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Trajectory Prediction for Vectored Area Navigation Arrivals

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To maximize the capacity of airports by optimally allocating available resources such as runways, the arrival times of individual aircraft need to be computed. However, accurately predicting arrival times is difficult because aircraft trajectories are frequently vectored off the standard approach procedures. This paper introduces a new framework for predicting aircraft arrival times by incorporating probabilistic information for the types of trajectory patterns that will be applied by human air traffic controllers. The major patterns of the trajectories are identified, and the probabilities of those patterns are computed based on the patterns of the preceding aircraft. The proposed method is applied to traffic scenarios in real operations to demonstrate its performance.

Nomenclature

$\boldsymbol{B}$	=	coefficient vector of the regression model $T(\boldsymbol{Z})$
$\boldsymbol{B}^i$	=	coefficient vector of regression model $T^i(\boldsymbol{Z})$
$\underline{\mathrm{ETA}}(\boldsymbol{Z})$	=	estimated time of arrival by a single linear regression model given input vector $\boldsymbol{Z}$
$\overline{\mathrm{ETA}}(\boldsymbol{Z})$	=	estimated time of arrival by the proposed method given input vector $\boldsymbol{Z}$
$\hat{f}(w_k\|\theta^i)$	=	kernel density of observed trajectory data w_k for ith trajectory pattern
H	=	bandwidth matrix
$K(\cdot)$	=	kernel function
$L(\theta^i\|w^k)$	=	likelihood of ith trajectory pattern of aircraft k given observed trajectory w^k
M_i	=	number of trajectories that participated in ith trajectory pattern
N	=	number of trajectory patterns
n	=	number of aircraft inside the terminal airspace
$p(\theta_i)$	=	probability of ith trajectory pattern
$T(\boldsymbol{Z})$	=	regression model of travel time given input vector $\boldsymbol{Z}$
T_e	=	entry time to terminal airspace
$T^i(\boldsymbol{Z})$	=	regression model of travel time for ith trajectory pattern
$W(\theta_i)$	=	set of historical trajectory data belonging to ith trajectory pattern
w^k	=	observed trajectory data of aircraft k
Θ	=	set of trajectory patterns
θ_i	=	ith trajectory pattern

I. Introduction

WITH increasing air traffic demand, the airspace surrounding major airports has already become saturated. Air traffic congestion around airports results in many problems, including delays in the flight schedule and a high workload level for the air traffic controllers. Several research efforts have been made to relieve traffic congestion by regulating arrival traffic, primarily through speed adjustments from cruise to the runway [1–5]. In those efforts, the aircraft arrival times need to be predicted to compute the necessary delay or speedup opportunities. Other research focuses on optimally scheduling available resources in the airport such as runways, taxiways, gates, and ground crews to maximize airport capacity [6–10]. In these studies, aircraft arrival times are considered as input variables to the optimization process. Therefore, improving the prediction accuracy for arrivals becomes critical. For example, in advanced surface management systems, aircraft surface operations are precisely planned and controlled and, to that end, the accurate prediction of aircraft landing times stands as a fundamental research problem [9,10].

One common approach for trajectory prediction is to explicitly model aircraft movement and compute the aircraft's future trajectory while still accounting for various procedural constraints in the airspace [11–14]. Different aircraft behavior models, including the Base of Aircraft Data, have been developed and used for trajectory prediction [15–17]. Various efforts to improve the prediction accuracy by incorporating additional information have also been made; several studies [18,19] have proposed prediction models with aircraft intent information, whereas another attempt [20] was made to reduce prediction error by adaptively estimating the aircraft's current weight.

Taking a different approach, machine learning techniques have been studied for trajectory prediction [21–24]. In these studies, a wide range of regression models was applied to deal with their specific prediction situations. Several studies focused on the trajectory prediction of climbing aircraft, where various nonlinear regression models, such as artificial neural networks, were intensively investigated [21,22]. Another study adopted generalized linear models (GLMs) to predict arrival times at fixes along the route, and the benefits to continuous descent operations were identified [23]. Another study has applied the functional regression for the midterm trajectory prediction by considering the trajectory data as functional [24].

Although the performance of the aforementioned trajectory prediction methods has been extensively verified, the usability is limited to aircraft under a minimum level of intervention by air traffic controllers. The trajectories of such aircraft do not significantly deviate from the fixed air route structure as sequences of predefined waypoints. Conversely, aircraft trajectories in the terminal airspace are random and difficult to define based

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on the air route structure. Although a fixed air route structure in terminal airspaces [e.g., standard terminal arrival route (STAR)] still exists, the aircraft paths are frequently modified by air traffic controllers to maintain proper separation from other aircraft or to shorten the flight distance to minimize fuel consumption. Such modifications of the aircraft trajectories by air traffic controllers are termed vectoring, which is one of the primary reasons for the difficulty in making trajectory prediction [25–27].

Several recent research efforts have attempted to improve the accuracy of trajectory prediction in vectored area navigation (RNAV) airspace. One study focused on aircraft trajectory in the final approach phase and attempted to predict the length of the upwind leg in the trombone flight procedure [28]. In this study, the leg length of the aircraft path was considered a random variable. The Bayes rule was then used to predict the probability distribution over the possible leg lengths given the separation distance to the preceding aircraft. Taking another approach, the possibility of predicting the trajectory pattern of an aircraft based on traffic complexity information was explored [29,30]. In these studies, the statistical relationship between the various traffic complexity factors, and trajectory patterns were investigated. Still another study investigated the trajectories of departure aircraft in an effort to reveal the air traffic controller motivation in shortcut instructions [27]. In a similar effort, a multiple-linear regression model with traffic density as one of the input variables was suggested to predict aircraft taxi time [31].

Building upon the previous research, this paper proposes a new method to predict the aircraft time of arrival to the destination airport by identifying the major trajectory patterns in a vectored RNAV terminal airspace. The probabilities of a target aircraft's possible trajectory patterns are computed based on information from the patterns of its preceding aircraft. The proposed method is applicable to vectored airspace in a congested traffic situation, where aircraft trajectory patterns are highly correlated with the patterns applied to their preceding aircraft. With the probabilistic information on possible trajectory patterns, the arrival time of the target aircraft is computed as the weighted sum of elemental arrival times from the regression models developed for each pattern of the vectored trajectories.

The remainder of the paper is organized as follows: Sec. II briefly explains the machine learning approach for trajectory prediction; the airspace of interest and traffic data used in this paper are described; and the limitation in the simple application of machine learning techniques to predict vectored trajectories is illustrated. In Sec. III, we explain the proposed approach for arrival time prediction. Among the different parts of the approach, particular attention is given to the method to obtain the probabilities of the trajectory patterns for the target aircraft. In Sec. IV, the proposed method is applied to arrival traffic into the Incheon International Airport (ICN) to demonstrate its performance. Finally, we discuss the results and describe future work.

II. Machine Learning Approach for Trajectory Prediction

Among the different approaches to trajectory prediction, the proposed method can be considered one that is based on machine learning techniques. Therefore, to highlight the technical contribution of the proposed method, the conventional approach using machine learning techniques in trajectory prediction is briefly explained in this section. Problem statements are also provided with an illustration of the airspace concerned and trajectory data used in this paper.

A. Airspace and Flight Procedure

Figure 1 shows the Seoul terminal airspace (TMA) and the RNAV STARs to the ICN under the south wind airport configuration. The Seoul TMA, which borders area control center (ACC) sectors and other TMAs, is responsible for the control of arrival and departure traffic to and from the ICN. The western and eastern boundaries of this sector are approximately 60 and 70 miles away from the ICN, respectively, whereas the northern and southern boundaries are situated about 25 and 55 miles from the ICN, respectively. There are four entry fixes at the boundary for the arrival traffic. When an arrival aircraft approaches the entry fix, the aircraft is handed off from the ACC controllers to a Seoul TMA controller, and it begins following the corresponding STAR. The STARs are defined laterally with a series of waypoints, which may have assigned speeds or altitude restrictions. Without a controller's instruction, the RNAV-capable aircraft can follow the STAR quite precisely by the aid of the onboard flight management system, which increases the predictability of the aircraft's arrival time.

