Competing for Time: A Study of Mobile Applications*

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April 11, 2023

Abstract

Apps compete for limited user time regardless of their functions. I develop a discrete-continuous demand model with a binding time constraint. I estimate the model on three pairs of apps (including substitutes and complements) with joint app usage in China. I use updates to disentangle complementarity from correlated preferences. The recovered competition patterns are realistic. I decompose competition into functional competition and budget competition, the latter of which captures the effects of a binding time constraint. Budget competition can dominate functional competition and a merger of complements can hurt consumers. A model-free metric is proposed to gauge budget competition.

*I thank Mo Xiao, Gautam Gowrisankaran, Yong Liu, Mauricio Varela, Antonio Galvao for their valuable guidance and suggestions and Yedda Wang at iResearch for her support with the data. I also want to thank Ao Wang, Daniel Ershov and other participants at China VIOS 2021, IIOC 2021, ISMS Marketing Science 2020, and YES 2020 and seminar participants at ShanghaiTech University, University of Arizona, Renmin University of China, Fudan University, Sun Yat-sen University, Luohan Academy, SHUFE, Jinan IESR for helpful comments. From December 2020 to June 2021, I was a resident scholar at Luohan Academy, which is an affiliate of Alibaba. All opinions and errors are my own.

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Instagram can hurt us.

–Mark Zuckerberg, Facebook CEO, 2012

Think about it, when you watch a show from Netflix [......], you stay up late at night. We're competing with sleep, on the margin.

–Reed Hastings, Netflix CEO, 2017

1 Introduction

Antitrust authorities have been struggling to deal with mergers of free apps (Wu, 2017).[1] None of the major acquisitions by tech giants were blocked in recent years in the United States (Cabral, 2021).[2] Scholars believe that the reason is that antitrust authorities lack the tools to analyze mergers of apps.[3] The UK Office of Fair Trading (OFT) approved the acquisition of Instagram by Facebook in 2012 partly on the ground that, in the market for camera apps, Facebook would still face competition from other photo apps after the merger.[4] The market definition that Instagram is a photo app and Facebook is not is certainly debatable. Zuckerberg would disagree. When contemplating the acquisition, Zuckerberg wrote to his CFO to explain the motivation:

> "There are network effects around social products and a finite number of different social mechanics to invent. Once someone wins at a specific mechanic, it's difficult for others to supplant them without doing something different."

Zuckerberg saw one social media app (Instagram) challenging another (Facebook) and that acquisition can "neutralize a potential competitor"[5]. In another email sent days before the announcement of the merger, he wrote: "Instagram can hurt us." From a different position, Tim Wu, the famous legal scholar, and Chris Hughes, a co-founder of Facebook, also believe Instagram and Facebook are competitors and antitrust

[1] See Scott Morton *et al.* (2019), Furman *et al.* (2019), European Commission. Directorate General for Competition. (2019), and Cabral (2021) for related discussions.

[2] The same is true for China until a recent wave of antitrust enforcement. In July 2021, a proposed merger of Douyu and Huya, two live-streaming apps, was blocked because of their vertical relationship with Tencent.

[3] In his opinion piece in the Washington Post, Tim Wu (Wu, 2018) argues:

> "Our standards for assessing mergers, fixated on consumer prices, were a poor match for the tech economy and are effectively obsolete."

In the report commissioned by the Stigler Committee on Digital Platforms, Scott Morton *et al.* (2019) proposes

> "The law needs better analytical tools to take into account the impact of potential and nascent competitors and competition. Market definition will vary according to what consumers are substituting between[......]."

[4] See Anticipated acquisition by Facebook Inc of Instagram Inc, ME/5525/12 (Office of Fair Trading, 22 August 2012) at https://assets.publishing.service.gov.uk/media/555de2e5ed915d7ae200003b/facebook.pdf.

[5] Those emails were revealed during the House antitrust subcommittee's hearing on antitrust issues in July 2020. See the story "'Instagram can hurt us': Mark Zuckerberg emails outline plan to neutralize competitors" at https://www.theverge.com/2020/7/29/21345723/facebook-instagram-documents-emails-mark-zuckerberg-kevin-systrom-hearing

authorities should reverse this and other similar decisions (Wu, 2018; Hughes, 2019). There are arguments for both sides. Considering that users can share Instagram photos to Facebook, the two apps are also complements.[6] Different assumptions about the competitive relationship between Facebook and Instagram (complements, substitutes, or independents) can lead to different antitrust decisions. What we need is a quantitative method that can estimate complementarity/substitutability between apps from data.

Competition among apps is further complicated by the binding time constraint. When asked about competition at an Netflix's earnings call in 2017, Reed Hastings replied with the opening quote (Hern, 2017). He explained it in another occasion (Raphael, 2017): "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and unwind, hang out, and connect–and we compete with all of that." Hastings highlighted the role of time constraint in competition analysis. In this paper, I refer to it as "budget competition" to distinguish it from "functional competition". Users have at most 24 hours per day. A minute spent on Tik Tok is a minute not spent on WeChat. Budget competition has been invoked in the high-profile antitrust case *Qihu v. Tencent* in 2013 to expand the relevant market. Tencent argued that its instant-messaging software QQ competes with all other Internet companies for user attention (time).[7] This claim is trivially true for all apps/software and offline activities regardless of complementarity or substitutability. What matters is the size of budget competition. When budget competition is large enough, a merger of complements can hurt consumers. To calculate the size of budget competition, we must estimate a structural model of demand with a binding time constraint, in addition to allowing for complementarity/substitutability.

The difficulty in estimating competitive relationships among apps is twofold. First, modeling competition without price is a daunting task. There are no price variations, and therefore we cannot cannot estimate price elasticities. Functional definitions do not work. WeChat, the flagship app of Tencent, is classified as "Social Networking" by the Apple App Store and "Communication" by the Google Play Store as of 2019. Users know that WeChat is more than the two definitions: it is also a mobile payment app, a publishing platform, a platform of mini programs, and so on. Adding a budget constraint would further complicate utility maximization problems and hence demand estimation.Second, estimating complementarity/substitutability is difficult. Taste for variety and correlated preferences can confound the estimation of complementarity. We need more than aggregate market share data to separate complements from substitutes.

In this paper, I propose and estimate a model of time allocation to apps. The model features a quadratic utility function to capture the discrete-continuous nature of app usage and allow for substitutes as well as

[6] In the public announcement of the acquisition, Zuckerberg said:"We believe these are different experiences that complement each other." See the public announcement at https://investor.fb.com/investor-news/press-release-details/2012/Facebook-to-Acquire-Instagram/default.aspx

[7] Hence the relevant market in this case should include all major Internet companies and their software. The Supreme People's Court rejected this argument with qualitative analysis in the final adjudication in 2013.

complements (Thomassen *et al.*, 2017)[8]. I add a binding time constraint to study budget competition. In this model, an app is described by a taste parameter, a satiation parameter, and interaction parameters with other apps. The taste parameter is the marginal utility at zero usage, and the satiation parameter determines how fast the marginal utility depreciates as an user spends more time on the app. The interaction parameters are cross partial derivatives that measure the interactions between apps: if the interaction parameter between a pair of apps is positive, then they are complements; otherwise, they are substitutes. Taste parameters have random components that can be correlated across apps. Users allocate their time to apps and offline activities subject to a time constraint. I allow for network effects in taste parameters. I use GMM estimation *a la* Berry *et al.* (1995) such that I could utilize instrument variables (IVs). Quadratic utility functions are second-order approximations to any reasonable utility functions. Therefore, my model nests the random coefficients discrete choice model of Berry *et al.* (1995) as a special case with only taste parameters.

I estimate the model using weekly market-level app usage data from China in the first quarter of 2017. This is the first academic paper to use this type of data, to the best of my knowledge.[9] Markets in this data set are demographic groups in China defined by age, gender, and province. I observe the number of active users and usage (time spent) of popular apps and Android smartphone. The active user data help me identify taste parameters of apps and the usage data help me identify satiation parameters. Network effects are identified with geographic variations in the date set(Weiergraeber, 2022). In addition, for each pair of apps, I observe the number of users who use both apps in a week. The common user data are informative, but these data are not sufficient for the identification of complementarity/substitutability.

The econometric challenge is to separate correlated preferences from complementarity/substitutability. The observation of common users in two apps can be the result of complementarity between the two apps, or the fact that the preferences of the two apps are positively correlated due to unobserved characteristics. My identification strategy is based on an "old definition" of complements (substitutes)[10]: if users spend more time on an app due to an exogenous increase in its utility, the (marginal) utility of its complements (substitutes) would increase (decrease). Updates of an app should affect the utility of this app but not the *utilities* of other apps. However, updates of an app could change the *usage* of other apps through complementarity/substitutability. This is similar to the strategy used in Gentzkow (2007).

My model can recover diverse competition relationships. I apply this model to three representative pairs of apps: a pair of substitutes *a priori* (Baidu Map and Amap), a pair of complements *a priori* (Baidu and Baidu Map), and a pair of apps with an ambiguous relationship (WeChat and Kwai)[11]. In each case, the

[8] Lewbel & Nesheim (2019) also use a quadratic utility model.

[9] A concurrent paper by Kawaguchi *et al.* (2022) uses national usage data of apps from Japan. One key difference is that they do not have common user data, which is crucial to my identification.

[10] Samuelson (1974) discusses various definitions of complements and substitutes.

[11] WeChat and Kwai did not offer similar functions at the time. The relationship is similar to that between Instagram and

estimated results are reasonable. WeChat and Kwai were weak substitutes though they did not offer similar functions at the time. This can be interpreted as one social media app competing with another social media app, as with Instagram and Facebook. IVs are crucial to my estimation. When I assume away correlated preferences and rely only on the common user data for identification, Baidu Map and Amap are estimated to be weak complements. I find significant network effects in all the apps.

I simulate counter-factuals to see what happens to the other app if one of the apps is shut down. Consulting firms publish reports analyzing market share of apps based on time spent rather than active users.[12] In Figure 1, I plot the market shares of tech giants in terms of time spent on their apps. It is only natural for Tencent to ask if ByteDance or other firms/apps competed time away from its apps. My simulations can answer this question. I find that Kwai users would increase their time spent on WeChat on average by 15 minutes if Kwai is shut down. In other words, Kwai competes 15 minutes away from WeChat each week. The competition pressure is significant considering that Kwai was a young app and they did not offer similar functions at the time. I calculate diversion ratios in terms of active users and time spent. In the WeChat and Kwai example, if Kwai is shut down, 15% of its time will be diverted to WeChat; if WeChat is shutdown, 3% of its time will be diverted to Kwai.

I then decompose competition effects of one app on another into two parts: "functional competition" and "budget competition". Functional competition captures the fact that if two apps offer similar functions, using one would reduce the (marginal) utility of the other. Budget competition captures the fact that all apps are competing for the limited time of users. I find that budget-competition effects are negligible (less than 0.03 hour for 1000 users) for the first two pairs of apps (Baidu Map and Amap, Baidu and Baidu Map). The reason is that users spend a small amount of time on Baidu Map and Amap. This suggests that if the apps of interest are "small" (in terms of time spent), researchers can model the demand of these apps without a time constraint and still capture virtually all the competitive effects. The budget-competition effect is orders-of-magnitude larger (3.8 hour for 1000 users) for WeChat and Kwai because their preferences are positively correlated, and a large number of users spend a substantial amount of time on both them. I propose a model-free metric to gauge budget competition of app 2 on app 1: $\frac{t_2 t_1}{T - t_2}$. Budget competition is increasing in $t_1 t_2$ and, by extension, the correlation between t_1 and t_2. This metric is only valid under restrictive assumptions. However, budget competition calculated using this model-free metric are reasonably close to the results from the estimated model in terms of order of magnitude. With this metric, we can put a ballpark figure on budget competition with aggregate usage data or a simple survey of users. We can also combine the metric with institutional knowledge to get more accurate estimates of budget competition.

Facebook before the merger.

[12]For example, Nielsen (https://www.nielsen.com/us/en/insights/article/2014/smartphones-so-many-apps-so-much-time/) and QuestMobile (https://www.questmobile.com.cn/research/report-new/118).

