

# Introduction to Smith-Waterman Algorithm

## 1.Input

### a.DNA sequences

Two DNA sequences in text consist of "A","C","G","T", for example:

ATCATGAGCTA  
ATGGGCCT

### b.Similarity scoring matrix

A 4 x 4 matrix, and each element denotes the similarity score of a pair, for example:

S	A	C	T	G
A	1	-1	-1	-1
C	-1	1	-1	-1
T	-1	-1	1	-1
G	-1	-1	-1	1

### c.Gap Penalty Function

A function compute the penalty for a gap of width k. Normally we, set it as a linear function, for example:

$$P(k) = ak + b, \text{ where } a \geq 0, a+b \geq 0, k \geq 1$$

## 2.Output

The output consists of two aligned sequences, which means the similar region of two original sequences are matched with a '|'. Also, the gaps (skipped in alignment) are denoted by "\_".

In this software, we also output a brief report about the alignment.

## 3.Algorithm

a. Let m and n denote the length of two input sequences. Let H be a (m+1) x (n+1) matrix.

b. Set the first row and first column of H to 0, i.e. if  $i=0$  or  $j=0$ ,  $H_{i,j}=0$ .

c. Iteratively compute the element of H, by:

$$H_{i,j} = \max\{ \\ S(\text{SequenceA}_{i-1}, \text{SequenceB}_{j-1}) + H_{i-1,j-1}, \\ \max\{ H_{i,j-k} - \text{penalty}(k) \mid k \geq 1 \}, \\ \max\{ H_{i-k,j} - \text{penalty}(k) \mid k \geq 1 \}, \\ \}, \quad i \geq 1 \text{ and } j \geq 1$$

d. Traceback from the maximum of H.

$$[x, y] = \arg \max_{i,j} H_{i,j}$$

For  $H_{i,j}$ , we call  $H_{i-1,j}$ ,  $H_{i,j-1}$ ,  $H_{i-1,j-1}$  the neighbor of  $H_{i,j}$

Traceback step by step, until meet a 0 in the neighbor.

```
While( none of  $H_{x,y}$ 's neighbor is 0 )
{
     $[x, y] = \arg \max_{i,j} H_{i,j}$ ,  $H_{i,j}$  is  $H_{x,y}$ 's neighbor.
    Record this step.
}
```

e. Determine the aligned sequences.

Determine the aligned sequences according to the traceback record, where diagonal steps represent matches and vertical or horizontal steps represent gaps.

#### 4. Improvement on the Algorithm

It's obvious that parallel computation of H will be far more time efficient.

Some heuristics can also improve the algorithm, like dynamically abandoning computing brinks and corners of H.