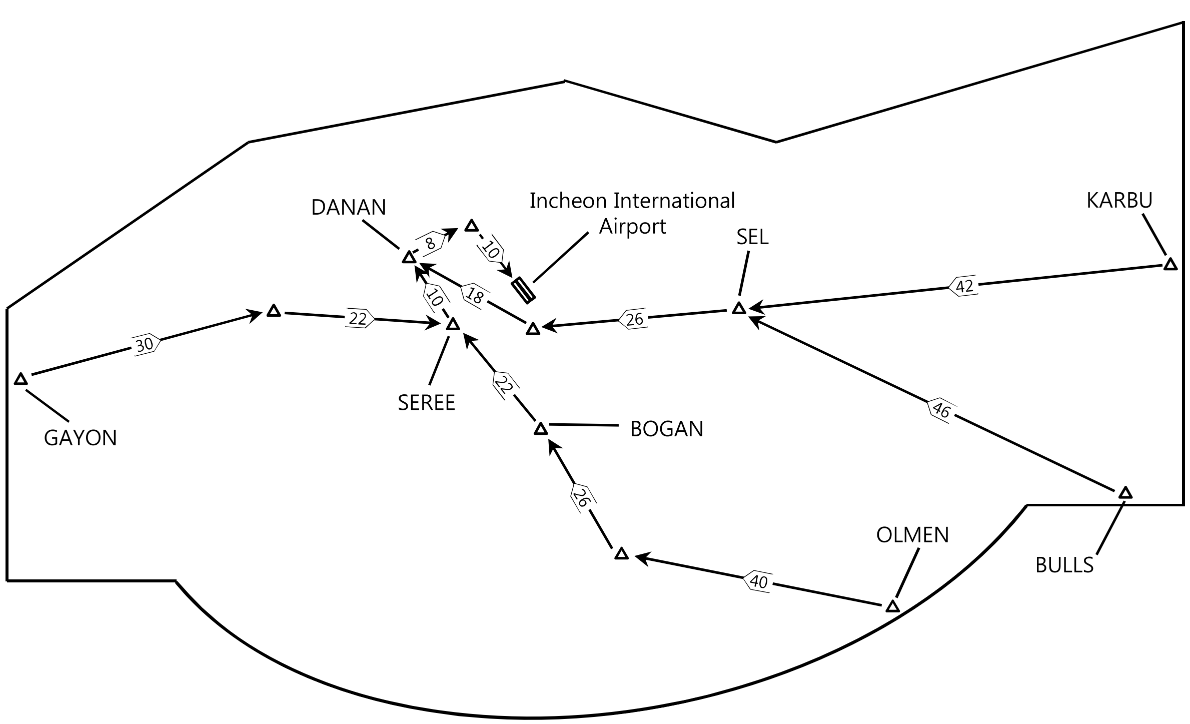


Fig. 1 RNAV STARs to the Incheon International Airport under the south wind airport configuration.

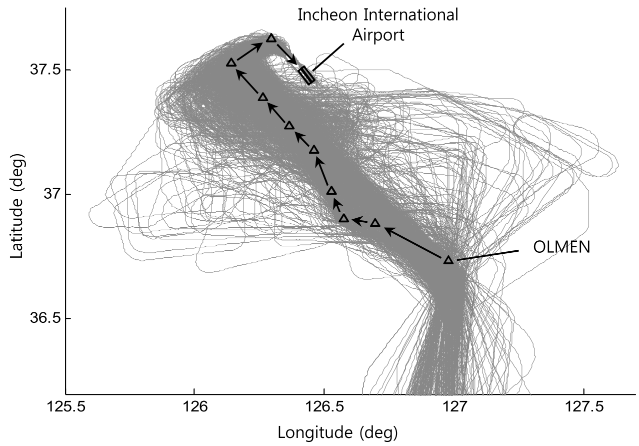


Fig. 2 Historical trajectories from the OLMEN entry fix to the Incheon International Airport.

Even in typical operations, an aircraft following a STAR is often vectored by the controller to maintain the proper separations from other aircraft or to shorten the flight distance to minimize fuel consumption. For example, the arrival route from the OLMEN entry fix has two merge points (SEREE and DANAN) along the path, as depicted in Fig. 1. Because there is little cooperation between the sectors feeding into the Seoul TMA, the spacing between the aircraft may not be suitable at the merging fixes, and intervention by air traffic controllers is unavoidable. This situation frequently occurs when arrival traffic demand is high. However, when there is a small number of aircraft in the terminal airspace, the aircraft are frequently instructed to fly directly to BOGAN to shorten the flight time.

B. Problem Statements

The time of arrival of an aircraft to the ICN is to be predicted. The estimated time of arrival (ETA) of an aircraft is to be computed when the aircraft is at the entry fix of the terminal airspace. Although, in real operations, the ETA continues to be updated as the aircraft moves through the terminal airspace, a specific situation of trajectory prediction must be considered in this paper to better illustrate the proposed method. Note that the prediction of the ETA at the entry fix is difficult because the controllers' decisions about how they will control the aircraft inside the terminal airspace have not yet been realized. Among four major inbound traffic flows, we focused on the aircraft arriving from the OLMEN entry fix. The arrival time to the airport through this fix is usually very difficult to estimate due to the diversity of the vectoring patterns, as shown in Fig. 2.

C. Aircraft Trajectory Data

The radar surveillance data for the arrival trajectories to the ICN through the OLMEN fix under the south wind airport configuration were collected between March and May 2012. The radar surveillance data provides aircraft state information (i.e., latitude, longitude, altitude and speed), call sign, aircraft type, actual landed runway, and other data. To eliminate the variability in arrival times due to differences in the aircraft types, only aircraft with the same wake vortex category are considered in this paper; based on the number of trajectories available, the aircraft in the "heavy" category, as defined by the International Civil Aviation Organization, were selected. A total of 1234 trajectories were used to demonstrate the performance of the proposed method.

D. Machine Learning Approach for Trajectory Prediction

Figure 3 shows a schematic view of the machine learning approach for trajectory prediction [23]. The historical trajectory and meteorological data are collected and used to train a prediction model for which the output are the times of arrival of an aircraft at significant points in the airspace. Different learning techniques, such as GLMs or artificial neural networks, can be applied for the prediction model. The input variables of the model need to be carefully determined based on their explanatory powers relative to the output variable.

To illustrate the challenges of using machine learning techniques for trajectory prediction in vectored RNAV airspace, a multiple-linear regression model was developed and applied to the arrival traffic in Fig. 2. The time of arrival to the airport is predicted, and the regression model for the aircraft travel time from the entry fix to the airport is constructed as follows:

$$T(\mathbf{Z}) = \beta_0 + \beta_1 z_1 + \cdots + \beta_m z_m \tag{1}$$

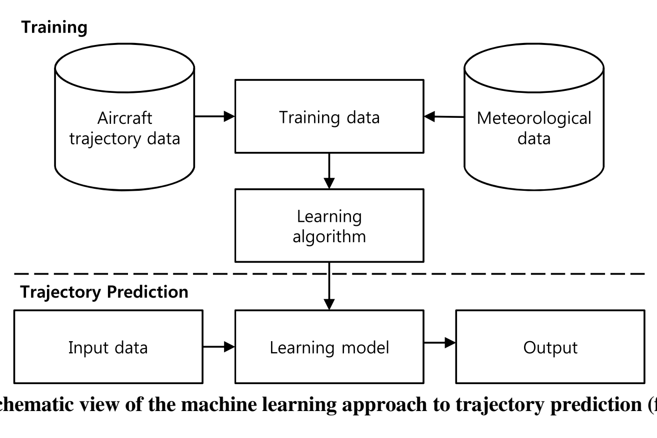


Fig. 3 Schematic view of the machine learning approach to trajectory prediction (from [23]).

where $T(\boldsymbol{Z})$ is the travel time given input variable $\boldsymbol{Z} = [z_1 \quad z_2 \quad \cdots \quad z_m]$, and its coefficient vector $\boldsymbol{B} = [\beta_0 \quad \beta_1 \quad \cdots \quad \beta_m]$ is determined to minimize the prediction errors against the training data. The groundspeed and altitude at the entry fix are selected as input variables; their statistical significance to the time of arrival was shown in a previous study [23]. Windspeed is another important class of input variables, but it cannot be considered in this paper due to the lack of available data. Note that variability in arrival times due to differences in the aircraft type should be minimal because only aircraft with the same wake vortex category (i.e., heavy) were considered. Based on the regression model of the travel time, the time of arrival to the airport can be computed as follows:

$$\mathrm{ETA}(\boldsymbol{Z}) = T_e + T(\boldsymbol{Z}) \tag{2}$$

where $\mathrm{ETA}(\boldsymbol{Z})$ represents the estimated time of arrival of the target aircraft given input vector $\boldsymbol{Z}$, and T_e is the time when the target aircraft is located at the entry fix.