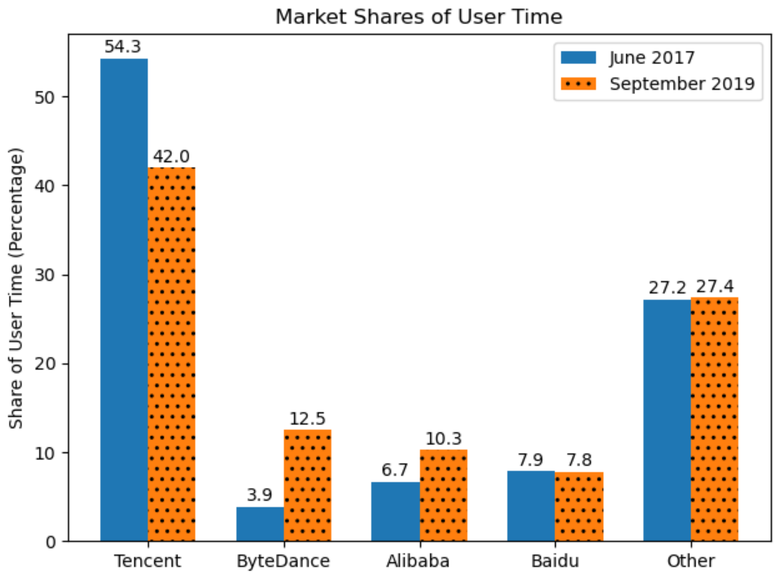


Figure 1: Market Shares of Tech Giants in China
Note: Market shares are calculated based on time spent on apps developed by each tech giants.
Data Source: Quest Mobile.

The decomposition suggests that budget competition can dominate functional competition and a merger of complementary apps can hurt consumers.

I combine the estimated demand with a stylized supply model to simulate mergers of apps. The demand and supply model highlights the fact that prices and ad load are colinear in both the utility function and the profit function and conceptually the same thing. I find when competing apps (functional competition or budget competition) merge, prices (ad load) increase and consumer welfare decreases. Despite the fact that WeChat and Kwai had no similar functions in the first quarter of 2017, the welfare of 1000 simulated smartphone users in one week could decrease by 300 yuan if the two apps merge. The results partly explain Tencent's 2 billion dollar investment in Kwai in late 2019.

This paper contributes to the emerging literature on mobile applications. Due to data limitations, researchers have mostly focused on the supply side of apps (Liu *et al.*, 2013; Yin *et al.*, 2014; Bresnahan *et al.*, 2014b,a; Liu, 2017; Wen & Zhu, 2017; Ershov, 2018; Leyden, 2019). The demand side for apps is either absent in the papers or described with aggregate ranking or downloads data from app stores (Carare, 2012; Ghose & Han, 2014; Li *et al.*, 2016; Yi *et al.*, 2017; Le Guel *et al.*, 2020; Deng *et al.*, 2020).[13] An immediate predecessor of this paper is Han *et al.* (2016). They adopt a multi-nominal discrete-continuous extreme value

[13]Both Wu *et al.* (2022) and Lee (2018) use a panel of individual usage of smartphone. However, both observe usage of categories rather than apps. Lee (2018) estimates the demand for smartphone. Wu *et al.* (2022) uses a hidden Markov model to analyze what motivates mobile app usage.

(MDCEV) model developed by Bhat (2005) and allow for correlation in utilities between different apps by adding a factor analytic structure. With individual level panel data from Nielsen KoreanClick, they estimate positive or negative correlations in preferences across apps. However, substitutes or complements are not modeled in their paper. As these authors have noted in their paper, the correlation of preferences between Naver and Daum and that between Kakao Talk and Kakao Story are estimated to be positive and large. However, common sense suggests that the first pair are substitutes and the second pair are complements. By contrast, my paper explicitly disentangles substitutability/complementarity from correlated preferences with the common user data and IVs. My model also differs from Han *et al.* (2016) in using market level data. Though these data are widely used in the industry, to the best of my knowledge, this is the first instance of using market-level data of app usage in academic research.

A concurrent paper by Kawaguchi *et al.* (2022) simulate mergers of apps. They estimate demand and supply for apps in two categories with usage and advertising data from Japan. The demand in their paper is more restrictive. The difference between their model and mine highlight the trade-off between flexible competition patterns and scalability. Their merger results are also different from mine. According to their results, a merger among the top 10 social media apps in Japan has negligible effects on surplus. Aridor (2022) studies addiction to social media apps with individual apps usage from an experiment. Aridor (2022) is complementary to my work because he focuses on the dynamic aspect of app usage that is not in my paper.

In terms of methodology, this paper is a second-order extension of Berry *et al.* (1995). My model is the first to combine four appealing features into a model of consumer demand: discrete-continuous decisions, interactions between products, budget constraint, and estimation with instruments. This paper relates to the literature on the demand of differentiated goods in economics and marketing especially when complementarity is of interest (Kim *et al.*, 2002; Nair *et al.*, 2005; Song & Chintagunta, 2006, 2007; Mehta, 2007; Gentzkow, 2007; Thomassen *et al.*, 2017; Ershov *et al.*, 2018; Vélez-Velásquez, 2019; Lewbel & Nesheim, 2019; Wang, 2020). Unlike Gentzkow (2007), this paper models not only the extensive margin (which products are chosen) but also the intensive margin (quantities of chosen products) of consumer decisions. This is especially important if we want to estimate complementarity. Consumers buy two boxes of cereal with different flavors because of taste for variety (decreasing marginal utility) rather than complementarity. A discrete choice model with bundles of different products cannot identify complementarity from taste for variety. Taste for variety is captured by satiation parameters and can be estimated with usage data in my paper. This paper also contributes to the study of time allocation in transportation research (Kitamura, 1984; Bhat, 2005; Pawlak *et al.*, 2015, 2017; Bhat, 2018) by directly estimating relationships between activities. Furthermore, this model is a flexible second-order approximation to consumer decisions and hence can be adapted to study other topics.

Finally, this paper contributes to the policy debate on regulating the digital economy (Furman *et al.*, 2019; European Commission. Directorate General for Competition., 2019; Scott Morton *et al.*, 2019). To the best of my understanding, this paper and Kawaguchi *et al.* (2020) are the first papers to simulate mergers of mobile applications. A key challenge in analyzing the digital economy is that the digital economy is characterized by free services and existing economic tools require prices. Complements and substitutes are defined with compensated cross-price elasticities and market power is defined with prices as well. Market power exists even though prices are zero. This paper extends economic tools to incorporate important features of apps, including zero (or negative) prices. In this paper, price and advertising are both (linear) components of quality. My analysis shows that market power exists and can be measured even when prices are zero. My decomposition of competition provides a new theory of harm to consumers: a merger of complementary apps can hurt users if budget competition dominates functional competition.

2 Data

There are two types of app usage data available in the mobile Internet industry: individual level data and market level data. The first type of data resembles traditional surveys: firms pay individuals for their permission to install an app or software in order to monitor the usage of their devices. The data sets used by Han *et al.* (2016) , Lee (2018), Wu *et al.* (2022), and Boik *et al.* (2016) fall into this category. The data set used in this paper is aggregate market data, which is estimated based on a large quantity of observations from different sources. Wireless carriers and app developers are the two major sources. For instance, China Unicom provides app usage data based on traffic data from its users.[14] App developers mostly use third-party libraries to analyze behaviors of their users.[15] Those data are then traded and matched based on unique device identifiers. In sum, market level data are estimated from snapshots of millions of devices, whereas individual data are 24×7 information from thousands of users. While both individual level data and market level data can be used to estimate relationships among apps, market level data are widely available in different countries and raise much less privacy concerns.[16]

Data used in this paper are from iResearch, a leading consulting firm in China with a focus on the mobile Internet industry. There are three parts of our data: the app usage data, the smartphone usage data, and the common user data. I introduce these data in the following subsections. All data are weekly data taken

[14] See https://www.cubigdata.cn

[15] For a story about how this works, see the report by the Wall Street Journal: https://www.wsj.com/articles/you-give-apps-sensitive-personal-information-then-they-tell-facebook-11550851636?mod=article_inline

[16] For example, Facebook shut down its “Facebook Research” app because of public anger. See https://www.wired.com/story/facebook-research-app-root-certificate

from the first quarter of 2017 in China.

2.1 App Usage Data

I acquired app usage data of the top 300 apps on Android cellphones of 290 demographic groups for 13 weeks in China. In this data set, a market is a demographic group defined by gender (male and female), age groups (below 24, 25-30, 31-35, 36-40, and above 40), and geographic areas (28 provinces and an "other" category). I do not have all the 300 apps' data as some apps have an estimated number of active users that is too small to be reliable. The threshold is 50,000. On average, I observe about 82 apps for each week-market pair. I observe more apps for large demographic groups in the data set. In total, I have 312,724 week-market-app observations. For each unit of observation, I observe the number of devices (per ten thousands) that used the app at least once during the week (henceforth, active user) and the average number of minutes spent on the app per device during the week (henceforth, average time spent). The summary statistics are in the upper panel of Table 1. The zeros in the table result from the technical difficulty in estimating usage of some apps, for example, input methods.

2.2 Smartphone Usage Data

iResearch provides total usage of Android devices, i.e., the smartphone usage data. Similarly, I have the number of active devices (per ten thousands) that are used at least once during the week (active users) and the average number of minutes spent on Android smartphones per device during the week (average time spent). With those data, I calculate market shares of apps in each market which is the number of active users of an app divided by the number of active users of Android smartphones in that market. The summary statistics are in the middle panel of Table 1.

2.3 Common User data

Most importantly, I have the common user data. For each pair of apps, I observe the number of Android smartphone users that used both apps at least once during the week (henceforth, common user). Again the 50000 threshold applies. On average, I observe about 110 apps each week. I only have common user data at the national level because they are small, and hence unreliable, at the demographic group level. The summary statistics are in the lower panel of Table 1.

3 Model

3.1 The Baseline Model

A consumer $i = 1, 2, , ..., I$ allocates her time T to J apps and an outside option denoted by $j = 0$. The utility from an allocation described by $\mathbf{t} = [t_{i0}, t_{i1}, t_{i2}, ..., t_{iJ}]'$ where t_{ij} is the amount of time allocated to option $j = 0, 1, 2, ..., J$ is given by

$$U(\mathbf{t}) = \boldsymbol{\mu}'\mathbf{t} + 0.5\mathbf{t}'\boldsymbol{\Gamma}\mathbf{t} \tag{1}$$

where

$$\boldsymbol{\mu} = [\mu_{i0}, \mu_{i1}, ..., \mu_{iJ}]'$$

and

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{i0} & \gamma_{i01} & \cdots & \gamma_{i0J} \\ & \gamma_{i1} & \cdots & \gamma_{i1J} \\ & & \ddots & \vdots \\ & & & \gamma_{iJ} \end{bmatrix}.$$

$\boldsymbol{\mu}$ is a $(J+1) \times 1$ vector of first order parameters and $\boldsymbol{\Gamma}$ is a $(J+1) \times (J+1)$ symmetric matrix of second order parameters. The marginal utility of app j is

$$MU_{ij} = \mu_{ij} + \gamma_{ij} t_{ij} + \sum_{j' \neq j} \gamma_{ijj'} t_{ij'}.$$

The marginal utility of app j consists of three components. The first term μ_{ij} is the marginal utility of app j at zero usage and it will be referred to as the taste parameter of app j. γ_{ij} in the second term determines how MU_{ij} changes as a user spends more time on app j. Therefore, γ_{ij} should be negative and will be referred to as the satiation parameter of app j. The last term captures the impact of app j' on app j: if parameter $\gamma_{ijj'} > 0$, then MU_{ij} is increasing in $t_{j'}$ and they are complements; otherwise, they are substitutes.[17] Therefore, the interaction parameter $\gamma_{ijj'}$ determines if j and j' are likely to be used together.

At the optimal level $\mathbf{t}^*$, the marginal utilities of apps that are used should be equalized. Denote this as λ. Zero usage arises naturally when the marginal utility at zero is too small, i.e., $\mu_{ij} < \lambda$. In Figure 2, I plot three apps with different combinations of μ_j and γ_j without considering $\gamma_{jj'}$ for now. Intuitively, μ_j determines if an app is used and conditional on being used, γ_j determines the time spent on app j.

