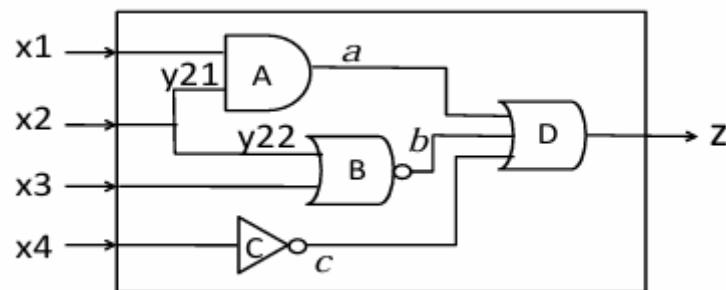


# HW1

Git repository URL: [https://github.com/yuanchung0521/VLSI\\_Testing](https://github.com/yuanchung0521/VLSI_Testing)

姓名	系所	學號	分工
王翊鴻	清大電機所	112061614	協助程式設計、驗證 coding 結果、報告製作
李元中	交大電機所	312511052	主要程式設計、演算法建構、報告製作



(a) The result of truth table

(a) Truth Table									
x1	x2	x3	x4	y21	y22	a	b	c	z
0	0	0	0	0	0	0	1	1	1
0	0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	0	0	1	1
0	0	1	1	0	0	0	0	0	0
0	1	0	0	1	1	0	0	1	1
0	1	0	1	1	1	0	0	0	0
0	1	1	0	1	1	0	0	1	1
0	1	1	1	1	1	0	0	0	0
1	0	0	0	0	0	0	1	1	1
1	0	0	1	0	0	0	1	0	1
1	0	1	0	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0	0
1	1	0	0	1	1	1	0	1	1
1	1	0	1	1	1	1	0	0	1
1	1	1	0	1	1	1	0	1	1
1	1	1	1	1	1	1	0	0	1

**(b) the number of stuck-at faults detected by each of the 16 input vectors.**

```
===== (b) deductive Fault Simulation =====
Input pattern (x1, x2, x3, x4) = (0 ,0 ,0 ,0) :
- #stuct-at-fault = 1
- faults = Node z(1) : z_0

Input pattern (x1, x2, x3, x4) = (0 ,0 ,0 ,1) :
- #stuct-at-fault = 5
- faults = Node z(1) : b_0 x2_1 x3_1 z_0 y22_1

Input pattern (x1, x2, x3, x4) = (0 ,0 ,1 ,0) :
- #stuct-at-fault = 3
- faults = Node z(1) : x4_1 c_0 z_0

Input pattern (x1, x2, x3, x4) = (0 ,0 ,1 ,1) :
- #stuct-at-fault = 6
- faults = Node z(0) : c_1 x4_0 b_1 a_1 z_1 x3_0

Input pattern (x1, x2, x3, x4) = (0 ,1 ,0 ,0) :
- #stuct-at-fault = 3
- faults = Node z(1) : c_0 x4_1 z_0

Input pattern (x1, x2, x3, x4) = (0 ,1 ,0 ,1) :
- #stuct-at-fault = 8
- faults = Node z(0) : c_1 x4_0 b_1 x2_0 y22_0 a_1 x1_1 z_1

Input pattern (x1, x2, x3, x4) = (0 ,1 ,1 ,0) :
- #stuct-at-fault = 3
- faults = Node z(1) : c_0 x4_1 z_0

Input pattern (x1, x2, x3, x4) = (0 ,1 ,1 ,1) :
- #stuct-at-fault = 6
- faults = Node z(0) : c_1 x4_0 b_1 a_1 x1_1 z_1

Input pattern (x1, x2, x3, x4) = (1 ,0 ,0 ,0) :
- #stuct-at-fault = 1
- faults = Node z(1) : z_0

Input pattern (x1, x2, x3, x4) = (1 ,0 ,0 ,1) :
- #stuct-at-fault = 4
- faults = Node z(1) : b_0 x3_1 y22_1 z_0

Input pattern (x1, x2, x3, x4) = (1 ,0 ,1 ,0) :
- #stuct-at-fault = 3
- faults = Node z(1) : c_0 x4_1 z_0

Input pattern (x1, x2, x3, x4) = (1 ,0 ,1 ,1) :
- #stuct-at-fault = 8
- faults = Node z(0) : c_1 x4_0 a_1 b_1 x2_1 y21_1 x3_0 z_1

Input pattern (x1, x2, x3, x4) = (1 ,1 ,0 ,0) :
- #stuct-at-fault = 1
- faults = Node z(1) : z_0

Input pattern (x1, x2, x3, x4) = (1 ,1 ,0 ,1) :
- #stuct-at-fault = 4
- faults = Node z(1) : a_0 y21_0 x1_0 z_0

Input pattern (x1, x2, x3, x4) = (1 ,1 ,1 ,0) :
- #stuct-at-fault = 1
- faults = Node z(1) : z_0

Input pattern (x1, x2, x3, x4) = (1 ,1 ,1 ,1) :
- #stuct-at-fault = 5
- faults = Node z(1) : a_0 y21_0 x2_0 x1_0 z_0
```