The regression model in Eq. (1) was trained with the historical trajectories, as explained in Sec. II.C. The trained model is represented by the surface shown in Fig. 4 and provides the travel times of the aircraft for all of the groundspeed and altitude combinations at the entry fix. In this figure, the travel times of the historical trajectories are also shown by the diamond symbols. The significance of the model was tested based on the null hypothesis $\beta_1 = \beta_2 = \cdots = \beta_m = 0$. With a confidence level of 95%, the null hypothesis was rejected; therefore, the model can be considered statistically valid. The correlation coefficient R^2 between the actual and estimated travel times was also computed and was 0.07; i.e., the regression model explains only 7% of the variance in the actual travel times. Compared to the previous study [23], this result emphasizes the level of difficulty in trajectory prediction for vectored RNAV airspace. Note that, unlike the previous studies, the trajectories of the aircraft that were vectored off the arrival route are still included in the training data.

The performance of the prediction model in Eqs. (1) and (2) was evaluated by Monte Carlo cross validation [32]. The validations were performed 10 times repeatedly; and for each validation, a set of 100 test flights from the available data was sampled. The ETAs of the test flights were predicted by the multiple-linear regression model, and the errors between the predicted and actual values of the arrival times were computed. The root mean square (rms) of the errors was 134.3 s, whereas the mean travel time from the entry fix to the airport was 1253.2 s. The prediction errors are significant, because a single regression model cannot successfully consider the diversity in the vectored trajectories and averages out the variations in flight time across the different trajectory patterns. A simple approach to reduce the prediction errors is to include traffic complexity factors in the regression model. The traffic density inside the terminal airspace was included, but the rms error of the estimated times of arrival was only marginally improved to 129.2 s. Indeed, previous research has identified the difficulty of predicting the trajectory pattern of an aircraft based on various traffic complexity factors (e.g., traffic density) in terminal airspace [27,29].

III. Estimated Time of Arrival via Trajectory Pattern Prediction

A. Overview

The proposed method consists of two stages: training and prediction. Unlike the conventional prediction method with machine learning techniques (as described in Fig. 3), the training stage is subdivided into two steps. The first step is to identify the major patterns of the vectored trajectories for a given airspace. The trajectory patterns are determined based on the clustering in historical traffic data. The second step of the training stage is to construct regression models of the travel time for each trajectory pattern. Unlike the previous studies with machine learning techniques, different regression models are developed for each trajectory pattern identified in the first step. Throughout the paper, $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ denotes a set of trajectory patterns identified in the first step, and $T^i(\boldsymbol{Z})$ denotes the regression model of the travel time for trajectory pattern θ_i.

In the prediction stage, the estimated time of arrival for the target aircraft is computed based on its current position and speed information. The important part of the prediction stage is determining the trajectory pattern that the target aircraft will take in the future. A new approach based on information from the patterns of the preceding aircraft is proposed. The probability distribution of the trajectory patterns $P(\theta_i)$ is computed by the Bayes rule, and the elemental arrival times for each possible trajectory pattern θ_i are computed based on the regression models $T^i(\boldsymbol{Z})$. The ETA of the target aircraft is then computed using the weighted sum of elemental arrival times with their associated probabilities. The flow diagram of the proposed method is shown in Fig. 5, and a more detailed explanation will be provided in Secs. III.B–III.E.

B. Trajectory Pattern Identification

The major patterns of the vectored trajectories are found by clustering historical trajectory data for the airspace of interest. The idea of clustering trajectory data can be briefly explained as follows. The "distances" between each pair of trajectories are computed. A trajectory is then clustered into one of several groups based on its relative distances from the others. Intuitively, two trajectories with a smaller distance between them are more

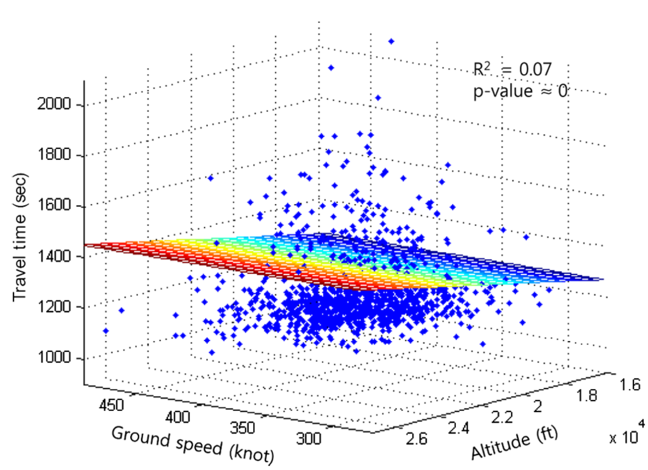


Fig. 4 Travel time from the OLMEN entry fix to the Incheon International Airport using a multiple-linear regression model with historical training trajectory data.

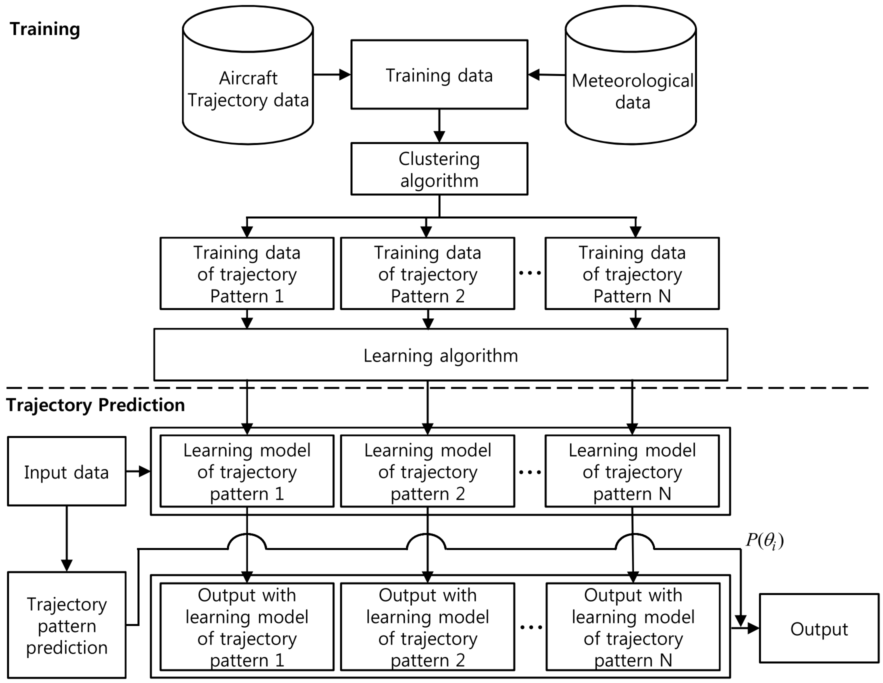


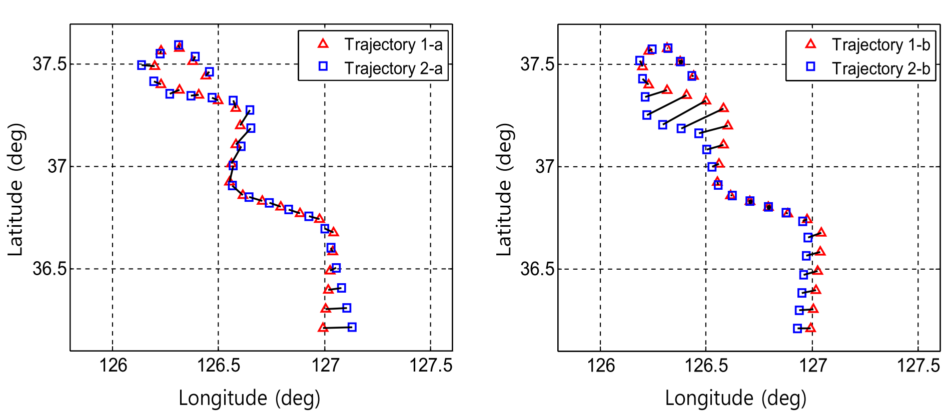
Fig. 5 Flow diagram of the proposed method for trajectory prediction.