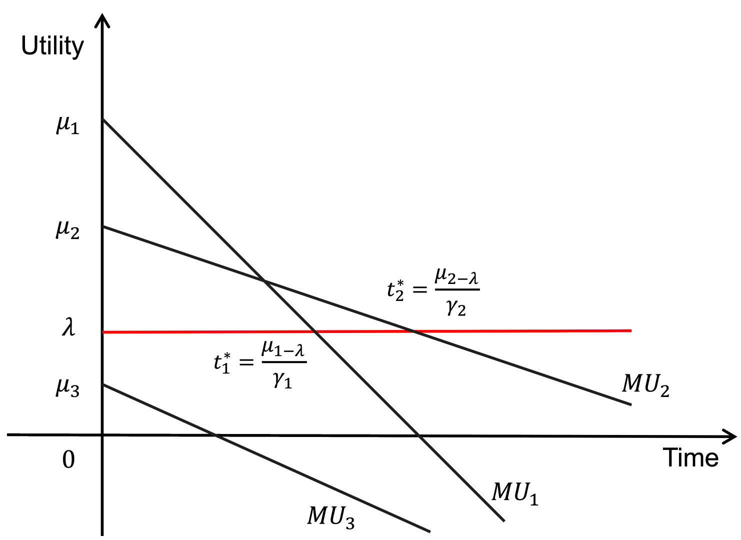
I choose the quadratic utility function because it naturally models the discrete-continuous nature of app

[17] The modern definition is based on compensated cross-price elasticities. Samuelson (1974) discusses various definitions of complements and substitutes.

Table 1: Summary statistics of app usage

Variables	Mean	Min	Max	StdDev	# Obs	Unit
App Usage Data						
Active user	28.05	5	1074.2	46.38	312724	ten thousands
Market share	0.1064	0.004	0.958	0.132	312724	-
Average time spent	58.65	0	800.17	74.86	312724	minutes
Smartphone Usage Data						
Active user	225.05	10.31	1238.75	193.92	3770	ten thousands
Average time spent	1006	561.5	1435.5	202.08	3770	minutes
Common User Data						
Common user	599.21	6.41	29979.13	1387.733	79809	ten thousands

Note:
1, The smartphone and app usage data are weekly observations at the demographic group level from the first 13 weeks of 2017 in China. The common user data are weekly aggregate data for each pair of apps.
2. Active user of an app is the number of devices that used the app at least once during the week. Active user of smartphone is the number of Android smartphones that are used at least once during the week. Average time spent is the average number of minutes spent on the app per device during the week. Market share of an app is the active user of this app divided by the active user of Android smartphones in that market. Common user is the number of Android smartphones that use both apps at least once during the week.
3, The zeros in app usage data result from the technical difficulty of estimating usage of some apps, for example, input methods.
Data Source: iResearch.



Note: This graph plots the marginal utility of three apps: app 1, app 2, and app 3. λ is the marginal utility at the optimal allocation. For simplicity, I ignore $\gamma_{jj'}$. MU is the marginal utility of each app.

Figure 2: Marginal Utilities and Optimal Allocation of Time

usage and the complementarity/substitutability between apps. Despite the advantages of the quadratic utility function, the size of $\mathbf{\Gamma}$ increases quadratically in J. Therefore, instead of analyzing 100 apps in one model, which involves a gigantic matrix $\mathbf{\Gamma}$, I analyze a smaller model with more assumptions and only two apps of interest. Two is certainly not an ideal number. However, many mergers in the mobile Internet industry are about two apps (for example, Facebook's acquisition of Instagram). The only limit on how many apps researchers can analyze is computational resources and "thickness" of data (common user data for all possible pairs of apps).[18]

3.2 A Simplified Model

In the model to be estimated, there are four options $j = 0, 1, 2, 3$, where $j = 1, 2$ are the two apps of interest and $j = 0$ is the option of not using a smartphone and $j = 3$ is a generic app which is to use any other apps. The utility maximization problem of consumer i in market $m = 1, 2, ..., M$ is

$$\max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} t_{i0m} - 0.0005 t_{i0m}^2 + \sum_{j=1}^{2} \mu_{ijm} t_{ijm} + 2 t_{i3m} + \frac{1}{2} \sum_{j=1}^{3} \gamma_{ijm} t_{ijm}^2 + \gamma_{12} t_{i1m} t_{i2m} \tag{2}$$

$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168$$

I add more assumptions compared to (1). To normalize the level of the utility function, I assume $\mu_{i0m} = 1$. I assume $\mu_{i3m} = 2 > \mu_{i0m}$ because the market shares of $j = 3$ are always 1. Because the time spent on $j = 0$ is a residual term ($t_0 = 168 - t_1 - t_2 - t_3$) in the model, I assume γ_{i0m} be a non-positive constant -0.001 ($\frac{1}{2} \times 0.001 = 0.0005$). I also assume $\gamma_{10} = \gamma_{20} = \gamma_{13} = \gamma_{23} = 0$ because those who use either app 1 or app 2 will always use the generic app and spend some time on offline activities.

168 is the total number of hours in a week and the time scope of this utility function. This is a choice imposed by the data structure: I happen to observe weekly usage. One can certainly consider utility functions defined over a month, a day, an hour, or a second if data permits. The estimated demand models will be different but valid under their respective time scope. For example, when we model usage for each second, all apps are substitutes.[19] We can often observe more dis-aggregate usage with survey data as we discussed in Section §2.

[18] One may also be concerned about the number of exclusion restrictions. This is not an issue if we use updates as IV. The update history of each app will be interacted with residuals of all other apps and *vice versa*. Therefore, the number of exclusion restrictions increases quadratically as well.

[19] I thank one of the referees to point this out.

3.3 Consumer Heterogeneity

Consumers have different preferences regarding apps. μ_{ijm} and γ_{ijm} are parameterized as

$$\mu_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^{\mu} + \zeta_1 s_{1m}^r + \xi_{1m}^{\mu} + \varepsilon_{i1m} = \delta_{1m}^{\mu} + \varepsilon_{i1m} \tag{3}$$

$$\mu_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^{\mu} + \zeta_2 s_{2m}^r + \xi_{2m}^{\mu} + \varepsilon_{i2m} = \delta_{2m}^{\mu} + \varepsilon_{i2m} \tag{4}$$

$$\gamma_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^{\gamma} + \xi_{1m}^{\gamma} = \delta_{1m}^{\gamma} \tag{5}$$

$$\gamma_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^{\gamma} + \xi_{2m}^{\gamma} = \delta_{2m}^{\gamma} \tag{6}$$

$$\gamma_{i3m} = \mathbf{x}_m \boldsymbol{\beta}_3^{\gamma} + \xi_{3m}^{\gamma} = \delta_{3m}^{\gamma} \tag{7}$$

where $\mathbf{x}_m$ is a set of exogenous market level variables. $\zeta_1 s_{1m}^r$ and $\zeta_2 s_{2m}^r$ are the terms of network effects. I follow Berry *et al.* (1995) and Nevo (1998) in denoting market-level parameters with $\boldsymbol{\delta} = (\delta_{1m}^{\mu}, \delta_{2m}^{\mu}, \delta_{1m}^{\gamma}, \delta_{2m}^{\gamma}, \delta_{3m}^{\gamma})$. ξ^{μ} and ξ^{γ} capture app-market specific idiosyncratic error terms. For example, a weather shock to market m may increase the marginal utility of Uber but not that of Google Docs. ε_{i1m} and ε_{i2m} are individual error terms that are iid across individuals but can be correlated across apps. ε_{i1m} and ε_{i2m} capture unobserved individual characteristics that affect utilities derived from apps. For example, users with cars, compared to those without cars, derive higher utilities from Google Maps and lower utilities from Uber. Therefore, the preference of Uber and the preference of Google Maps can be negatively correlated. As discussed in Train (2009), the variance of μ_{ijm} cannot be separately identified from the mean of μ_{ijm}. I assume $(\varepsilon_{i1m}, \varepsilon_{i2m})$ follows a normal distribution $N(\mathbf{0}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

ρ captures correlated preferences. As we add more controls in $\mathbf{x}_m$, ρ may be closer to zero. Given that we can never control for all relevant factors at the individual level, we should not assume $\rho = 0$. γ_{12} and ρ together explains the common user between app 1 and app 2. An econometric challenge is to disentangle γ_{12} from ρ, which will be discussed in Section §4.

3.4 Network Effects

Network effects are ubiquitous in digital markets and a possible reason to support mergers of competing apps. Thanks to the data structure, I can incorporate network effects and construct instruments *a la* Weiergraeber (2022). The contemporaneous market share of an app within a user's reference group enters the marginal utility of that app and captures the network effect. For user i in market m, the market share of app 1 in her reference group s^r_{1m} is a linear component of δ^μ_{1m}. A user's reference group consists of all users in her local market (province). To deal with the reflection problem, I use the lagged market share in the same local market *other than m* itself, $s^{r-m}_{1m(w-1)}$ as IV for s^r_{1mw} where w denotes week (Weiergraeber, 2022). $s^{r-m}_{1m(w-1)}$ could affect s^r_{1mw} through switching costs, inertia, or other channels. However, the lagged share of other users, $s^{r-m}_{1m(w-1)}$, should not directly enter the contemporaneous mean utility, δ^μ_{1mw}. Therefore, we can identify the coefficient of s^r_{1mw}, ζ_1. A positive ζ_1 means users derive utility from other people using the same app.

4 Estimation

I use GMM to match moments predicted by the model with moments calculated from the data. The full set of parameters to be estimated are $\boldsymbol{\theta} = (\boldsymbol{\beta}^\mu_1, \boldsymbol{\beta}^\mu_2, \boldsymbol{\beta}^\gamma_1, \boldsymbol{\beta}^\gamma_2, \boldsymbol{\beta}^\gamma_3, \zeta_1, \zeta_2, \gamma_{12}, \rho)$. As in Nevo (1998), denote the linear parameters with $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}^\mu_1, \boldsymbol{\beta}^\mu_2, \boldsymbol{\beta}^\gamma_1, \boldsymbol{\beta}^\gamma_2, \boldsymbol{\beta}^\gamma_3, \zeta_1, \zeta_2)$ as they will enter the GMM function linearly and the nonlinear parameters with $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$. I observe a set of markets, which are defined to be demographic groups, for 12 weeks. Denote weeks with w. For each market-week unit, I observe s^*_{1mw} and s^*_{2mw}, the share of users who uses app 1 and app 2, t^*_{1mw}, t^*_{2mw} and t^*_{3mw}, the average time spent on app 1, app 2, and all other apps in hours. For each week, I also observe the total number of common user between app 1 and app 2, c^*_{12w}. The asterisks indicate that they are observed variables. Hence the endogenous variables to be explained are $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$, c^*_{12w}, and $\mathbf{s}^r_{mw}$. The exogenous variables include $\mathbf{x}_{mw}$, a set of week and market fixed effects, and $\mathbf{s}^r_{mw}$, the market shares in the reference group. Note that $\boldsymbol{\delta} = \mathbf{x}_{mw}\boldsymbol{\beta} + \mathbf{s}^r_{mw}\boldsymbol{\zeta} + \boldsymbol{\xi}$.

With those notations, the model can be succinctly summarized as

$$(\mathbf{y}^*_{mw}, c^*_{12w}) = f(\boldsymbol{\delta}, \gamma_{12}, \rho) = f(\mathbf{x}_{mw}\boldsymbol{\beta} + \mathbf{s}^r_{mw}\boldsymbol{\zeta} + \boldsymbol{\xi}, \gamma_{12}, \rho)$$

where $f(\cdot)$ is the nonlinear model described in the previous section and $\boldsymbol{\xi}$ is the stack of all market level error terms. Note that there are five components in $\boldsymbol{y}^*_{mw}$ and five components in $\boldsymbol{\delta}$. At the market level, we have six outcome variables but seven parameters. The model is not identified with the observed variables we have.

