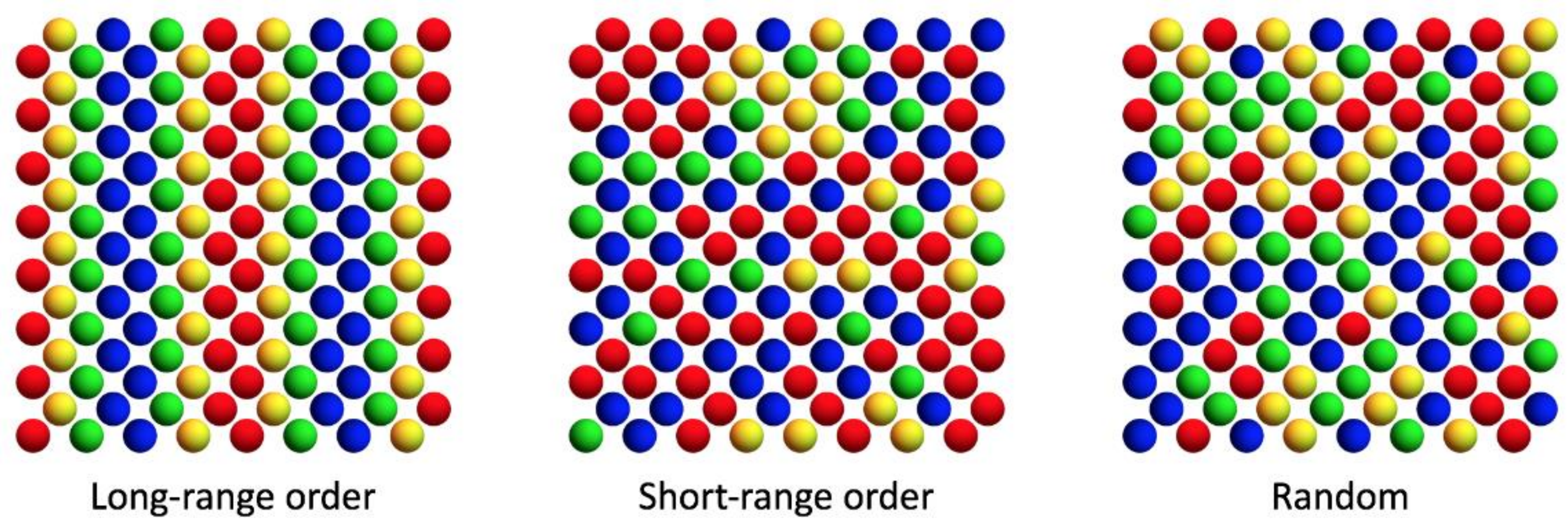
Computing with long-range order:
From AI to combinatorial optimization

Yuanhang Zhang

12/24/2024

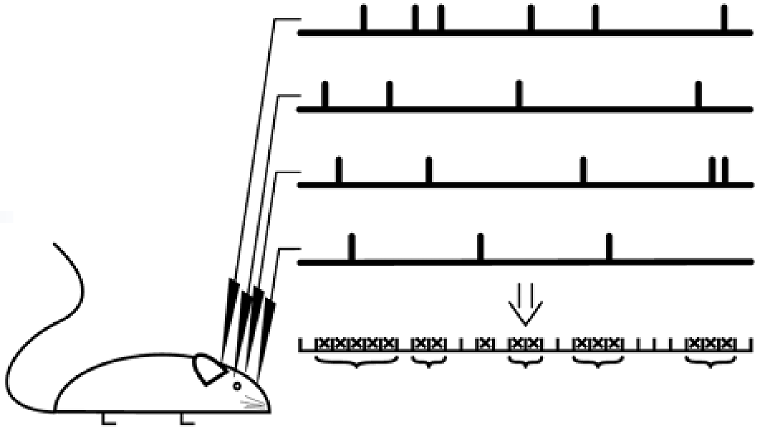
Long-range order

- Long-range order: $\langle x(\boldsymbol{r})x(\boldsymbol{r}+\boldsymbol{R})\rangle \propto \boldsymbol{R}^{-\alpha}$
- Short-range correlations: $\langle x(\boldsymbol{r})x(\boldsymbol{r}+\boldsymbol{R})\rangle \propto e^{-\beta\boldsymbol{R}}$



Liu, Xianglin, et al. "Machine learning modeling of high entropy alloy: the role of short-range order." *arXiv preprint arXiv:1906.02889* (2019).

Criticality in brains?



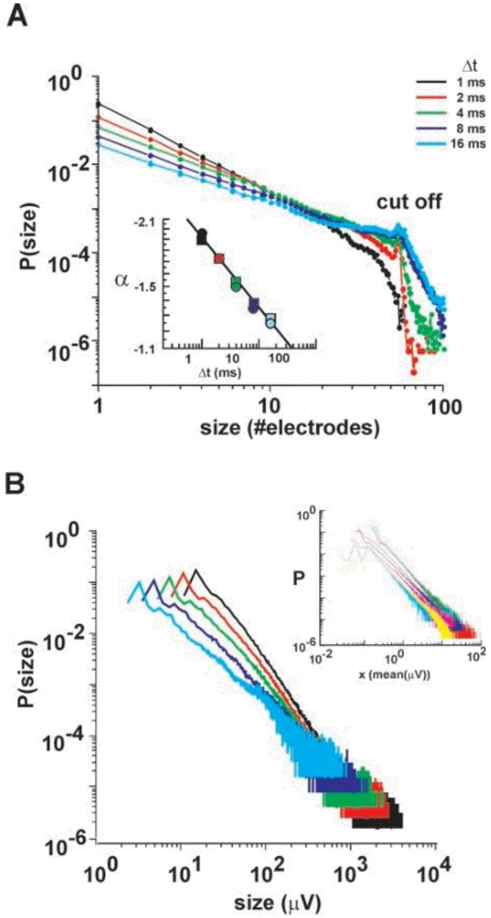
Behavioral/Systems/Cognitive

Neuronal Avalanches in Neocortical Circuits

John M. Beggs and Dietmar Plenz
Unit of Neural Network Physiology, Laboratory of Systems Neuroscience, National Institute of Mental Health, Bethesda, Maryland 20892

Networks of living neurons exhibit diverse patterns of activity, including oscillations, synchrony, and waves. Recent work in physics has shown yet another mode of activity in systems composed of many nonlinear units interacting locally. For example, avalanches, earthquakes, and forest fires all propagate in systems organized into a critical state in which event sizes show no characteristic scale and are described by power laws. We hypothesized that a similar mode of activity with complex emergent properties could exist in networks of cortical neurons. We investigated this issue in mature organotypic cultures and acute slices of rat cortex by recording spontaneous local field potentials continuously using a 60 channel multielectrode array. Here, we show that propagation of spontaneous activity in cortical networks is described by equations that govern avalanches. As predicted by theory for a critical branching process, the propagation obeys a power law with an exponent of −3/2 for event sizes, with a branching parameter close to the critical value of 1. Simulations show that a branching parameter at this value optimizes information transmission in feedforward networks, while preventing runaway network excitation. Our findings suggest that "neuronal avalanches" may be a generic property of cortical networks, and represent a mode of activity that differs profoundly from oscillatory, synchronized, or wave-like network states. In the critical state, the network may satisfy the competing demands of information transmission and network stability.