likely to be grouped into the same cluster in comparison to those with a larger distance. Various approaches for trajectory clustering have been proposed, depending on how the distances between the two trajectories are defined and how each cluster is created and built up [33–37].

Gariel et al. [33] and Salaün et al. [34] proposed a method of trajectory clustering by using principal component analysis (PCA) for dimension reduction followed by density-based spatial clustering of applications with noise [38]. In their study [33,34], each trajectory was first quantified as a time sequence of the various aircraft state variables, such as the position, heading angle, and distance from a specific point in space. A few of these explanatory variables, each of which is the linear combination of the originally chosen variables, are then selected by using PCA; and based on those explanatory variables, the Euclidian distance between the two trajectories is computed. A set of clusters is then created by collecting any trajectories within a certain distance. Taking another approach, a trajectory clustering method based on the Fourier and wavelet coefficients of each trajectory has been proposed [35,36]. Another study applied a hierarchical algorithm for trajectory clustering, in which a trajectory was quantified as a time sequence of aircraft position vectors [37].

Since the primary purpose of trajectory clustering in this paper is to identify the vectoring patterns of air traffic controllers, the spatial shape of the trajectory is more important than the speed or acceleration of the aircraft along the trajectory. Note that, in the proposed method, the speed variation of the aircraft movements along the trajectory can be considered by the regression models of travel time, which will be discussed later in the paper. The previously suggested clustering approaches are inappropriate for the proposed method because, in those approaches, a small misalignment in time would result in a large distance between the trajectories. As an example, two sets of trajectory pairs are shown in Fig. 6. In this figure, each trajectory is represented by a time sequence of aircraft positions, and the lengths of all sequences are identical. Although the trajectories in Fig. 6a are more similar in shape to each other than the trajectories in Fig. 6b, the simple application of the Euclidian distance between the sequences results in less similarity (i.e., a larger distance) between the trajectories in Fig. 6a than between those in Fig. 6b. In fact, a small misalignment in the early parts of the two trajectories in Fig. 6a introduces the continuing discrepancy in subsequent points of the sequences.

To address this problem, the dynamic time warping (DTW) algorithm [39] is applied. With the DTW algorithm, the optimal alignment of two trajectories as sequences can be found, and the distance between those trajectories is computed based on that alignment. Once the distances between all pairs of trajectories are computed, the hierarchical clustering method is applied based on those distances. The hierarchical clustering method was chosen due to the advantage that any valid criterion for clustering could be used. The Ward minimum variance criterion [40] is applied in this paper after comparing the results with various criteria.



a) Trajectory pair 1 **b) Trajectory pair 2**

Fig. 6 Example to illustrate the necessity of optimal alignment of the trajectory data.

C. Regression Models of Travel Time

The multiple-linear regression models for travel time are constructed for each trajectory pattern identified in the previous step. The travel time between the entry fix and the destination airport for trajectory pattern θ_i is given as follows:

$$T^i(\boldsymbol{Z}) = \beta_0^i + \beta_1^i z_1 + \cdots + \beta_m^i z_m \tag{3}$$

where $T^i(\boldsymbol{Z})$ is the travel time for trajectory pattern θ_i with input variable $\boldsymbol{Z} = [z_1 \quad z_2 \quad \cdots \quad z_m]$, and the coefficient vector $\boldsymbol{B}^i = [\beta_0^i \quad \beta_1^i \quad \cdots \quad \beta_m^i]$ is determined based on the historical trajectories that belong to pattern θ_i. Only the groundspeed and altitude at the entry fix are used as input variables, but other types of variables such as wind information or aircraft type are not excluded in the proposed method. Among the various factors affecting accuracy in trajectory prediction, this paper focuses on the vectoring by human traffic controllers, and a simple regression model for aircraft travel time is considered. However, further investigation of input variables, such as wind information, in the regression models should be done in future work. Similarly, different kinds of regression models such as artificial neural networks could be applied in the proposed method.

D. Trajectory Pattern Prediction

The trajectory pattern of the target aircraft is predicted when the aircraft is at the entry fix of the terminal airspace. The trajectory pattern of the target aircraft is difficult to predict because the trajectory up to the entry fix contains little information about its trajectory inside the terminal airspace, which will be realized in its future. On the other hand, the trajectory patterns of the aircraft that preceded the target aircraft are easier to predict because the observed parts of their trajectories already reflect the controllers' decisions on how to control those aircraft inside the terminal airspace. In general, the more the aircraft trajectory has progressed towards the destination airport, the easier its pattern can be predicted.

In the proposed method, the trajectory pattern of an aircraft is predicted based on the pattern of its preceding aircraft. Consider a sequence of n aircraft (AC) inside the terminal airspace (e.g., $\mathrm{AC}^1 - \mathrm{AC}^2 - \ldots - \mathrm{AC}^n$), as shown in Fig. 7, where the large circle represents the boundary of the terminal airspace. Every aircraft has entered the terminal airspace individually through the same entry fix and is flying toward the destination airport. In this paper, we assign a number to each aircraft, from the aircraft that entered the airspace first to the aircraft that entered last and is currently at the entry fix. The probabilities of the possible trajectory patterns for each aircraft are determined in the following way:

1) Let $P^0(\theta_i) = 1/N$ for $i = 1, 2, \cdots, N$, where N is the number of possible trajectory patterns.

2) The probability of trajectory pattern θ_i for AC^1 is computed by the Bayes rule as follows:

$$P^1(\theta_i) = \frac{L(\theta_i|w^1)}{\sum_{j=1}^{N} L(\theta_j|w^1)P^0(\theta_j)} P^0(\theta_i) \quad \text{for } i = 1, 2, \cdots, N \tag{4}$$

where $w^1 = \{w_1^1, w_2^1, w_3^1, \cdots\}$ represents the observed trajectory data for AC^1, $w_j^1 = [x_j^1 \quad y_j^1]^T$ is the position of AC^1 in the east and north directions measured at time step j; and $L(\theta_i|w^1)$ is the likelihood of θ_i for AC^1 given observed trajectory w^1. The method for computing the likelihoods of θ_i will be explained in detail later in this section.