The econometric challenge is to identify γ_{12} from ρ. Intuitively, both γ_{12} and ρ can explain c^*_{12w}. If one observes that many users use both NYTimes and WSJ, it could be the case that NYTimes and WSJ are complements as they offer different perspectives on the same events, or that users have a strong demand of news in general. In the first case, $\gamma_{12} > 0$. In the second case, $\rho > 0$. In economic textbooks, complements and substitutes are defined with compensated cross-price elasticities of demand: if an exogenous increase in the price of product A leads to a decrease in the compensated demand of product B, then they are complements; otherwise, they are substitutes. When there is no price, one can extend the definition: if users spend more time on an app due to an exogenous increase in its utility, the (marginal) utility of its complements (substitutes) would increase (decrease). This definition is based on cross derivatives of the utility functions and the basis of my identification strategy with updates as instruments. Updates of app 1 should change the *utility* of app 1 but not that of app 2. However, updates of app 1 can change the *usage* of app 2 through γ_{12}. Therefore, I use the following moments to identify nonlinear parameters γ_{12} and ρ

$$E(c^*_{12} - c_{12}) = 0 \tag{8}$$

$$E(update_{2w} \cdot \xi^{\mu}_{1mw}) = 0 \tag{9}$$

$$E(update_{1w} \cdot \xi^{\mu}_{2mw}) = 0 \tag{10}$$

The moment in (8) matches the observed common user and the predicted common user given γ_{12} and ρ. The moments in (9) and (10) are based on the assumption that the update history of app 1 (app 2) should not enter the utility of app 2 (app 1) directly. I use update history of the iOS version of the same app because update history of Android apps is not reliable.[20] Updates of the iOS version cannot possibly change utilities of any other Android app. Specifically, update history is described by three variables: the cumulative numbers of small updates, medium updates, and major updates.[21] Because I use cumulative number of updates, my IVs can capture the effects of updates even if users do not adopt updates immediately. As shown in the subscript of $update_{1w}$, this update history is common to all users in China and co-linear with time fixed effects. To circumvent this problem, I create market-specific update history variables, which allows each market to respond to the updates differently. Therefore, there are at most $3 \times M$ moments implied by (9).

[20] One reason is that developers can publish Android apps outside mainstream app stores.

[21] "Small", "medium", and "major" are defined by the digits of version numbers.

The identification of linear parameters $\boldsymbol{\beta}$ is straightforward and relies on the following moment conditions:

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{1mw}) = 0 \tag{11}$$

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{2mw}) = 0 \tag{12}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{1mw}) = 0 \tag{13}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{2mw}) = 0 \tag{14}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{3mw}) = 0 \tag{15}$$

To estimate network effects $\boldsymbol{\zeta}$, I use the following moment conditions as discussed in section 3.4:

$$E(s^{r-m}_{1m(w-1)} \cdot \xi^{\mu}_{1mw}) = 0 \tag{16}$$

$$E(s^{r-m}_{2m(w-1)} \cdot \xi^{\mu}_{2mw}) = 0 \tag{17}$$

Based on the above moments from (8) to (17), the GMM estimation is to minimize

$$\min_{\boldsymbol{\theta}} \boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi} + (c^*_{12} - c_{12})^2 \tag{18}$$

where $\boldsymbol{\xi}$ is the stack of all market level error terms and $\mathbf{z}_{mw} = (\mathbf{x}_{mw}, updata_{1w}, updata_{2w}, s^{r-m}_{1m(w-1)}, s^{r-m}_{2m(w-1)})$ collects all the exogenous variables. I separate $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ from $(c^*_{12w} - c_{12w})^2$ to highlight the fact that $\boldsymbol{\theta}_1$ enters $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ linearly and does not enter $(c^*_{12w} - c_{12w})^2$ given $\boldsymbol{\delta}$. Therefore, we can limit the globe search to $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$ as $\boldsymbol{\theta}_1$ is a linear function of $\boldsymbol{\delta}$.

This estimation follows Berry *et al.* (1995) with an inversion step and a global search step. I need to find the values of $\boldsymbol{\delta}$ that match the five observed market outcomes $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ given (γ_{12}, ρ). This is to solve the following system of nonlinear equations,

$$\mathbf{y}^*_{mw} = \mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho). \tag{19}$$

Note that each component in $\mathbf{y}_{mw}$ is monotonically increasing in the corresponding component in $\boldsymbol{\delta}$. For example, given $(\delta^{\mu}_{2mw}, \delta^{\gamma}_{1mw}, \delta^{\gamma}_{2mw}, \delta^{\gamma}_{3mw})$ and (γ_{12}, ρ), s_{1mw} is increasing in δ^{μ}_{1mw}. I solve (19) by iterating on $\boldsymbol{\delta}$ analogously to the contraction mapping used by Berry *et al.* (1995) and Gowrisankaran & Rysman (2012):

$$\boldsymbol{\delta}^{new} = \boldsymbol{\delta}^{old} + \boldsymbol{\phi} \cdot \{ln(\mathbf{y}^*_{mw}) - ln(\mathbf{y}_{mw}(\boldsymbol{\delta}^{old}, \gamma_{12}, \rho))\} \tag{20}$$

where $\boldsymbol{\phi}$ are five positive tuning parameter used in the iterations.

Despite the appealing features of quadratic utility functions, there is no analytical solution to quadratic optimization problems. Therefore, I use numerical integration to form expectations of $\mathbf{y}_{mw}$. Let N_s be the number of simulations used for integration. We have

$$\mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho) = \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{y}_{nmw}(\boldsymbol{\delta}, \gamma_{12}, \varepsilon_{n1mw}, \varepsilon_{n2mw}) \tag{21}$$

where $\mathbf{y}_{nmw}$ are the individual outcomes for the nth draw of $(\varepsilon_1, \varepsilon_2)$. In practice, I use 1000 Halton draws in the integration.

To summarize, the estimation consists of the following steps:

1. For a pair of (γ_{12}, ρ), invert out $\boldsymbol{\delta}(\gamma_{12}, \rho)$ with the mapping described in (20).
2. Calculate $c_{12}(\boldsymbol{\delta}(\gamma_{12}, \rho), \gamma_{12}, \rho)$ and $\boldsymbol{\xi}(\boldsymbol{\delta}(\gamma_{12}, \rho), \mathbf{z})$. Based on them, calculate the value of GMM function in (18).
3. Find (γ_{12}, ρ) that minimizes the GMM value calculated in step 2.

5 Estimation Results

I estimate the model on three representative pairs of apps to see how the model performs in different situations. For the first two pairs, I choose them because they are obviously a pair of substitutes (Baidu Map and Amap) and a pair of complements (Baidu and Baidu Map). A satisfactory model can infer the relationships from data. I study WeChat and Kwai because users spend a lot of time on them so that budget competition may be salient and the relationship between the two is *a priori* ambiguous. For each pair of apps, I estimate the model with a balanced panel of 12 weeks.[22]

5.1 Substitutes

The first pair of apps are Baidu Map (app 1) and Amap (app 2), two dominant players in the mobile map market in China. During the 13 weeks, the number of active users of Baidu Map increased from 90 million to 110 million and that of Amap increased from 75 million to 100 million. The number of common users between Baidu Map and Amap increased from 11 million to 18 million. The summary statistics of market level variables are in Table 2.

[22] I use lagged variables in the moment conditions of network effects.

Table 2: Summary Statistics of Baidu Map and Amap

Variables	Mean	StdDev	Min	Max	Unit	Obs
$s^*_{BaiduMap}$	0.1541	0.0352	0.0641	0.2988	-	3095
s^*_{Amap}	0.1322	0.0318	0.0656	0.2607	-	3095
$t^*_{BaiduMap}$	0.0373	0.0136	0.0022	0.1238	hour	3095
t^*_{Amap}	0.0744	0.0304	0.011	0.2877	hour	3095
t^*_{3mw}	17.357	3.0635	9.56	23.8218	hour	3095

Note: $s^*_{BaiduMap}$ (s^*_{Amap}) is the number of active users of Baidu Map (Amap) divided by the number of active users of Android cellphones. $t^*_{BaiduMap}$ (t^*_{Amap}, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphone.
Data Source: iResearch.

The first three columns of Table 3 present the estimates of γ_{12} and ρ with different IVs. These estimates have the same signs. I use column (3) as my main results because both IVs are used. Baidu Map and Amap are estimated to be substitutes ($\hat{\gamma}_{12} = -0.923$), which confirms our prior belief. $\hat{\rho} = 0.59$ suggests that Baidu map and Amap target the same group of users. Because the two apps offer similar functions, users who need Baidu Map will also find Amap useful. For the same reason, users who already use one would find the other redundant. A negative γ_{12} and a large ρ are what characterize a pair of direct competitors. For comparison, I also estimate γ_{12} with the assumption $\rho = 0$ in column (4) of Table 3. In this specification, Baidu Map and Amap are estimated to be almost independent apps. γ_{12} and ρ "substitute" each other in explaining the common user data: from column (3) to column (4), as ρ decreases from 0.59 to 0, γ_{12} increases from -0.923 to 0.04.

Table 3: Estimates for Baidu Map and Amap

	(1)	(2)	(3)	(4)
γ_{12}	-0.189	-0.935	-0.923	0.04
	(0.006)	(0.003)	(0.002)	(0.001)
ρ	0.1	0.6	0.59	0
	(0.004)	(0.002)	(0.005)	-
ζ_1	3.94	4.032	4.02	3.871
	(0.0084)	(0.0088)	(0.0085)	(0.0085)
ζ_2	4.65	4.414	4.665	4.549
	(0.0123)	(0.0122)	(0.0121)	0.0119
$update_1$ as IV	No	Yes	Yes	No
$update_2$ as IV	Yes	No	Yes	No

Note: Standard errors are in parentheses.
Data Source: The author's calculations.

γ_{12} and network parameters are structural parameters in a utility function. To understand the economic significance of γ_{12}, I quantify the value of substitutability/complementarity with compensating variation (CV). I calculate the compensating variations (CVs) of apps and pairs of apps. More specifically, I increase the total amount of time a user has to compensate for the loss of an app (or a pair of apps), such that the

Table 4: Compensating Variations and Network Elasticities of Baidu Map and Amap

	(1)	(2)	(3)
CV of Baidu Map	20.179	18.621	18.7903
CV of Amap	38.6129	35.8482	35.9233
CV of Both	59.4439	62.0703	63.019
Substitutability (Complementarity)	-0.6519	-8.6012	-8.3054
Network Elasticity 1	1.1	1.2	1.4
Network Elasticity 2	0.9	1.3	1.2
$(\hat{\gamma}_{12}, \hat{\rho})$	(-0.189, 0.1)	(-0.935, 0.6)	(-0.923, 0.59)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The results are based on a hypothetical market whose market outcomes are the mean of the variables in the whole balanced sample used in the estimation.
2, CV cells are the sum of CV in hours for all 1000 simulated users.
3, The network elasticities are calculated with simulated market outcomes. Therefore, the increments of market shares are 0.1 percentage points.
Data Source: The author's calculations.

maximized utilities are the same before and after shutting down the app (the pair). The difference between the sum of CVs of each app and the CV of the pair of the apps is the value of substitutability/complementarity. This is the utility specification in the discrete model of Gentzkow (2007). This means that discrete choice models as in Gentzkow (2007) are a special case of my model. The CVs of Baidu Map, Amap, and the pair and the value of substitutability/complementarity are in Table 4. Substitutability is economically significant. Using the results in the last column in Table 4, the CV of Baidu Map is 18.8 hours for 1000 users when Amap is available. If Amap exits the market, consumers have one fewer option and the CV of Baidu Map would be 18.8+8.3=27.1 hours.

To understand the economic significance network parameters, I calculate network elasticities for Baidu Map and Amap. I increase the market share of Baidu Map (Amap) in the reference group by 1 percentage point and simulate the market outcomes. The network elasticity of Baidu Map (Amap) is the corresponding increase of market share in percentage points. From Table 4, the elasticities are large and fluctuate around 1 across specifications. Note that the reference group is a province with ten demographic groups and the focal market is just one of the ten groups.

5.2 Complements

The second pair of apps are Baidu (app 1) and Baidu Map (app 2). As the names suggest, they are developed by the same company, Baidu, Inc. The core functions of Baidu app are searching and news stream. I expect search engines and maps, and hence Baidu and Baidu Map, are complements. For example, when users search for locations, the first results often direct users to map apps. During the 13 weeks, the number of active users of Baidu fluctuated around 177 million. The number of common users between Baidu and Baidu Map increased from 30 million to 37 million. The summary statistics of market level variables are in Table 5. Note that there are slight differences between the summary statistics of s^*_{2mw} in Table 5 and the summary statistics of s^*_{1mw} in Table 2. This arises because the balanced panels of the two pairs are slightly different.