Key words: cortex; organotypic culture; branching process; self-organized criticality; multielectrode array; power law



Beggs, John M., and Dietmar Plenz. "Neuronal avalanches in neocortical circuits." *Journal of neuroscience* 23.35 (2003): 11167-11177.

Background: Self-organized criticality

Sandpile model:
Self-organized criticality

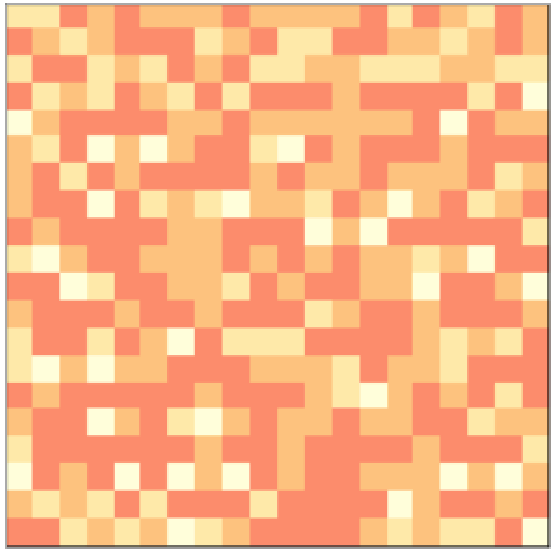
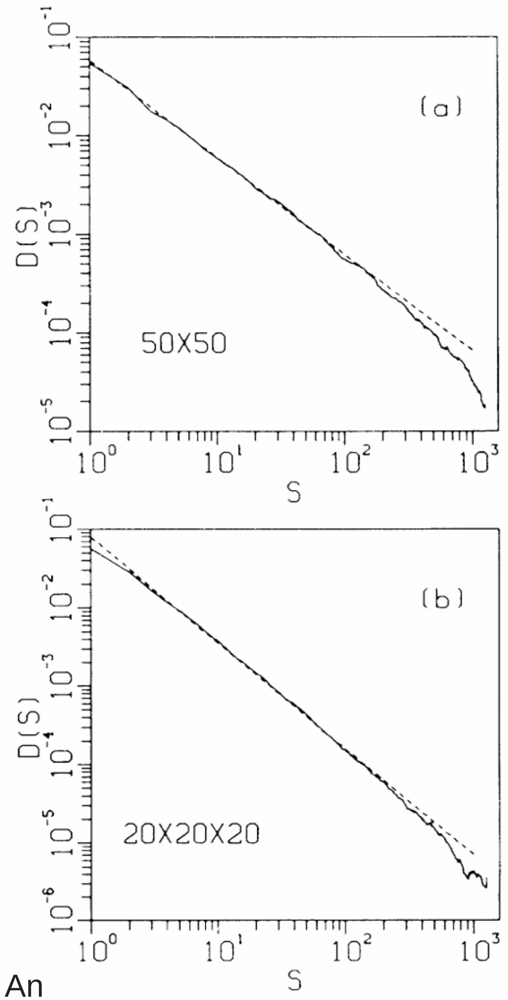


Image credit:
https://runestone.academy/ns/books/published/complex/SelfOrganizedCriticality/ImplementingTheSandPile.html

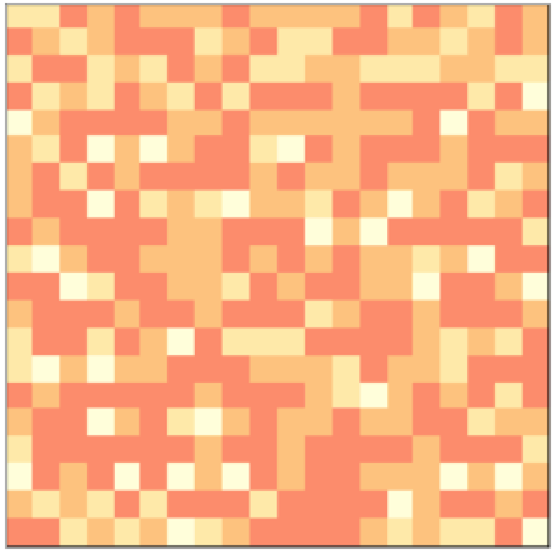


Bak, Per, Chao Tang, and Kurt Wiesenfeld. "Self-organized criticality: An explanation of the 1/f noise." *Physical review letters* 59.4 (1987): 381.

Background: Self-organized criticality

- Slow addition of sand
- Instantaneous toppling
- Power-law distribution of avalanches

Sandpile model:
Self-organized criticality



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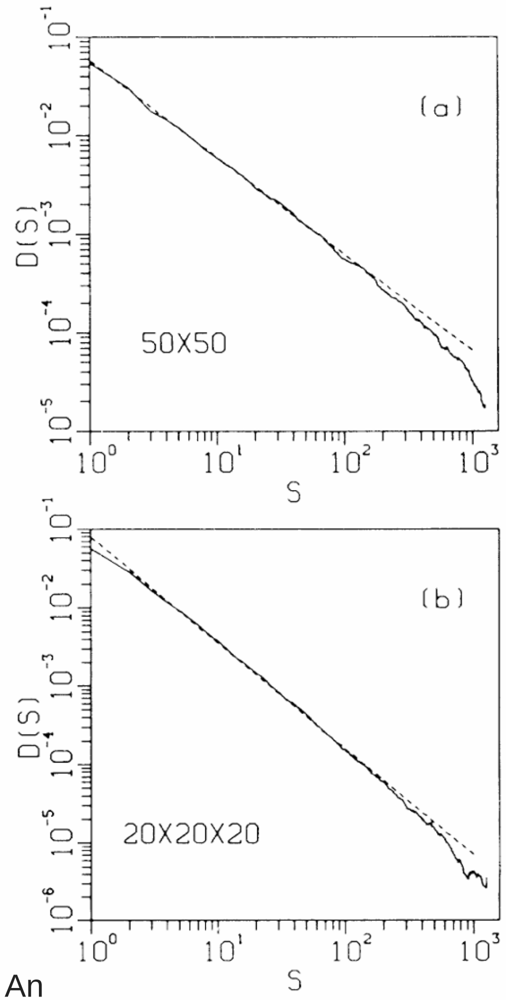