3) One of the main ideas of the proposed method is that, under a congested traffic situation, the trajectory pattern of an aircraft in vectored RNAV airspace is highly correlated with the pattern of its preceding aircraft. The probabilities of the trajectory patterns of AC^{k-1} can therefore be used as prior knowledge for the probabilities of the trajectory patterns for AC^k. The probability of trajectory pattern θ_i for AC^k is computed by the Bayes rule as follows:

$$P^k(\theta_i) = \frac{L(\theta_i|w^k)}{\sum_{j=1}^{N} L(\theta_j|w^k)P^{k-1}(\theta_j)} P^{k-1}(\theta_i) \quad \text{for } i = 1, 2, \cdots, N \quad \text{and} \quad k = 2, 3, \cdots, n-1 \tag{5}$$

where $w^k = \{w_1^k, w_2^k, w_3^k, \cdots\}$ represents the observed trajectory data for AC^k; and $L(\theta_i|w^k)$ is the likelihood of θ_i for AC^k given trajectory data w^k. Note that the uniform prior was used for the first aircraft, as in Eq. (4). A transition matrix could be multiplied to $P^{k-1}(\theta_i)$ before it is adopted as

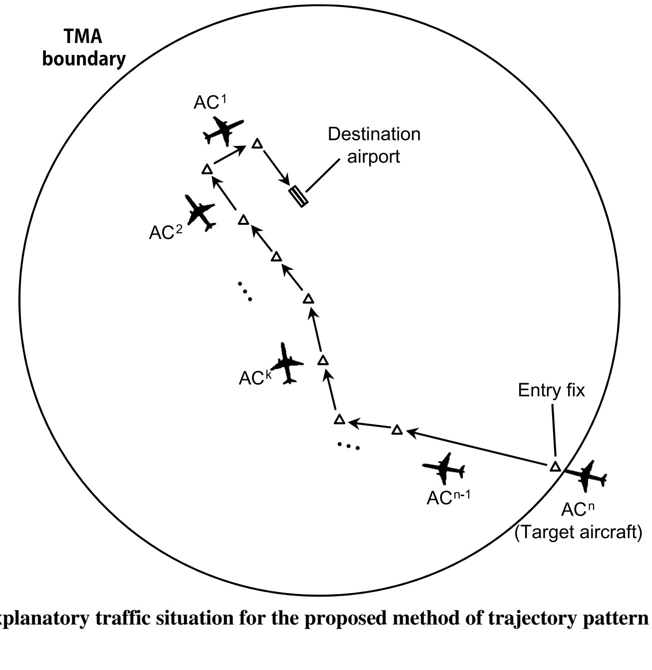


Fig. 7 Explanatory traffic situation for the proposed method of trajectory pattern prediction.

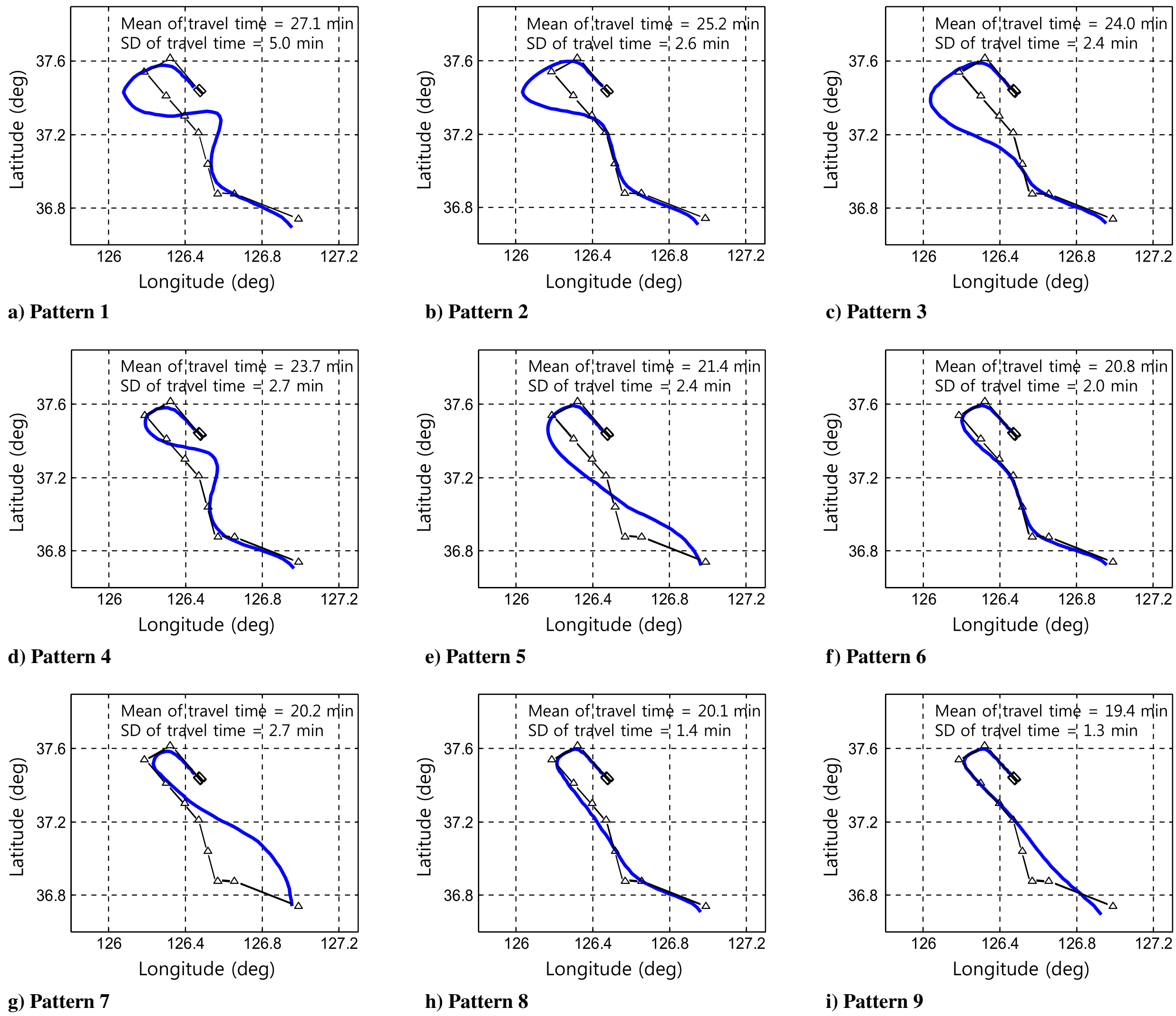


Fig. 8 Trajectory patterns identified for the arrival traffic through the OLMEN entry fix in Fig. 2 (SD denotes standard deviation).

prior probabilities in Eq. (5). However, as will be shown in Sec. IV.A, there exist negative correlations between some trajectory patterns that cannot be easily treated by a transition matrix. Furthermore, an inadequate estimation of the transition matrix would result in significant loss of performance [41]; therefore, this should be more carefully addressed in future research.

4) When AC^n (i.e., the target aircraft) is located at the entry fix, its trajectory inside the terminal airspace is not yet realized. In this case, no trajectory data are available for computing the likelihoods $L(\theta_i|\cdot)$; therefore, the probabilities of the trajectory patterns for AC^n are set to equal its prior probabilities, which are adopted from the probabilities of AC^{n-1}:

$$P^n(\theta_i) = P^{n-1}(\theta_i) \quad \text{for } i = 1, 2, \cdots, N \tag{6}$$

The dependency upon aircraft arrival order might be a possible pitfall of the proposed method. However, the prior probabilities are only adopted from the preceding aircraft, and the likelihood information given by the observed trajectory data is still reflected in the pattern probabilities, as shown in Eq. (5). Therefore, in a real implementation of this method, the predicted pattern can converge into the correct one because the likelihood information becomes dominant as the target aircraft proceeds through the airspace, as will be shown in Sec. IV.B.

To determine the probability of each trajectory pattern θ_i, its likelihood function $L(\theta_i|w)$, given observed trajectory data $w = \{w_1, w_2, w_3, \cdots\}$, needs to be constructed. Among various techniques, the kernel density estimation method is used in this paper [42]. Let $W(\theta_i) = \{W_1(\theta_i), W_2(\theta_i), W_3(\theta_i), \cdots\}$ be the set of historical trajectory data that belong to the trajectory pattern θ_i, where $W_j(\theta_i) = [X_j(\theta_i) \quad Y_j(\theta_i)]$ is the aircraft position in the east and north directions of the jth data in the set. By assuming each observation is independent and identically distributed, the kernel density of the trajectory pattern θ_i for an observation w_k is computed as follows:

$$\hat{f}(w_k|\theta_i) = \frac{1}{M_i|H|}\sum_{j=1}^{M_i} K(H^{-1}(w_k - W_j(\theta_i))) \tag{7}$$

where M_i is the size of $W(\theta_i)$ (i.e., the number of trajectories participating in the ith trajectory pattern); H is the bandwidth matrix that defines the smoothness of the density function $\hat{f}$; and $K(\cdot)$ is the kernel function, which is the multivariate Gaussian in this paper. It is important to choose proper values for the bandwidth matrix and, in this paper, the optimal values that minimize the asymptotic mean integrated squared error were derived by the Scott rule [42]. The likelihood $L(\theta_i|w)$ can then be computed by multiplying the density of each trajectory observation as follows:

$$L(\theta_i|w) = \prod_k \hat{f}(w_k|\theta_i) \tag{8}$$

where w_k is the kth observation in trajectory data w; and $\hat{f}(w_k|\theta_i)$ is the density estimation of the trajectory pattern θ_i for w_k, which can be computed by Eq. (7). Note that, in this paper, the probabilities of the trajectory patterns are predicted based only on the positional data of the aircraft, but other types of trajectory data such as heading angle can also be included.