(c) the average number of stuck-at faults detected by an input vector.

$$\frac{1 + 5 + 3 + 6 + 3 + 8 + 3 + 6 + 1 + 4 + 3 + 8 + 1 + 4 + 1 + 5}{16} = 3.875$$

```
===== (c) Average #fault per intput vector =====
avg fault per input = 3.875
```

(d) the input vector that detects the maximum number of faults. List the faults detected.

```
Input pattern (x1, x2, x3, x4) = (0 ,1 ,0 ,1) :
- #stuct-at-fault = 8
- faults = Node z(0) : c_1 x4_0 b_1 x2_0 y22_0 a_1 x1_1 z_1
```

```
Input pattern (x1, x2, x3, x4) = (1 ,0 ,1 ,1) :
- #stuct-at-fault = 8
- faults = Node z(0) : c_1 x4_0 a_1 b_1 x2_1 y21_1 x3_0 z_1
```

(e) the average number of test patterns for a fault.

```
===== (e) Average #pattern per fault =====
#Pattern for z_1 = 4
#Pattern for z_0 = 12
#Pattern for c_0 = 4
#Pattern for b_1 = 4
#Pattern for b_0 = 2
#Pattern for a_1 = 4
#Pattern for x2_1 = 2
#Pattern for a_0 = 2
#Pattern for x3_1 = 2
#Pattern for x3_0 = 2
#Pattern for x2_0 = 2
#Pattern for x1_0 = 2
#Pattern for x1_1 = 2
#Pattern for y21_0 = 2
#Pattern for y22_1 = 2
#Pattern for x4_0 = 4
#Pattern for x4_1 = 4
#Pattern for c_1 = 4
#Pattern for y21_1 = 1
#Pattern for y22_0 = 1

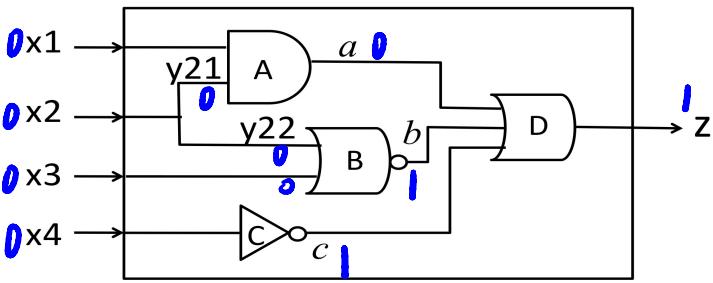
avg pattern per fault = 3.1
```

$$\frac{4 + 12 + 4 + 4 + 2 + 4 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 4 + 4 + 4 + 1 + 1}{20} = \frac{62}{20} = 3.1$$