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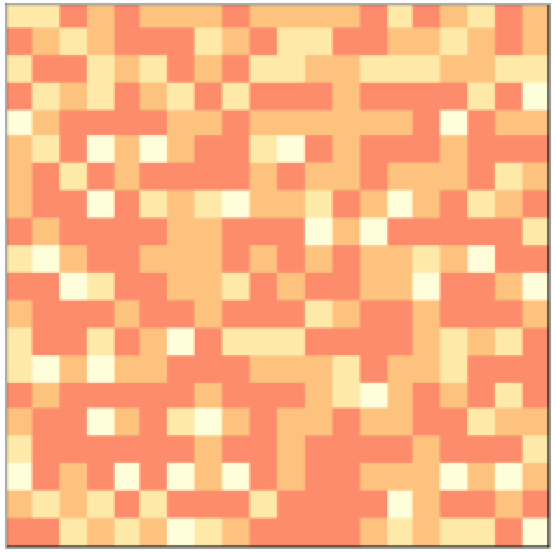

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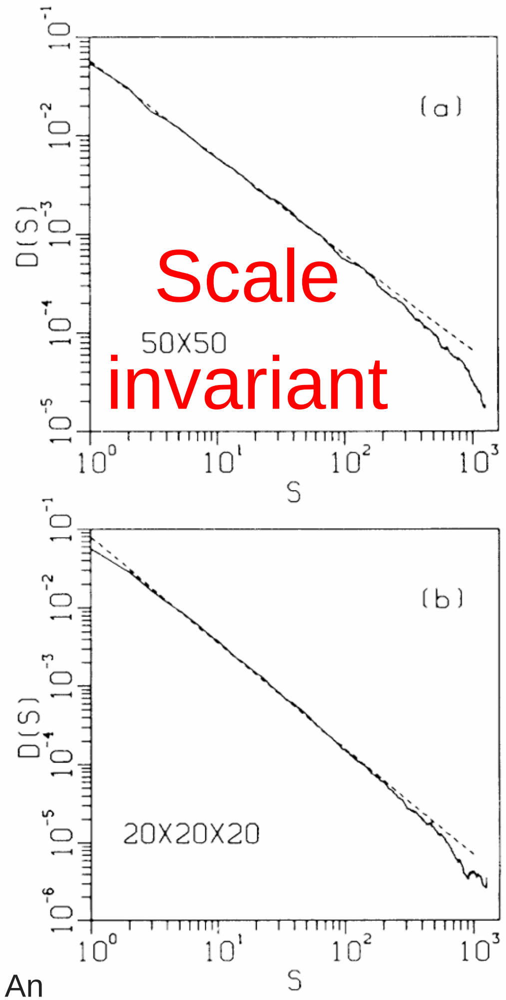
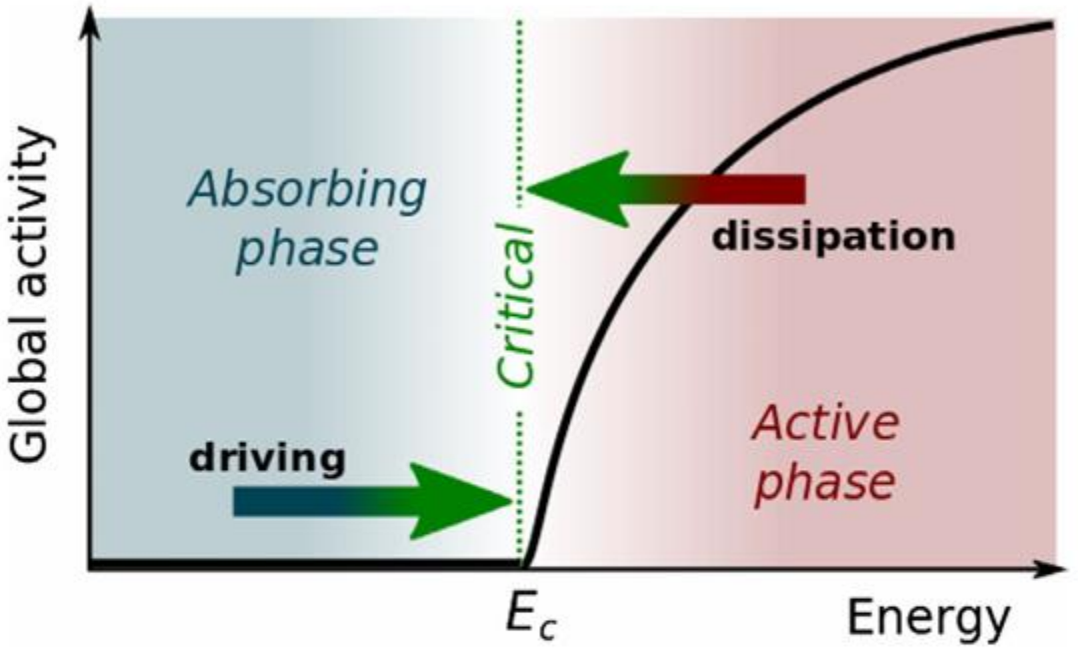


Bak, Per, Chao Tang, and Kurt Wiesenfeld. "Self-organized criticality: An explanation of the 1/f noise." *Physical review letters* 59.4 (1987): 381.

What is being "critical?"

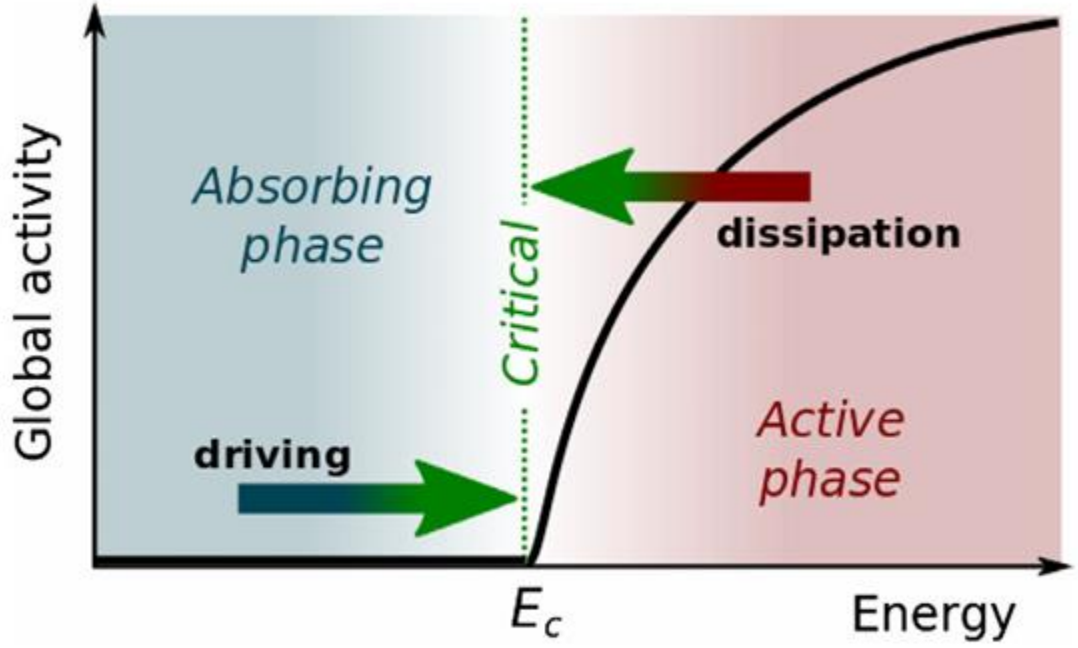
- What are the phases?
  - Neural synchronization (Deco et al. 2014, Palmigiano et al. 2017, ……)
  - Chaos
  - Percolation (Tagliazucchi *et al*. 2012)
  - Ising model (Tkacik et al., 2013, 2014, 2015)
  - ……



Munoz, Miguel A. "Colloquium: Criticality and dynamical scaling in living systems." *Reviews of Modern Physics* 90.3 (2018): 031001.