Table 5: Summary Statistics of Baidu and Baidu Map

Variables	Mean	StdDev	Min	Max	Unit
s^*_{Baidu}	0.2631	0.0483	0.1105	0.4267	-
$s^*_{BaiduMap}$	0.1531	0.0352	0.0641	0.2989	-
t^*_{Baidu}	0.3149	0.083	0.0464	0.719	hour
$t^*_{BaiduMap}$	0.0369	0.0137	0.0022	0.1238	hour
t^*_{3mw}	16.9487	3.103	9.3517	23.5481	hour

Note: s^*_{Baidu} ($s^*_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t^*_{Baidu} ($t^*_{BaiduMap}$, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in Table 6. The coefficients in the first column are quite different from those in the other two columns. This suggests that my IV is not ideal. As before, I treat results in column (3) as the main results. The main results confirm our prior belief that Baidu and Baidu Map are complements. As in the previous subsection, I calculate the CVs of Baidu Map, Amap, and the pair and the value of substitutability/complementarity as well as network elasticities in Table 7. The CVs of Baidu Map are in line with those in Table 4. The complementarity is small in magnitude but large as a percentage of the CV of Baidu Map for columns (2) and (3). This suggests that Baidu Map relies on Baidu but not the reverse. Baidu has other apps named after Baidu and they are in the "Baidu Core" business unit. It is reasonable to assume that those apps are also complements with the Baidu app.

Table 6: Estimates for Baidu and Baidu Map

	(1)	(2)	(3)
γ_{12}	0.936	-0.259	0.096
	(0.0003)	(0.0002)	(0.0003)
ρ	-0.54	0.38	0.04
	(0.0005)	(0.0001)	(0.0001)
ζ_1	3.512	3.4894	3.487
	(0.0064)	(0.0065)	(0.0065)
ζ_2	1.504	4.1123	3.582
	(0.0178)	(0.0111)	(0.0114)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note: Standard errors are in parentheses.
Data Source: The author's calculations.

Table 7: Compensating Variations and Network Elasticities of Baidu and Baidu Map

	(1)	(2)	(3)
CV of Baidu	180.704	188.36	189.903
CV of Baidu Map	19.833	18.904	19.6514
CV of Both	189.382	214.211	207.972
Substitutability (Complementarity)	11.155	-6.947	1.582
Network Elasticity 1	1.1	0.7	1.1
Network Elasticity 2	0.4	1.1	0.6
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(0.936, -0.54)	(-0.259, 0.38)	(0.096, 0.04)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The results are based on a hypothetical market whose market outcomes are the mean of the variables in the whole balanced sample used in the estimation.
2, CV cells are the sum of CV in hours for all 1000 simulated users.
3, The network elasticities are calculated with simulated market outcomes. Therefore, the increments of market shares are 0.1 percentage points.
Data Source: The author's calculations.

5.3 Independent Apps

The last pair of apps I studied are WeChat (app 1) and Kwai (app 2). WeChat is the flagship app of Tencent, which was first released in 2011. By the first quarter of 2017, the main functions include instant messaging, social media ("Moments"), mobile payment ("WeChat Pay"), content distribution ("Subscriptions"), and app store ("mini program"). It is a super-app used by almost all smartphone users in China. From Table 8, users spent about one fourth of their smartphone time on WeChat. Given it market dominance, WeChat is competing with all other apps for user time. Kwai is a video-sharing app that features short videos and live-streaming. Thanks to the recommendation algorithms, short video apps like Kwai and Tik Tok are often referred to as "a black hole of time". In terms of their functions, WeChat and Kwai seems to be independent or weak substitutes in the broad sense of social networking. However, WeChat and Kwai share a lot of common users. During the 13 weeks, the number of active users of WeChat fluctuated around 555 million, and those of Kwai increased from 78 million to 81 million. The number of common users between WeChat and Kwai is about 70 million. It is tempting to conjecture that the two apps are complements based on the number of common users. Overall, the competitive relationship between WeChat and Kwai is ambiguous.

Table 8: Summary Statistics of WeChat and Kwai

Variables	Mean	StdDev	Min	Max	Unit
s^*_{WeChat}	0.8498	0.0521	0.6958	0.958	-
s^*_{Kwai}	0.1224	0.0282	0.0543	0.2652	-
t^*_{WeChat}	4.5517	0.8122	2.5735	7.7646	hour
t^*_{Kwai}	0.1952	0.0553	0.0297	0.4806	hour
t^*_{3mw}	12.7142	2.5392	5.9482	17.2365	hour

Note: s^*_{WeChat} (s^*_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t^*_{WeChat} (t^*_{Kwai}, t^*_{3mw}) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in Table 9. As before, I treat results in column (3) as the main results. $\hat{\gamma}_{12} = -0.036$ refutes the conjecture that WeChat and Kwai are complements. The large number of common users is explained by the positive correlation between the preference for WeChat and that for Kwai ($\hat{\rho} = 0.504$). This suggests that the "budget-competition" effect of one app on the other may be large. With these estimates, I calculate the CVs of WeChat, Kwai, and the pair and the value of substitutability/complementarity in Table 10. When WeChat is shut down, the CV of Kwai would increase by about 100%. If Kwai is shut down, the CV of WeChat would also increase significantly. WeChat and Kwai are competing for user time despite a large number of common users and seemingly independent functions.

Comparing the results of the three pairs of apps, the substitutability/complementarity term is positive if and only if γ_{12} is positive. Estimated γ_{12} cannot be compared across models. $\hat{\gamma}_{12} = -0.923$ in column (3) of Tables 4 and $\hat{\gamma}_{12} = -0.036$ in column (3) of Table 10. By contrast, the substitutability/complementarity term is -8.3054 hours in the first case and -91.474 hours in the second case.

Table 9: Estimates for WeChat and Kwai

	(1)	(2)	(3)
γ_{12}	0.351	-0.036	-0.036
	(0.0001)	(0.0001)	(0.0002)
ρ	-0.68	0.504	0.504
	(0.0001)	(0.0002)	(0.0002)
ζ_1	4.236	4.271	4.271
	(0.0043)	(0.0043)	(0.0043)
ζ_2	3.509	3.845	3.845
	(0.0157)	(0.0057)	(0.0057)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note: Standard errors are in parentheses.
Data Source: The author's calculations.

Table 10: Compensating Variations and Network Elasticities of WeChat and Kwai

	(1)	(2)	(3)
CV of WeChat	4698.878	4836.637	4836.637
CV of Kwai	95.741	92.277	92.277
CV of Both	4699.046	5020.388	5020.388
Substitutability (Complementarity)	95.573	-91.474	-91.474
Network Effects1	0.8	0.9	0.9
Network Effects2	0.6	0.6	0.6
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(0.351, -0.68)	(-0.036,0.504)	(-0.036,0.504)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The results are based on a hypothetical market whose market outcomes are the mean of the variables in the whole balanced sample used in the estimation.
2, CV cells are the sum of CV in hours for all 1000 simulated users.
3, The network elasticities are calculated with simulated market outcomes. Therefore, the increments of market shares are 0.1 percentage points.
Data Source: The author's calculations.

6 Functional Competition and Budget Competition

To better understand the competitive relationship between apps, I perform two sets of exercises. In the first exercise, I shut down one of the two apps to see how the usage of the other app would change. I calculate diversion ratios based on the counterfactual results: if one app exits the market, how much of its usage will be diverted to another app?[23] With these simulations, app developers would know who competes time away from their apps. In the second part, I decompose the competitive effects of one app on another into two parts: "functional competition" and "budget competition". The decomposition clarifies the references to "competition for time/attention" in the business community and informs the legal debate on the definition of "relevant market" in antitrust cases. In addition, I propose a model-free metric to quickly gauge the magnitude of budget competition. This metric can aid antitrust authorities in assessing the size of budget competition and determining whether a proposed merger should be contested.

6.1 Competitive Effects

In this subsection, I simulate counter-factuals in which one of the apps is shut down. For each pair of apps, I simulate market outcomes for a hypothetical market whose market outcomes are the mean of the whole panel with different sets of (γ_{12}, ρ).[24] In Table 11, columns (2) and (3) present counter-factuals for the baseline

[23]Diversion ratios are an important tool of antitrust authorities to analyze horizontal mergers. The 2010 Horizontal Merger Guidelines note:"Diversion ratios between products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects, with higher diversion ratios indicating a greater likelihood of such effects."

[24]Specifically, I invert out $\boldsymbol{\delta}$ for the two pairs of (γ_{12}, ρ) and then set δ^{μ}_{2mw} (δ^{μ}_{1mw}), the mean marginal utilities of app 2 (app 1), to be a very small number, -20, and simulate the market outcomes.

estimates and columns (4) and (5) present counter-factuals for the estimates in the last column of Table 3 where we assume $\rho = 0$.

Table 11: Counter-factuals of Baidu Map and Amap

	Observed Outcomes	Baseline		Assume $\rho = 0$	
		No Baidu Map	No Amap	No Baidu Map	No Amap
	(1)	(2)	(3)	(4)	(5)
$s_{BaiduMap}$	0.1541	0	0.19	0	0.154
s_{Amap}	0.1322	0.156	0	0.133	0
$t_{BaiduMap}$	0.0373	0	0.0483	0	0.0372
t_{Amap}	0.0744	0.0836	0	0.074	0
t_3	17.357	17.357	17.358	17.357	17.358
Diversion Ratio (Share)	-	0.1778	0.3186	0	0
Diversion Ratio (Time)	-	0.2428	0.1478	-0.0062	-0.0027
$(\hat{\gamma_{12}}, \hat{\rho})$		(-0.923, 0.59)		(0.04,0)	

Note:
1, The observed outcomes in column (1) are the mean of the variables in the whole balanced sample used in the estimation.
2, $s_{BaiduMap}$ (s_{Amap}) are the number of active users of Baidu Map (Amap) divided by the number of active users of android cellphones. $t_{BaiduMap}$ (t_{Amap}, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphones.
3, Diversion ratio (Share) is the increase in the market share of the remaining app divided by the market share of *unique* user of the exit app.
4, Diversion ratio (Time) is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

The two sets of simulated outcomes are vastly different. When $\gamma_{12} = 0.04$ and $\rho = 0$, shutting down one app has almost no effect on the other. By contrast, when $\gamma_{12} = -1.15$ and $\rho = 0.7711$, the market share of Baidu Map would increase by 3.6 percentage points if I shut down Amap, and the market share of Amap would increase by 2.4 percentage points if I shut down Baidu Map. Consider the case of shutting down Amap when $(\gamma_{12}, \rho) = (-0.923, 0.59)$. The inversion process reveals that there are 1.9% users that use both Baidu Map and Amap. Therefore, there are 11.3% users using Amap but not Baidu Map. When Amap is shut down, 3.6% out of the 11.3% unique users turn to Baidu Map. This is how we calculate diversion ratio in terms of market share: $\frac{\Delta s_1}{s_2^{unique}} = \frac{0.19-0.154}{0.132-0.019} = 0.3186$. When we focus on time spent, diversion ratio is simply the increase in the total time spent on Baidu Map divided by the total time spent on Amap before its exit: $\frac{\Delta t_1}{t_2} = \frac{0.0483-0.0373}{0.0744} = 0.1478$. In other words, when Amap exits the market, 15% of its time goes to Baidu Map and 85% of its time goes to offline activities and the generic app. As Baidu Map and Amap are close competitors, we may expect the diversion ratio to be higher. However, there are other map apps consumers can use such as Tencent Map, which is included in the generic app (app 3). Note that the effects of shutting down Amap on Baidu Map are larger than the reverse. When Baidu Map is not available, the market share of Amap would only increase by 2.4 percentage points.

The counter-factual results for Baidu and Baidu Map with the baseline estimates are in Table 12. Shutting down Baidu Map has negligible effects on Baidu.[25] However, shutting down Baidu would reduce the market share of Baidu Map by 4.2 percentage point and the time spent on Baidu Map by more than 50% of the observed level. It is therefore no surprise that the company Baidu treats the Baidu app as its core business.[26] A caveat to my findings is that the discovery process of apps is not modeled in this paper. Cross-promotion between apps developed by the same company is a widely used marketing strategy.[27] Promoting Baidu Map with Baidu will lead to a persistent large number of common users if there are significant switching costs. Diversion ratio is -0.9 when Baidu Map exits the market. Therefore, even if Baidu Map is unprofitable, Baidu may choose to maintain it's operation due to its contribution to the usage of the Baidu app.

