffusive flux J is:

$$J = DA \frac{dc}{dx}$$

1.3 Heat Transfer

For heat transfer through a material with length l , area A and thermal conductivity K between two heat reservoirs $T_1 > T_2$:

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{l}$$

For a blackbody at temperature T radiating heat away:

$$\frac{dQ}{dt} = \sigma AT^4$$

The heat transferred by changing the temperature of a solid of mass m with heat capacity c is:

$$\Delta Q = mc\Delta T$$

1.4 Thermodynamic Processes

In all the process described below, the heat Q that goes into the gas is positive, and the work done on the gas W is positive. The first law of thermodynamics states that the change of internal energy U is:

$$U = Q + W$$

$$U(\gamma - 1) = NkT$$

Where $\gamma = C_p/C_v$ is the ideal gas constant and $C_v = C_p - k$.

1.4.1 Isochoric

In this constant volume process:

$$W = 0$$

$$Q = NC_v \Delta T$$

$$U = Q$$

1.4.2 Isobaric

In a constant pressure volume expansion from V_1 to V_2 :

$$W = P(V_1 - V_2)$$

$$Q = NC_p \Delta T$$

$$U = NC_v \Delta T$$

1.4.3 Isothermal

For an isothermal expansion from V_1 to V_2 :

$$W = NkT \ln \left(\frac{V_1}{V_2}\right)$$

$$Q = -W$$

$$U = 0$$

1.4.4 Adiabatic

For an adiabatic process,

$$W = - \int P dV$$

$$Q = 0$$

$$U = W$$

Integrating the work done, we get the following relation:

$$PV^\gamma = \text{constant}$$

1.5 Heat Engines

The efficiency of a heat engine that takes in Q_H and gives out Q_L while doing work W , its efficiency is given by:

$$\eta = \frac{|W|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$

The efficiency of a heat pump that uses W to pump Q_L from the col reservoir is:

$$\eta = \frac{|Q_L|}{|W|}$$

All reversible engines operating between the same two temperatures have the same efficiency as a Carnot engine, as you can fit many infinitesimally small Carnot cycles into any reversible cycle:

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H}$$

1.6 Second Law

- A process whose only net result is to take heat from a reservoir and convert it to heat is impossible.
- No heat engine can working between two temperatures T_1 and T_2 can have a higher efficiency than a reversible engine.

1.7 Entropy

1.7.1 Macroscopic Def.

Entropy is the measure of disorder. If heat is added reversibly into a system at temperature T , the increase in entropy in the system is:

$$dS = \frac{dQ}{T}$$

Entropy is a state function that doesn't depend on the path travelled. The total entropy change in the system and surroundings for a reversible process is zero. For an irreversible process, the total entropy change is always positive. At $T = 0$, $S = 0$. This is the third law of thermodynamics.

1.7.2 Microscopic Def.

Boltzmann defined entropy of a system by counting the number of indistinguishable microstates w inside:

$$S = k \ln w$$