What is being "critical?"

- What if there is no transition at all?

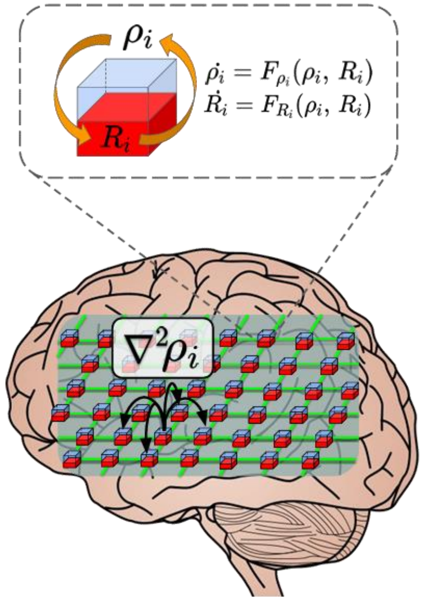


Hesse, Janina, and Thilo Gross. "Self-organized criticality as a fundamental property of neural systems." *Frontiers in systems neuroscience* 8 (2014): 166.

Memory-induced long-range order (MILRO)

- Memory, or time non-local interactions, induces spatial long-range order (LRO)

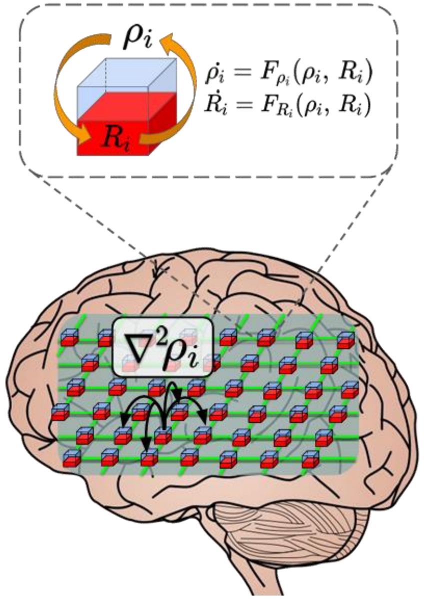
Biological neurons



$$\dot{\rho}_i = (-a + R_i)\rho_i + b\rho_i^2 - c\rho_i^3 + h + D\nabla^2\rho_i + \sigma\eta_i$$

$$\dot{R}_i = \delta - \frac{1}{\tau_D}(R_i\rho_i + \sigma\eta_i)$$

Biological neurons



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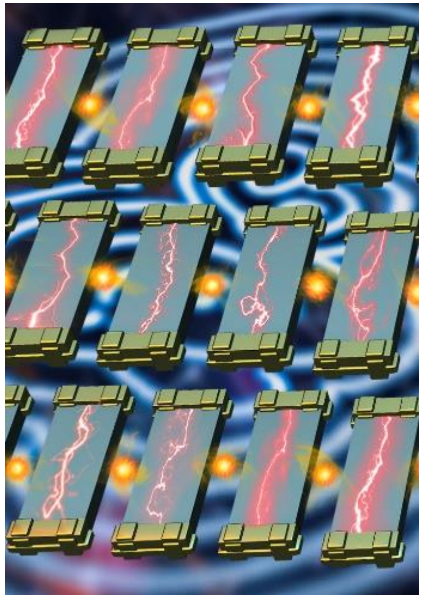
Artificial spiking neurons



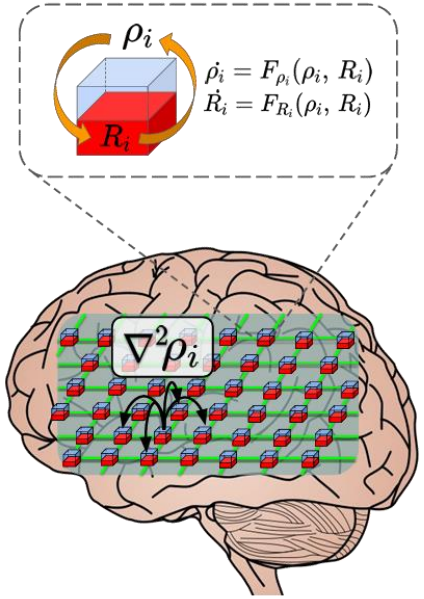
$$C\dot{V}_i = \frac{V_i^{in}}{R_i^{load}} - V_i\left(\frac{1}{R_i} + \frac{1}{R_i^{load}}\right)$$

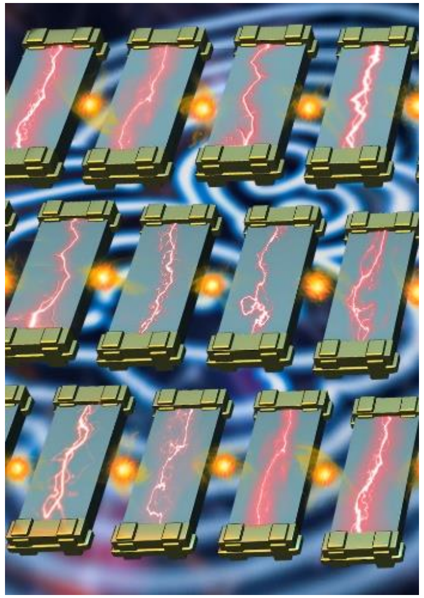
$$C_{th}\dot{T}_i = \frac{V_i^2}{R_i} - S_e(T_i - T_0) + S_c\nabla^2 T_i + \sigma\eta_i$$

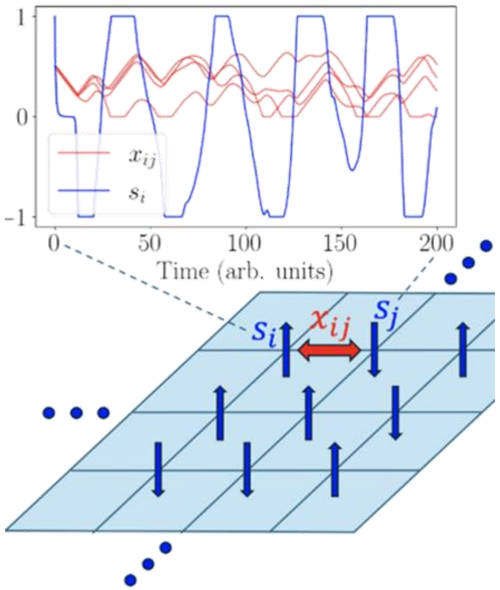
Biological neurons

Artificial spiking neurons

Spin glass with memory







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$$\dot{s}_i = \sum_{\langle ij \rangle} J_{ij}s_j - g\sum_{\langle ij \rangle} x_{ij}s_i$$
$$\dot{x}_{ij} = \gamma(C_{ij} - \delta)$$