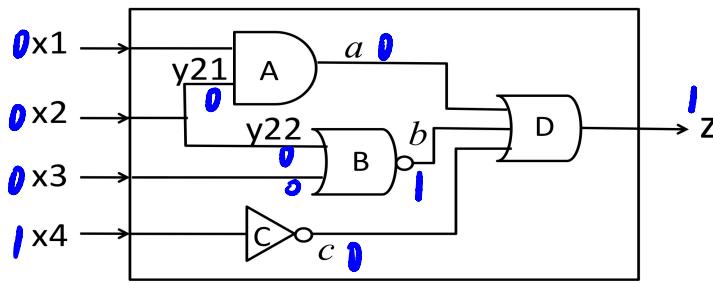
**(f) the hardest-to-detect fault(s)**

```
===== (e) Average #pattern per fault =====
#Pattern for z_1 = 4
#Pattern for z_0 = 12
#Pattern for c_0 = 4
#Pattern for b_1 = 4
#Pattern for b_0 = 2
#Pattern for a_1 = 4
#Pattern for x2_1 = 2
#Pattern for a_0 = 2
#Pattern for x3_1 = 2
#Pattern for x3_0 = 2
#Pattern for x2_0 = 2
#Pattern for x1_0 = 2
#Pattern for x1_1 = 2
#Pattern for y21_0 = 2
#Pattern for y22_1 = 2
#Pattern for x4_0 = 4
#Pattern for x4_1 = 4
#Pattern for c_1 = 4
#Pattern for y21_1 = 1
#Pattern for y22_0 = 1
```

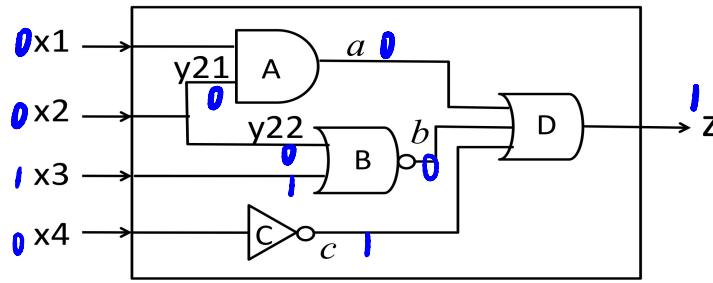
The hardest-to-detect fault is y21 stuck-at-1 and y22 stuck-at-0. They only have one test pattern to detect it.



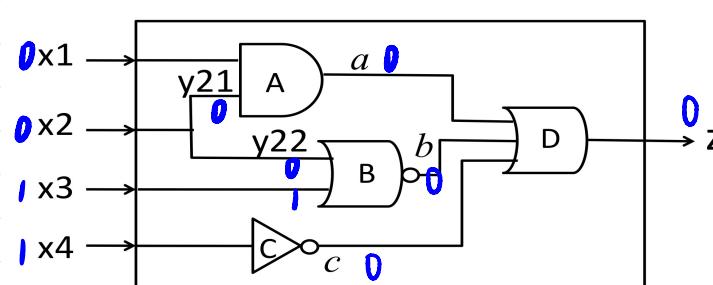
$$\begin{aligned}
 Lx_1 &= \{x_1/1\} & Lx_2 &= \{x_2/1\} \\
 Lx_3 &= \{x_3/1\} & Lx_4 &= \{x_4/1\} \\
 Ly_{21} &= \{x_2/1, y_2/1\}, Ly_{22} = \{x_2/1, y_2/1\} \\
 La &= Lx_1 \cdot Ly_{21} + \{a/1\} = \{a/1\} \\
 Lb &= Ly_{22} + Lx_3 + \{b/0\} = \{b/0, x_2/1, y_2/1, x_3/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/1, c/0\} \\
 Lz &= \bar{La} \cdot Lb \cdot Lc + \{z/0\} \\
 &= \{z/0\}_a
 \end{aligned}$$