Table 12: Counter-factuals of Baidu and Baidu Map

	Observed Outcomes (1)	No Baidu (2)	No Baidu Map (3)
s_{Baidu}	0.263	0	0.26
$s_{BaiduMap}$	0.153	0.11	0
t_{Baidu}	0.3149	0	0.2815
$t_{BaiduMap}$	0.0369	0.0176	0
t_3	16.949	16.953	16.949
Diversion Ratio (Share)	-	-0.2028	-0.0194
Diversion Ratio (Time)	-	-0.0613	-0.9052
$(\hat{\gamma}_{12}, \hat{\rho})$	(0.096,0.04)		

Note:
1, The observed outcomes in column (1) are the mean of the variables in the whole balanced sample used in the estimation.
2, s_{Baidu} ($s_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t_{Baidu} ($t_{BaiduMap}$, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio (Share) is the increase in the market share of the remaining app divided by the market share of unique user of the exit app.
4, Diversion ratio (Time) is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

The counter-factuals for WeChat and Kwai with the baseline estimates are in Table 13. As expected, the competitive effects of WeChat on Kwai are larger than the reverse. Given that WeChat is the dominant player and Kwai is the entrant, it would be more interesting to focus on the effect of Kwai on WeChat. The market share of WeChat does not change in response to the exit of Kwai. This is because almost all Kwai users (11.7% out of 12.2%) already use WeChat. A diversion ratio of 15% means that about 15% of the time spent on Kwai comes from WeChat. The remaining 85% mostly comes from offline activities.[28] 0.03 (4.58-4.55) hours seem to be small for a representative Android smartphone user. For the 122 users who use

[25] The difference between 0.263 and 0.26 is due to simulation error.
[26] In the annual reports of Baidu, Inc., Baidu app, Baidu Map and other apps named after Baidu are in the "Baidu Core" business group.
[27] See The Ultimate Mobile Marketing Playbook by App Annie at https://www.appannie.com/en/insights/aso-app-store-optimization/ultimate-mobile-marketing-playbook/
[28] I assume away complementarity and correlated preference between app 1 (app 2) and the generic app.

Kwai among the 1000 simulations, their time spent on WeChat would increase on average by 15 minutes if Kwai is shut down. For WeChat, competition from Kwai is significant. A caveat is that the competitive effects of Kwai on WeChat I estimated with data from 2017 is likely to be a lower bound. Usage of WeChat and that of Kwai have grown significantly since then. Tencent also added short-video and live streaming to WeChat to directly compete with Kwai and Tik Tok in the first quarter of 2020. The competitive effects will be much larger now.

Table 13: Counter-factuals of WeChat and Kwai

	Observed Outcomes (1)	No WeChat (2)	No Kwai (3)
s_{WeChat}	0.85	0	0.851
s_{Kwai}	0.1224	0.177	0
t_{WeChat}	4.5517	0	4.5802
t_{Kwai}	0.1952	0.3247	0
t_3	12.7142	12.7625	12.716
Diversion Ratio (Share)	-	0.0763	0
Diversion Ratio (Time)	-	0.0284	0.1479
$(\hat{\gamma}_{12}, \hat{\rho})$		(-0.036,0.504)	

Note:
1, The observed outcomes in column (1) are the mean of the variables in the whole balanced sample used in the estimation.
2, s_{WeChat} (s_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t_{WeChat} (t_{Kwai}, t_3) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio (Share) is the increase in the market share of the remaining app divided by the market share of unique user of the exit app.
4, Diversion ratio (Time) is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

6.2 Decomposition

In the previous sections, I simulate the competitive effects of one app on another. For example, if Kwai is shut down, Kwai users will increase their time spent on WeChat on average by about 15 minutes. In this subsection, I decompose the competitive effects of one app on another app into two parts: "functional competition" and "budget competition". When an app is shut down,[29] users will reallocate their time to the remaining apps. There are two reasons the exit of an app could affect other apps. First, because of substitutability (complementarity), users find the remaining apps more (less) appealing. Hence users will spend more (less) time on the remaining apps. This is the functional-competition effect. Second, the exit of an app means time that used to be spent on that app is now "free", and users can allocate it to the remaining apps. This is the budget-competition effect.

Consider the original bundle, $\mathbf{t}^o = \arg\max U(\mathbf{t})$, and the final bundle, $\mathbf{t}^f = \arg\max U(\mathbf{t}|\mu_j = -\infty)$,

[29]The following analysis applies to entry and price changes as well.

subject to the same time constraint $\sum_{k=0}^{J} t_k = T$. $\mathbf{t}^f - \mathbf{t}^o$ summarizes the effects of the exit of app j. To formally separate budget competition and functional competition, I introduce an intermediate step. In the intermediate step, the consumer chooses an intermediate bundle, $\mathbf{t}^i$, such that the marginal utilities of $\mathbf{t}^i$ equal to the marginal utility of $\mathbf{t}^o$ except for app j. That is, $\mathbf{t}^i$ is the solution to the following system of linear equations:

$$\frac{\partial U(\mathbf{t}^i|\mu_j = -\infty)}{\partial t_k^i} = \frac{\partial U(\mathbf{t}^o)}{\partial t_k^o} \quad \forall k \neq j \& t_k^i \geq 0 \tag{22}$$
$$t_j^i = 0.$$

Note that the time constraint is irrelevant in this step. $\mathbf{t}^i - \mathbf{t}^o$ is the functional-competition effect because the difference is entirely due to complementarity or substitutability among apps. $\mathbf{t}^f - \mathbf{t}^i$ is therefore the budget-competition effect. With estimated models in the previous section, the numerical calculation of budget competition is simple and the results are in Table 15.

As expected, the budget-competition effects are negligible for the first two pairs because users spend a small amount of time on Baidu Map and Amap. If apps of interest are small in terms of time spent, researchers can estimate a model with no budget constraint as in Thomassen *et al.* (2017), and capture virtually all competitive effects. For WeChat and Kwai, the budget-competition effects are larger because their preferences are positively correlated, and users spend a substantial amount of time on them. Market shares and usage of the two apps increased significantly over the past years. The budget-competition effects would be much larger now.

6.3 Analytical Characterization of Budget Competition

In the model I estimated, (22) can be reduced to one single equation when app 2 exits the market[30]:

$$\mu_1 + \gamma_1 t_1^o + \gamma_{12} t_2^o = \mu_1 + \gamma_1 t_1^i.$$

In Table 14, I provide general analytic solutions of functional competition and budget competition depending on if t_1^o and t_1^i are strictly positive. Note that for app 2 to have any competitive effect, be it budget competition or functional competition, t_2^o must be strictly positive, which is implicitly assumed in Table 14. The results in Table 14 are intuitive. Let us focus on the first row and assume $\gamma_{12} \leq 0$, $t_1^o > 0$ and $t_1^i > 0$. In the intermediate step, $\frac{\gamma_{12}}{\gamma_1} t_2^o$ are diverted to app 1 due to functional competition. That leaves the user

[30] The marginal utilities of app 3 and the offline option would not change with the exit of app 2 because I assume $\gamma_{20} = \gamma_{23} = 0$.

Table 14: Analytical Decomposition

t_1^o	t_1^i	Functional Competition ($t_1^i - t_1^o$)	Budget Competition ($t_1^f - t_1^i$)
$t_1^o > 0$	$t_1^i > 0$	$\frac{\gamma_{12}}{\gamma_1} t_2^o$	$(1 - \frac{\gamma_{12}}{\gamma_1}) t_2^o \frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$
$t_1^o > 0$	$t_1^i = 0$	$-t_1^0$	$\max\{0, \frac{T - \frac{\mu_1 - \mu_0}{\gamma_0} - \frac{\mu_1 - \mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}\}$
$t_1^o = 0$	$t_1^i > 0$	$\frac{\mu_2 - \mu_1}{\gamma_1} + \frac{\gamma_2}{\gamma_1} t_2^o$	$[(1 - \frac{\gamma_2}{\gamma_1}) t_2^o - \frac{\mu_2 - \mu_1}{\gamma_1}] \frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$
$t_1^o = 0$	$t_1^i = 0$	0	$\max\{0, \frac{T - \frac{\mu_1 - \mu_0}{\gamma_0} - \frac{\mu_1 - \mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}\}$

Note: This table presents analytical characterization of functional competition and budget competition. The calculations are in Appendix B.

with $(1 - \frac{\gamma_{12}}{\gamma_1}) t_2^o$ of free time, which is allocated to the remaining options *proportional to the inverse of their satiation parameters*. The intuition is similar for complements. When $\gamma_{12} > 0$, t_1^i decreases by $|\frac{\gamma_{12}}{\gamma_1}| t_2^o$. Therefore, the free time is $(1 + |\frac{\gamma_{12}}{\gamma_1}|) t_2^o$ and larger than t_2^o. For *users* with $t_1^o > 0$ and $t_1^i > 0$, the gross diversion ratio consists of two parts: diversion because of functional competition ($\frac{\gamma_{12}}{\gamma_1}$) and diversion because of budget competition ($(1 - \frac{\gamma_{12}}{\gamma_1}) \frac{1}{\gamma_1} \frac{1}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}$):

$$\text{Diversion Ratio} = \frac{\gamma_{12}}{\gamma_1} + (1 - \frac{\gamma_{12}}{\gamma_1}) \frac{1}{\gamma_1} \frac{1}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}. \tag{23}$$

From Table 14 and (23), budget competition is increasing in γ_{12} and γ_1. Another immediate implication of (23) is that budget competition may dominate functional competition and apps with a positive γ_{12} can be gross substitutes. How likely is this event? The answer depends on $\frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$, which describes how free time is allocated to remaining options. It is a nonlinear function of deep parameters in a structural model. To calculate the size of budget competition, we must estimate a full model as in Section §5. However, with some assumptions, we can transform $\frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$ into ratios of real-world variables.

6.4 A Model-Free Metric of Budget Competition

I propose a model-free metric to gauge budget competition. To motivate this metric, consider two apps of interest, app 1 and app 2. Suppose we observe a user and her time spent on app 1 and app 2 by this user,

t_1^* and t_2^*. The utility function of this user is

$$\max_{t_0,t_1,t_2\geq 0} t_0 + t_1 + t_2 - 0.0005t_0^2 + \frac{1}{2}\gamma_1 t_1^2 + \frac{1}{2}\gamma_2 t_2^2 \tag{24}$$
$$s.t. \quad t_0 + t_1 + t_2 = T$$

where t_0 is the time spent on any other activities, online or offline. I assume $\mu_1 = \mu_2 = 1 = \mu_0$ and $\gamma_{12} = 0$ to avoid complicated estimation and obtain a closed-from metric. The solution to this maximization problem is $(t_0, t_1, t_2) = (T - t_1^* - t_2^*, t_1^*, t_2^*)$. We can solve for γ_1 and γ_2 with the FOCs. Consider the exit of app 2:

$$\max_{t_0,t_1\geq 0} t_0 + t_1 - 0.0005t_0^2 + \frac{1}{2}\gamma_1 t_1^2 \tag{25}$$
$$s.t. \quad t_0 + t_1 = T$$

Let $(T - t_1', t_1')$ be the optimal time allocation when app 2 is absent. I prove in the appendix that

$$\Delta t_1 = t_1' - t_1^* = t_2^*(\frac{t_1^*}{t_0^* + t_1^*}) = t_2^*(\frac{t_1^*}{T - t_2^*}). \tag{26}$$

When app 2 exits the market, time that used to be spent on app 2 will be reallocated to the remaining apps *proportional to their time share before the exit.* $\frac{1}{\gamma_1(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}$ in our previous analysis becomes $\frac{t_1^*}{t_0^*+t_1^*} = \frac{t_1^*}{T-t_2^*}$.[31] The budget-competition effect of app 2 on app 1 is increasing in t_1^* and t_2^* if $t_2^* < 0.5T$. The numerator in (26) are the same for app 1 and app 2. Therefore, budget competition is largely symmetric between app 1 and app2 unless $T - t_1 \gg T - t_2$ or the reverse. Adding more independent options (apps) would not change (26).

When we observe a group of users, the overall budget-competition effect is

$$\sum_i \Delta t_{i1} = \sum_i t_{i2}^*(\frac{t_{i1}^*}{T - t_{i2}^*}). \tag{27}$$

The total size of budget competition increases with the correlation between t_{i1}^* and t_{i2}^*. Alternatively, if we only observe aggregate usage of a set of markets, we can adopt a representative user approach and plug in average usage of app 1 and app 2 into (26).

Two assumptions are crucial to this metric. Firstly, I assume all options are independent to avoid estimating γ_{12} in a full model. Therefore this metric is more accurate when we expect $\mid \gamma_{12} \mid$ to be close

[31] Adding more independent options would not change the budget competition effect of app 2 on app 1. t_3 merges into the outside option.

to zero. However, we can combine our belief of γ_{12} with the metric using 23. We will see an example in two paragraphs. Secondly, I assume $\mu_1 = \mu_2 = 1 = \mu_0$ to have a closed-form metric of budget competition. Therefore, any difference in t_1 and t_2 is attributed to γ_1 and γ_2. This would be a serious concern if $s_1 \gg s_2$ or $s_1 \ll s_2$.