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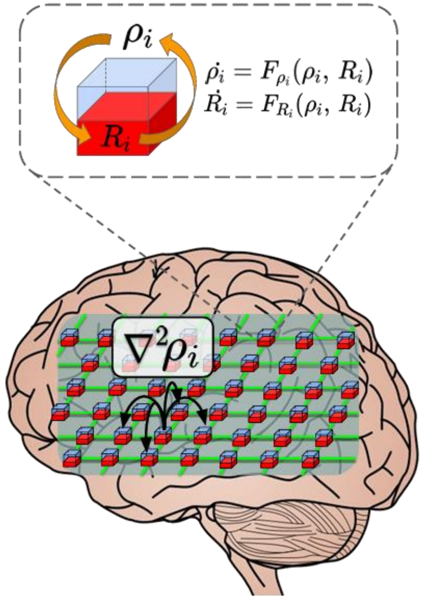
Biological neurons



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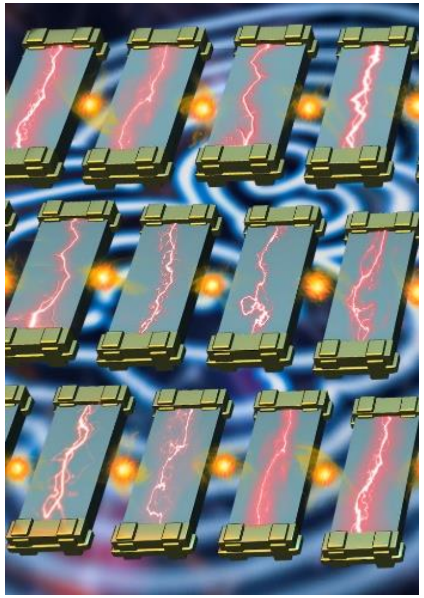
Artificial spiking neurons



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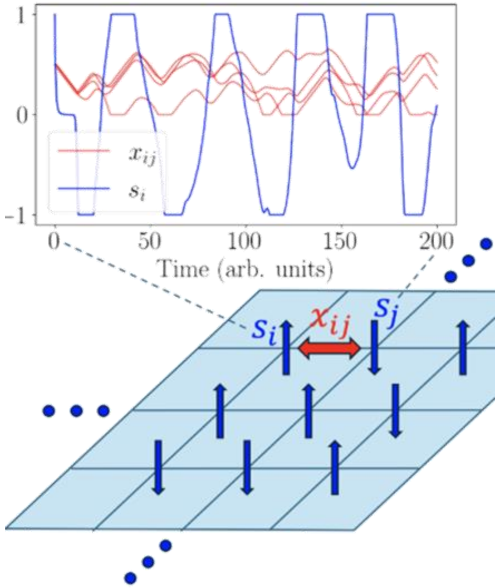
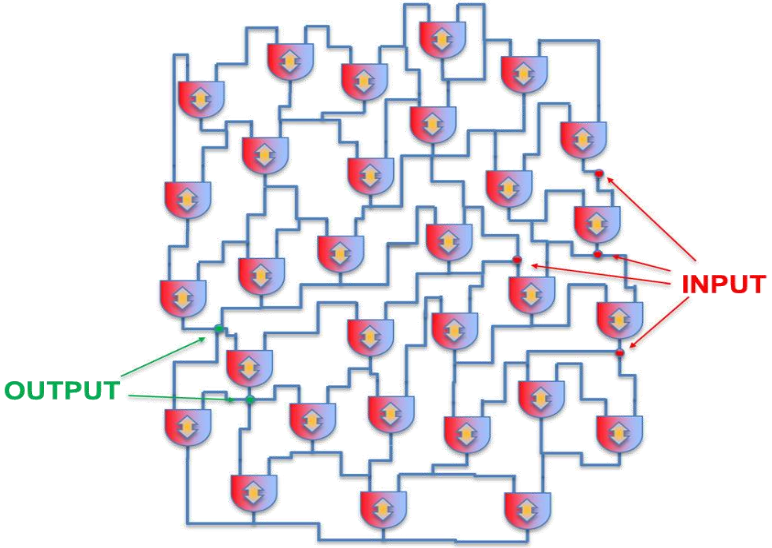
Spin glass with memory



$$\dot{s}_i = \sum_{\langle ij\rangle} J_{ij}s_j - g\sum_{\langle ij\rangle} x_{ij}s_i$$

$$\dot{x}_{ij} = \gamma(C_{ij} - \delta)$$

MemComputing machine: dynamical system with memory

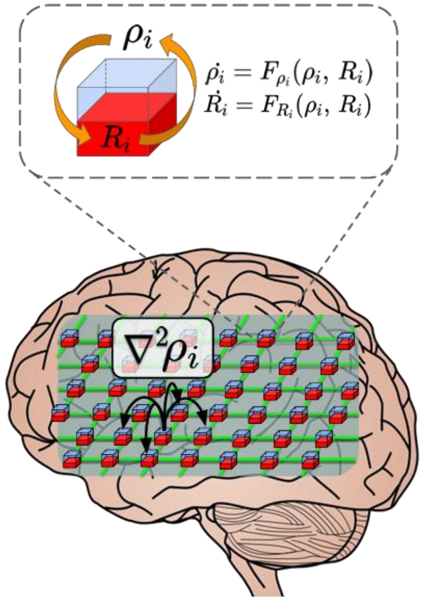


$$\dot{v}_n = \sum x_{l,m}x_{s,m}G_{n,m}(v_n, v_j, v_k) + (1 + \zeta x_{l,m})(1 - x_{s,m})R_{n,m}(v_n, v_j, v_k)$$