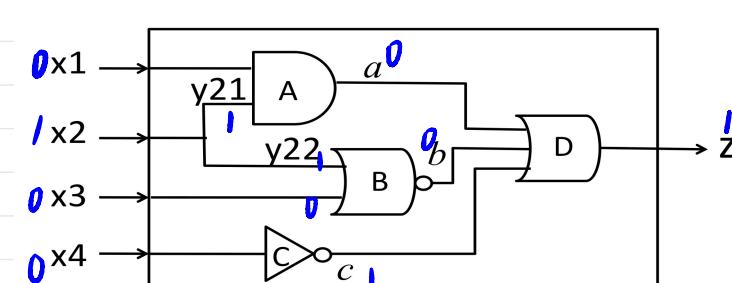
$$\begin{aligned}
 Lx_1 &= \{x_1/1\} & Lx_2 &= \{x_2/1\} \\
 Lx_3 &= \{x_3/1\} & Lx_4 &= \{x_4/0\} \\
 Ly_{21} &= \{x_2/1, y_2/1\}, Ly_{22} = \{x_2/1, y_2/1\} \\
 La &= Lx_1 \cdot Ly_{21} + \{a/1\} = \{a/1\} \\
 Lb &= Ly_{22} + Lx_3 + \{b/0\} = \{b/0, x_2/1, y_2/1, x_3/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/0, c/0\} \\
 Lz &= \bar{La} \cdot Lb \cdot Lc + \{z/0\} \\
 &= \{z/0, b/0, x_2/1, y_2/1, x_3/1\}_a
 \end{aligned}$$



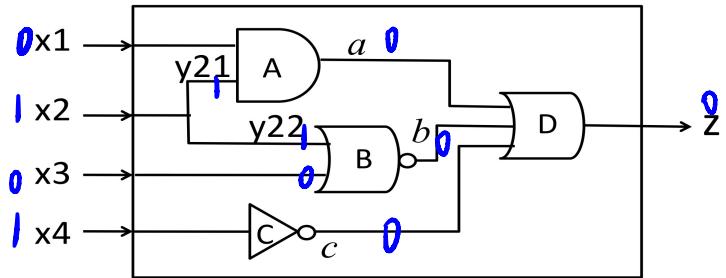
$$\begin{aligned}
 Lx_1 &= \{x_1/1\} & Lx_2 &= \{x_2/1\} \\
 Lx_3 &= \{x_3/0\} & Lx_4 &= \{x_4/1\} \\
 Ly_{21} &= \{x_2/1, y_2/1\}, Ly_{22} = \{x_2/1, y_2/1\} \\
 La &= Lx_1 \cdot Ly_{21} + \{a/1\} = \{a/1\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{b/1, x_2/1, y_2/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/1, c/0\} \\
 Lz &= \bar{La} \cdot \bar{Lb} \cdot Lc + \{z/0\} \\
 &= \{z/0, c/0, x_4/1\}_a
 \end{aligned}$$



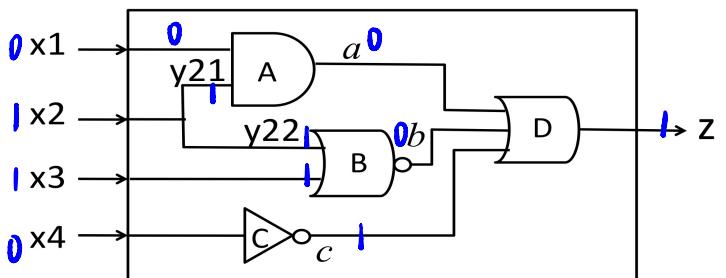
$$\begin{aligned}
 Lx_1 &= \{x_1/1\} & Lx_2 &= \{x_2/1\} \\
 Lx_3 &= \{x_3/0\} & Lx_4 &= \{x_4/0\} \\
 Ly_{21} &= \{x_2/1, y_2/1\}, Ly_{22} = \{x_2/1, y_2/1\} \\
 La &= Lx_1 \cdot \bar{y}_{21} + \{a/1\} = \{a/1\} \\
 Lb &= Lx_3 \cdot \bar{y}_{22} + \{b/1\} = \{x_3/0, b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_4/0, c/1\} \\
 Lz &= La + Lb + Lc + \{z/1\} \\
 &= \{a/1, x_3/0, b/1, x_4/0, c/1, z/1\}
 \end{aligned}$$