For a large dataset, the kernel method requires extensive computational time, which could hinder the real-time applicability of the proposed algorithm; given M_i data points for the pattern θ_i, the computational complexity for evaluating the densities of n aircraft is $\mathcal{O}(nM_i)$ [43]. However, the proposed method does not preclude the use of other techniques for the likelihood computation, such as the kernel method with a fast Gauss transform [44].

E. Estimated Time of Arrival

The elemental travel times of the target aircraft (AC^n) are computed by Eq. (3) for each trajectory pattern θ_i. Those elemental travel times are then combined with the probabilities of each of the trajectory patterns, which are computed by Eq. (6). Finally, the arrival time, which minimizes the mean-square prediction error over Θ, can be computed as follows:

$$\overline{\mathrm{ETA}}(\mathbf{Z}) = T_e + \sum_{i=1}^{N} T^i(\mathbf{Z}) \times P^n(\theta_i) \tag{9}$$

where $\overline{\mathrm{ETA}}(\mathbf{Z})$ represents the estimated time of arrival by the proposed method given input variable $\mathbf{Z}$; $P^n(\theta_i)$ is the probability of the ith trajectory pattern, which is given by Eq. (6); and $T^i(\mathbf{Z})$ is the travel time from the entry fix to the airport given by the linear regression models in Eq. (3). Note that, instead of the mean value of the trajectory pattern [as in Eq. (9)], with the exception of the maximum likelihood estimation (MLE), other types of estimators such as the maximum a posteriori probability can be used for the ETA computation; the MLE cannot be used because there are no trajectory data available to compute the likelihoods for the target aircraft when it is located at the entry fix.

IV. Numerical Examples

The performance of the proposed method was demonstrated with real operational data from the arrival traffic to the ICN, as shown in Fig. 2. The ETA of aircraft through the OLMEN entry fix was predicted by the proposed method, and the prediction errors against the true measured values were computed. The historical trajectory data used to identify the trajectory patterns and to train the models are explained in Sec. II.C.

A. Training Stage

1. Trajectory Patterns

By applying the proposed clustering method to the historical trajectory data, nine different trajectory patterns were identified. Trajectory patterns with less than 10 participants were ignored in this paper, because the sample sizes for those patterns were considered statistically insufficient. The centroids of each pattern are shown in Fig. 8, with the mean and standard deviation of the travel time. The patterns are arranged in descending order of the mean travel time.

As shown in Fig. 8f, trajectory pattern 6 corresponds to the aircraft following the STAR through the OLMEN fix, whereas the other trajectory patterns represent aircraft with levels of vectoring that either extend or shorten the flight distance. Trajectory patterns 1–5 correspond to extending the flight paths to maintain a proper separation at the merge points, whereas trajectory patterns 7–9 correspond to taking a shortcut to the airport. It is interesting to note that trajectory pattern 5 should be considered a path extension, even though its shape in Fig. 8e appears to be a pattern with a shortcut instruction; in fact, its mean travel time is larger than that of trajectory pattern 6. The proportion of trajectories for each trajectory pattern is shown in Fig. 9.

A Pearson chi-square test of independence was conducted to investigate the correlation in the trajectory patterns between aircraft [45]. Each aircraft is categorized based on its trajectory pattern, and the aircraft in each category are then subcategorized based on the trajectory patterns of their preceding aircraft. The result is summarized by the contingency table, shown in Table 1, where the correlated proportions of the sample aircraft trajectory patterns are shown with the patterns of the preceding aircraft. For example, as shown in Table 1, 20.3% of the sample trajectories correspond to pattern 9, of which 50.0% have preceding aircraft with the same trajectory pattern. Note that only aircraft with at least one preceding aircraft are considered; therefore, the percentages of the trajectory patterns in Table 1 might be slightly inconsistent with the percentages shown in Fig. 9.

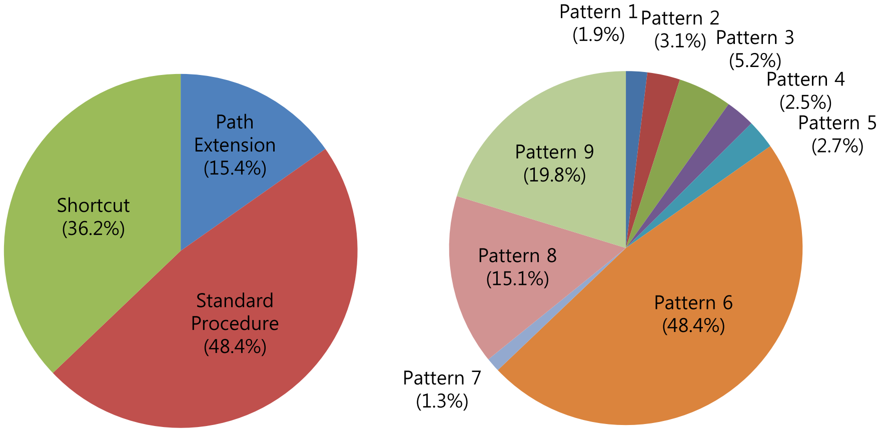


Fig. 9 Trajectories per trajectory pattern.

Table 1 Contingency table to investigate the trajectory pattern correlations

Pattern of sample AC	Pattern of preceding AC 1	2	3	4	5	6	7	8	9	Total, %
1	37.5% (13.7)	0.0% (−0.9)	12.5% (1.7)	12.5% (2.6)	0.0% (−0.7)	33.3% (−1.6)	0.0% (−0.6)	0.0% (−2.1)	4.2% (−1.9)	1.9
2	2.7% (0.5)	40.5% (13.8)	13.5% (2.4)	2.7% (−0.2)	2.7% (0.3)	29.7% (−2.4)	0.0% (−0.8)	0.0% (−2.6)	8.1% (−1.7)	3.0
3	4.9% (2.0)	11.5% (4.1)	31.1% (9.6)	6.6% (1.5)	3.3% (0.7)	27.9% (−3.4)	0.0% (−1.0)	11.5% (−0.9)	3.3% (−3.2)	4.9
4	12.1% (4.7)	0.0% (−1.0)	6.1% (0.3)	18.2% (4.9)	0.0% (−0.8)	48.5% (−0.1)	0.0% (−0.7)	15.2% (0.0)	0.0% (−2.8)	2.7
5	0.0% (−0.8)	3.0% (0.0)	3.0% (−0.5)	0.0% (−1.1)	21.2% (7.9)	27.3% (−2.6)	0.0% (−0.7)	15.2% (0.0)	30.3% (1.7)	2.7
6	0.7% (−2.6)	1.4% (−3.1)	3.6% (−2.2)	3.2% (0.0)	0.3% (−4.0)	69.7% (13.8)	0.5% (−2.7)	11.7% (−3.3)	8.8% (−8.8)	47.6
7	0.0% (−0.5)	6.3% (0.8)	0.0% (−0.9)	0.0% (−0.7)	0.0% (−0.6)	12.5% (−3.0)	37.5% (12.1)	18.8% (0.4)	25.0% (0.6)	1.3
8	0.0% (−2.0)	0.5% (−2.1)	3.6% (−1.0)	2.1% (−1.0)	2.6% (0.6)	33.9% (−4.6)	2.1% (0.8)	34.9% (8.2)	20.3% (0.5)	15.6
9	0.0% (−2.3)	1.2% (−1.8)	1.6% (−2.8)	1.2% (−2.0)	3.2% (1.5)	27.6% (−7.6)	2.0% (0.8)	13.2% (−1.0)	50.0%[a] (13.9)	20.3[a]
Total	1.7%	2.9%	5.0%	3.2%	2.0%	49.2%	1.5%	15.3%	19.1%	100.0

[a]20.3% of the sample trajectories correspond to pattern 9, of which 50.0% have preceding aircraft with the same trajectory pattern.