To what extend can we trust this metric? I compare this metric with the results calculated from the full model in Table 15. This simple metric produces estimates reasonably close to the results from the full model despite the simplifying assumptions. One exception is the budget competition of Kwai on WeChat. 5.2948 is 8 times larger than 0.657. The key reason is that the market share of WeChat is about 7 times that of Kwai, which violates the assumption that $\mu_1 = \mu_2$. This is less of a concern now.

Time spent on WeChat and Kwai and their market shares increased significantly since 2017. $\mu_1 = \mu_2$ is a more realistic assumption now. A reasonable guess is that for all smartphone users in China, $t_1^* = 20$ and $t_2^* = 10$ for a typical week in 2023. If Kwai exits the market, the budget-competition effect on WeChat is $10 \times \frac{20}{168-10} = 1.27$ for each one of the about 1 billion smartphone users. The gross diversion ratio in (23) can be positive for a pair of complements. For the sake of argument, let us assume $\frac{\gamma_{12}}{\gamma_1} = -0.05$ for WeChat and Kwai and we approximate $\frac{1}{\gamma_1} \frac{1}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}$ with $\frac{20}{168-10} = 0.127$. The gross diversion ratio is $-0.05 + 1.05 \times 0.127 = 0.083$. Therefore, a pair of complementary apps can be gross substitutes. Another way to use (23) is to calculate the threshold of $\frac{\gamma_{12}}{\gamma_1}$ above which the two apps are gross substitutes. In this case, the threshold is $\frac{-0.127}{1-0.127} = -0.145$. In other words, the diversion ratio implied by complementarity must be larger than 14.5% for WeChat and Kwai to be gross complements. This is highly improbable for WeChat and Kwai. Our analysis suggests that a merger of complementary apps can hurt users because of budget competition. Budget competition implies that being too "large" *per se* is a source of antitrust concern when analyzing merges of apps.

Rather than taking the metric at face value, we can use this metric to gauge the order of magnitude of budget competition, which is often exaggerated or downplayed in business and legal settings. In the high-profile antitrust lawsuit filed by Qihu against Tencent in 2012, Tencent was accused of abusing its market dominance in the instant messaging market, wherein its software QQ had a market share of 80%–95% according to different measures. Tencent countered that the relevant market should include virtually all Internet companies and their software because they were all competing for user time.[32] Tencent exaggerated budget competition to obfuscate its market dominance in the instant messaging market. With this metric and some aggregate usage data, we can easily dismiss this argument.

Netflix CEO Reed Hastings stated in 2017 that "We're competing with sleep" (Hern, 2017). Using this

[32] The court did not accept this market definition and stick to the market definition based on functions.

Table 15: Functional Competition and Budget Competition

	Baidu Map and Amap	Baidu and Baidu Map	WeChat and Kwai
The Exit of App 1			
Budget Competition	0.0046	0.027	3.846
Functional Competition	9.1469	-2.672	125.594
Total Effects on App 2	9.1514	-2.645	129.44
The Model-Free Metric	0.0165	0.0693	5.4359
The Exit of App 2			
Budget Competition	0.0137	0.0296	0.657
Functional Competition	10.6453	-2.476	28.223
Total Effects on App 1	10.6591	-2.447	28.881
The Model-Free Metric	0.0165	0.0692	5.2948
t_1^*	0.0373	0.3149	4.5517
t_2^*	0.0744	0.0369	0.1952
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.923, 0.59)	(0.096, 0.04)	(-0.036,0.504)

Notes:
1, This table is based on a hypothetical market whose market outcomes are the mean in the respective balanced samples.
2, The cells corresponding to the model-free metric is calculated according to (26) and then times 1000 so that they are comparable to results from the full model.

metric, we can put a ballpark figure on this claim. Assume Netflix users spend 0.5 hours per day[33] on Netflix and 7 hours on sleep. Then the budget competition effect of Netflix on sleep is $\frac{0.5\times7}{24-0.5} \approx 0.15$, which is about 9 minutes per day. This did not take into account the binge-watching habit of Netflix users. On a binge-watching day, assume a user spend 4 hours on Netflix and 5 hours on sleep. The budget competition effect of Netflix on sleep is larger: $\frac{4\times5}{24-4} = 1$. In other words, if she does not binge-watch, her sleep would increase by 1 hour due to budget competition. This is more reasonable than 9 minutes. We can do even better. Recall the quote from Reed Hastings: "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and unwind, hang out, and connect– and we compete with all of that."(Raphael, 2017) Assume a user spend on 12 hours on work and related activities and she can only watch Netflix/sleep/connect in the remaining 12 hours. In this case, budget competition is $\frac{4\times5}{12-4} = 2.5$. This number should be closer to what Reed Hastings had in mind when he said "We're competing with sleep". This is essentially a multi-stage budgeting model. As shown in the Netflix example and the WeChat/Kwai example, this metric can be combined with institutional knowledge ($\frac{\gamma_{12}}{\gamma_1}$, binge-watching, multi-stage budgeting) to more accurately gauge budget competition and assess overall competition.

[33]See the estimates by eMarketer at https://www.insiderintelligence.com/chart/232130.

7 Mergers

In the previous section, I quantify competition with counter-factuals and diversion ratios. Competition is then decomposed into functional competition and budget competition both analytically and quantitatively. However, we still do not know how competition, or the lack of it, can affect consumer welfare. In other words, how can consumers be hurt if prices of apps are zero anyway? In this section, I use stylized merger simulations to show that users will see more ads when competing apps merge.

7.1 Price and Profit Functions

To evaluate the effects of a merger of apps on firms and consumers, we need firms to have a decision variable which affects both consumer demand and profit. Traditionally, price is the decision variable. However, most apps are free to use. Apps developers make a profit by selling user attention to advertisers (Prat & Valletti, 2022). Users dislike prices and ads whereas firms prefer higher prices and ad load whenever possible. Conceptually, price and ad load are co-linear in demand and supply models of apps. In the following model, I still refer to the decision variable as "price" on the understanding that it is ad load for most free apps.

To simulate mergers of free apps, I need to specify the profit function and how money enters the utility function. For simplicity, I assume the utility function is linear in money:

$$\begin{aligned} \max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} & U(\mathbf{t}) - \alpha(p_1 \mathbb{I}\{t_{i1m} > 0\} + p_2 \mathbb{I}\{t_{i2m} > 0\}) \\ s.t. \quad & t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168 \end{aligned} \tag{28}$$

where $U(\mathbf{t})$ is from (2). p_1 and p_2 are the weekly subscription prices of app 1 and app 2. Alternatively, if firms charge users per use, the utility function becomes

$$\begin{aligned} \max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} & U(\mathbf{t}) - \alpha(p_1 t_{i1m} + p_2 t_{i2m}) \\ s.t. \quad & t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168 \end{aligned} \tag{29}$$

Note that in both (28) and (29), the marginal value of time in terms of money is $\frac{1-0.001t_0^*}{\alpha}$. This ratio is estimated to be \$1.65 per hour for Taiwan in 2001 (Shiaw, 2004). Therefore, I assume $\frac{1}{\alpha} = 8.5$ yuan.[34] It is important to note that this number would not change any qualitative results. We can also have different αs for different demographic groups and simulate merger effects separately.

[34]Considering the GDP per capita of Taiwan in 2001 and that of Mainland China in 2017, a comparable estimate for $\frac{1-0.001t_0^*}{\alpha}$ in our context is 7.2322 yuan. As the average of t_0 is 16.76 hours, $\frac{1-0.001t_0^*}{\alpha} \approx \frac{0.85}{\alpha} = 7.2322$. Therefore, $\frac{1}{\alpha} = 8.5$ yuan.

Corresponding to the two pricing strategies, the profit functions of a monopolist with an app $j = 1, 2$ in market m are

$$\Pi_{jm} = p_j \sum_{i}^{I_m} \mathbb{I}\{t_{ij} > 0\} + r_{jm} \sum_{i}^{I_m} \mathbb{I}\{t_{ij} > 0\} - \Psi \tag{30}$$

and

$$\Pi_{jm} = p_j \sum_{i}^{I_m} t_{ij} + r_{jm} \sum_{i}^{I_m} t_{ij} - \Psi \tag{31}$$

where Ψ is the fixed cost and r_{jm} is the advertising revenue per hour (per user). If we expand $U(\mathbf{t})$ in (29), the "net marginal utility" is now $\mu_j - \alpha P_j$. Therefore, a price change is equivalent to a change in μ_j for consumers. If consumers dislike advertisements, an increase in price is equivalent to an increase in ad load, which reduces μ_j. In (30), the "total revenue per user" is $p_j + r_{jm}$, which becomes the "total revenue per hour" in (31). Hence, a price change is equivalent to a change in r_{jm} for firms. In other words, an increase in the price of an app is equivalent to a decrease in quality (more specifically, an increase in ad load).

All the three components in the profit functions can change after a merger. If there are cost synergies, Ψ would change. Prat & Valletti (2022) are concerned about the increased market power in the advertising market after a merger, and r_{jm} would change in that case. In the following analysis, I assume $r_{jm} = \Psi = 0$ and focus on how p_j would change after a merger.

7.2 A Merger of Substitutes

The market outcomes before and after a merger of Baidu Map and Amap are in Table **??**. Column (1) displays the results when firms charge users weekly subscription prices. The results in column (1) are unusual: the prices are almost the same and the total profit increases. The post-merger monopolist exploits a discontinuity in the demand curve and induces one more user to use Amap with slightly lower prices. Because I only simulate for 1000 users, the demand of an app in terms of active user is a step function of price and not responsive to a small change in own price. However, the slope of this demand function can change a lot in response to a small change of the price of the other app. This is the result of simulation error.

Column (2) displays the results when firms charge users per use. The results in column (2) are intuitive for a pair of substitutes: prices and profits increase and consumer surplus decreases. As with other anti-competition mergers, the gain in profits is orders-of-magnitude smaller than the loss in consumer surplus. Such mergers should be blocked if antitrust agencies want to maximize total surplus.

Table 16: Mergers of Baidu Map and Amap

	Subscription		Pay-per-Use	
	Baidu Map	Amap	Baidu Map	Amap
	(1)		(2)	
Pre-Merger				
Prices	1.739	4.227	3.656	3.78
Active User	30	24	83	62
Total Usage	17.558	32.925	17.511	31.065
Consumer Surplus	110.2		168.3	
Profits	153.6		181.451	
Total Surplus	263.8		349.75	
Post-Merger				
Prices	1.739	4.227	4.2	4.04
Active User	30	25	74	60
Total Usage	17.558	33.94	15.244	29.31
Consumer Surplus	110.2		151.47	
Profits	157.8		182.521	
Total Surplus	268.03		333.99	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

7.3 A Merger of Complements

The market outcomes before and after a merger of Baidu and Baidu Map are in Table 17. Note that Baidu and Baidu Map are developed by the same company. Therefore, this merger analysis can be seen as a divestment analysis. Column (1) displays the results when firms charge users weekly subscription prices. Due to simulation error, prices do not change after a merger. Column (2) displays the results when firms charge users per use. The results in column (2) are intuitive. After a merger of a pair of complements, we expect the monopolist to internalize the complementarity between Baidu and Baidu Map by lowering prices. The prices of Baidu and Baidu Map are lower after the merger. Consumers benefit more from the merger: for 1000 users, consumer surplus increases by 2 yuan per week whereas profits increase by 0.03.[35] Therefore, the results in column (2) suggest that developing complementary apps around a flagship app can be a profitable strategy for tech firms. Baidu has other apps named after Baidu (for example, Baidu Browser and Baidu Netdisk). The "Baidu Core" in Baidu's financial reports consists of those complementary apps and notably Baidu.com.[36] The overall complementarity between Baidu and its satellite apps can be large.

We can compare profits calculated in Table 17 with actual data in Baidu's financial reports. The total

[35]The increase in profits should be a lower bound of the benefits of having the two apps in the same firm as I assume $r_{jm} = \Psi = 0$.

[36]For a detailed explanation of "Baidu Core", see the Form 20-Fs of Baidu.