$$\dot{x}_{s,m} = \beta(x_{s,m} + \epsilon)(C_m(v_i, v_j, v_k) - \gamma)$$

$$\dot{x}_{l,m} = \alpha(C_m(v_i, v_j, v_k) - \delta)$$

Biological neurons



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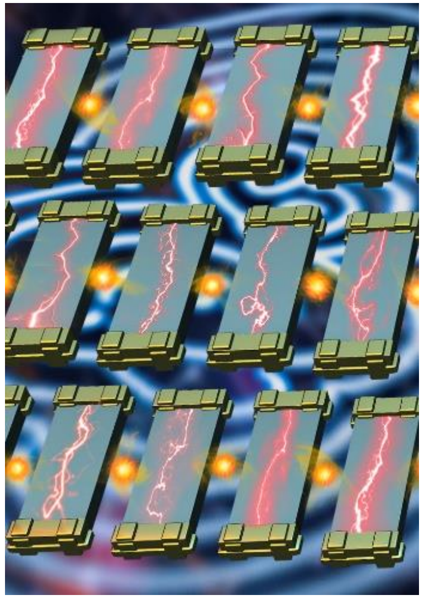
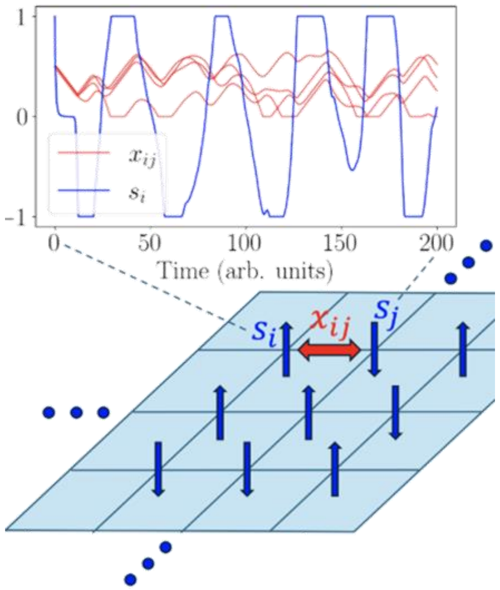
$$\dot{R}_i = \delta - \frac{1}{\tau_D}(R_i\rho_i + \sigma\eta_i)$$

Artificial spiking neurons



Spin glass with memory



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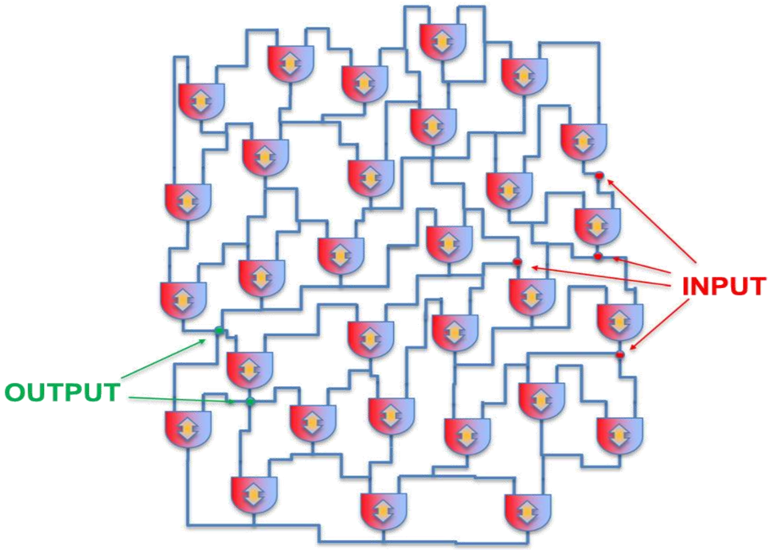
MemComputing machine: dynamical system with memory



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Fast activity dynamics

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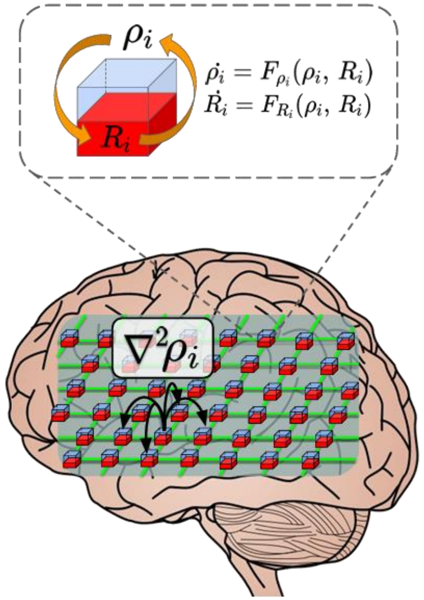
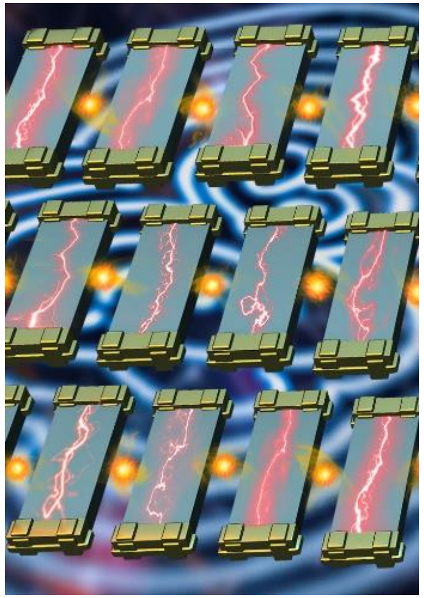
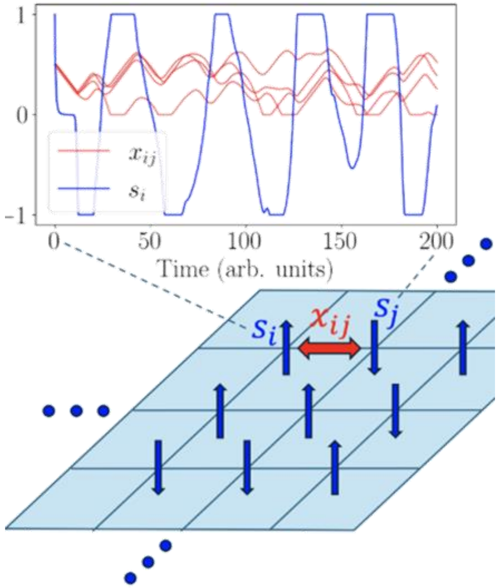
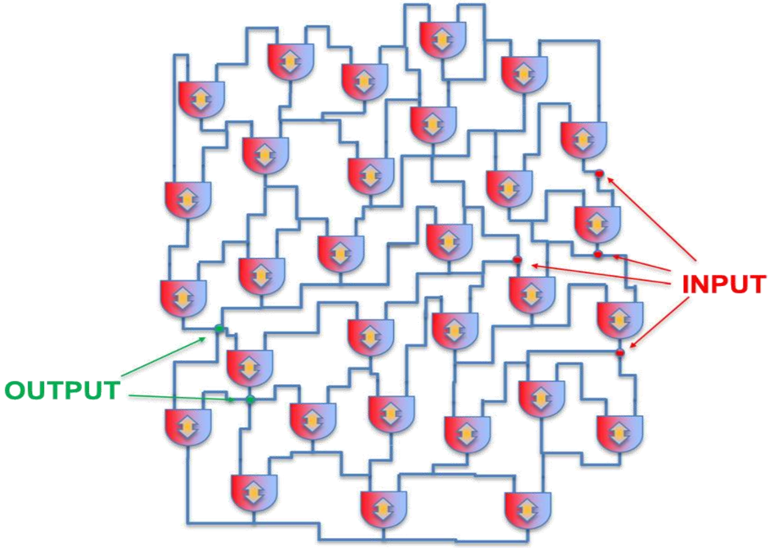
Biological neurons	Artificial spiking neurons	Spin glass with memory	MemComputing machine: dynamical system with memory