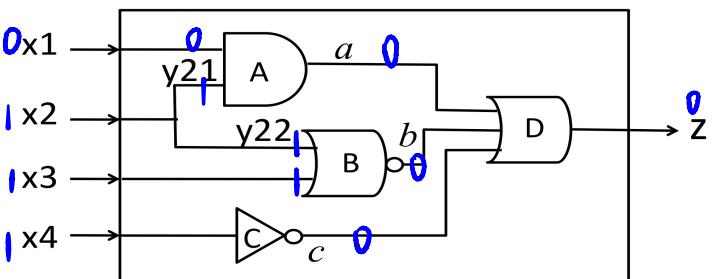
$$\begin{aligned}
 Lx_1 &= \{x_1/1\} & Lx_2 &= \{x_2/0\} \\
 Lx_3 &= \{x_3/1\} & Lx_4 &= \{x_4/1\} \\
 Ly_{21} &= \{x_2/0, y_2/1\}, Ly_{22} = \{x_2/0, y_2/1\} \\
 La &= Lx_1 \cdot \bar{y}_{21} + \{a/1\} = \{x_1/1, a/1\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{x_2/0, y_2/1, b/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/1, c/0\} \\
 Lz &= \bar{La} \cdot \bar{Lb} \cdot Lc + \{z/0\} = \{x_4/1, c/0, z/0\}
 \end{aligned}$$



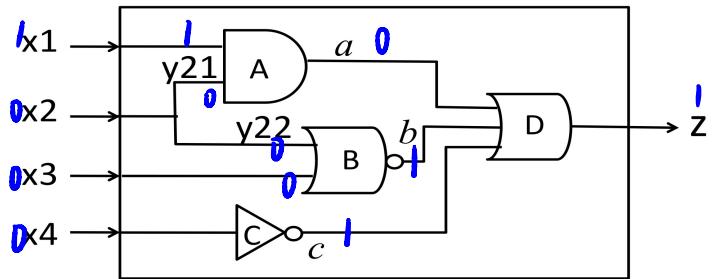
$$\begin{aligned}
 Lx_1 &= \{x_{1/1}\} \quad Lx_2 = \{x_{2/0}\} \\
 Lx_3 &= \{x_{3/1}\} \quad Lx_4 = \{x_{4/0}\} \\
 Ly_{21} &= \{x_{2/0}, y_{21/0}\}, Ly_{22} = \{x_{2/0}, y_{22/0}\} \\
 La &= Lx_1 \cdot \overline{Ly}_{21} + \{a/1\} = \{x_{1/1}, a/1\} \\
 Lb &= Ly_{22} \cdot \overline{Lx}_3 + \{b/1\} = \{x_{2/0}, y_{22/0}, b/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_{4/0}, c/0\} \\
 Lz &= La + Lb + Lc + \{z/1\} \\
 &= \{x_{1/1}, a/1, x_{2/0}, y_{22/0}, b/1, x_{4/0}, c/0, z/1\}
 \end{aligned}$$