Table 17: Mergers of Baidu and Baidu Map

	Subscription		Pay-per-Use	
	Baidu	Baidu Map	Baidu	Baidu Map
	(1)		(2)	
Pre-Merger				
Prices	8.147	2.055	4.243	3.845
Active User	53	23	128	65
Total Usage	148.2	14.6	126.58	13.746
Consumer Surplus	560.96		518.26	
Profits	479.05		589.95	
Total Surplus	1040		1108.21	
Post-Merger				
Prices	8.147	2.055	4.235	3.77
Active User	53	23	128	65
Total Usage	148.2	14.6	126.82	14.01
Consumer Surplus	560.96		520.25	
Profits	479.05		589.98	
Total Surplus	1040		1110.22	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

revenue from Baidu and Baidu Map is about 600 yuan for 1000 smartphone users in one week. The implied revenue is about 31.2 billion RMB.[37] The actual revenue of the Baidu Core in 2017 is 67.7 billion RMB. The two numbers are close considering that a significant part of the total revenue should come from Baidu.com and there are other Baidu apps not in Table 17.

7.4 A Merger of Seemingly Independent Apps

The market outcomes before and after a merger of WeChat and Kwai are in Table 18. Column (1) displays the results when firms charge users weekly subscription prices. Column (2) displays the results when firms charge users per use. Let us now focus on column (2) because pay-per-use is less prone to simulation error. As we see in section 6.2, WeChat and Kwai are competing not only in functions but also in the time budget. After the merger, the prices of WeChat and Kwai increase by 0.06% and 27.5%. As a result, profits increase by about 9 yuan per week whereas consumers surplus decreases by 169 yuan.By contrast, for 1000 users, consumer welfare decreases by 17 yuan per week after a merger of Baidu Map and Amap, a pair of direct competitors. The harm to consumers is large for a pair of seemingly independent apps.

The results suggest that without considering any dynamics, Tencent has incentive to acquire Kwai in

[37] Assume that there are 1 billion smartphone users in 2017. $600 \times 52 \div 1000$=31.2 billion.

Table 18: Mergers of WeChat and Kwai

	Subscription		Pay-per-Use	
	WeChat	Kwai	WeChat	Kwai
	(1)		(2)	
Pre-Merger				
Prices	46.15	10.17	7.316	3.692
Active User	276	27	570	63
Total Usage	2533.12	101.1	2016.67	95.68
Consumer Surplus	12832.25		11826.86	
Profits	13012.46		15107.52	
Total Surplus	25844.7		26934.38	
Post-Merger				
Prices	47.72	18.64	7.358	4.706
Active User	269	14	568	47
Total Usage	2485.95	63.79	2008.196	72.14
Consumer Surplus	12219.76		11657.73	
Profits	13097.91		15116.24	
Total Surplus	25317.67		26773.97	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

the first quarter of 2017. After failed attempts to promote its own short-video app WeSee, Tencent invested \$2 billion in Kwai in December 2019[38], and is now the largest institutional investor of Kwai after its IPO. Tencent also added short-video function to WeChat to directly compete with Kwai and Tik Tok in the first quarter of 2020.

7.5 Discussions

Some patterns in Tables 16, 17, and 18 warrant further discussions.

The prices of Baidu Map in Table 16 and Table 17 are different. The first reason is that the market fundamentals corresponding to the two tables are slightly different. The two different prices are both correct because they are calculated with different assumptions. The price of Baidu Map in Table 16 is the equilibrium price when Baidu is part of the generic app. The price of Baidu Map in Table 17 is the equilibrium price when Amap is part of the generic app. The price of Baidu and the price of Amap can both shift the demand of Baidu Map. Ideally, I should include all relevant factors (substitutes, complements, and any other products in the same budget constraint) in a demand model. However, this is not realistic due to data availability and computational feasibility. This critique applies to all demand estimation.[39]

[38]See https://www.scmp.com/tech/apps-social/article/3041747/tencent-said-invest-us2-billion-short-video-app-kuaishou
[39]See I.C in Gentzkow (2007) for a related discussion.

From Tables 16, 17, and 18, we can find that profits and total surplus are always higher with pay-per-use. However, consumer surplus can be lower with pay-per-use. This is because pay-per-use is a price discrimination tool that enables firms to discriminate users based on usage. If a user has a higher μ_{ij}, then she uses app j more and pays more with pay-per-use. By contrast, users pay the same price with subscription pricing. We do not often see firms employing pay-per-use because processing small payments is costly. We can also find that subscription suppresses the number of active users whereas pay-per-use suppresses usage. Users with higher μ_{ij} will benefit from subscription and users with lower μ_{ij} will benefit from pay-per-use.

8 Conclusion

The rapid development of the mobile Internet industry and its profound influence on our society warrant further understanding of this industry. This paper informs the public debate on antitrust issues in the mobile Internet industry. In this paper, I develop a discrete-continuous model of consumer demand for apps that allows for complements as well as substitutes, and incorporates a binding time constraint. I estimate the model with a weekly panel of app usage in the first quarter of 2017 in China. I separate complements from substitutes with the help of IVs. The model also incorporates network effects. I validate the model by applying it to three representative pairs of apps: each featured an important aspect of the competition landscape in this industry (*a priori* substitutes, *a priori* complements, and a pair with an ambiguous relationship). The estimation results recover realistic competition patterns.

I use counter-factuals and diversion ratios to measure the competition effects of one app on another. A distinctive feature of the app economy is the salient role of the binding time constraint. I decompose the overall competition effects into functional competition and budget competition. I show the determinants of budget competition both analytically and quantitatively. The decomposition reveals that budget competition can dominate functional competition and a merger of complementary apps can hurt consumers. I propose a simple model-free metric to gauge budget competition. Merger simulations show that price and ad load are colinear in both the utility function and the profit function and that consumer will see more ads when competing apps merge.

The demand model in this paper incorporates four desirable features: discrete-continuous decisions, interactions between products, budget constraints, and estimation with instruments. This model can further incorporate other notable features in the mobile Internet industry (for example, advertisement and two-sidedness) or be adapted to study consumer demand for other goods and services. One shortcoming of the demand model is that it does not accommodate dynamics. Future work could consider modeling dynamics of apps.

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Table 19: Covariates of Baidu Map and Amap in Taste Parameters

Covariates	Baidu Map	Standard Error	Amap	Standard Error
Week (03)	0.0187	0.0001	0.01	0.0001
Week (04)	-0.0463	0.0000	0.0002	0.0000
Week (05)	0.027	0.0002	0.0146	0.0002
Week (06)	0.008	0.0002	0.0103	0.0002
Week (07)	0.01	0.0001	0.009	0.0002
Week (08)	0.0006	0.0001	0.0068	0.0002
Week (09)	-0.0087	0.0001	0.0021	0.0001
Week (10)	-0.0005	0.0002	0.0038	0.0002
Week (11)	-0.0194	0.0002	0.0049	0.0002
Week (12)	0.0013	0.0002	0.0037	0.0003
Week (13)	0.005	0.0003	0.0069	0.0003
Female	-0.73	-	-0.84	-
Male	-0.643	-	-0.732	-
Age (<=24)	-0.793	-	-0.897	-
Age (25~30)	-0.643	-	-0.766	-
Age (31~35)	-0.489	-	-0.612	-
Age (36~40)	-0.728	-	-0.799	-
Age (>=40)	-0.902	-	-0.932	-

Notes:
1, This table provides parameters of covariates in μ_1 and μ_2 corresponding to the column (3) of Table 3.
2, The coefficients of gender and age groups are the simple average of market fixed effects with corresponding characteristics.
Data Source: The author's calculations.

A Covariates in μ_1 and μ_2

The covaraites in μ_1 and μ_2 are the market fixed effects, week fixed effects, and network effects. We have reported network parameters in the main text. In Table 19, I provide covariates from the main specification of Baidu Map and Amap (Column (3) in Table 3). In the following table, I report week fixed effects and aggregate market fixed effects to gender and age groups because there are more than 200 markets. The results are resonable: users between 31 and 35 and male users derive higher utility from map apps because they are more likely to own and drive a car.

B Budget Competition

The intermediate bundle (t_0^i, t_1^i, t_3^i) defined by (22) is easy to calculate. The functional competition in Table 14 is $t_1^i - t_1^o$. After this step, we can calculate how much time is left to be allocated as $\Delta T = T - t_0^i - t_1^i - t_3^i$. The intermediate bundle can be seen as the result of utility maximization over t_0, t_1, and t_3 subject to a time budget of $t_0^i + t_1^i + t_3^i$. Note that apps 0, 1, and 3 are independent. We can solve for the final bundle as the same utility maximization problem subject to a time budget of $t_0^i + t_1^i + t_3^i + \Delta T$. The following two lemmas are useful when calculating the final bundle. The budget competition effect of app 2 on app 1 is $t_1^f - t_1^i$.

Lemma 1. *For J independent apps that are used, when there are extra time ΔT, the increase in time spent on app j is $\Delta t_j = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^{J} \frac{1}{\gamma_k}}$.*

Proof. From the FOCs of the old bundle, we have

$$\mu_j + \gamma_j t_j^0 = \mu_k + \gamma_k t_k^0 \Rightarrow t_k^0 = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^0.$$

Similarily, we have $t_k^1 = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^1$. Because of the time constraint, we have

$$\sum_{l=1}^{J} t_l^0 = T \Rightarrow \sum_{k=1}^{J} (\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^0) = T \Rightarrow t_j^0 = \frac{T - \sum_k \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^{J} \frac{1}{\gamma_k})}$$

The budget constraint with extra time ΔT is

$$\sum_{l=1}^{J} t_l^1 = T + \Delta T \Rightarrow \sum_{k=1}^{J} (\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^1) = T + \Delta T \Rightarrow t_j^1 = \frac{T + \Delta T - \sum \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^{J} \frac{1}{\gamma_k})}$$

Therefore we have

$$\Delta t_j = t_j^1 - t_j^0 = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^{J} \frac{1}{\gamma_k}}$$

□

Lemma 2. *When an app q is used because of the extra time ΔT, $t_q^1 = \frac{T + \Delta T - \sum_k \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q (\sum_k \frac{1}{\gamma_k})}$*

Proof. Because q was not used ($t_q^0 = 0$), we have

$$\mu_q \leq \mu_k + \gamma_k t_k^0 \Rightarrow \frac{\mu_q - \mu_k}{\gamma_k} \geq t_k^0 \Rightarrow T \leq \sum_k \frac{\mu_q - \mu_k}{\gamma_k}$$

The FOCs of the new bundle are

$$\mu_q + \gamma_q t_q^1 = \mu_k + \gamma_k t_k^1 \Rightarrow t_k^1 = \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1$$

Combined with the new time constraint, we have

$$T + \Delta T = \sum_k \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1 \Rightarrow t_q^1 = \frac{T + \Delta T - \sum_{k \neq q} \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})}$$

Because $T \leq \sum_k \frac{\mu_j - \mu_k}{\gamma_k}$, we have

$$\frac{T + \Delta T - \sum_k \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q(\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q(\sum_k \frac{1}{\gamma_k})}.$$

$\square$

C Derivation of the Model-Free Metric

The FOCs of this user at the observed usage level are

$$1 - 0.001t_0^* = 1 - 0.001(T - t_1^* - t_2^*) = 1 + \gamma_1 t_1^* = 1 + \gamma_2 t_2^* \tag{32}$$

We have

$$t_1^* = \frac{0.001(t_2^* - T)}{\gamma_1 - 0.001} = \frac{\gamma_2}{\gamma_1} t_2^*$$

When app 2 exits the market, the new FOCs are

$$1 - 0.001t_0' = 1 - 0.001(T - t_1') = 1 + \gamma_1 t_1'$$

We have

$$t_1' = \frac{-0.001T}{\gamma_1 - 0.001}$$

$$\begin{aligned} t_1' = \frac{-0.001T}{\gamma_1 - 0.001} &= \frac{-0.001T + 0.001t_2^*}{\gamma_1 - 0.001} - \frac{0.001t_2^*}{\gamma_1 - 0.001} \\ &= t_1^* - \frac{0.001t_2^*}{\gamma_1 - 0.001} \\ &= t_1^* + t_2^*(\frac{-0.001}{\gamma_1 - 0.001}) \\ &= t_1^* + t_2^*(\frac{t_1^*}{t_0^* + t_1^*}) \end{aligned}$$

where the last equality is from $-0.001t_0^* = \gamma_1 t_1^* = \gamma_2 t_2^*$.