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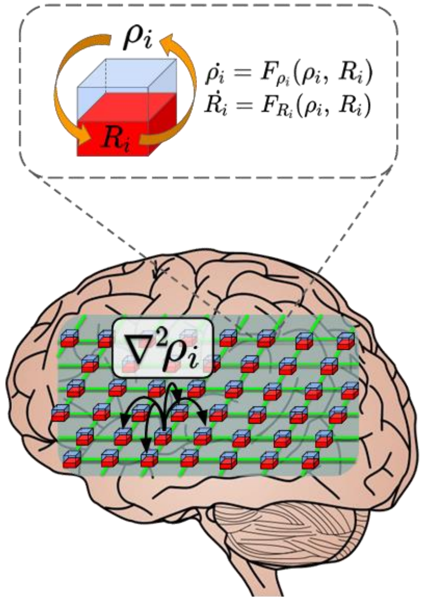
Slow memory dynamics

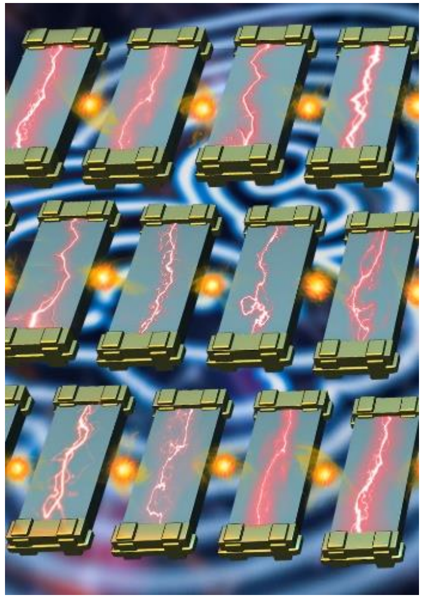
$$\dot{x}_{s,m} = \beta(x_{s,m} + \epsilon)(C_m(v_i, v_j, v_k) - \gamma)$$

$$\dot{x}_{l,m} = \alpha(C_m(v_i, v_j, v_k) - \delta)$$

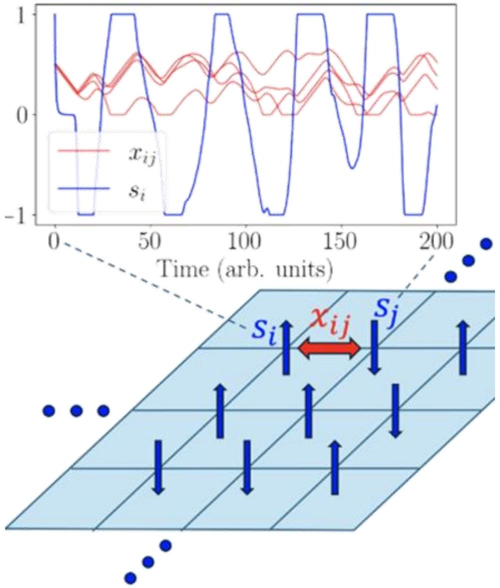
Biological neurons



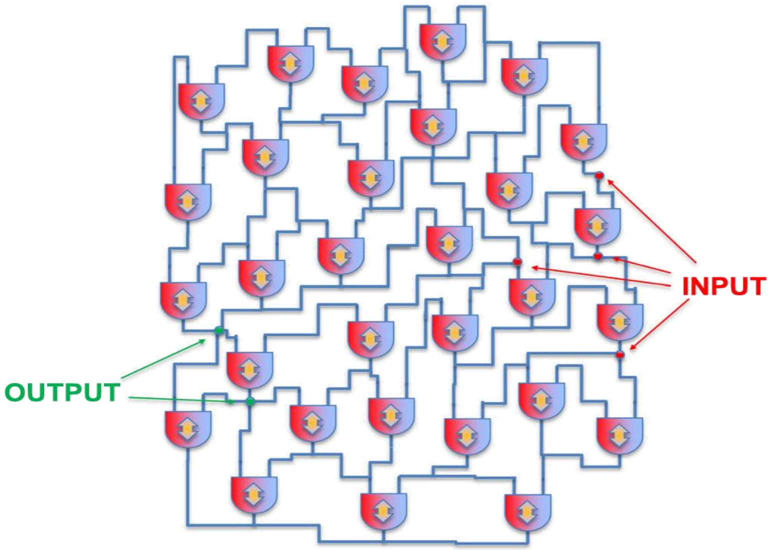
Artificial spiking neurons



Spin glass with memory



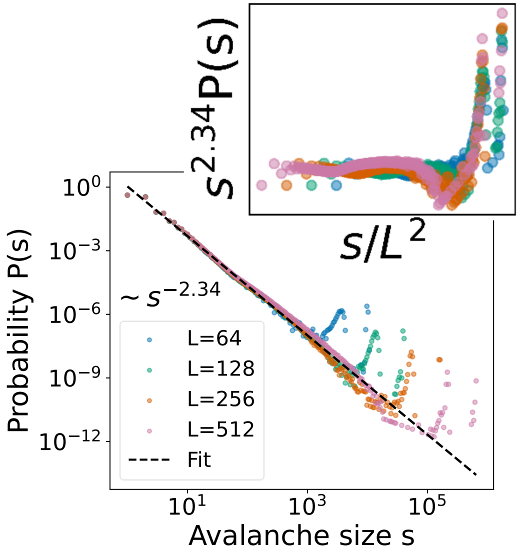
MemComputing machine: dynamical system with memory



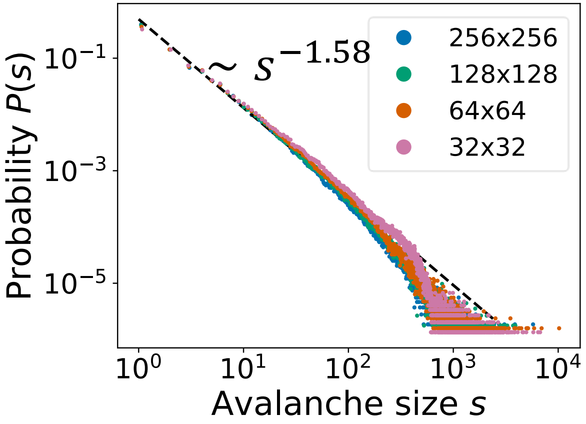
$$\dot{v}_i = \frac{1}{\tau_{\text{fast}}} F_v(\boldsymbol{v}, \boldsymbol{x})$$ Fast activity dynamics

$$\dot{x}_i = \frac{1}{\tau_{\text{slow}}} F_x(\boldsymbol{v}, \boldsymbol{x})$$ Slow memory dynamics