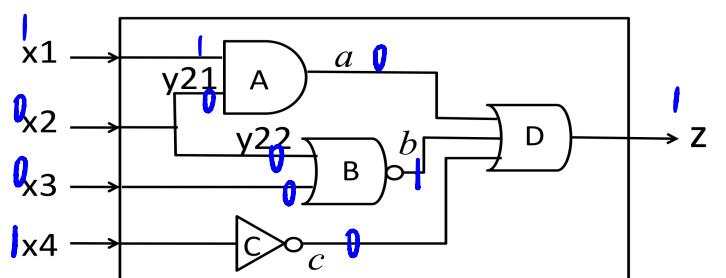
$$\begin{aligned}
 Lx_1 &= \{x_{1/1}\} \quad Lx_2 = \{x_{2/0}\} \quad Lx_3 = \{x_{3/0}\} \quad Lx_4 = \{x_{4/1}\} \\
 Ly_{21} &= \{x_{2/0}, y_{21/0}\}, Ly_{22} = \{x_{2/0}, y_{22/0}\} \\
 La &= Lx_1 \cdot \overline{Ly}_{21} + \{a/1\} = \{x_{1/1}, a/1\} \\
 Lb &= Ly_{22} \cdot \overline{Lx}_3 + \{b/1\} = \{b/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_{4/1}, c/0\} \\
 Lz &= \overline{La} \cdot \overline{Lb} \cdot Lc + \{z/1\} = \{x_{4/1}, c/0, z/1\}
 \end{aligned}$$



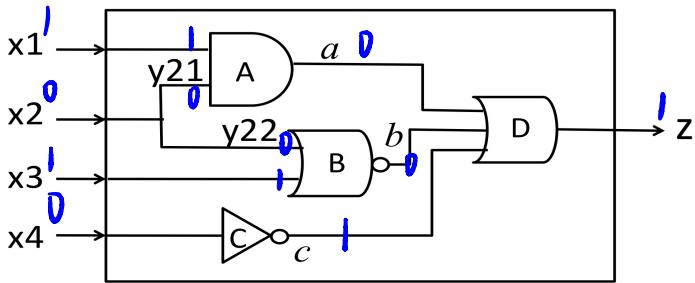
$$\begin{aligned}
 Lx_1 &= \{x_{1/1}\} \quad Lx_2 = \{x_{2/0}\} \quad Lx_3 = \{x_{3/0}\} \quad Lx_4 = \{x_{4/0}\} \\
 Ly_{21} &= \{x_{2/0}, y_{21/0}\}, Ly_{22} = \{x_{2/0}, y_{22/0}\} \\
 La &= Lx_1 \cdot \overline{Ly}_{21} + \{a/1\} = \{x_{1/1}, a/1\} \\
 Lb &= Ly_{22} \cdot \overline{Lx}_3 + \{b/1\} = \{b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_{4/0}, c/1\} \\
 Lz &= La + Lb + Lc + \{z/1\} \\
 &= \{x_{1/1}, a/1, b/1, x_{4/0}, c/1, z/1\}
 \end{aligned}$$



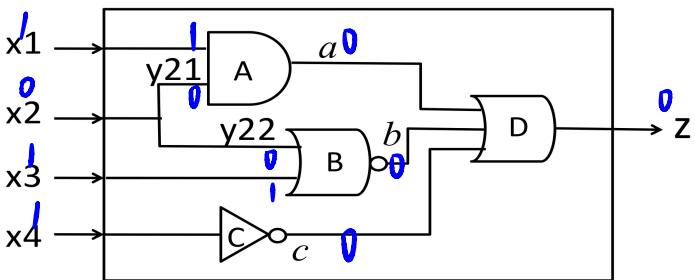
$$\begin{aligned}
 Lx_1 &= \{x_{1/0}\} \quad Lx_2 = \{x_{2/1}\} \quad Lx_3 = \{x_{3/1}\} \quad Lx_4 = \{x_{4/1}\} \\
 Ly_{21} &= \{x_{2/1}, y_{21/1}\}, Ly_{22} = \{x_{2/1}, y_{22/1}\} \\
 La &= \overline{Lx}_1 \cdot Ly_{21} + \{a/1\} = \{x_{2/1}, y_{21/1}, a/1\} \\
 Lb &= Lx_3 + Ly_{22} + \{b/0\} = \{x_{2/1}, x_{3/1}, y_{22/1}, b/0\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_{4/1}, c/0\} \\
 Lz &= \overline{La} \cdot \overline{Lb} \cdot Lc + \{z/0\} = \{z/0\}
 \end{aligned}$$