The dependency of the trajectory patterns was tested under the null hypothesis that the trajectory patterns between two consecutive aircraft are independent. The computed p value for the null hypothesis was almost zero, which was small enough to reject the null hypothesis at a significance level of $\alpha = 0.01$. We can therefore conclude that there is enough statistical evidence regarding the dependency of the trajectory patterns of the aircraft. The standardized Pearson residuals were also computed to determine the positivity or negativity of the dependency. The residual is defined as the difference between the observed value of the cells in the contingency table and the expected value based on the null hypothesis. The residuals are denoted inside the parentheses in each cell of Table 1. A positive value implies that the corresponding trajectory patterns are positively related, whereas negative values imply they are negatively related. The diagonal cells of the contingency table clearly show highly positive residuals, allowing us to conclude that the trajectory pattern of an aircraft is positively correlated with the pattern of its preceding aircraft.

2. *Models of Travel Time*

Multiple-linear regression models for aircraft travel time from the OLMEN fix to the airport were developed for each trajectory pattern in Fig. 8. The regression models trained by the historical data are summarized in Table 2. As shown in the table, the correlation coefficients R^2 were significantly improved overall in comparison to those of the single regression model used in Fig. 4. The correlation coefficients for trajectory patterns 4 and 5 are slightly lower than or equal to those with the single regression model, but they are not statistically significant (p value) due to the small sample sizes. The sample size for trajectory pattern 7 is also small, and the corresponding statistical significance is low. For those trajectory patterns, we can use the sample mean as a predicted travel time for the aircraft. Note that we may improve R^2 values for travel time by increasing the number of trajectory patterns in the hierarchical clustering algorithm, but the overall prediction performance could get worse because the accuracy in trajectory pattern prediction will be lower. Further investigation is needed in future research to determine the tradeoffs between R^2 for travel time and the overall accuracy in ETA prediction.

3. *Likelihood Functions of the Trajectory Patterns*

The probability density functions $f(\cdot\,|\theta_i)$ for each trajectory pattern were constructed by the kernel density estimation. Figure 10 shows the estimated densities for each trajectory pattern as a function of the aircraft position in the longitudinal and latitudinal directions. Based on the density functions in Fig. 10, the likelihoods of the trajectory patterns given the observed trajectory data can be computed by Eq. (8).

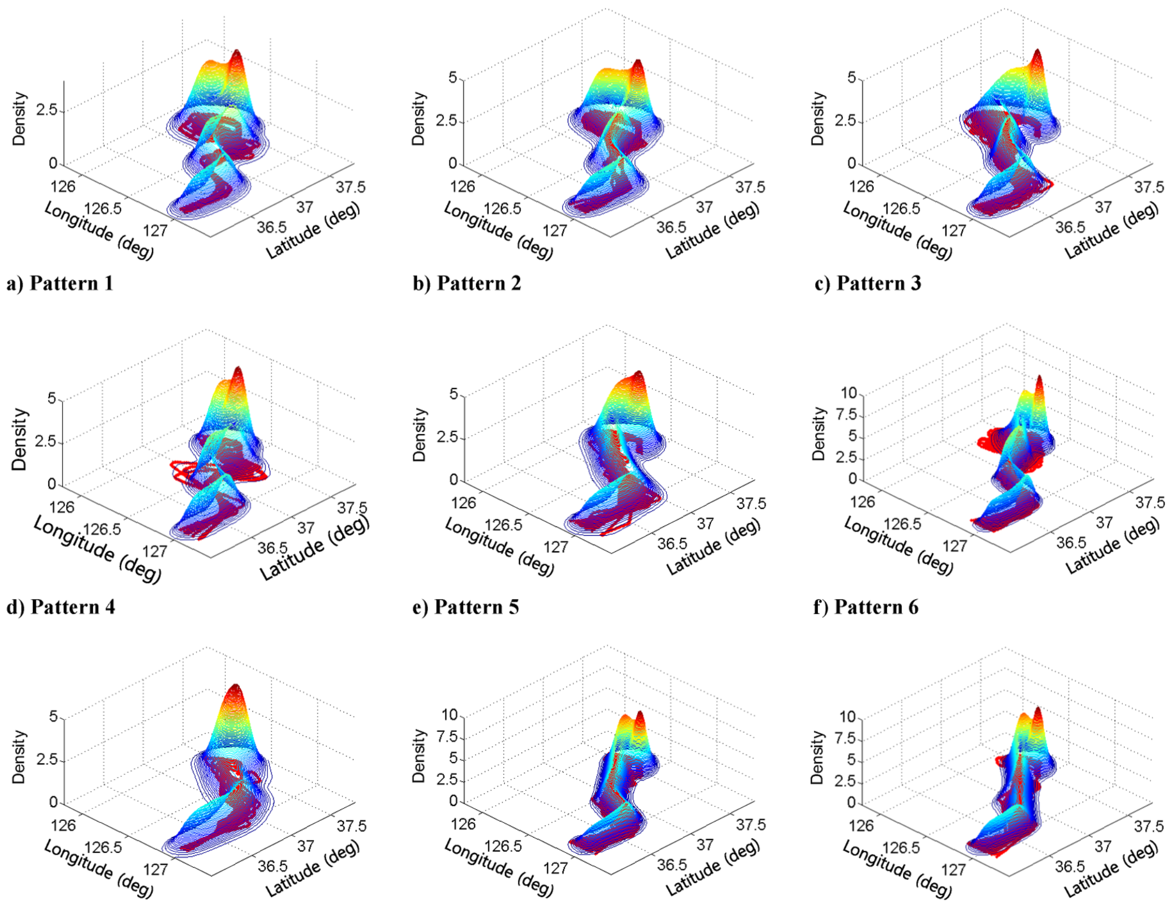
B. Prediction Stage

The performance of the proposed method was analyzed by Monte Carlo cross validation [32]. The validations were performed 10 times and, for each validation, a set of 100 test flights was independently sampled from the available data. The flights that had preceding aircraft in the air at the time of the prediction were conditionally sampled. Recall that the proposed method aims to predict the arrival time in vectored airspace with a sufficient level of traffic demand. Note also that, since the sampled data were excluded in the training stage, the training results shown in Sec. IV.A were only for the first validation set for illustrative purposes. The ETA of each aircraft was predicted, and the prediction errors (which are the differences between the predicted and true measured values of the arrival times) were computed.

A histogram of the prediction errors for the proposed method is shown in Fig. 11a. The prediction errors for the same flights based on the comparative method with a single regression model were also computed, and they are shown in Fig. 11b. The asymmetric features in the histograms are primarily a result of the fact that the flight time lower bound is limited by the straight line between the entry fix and the destination airport, whereas the flight distances in the upper direction are spread more widely. As shown in Fig. 11, the rms error of the proposed method was reduced by 20.5% (i.e., from 134.3 to 106.8 s) relative to the method without the trajectory pattern prediction. The prediction of arrival times by the comparative method [i.e., Eqs. (1) and (2)] is erroneous (note that mean travel time from the entry fix to the airport was 1253.2 s) because the single regression model averages out the variation in flight times resulting from diversity in the patterns of the vectored trajectories; in contrast, the proposed method successfully mitigates this problem by estimating probabilities for the possible patterns of the aircraft trajectories, and it applies the different regression models for each trajectory pattern. The maximum error of the proposed method was also significantly less: it was 426.9 s, which is a 30.0% improvement over the 609.7 s seen with the comparative method. Noting that a typical separation standard between consecutive

Table 2 Statistical significance (p value) and correlation coefficient R^2 for the regression models of travel time per trajectory pattern

	Pattern number 1	2	3	4	5	6	7	8	9
Number of trajectories	21	35	59	29	31	549	14	171	225
R^2	0.32	0.25	0.08	0.06	0.07	0.08	0.24	0.19	0.23
p value	0.03	0.01	0.09	0.43	0.38	≈0	0.22	≈0	≈0



a) Pattern 1 **b) Pattern 2** **c) Pattern 3**

d) Pattern 4 **e) Pattern 5** **f) Pattern 6**

g) Pattern 7 **h) Pattern 8** **i) Pattern 9**

Fig. 10 Probability density functions for the aircraft positions in the trajectory patterns in Fig. 8.