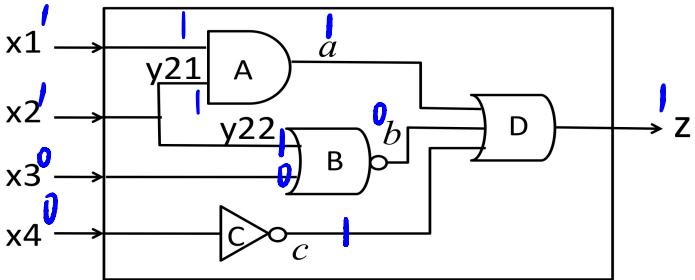
$$\begin{aligned}
 Lx_1 &= \{x_{1/0}\} \quad Lx_2 = \{x_{2/1}\} \quad Lx_3 = \{x_{3/1}\} \quad Lx_4 = \{x_{4/0}\} \\
 Ly_{21} &= \{x_{2/1}, y_{21/1}\}, Ly_{22} = \{x_{2/1}, y_{22/1}\} \\
 La &= \overline{Lx}_1 \cdot Ly_{21} + \{a/1\} = \{x_{2/1}, y_{21/1}, a/1\} \\
 Lb &= Lx_3 + Ly_{22} + \{b/0\} = \{x_{2/1}, x_{3/1}, y_{22/1}, b/0\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_{4/0}, c/1\} \\
 Lz &= \overline{La} \cdot \overline{Lb} \cdot Lc + \{z/0\} = \{x_{3/1}, y_{22/1}, b/0, z/0\}
 \end{aligned}$$



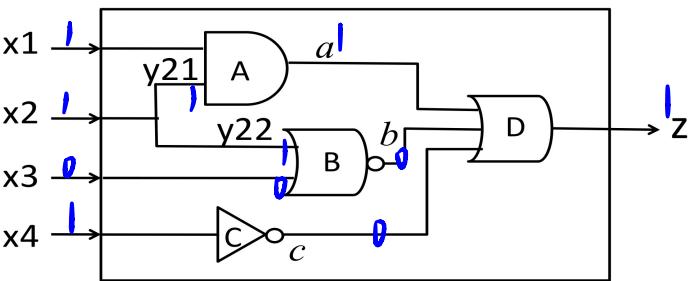
$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/1\} \quad Lx_3 = \{x_3/0\} \quad Lx_4 = \{x_4/1\} \\
 Ly_{21} &= \{x_2/1, y_2/1\} \quad Ly_{22} = \{x_2/1, y_2/0\} \\
 La &= \bar{Lx}_1 \cdot Ly_{21} + \{a/1\} = \{x_2/1, y_2/1, a/1\} \\
 Lb &= \bar{Ly}_{22} \cdot Lx_3 + \{b/1\} = \{x_3/0, b/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/1, c/0\} \\
 Lz &= \bar{La} \cdot \bar{Lb} \cdot Lc + \{z/0\} = \{x_4/1, y_2/0, z/0\}
 \end{aligned}$$



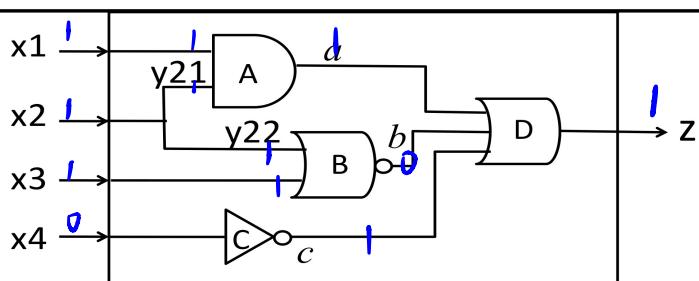
$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/1\} \quad Lx_3 = \{x_3/0\} \quad Lx_4 = \{x_4/0\} \\
 Ly_{21} &= \{x_2/1, y_2/1\} \quad Ly_{22} = \{x_2/1, y_2/0\} \\
 La &= \bar{Lx}_1 \cdot Ly_{21} + \{a/1\} = \{x_2/1, y_2/1, a/1\} \\
 Lb &= \bar{Ly}_{22} \cdot Lx_3 + \{b/1\} = \{x_3/0, b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_4/0, c/1\} \\
 Lz &= La + Lb + Lc + \{z/1\} \\
 &= \{x_2/1, y_2/1, a/1, x_3/0, b/1, x_4/0, c/1, z/1\}
 \end{aligned}$$



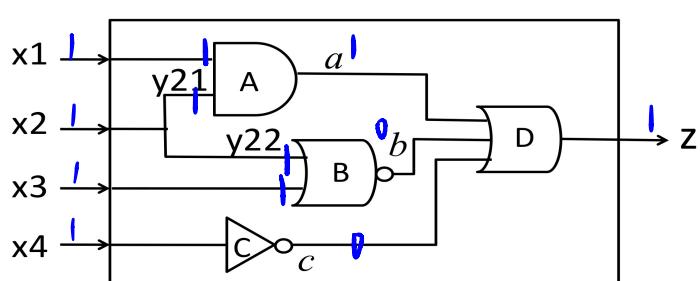
$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/0\} \quad Lx_3 = \{x_3/1\} \quad Lx_4 = \{x_4/1\} \\
 Ly_{21} &= \{x_2/0, y_2/0\}, Ly_{22} = \{x_2/0, y_2/0\} \\
 La &= Lx_1 + Ly_{21} + \{a/0\} = \{x_1/0, x_2/0, y_2/0, a/0\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{x_2/0, y_2/0, b/1\} \\
 Lc &= Lx_4 + \{c/0\} = \{x_4/1, c/0\} \\
 Lz &= La \cdot \bar{Lb} \cdot Lc + \{z/0\} \\
 &= \{z/0\}
 \end{aligned}$$



$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/0\} \quad Lx_3 = \{x_3/1\} \quad Lx_4 = \{x_4/0\} \\
 Ly_{21} &= \{x_2/0, y_2/0\}, Ly_{22} = \{x_2/0, y_2/0\} \\
 La &= Lx_1 + Ly_{21} + \{a/0\} = \{x_1/0, x_2/0, y_2/0, a/0\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{x_2/0, y_2/0, b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_4/0, c/1\} \\
 Lz &= La \cdot \bar{Lb} \cdot Lc + \{z/0\} \\
 &= \{x_1/0, x_2/0, y_2/0, c/0, z/0\}
 \end{aligned}$$



$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/0\} \quad Lx_3 = \{x_3/0\} \quad Lx_4 = \{x_4/0\} \\
 Ly_{21} &= \{x_2/0, y_2/0\}, Ly_{22} = \{x_2/0, y_2/0\} \\
 La &= Lx_1 + Ly_{21} + \{a/0\} = \{x_1/0, x_2/0, y_2/0, a/0\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_4/1, c/1\} \\
 Lz &= La \cdot \bar{Lb} \cdot Lc + \{z/0\} = \{z/0\}
 \end{aligned}$$



$$\begin{aligned}
 Lx_1 &= \{x_1/0\} \quad Lx_2 = \{x_2/0\} \quad Lx_3 = \{x_3/0\} \quad Lx_4 = \{x_4/0\} \\
 Ly_{21} &= \{x_2/0, y_2/0\}, Ly_{22} = \{x_2/0, y_2/0\} \\
 La &= Lx_1 + Ly_{21} + \{a/0\} = \{x_1/0, x_2/0, y_2/0, a/0\} \\
 Lb &= Ly_{22} \cdot \bar{Lx}_3 + \{b/1\} = \{b/1\} \\
 Lc &= Lx_4 + \{c/1\} = \{x_4/0, c/1\} \\
 Lz &= La \cdot \bar{Lb} \cdot Lc + \{z/0\} = \{x_1/0, x_2/0, y_2/0, a/0, z/0\}
 \end{aligned}$$