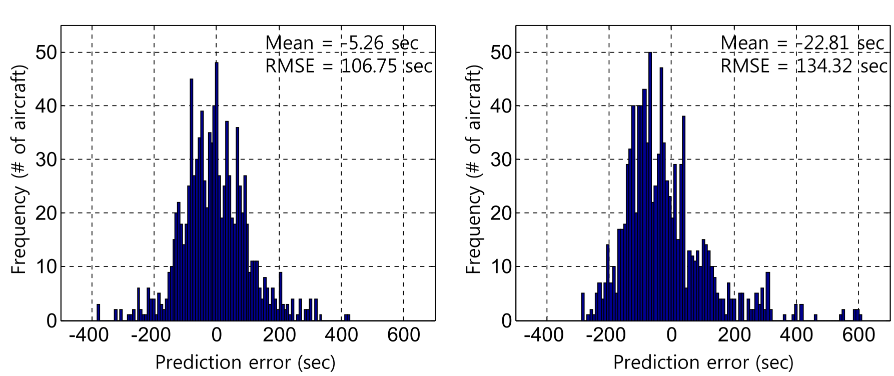
arrivals at the runway threshold is about 2 min, this reduction could imply practical benefits in airport operations. Further investigation is needed in future research to determine operational benefits, which might require extensive experimental work, including the human-in-the-loop simulation.

The improvement in the prediction errors was further investigated for each trajectory pattern. To categorize the test flights based on their trajectory patterns, the maximum likely trajectory pattern $\hat{\theta}$ of each flight is determined by its full trajectory from the entry fix to the runway threshold as follows:

$$\hat{\theta} = \arg\max_{\theta} L(\theta_i|\tilde{w}) \tag{10}$$

where $\tilde{w}$ represents the full trajectory data for each test flight. Note that the full trajectory data are necessary to categorize the prediction errors in Fig. 11, and the ETAs of those flights are still computed based only on their positions and speeds at the entry fix. The prediction errors for the trajectory patterns by the proposed and comparative methods are summarized in Table 3. As shown in the table, the prediction performances were improved by the proposed method for all trajectory patterns. In particular, the prediction errors of the proposed method for the flights with a path extension (trajectory patterns 1–5) are significantly better than those of the comparative method.

The evolution of the probability for the correct trajectory pattern over time was investigated. The correct trajectory pattern for each flight is identified as the maximum likely $\hat{\theta}$ for its full trajectory, as in Eq. (10). Figure 12 shows the time evolution of the mean probability of the correct trajectory pattern over all test flights. As shown in this figure, the probability of predicting the correct trajectory pattern increases as the target



a) Proposed method **b) Comparitive method**

Fig. 11 Histograms of the ETA prediction errors (RMSE denotes rms error).

Table 3 ETA prediction errors for the different trajectory patterns

Pattern number	RMS error in ETAs (proposed method)	RMS error in ETAs (comparative method)	Reduction in rms error by proposed method, %
Air traffic controller intervention (path extension)	124.9	179.7	30.5
1	224.2	254.0	11.7
2	147.5	171.5	14.0
3	82.2	119.2	31.0
4	123.1	217.2	43.3
5	68.0	71.3	4.7
Standard procedure	107.0	131.3	18.5
6	107.0	131.3	18.5
Air traffic control intervention (shortcut)	94.7	114.1	17.0
7	129.3	149.5	13.5
8	73.1	86.2	15.2
9	98.4	119.0	17.3
Total	106.8	134.3	20.5

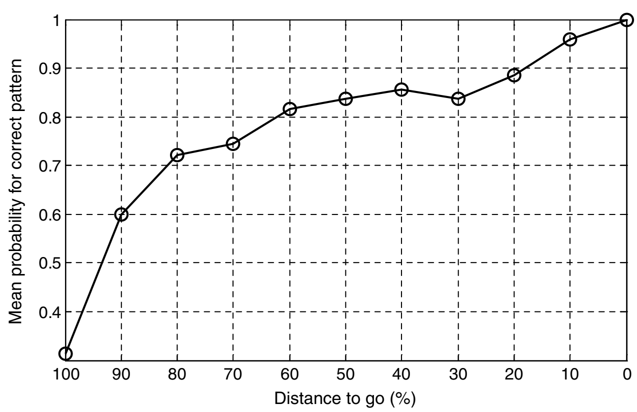


Fig. 12 Evolution of probability for the correct trajectory pattern.

aircraft moves from the entry fix to the destination airport; the horizontal axis of the figure represents the relative distance remaining along the trajectory to the destination, with 100% denoting aircraft at the entry fix and 0% denoting aircraft at the runway threshold. As discussed in Sec. III.D, in real implementation of the proposed method, the predicted flight pattern can converge into the correct one, as the likelihood information becomes dominant as the target aircraft proceeds.

V. Conclusions

As movement is made toward the next generation of air traffic management environments, close cooperation between the human controllers and automated support systems becomes more important. To design automated support systems that provide effective advisories for controllers, it must be understand how those controllers behave in the face of various air traffic situations. One of the key areas that can benefit from such an understanding of human behavior is the prediction of aircraft trajectories.

This paper proposed a new method to predict the time of arrival of aircraft through vectored RNAV airspace to the destination airport. Accurately predicting arrival times is challenging due to the diverse patterns of the trajectories vectored by the air traffic controllers. In the proposed method, more accurate predictions for the arrival times can be achieved by applying different regression models for each possible trajectory pattern of the target aircraft. The major patterns of the vectored trajectories are identified by clustering historical trajectory data, and the probabilities of those patterns are computed based on the trajectory patterns of the preceding aircraft. The proposed method was applied to real traffic data to demonstrate its performance.

Future work should extend the method by exploring various algorithms for each step in the learning process and calibrating associated parameters. Other types of clustering techniques could be applied to more effectively identify an appropriate set of trajectory patterns. Similarly, different regression models should be investigated to further improve the performance of the proposed method. In particular, wind information could significantly improve the performance of the proposed method and should be considered in future research. Room for improvement exists in the density estimation for the trajectory pattern prediction. Different types of density functions should be explored, whereas the parameters inside the density function should be fit to historical data. An extensive tradeoff analysis and experimental work are needed to further optimize the overall prediction performance of the proposed method.

In the proposed method, the trajectory pattern of an aircraft is predicted based on the patterns of the preceding aircraft through the same entering fix. In real operation, however, air traffic controllers determine how to control each aircraft with a broader view of the traffic in the airspace as a whole. Therefore, continuous efforts are needed to better understand the strategies used by human controllers for air traffic control in various airspace and traffic environments.

Although this study focuses on presenting a new framework for trajectory prediction, work remains to be done to identify additional use cases for the method in a variety of applied contexts in air traffic management. Possible applications for the proposed method include various precision air traffic operations, both on the airport surface and in the airspace. In general, the proposed method would most benefit operational situations under high traffic controller vectoring conditions, which include those with a missed-approach aircraft or under airport configuration changes. It could also be applied to trajectory prediction for departure aircraft. One example is when air traffic controllers must accommodate the departure aircraft from a nearby airport into the arrival stream by estimating its transit time through the terminal airspace. A more comprehensive performance validation of the proposed method also needs to be done, which will require extensive experimental work, including human-in-the-loop simulations.

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