Biological neurons



Artificial spiking neurons



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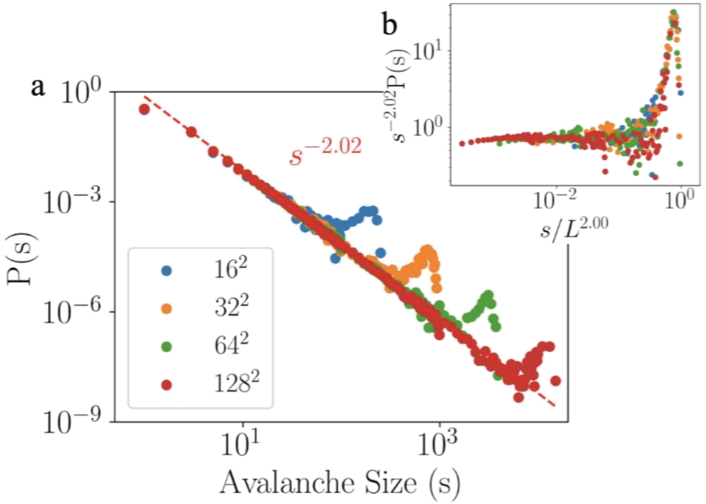
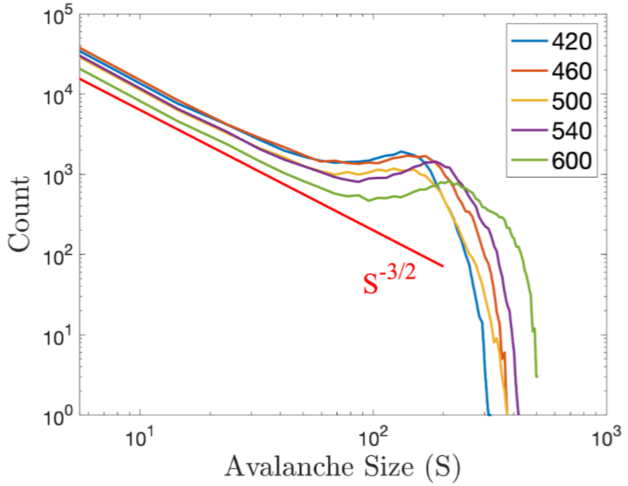


MemComputing machine: dynamical system with memory



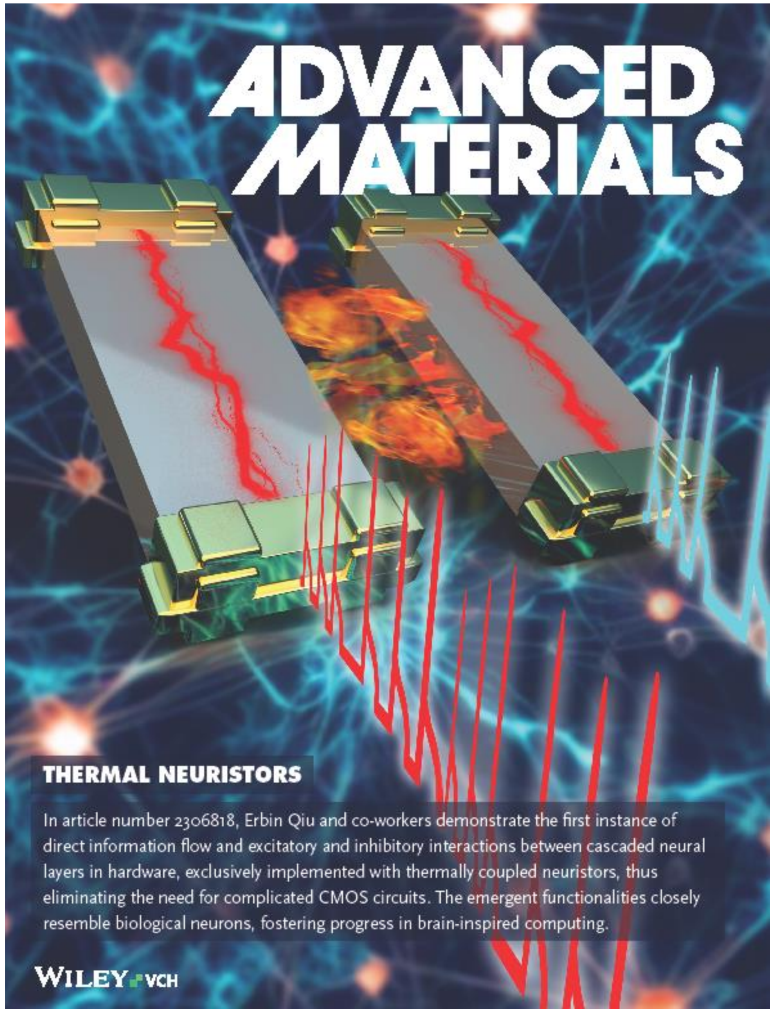
$$\dot{v}_i = \frac{1}{\tau_{\text{fast}}} F_v(\boldsymbol{v}, \boldsymbol{x})$$

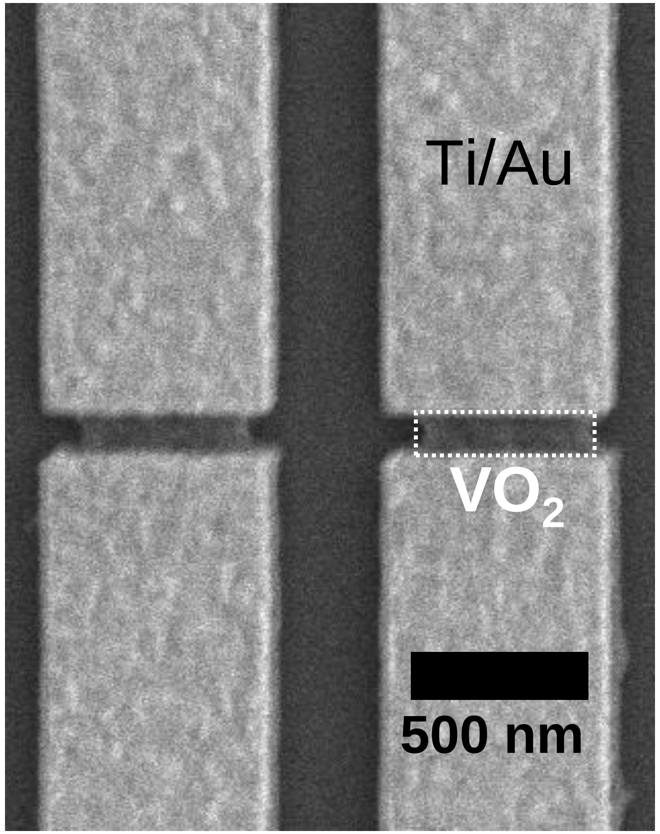
Fast activity dynamics

$$\dot{x}_i = \frac{1}{\tau_{\text{slow}}} F_x(\boldsymbol{v}, \boldsymbol{x})$$

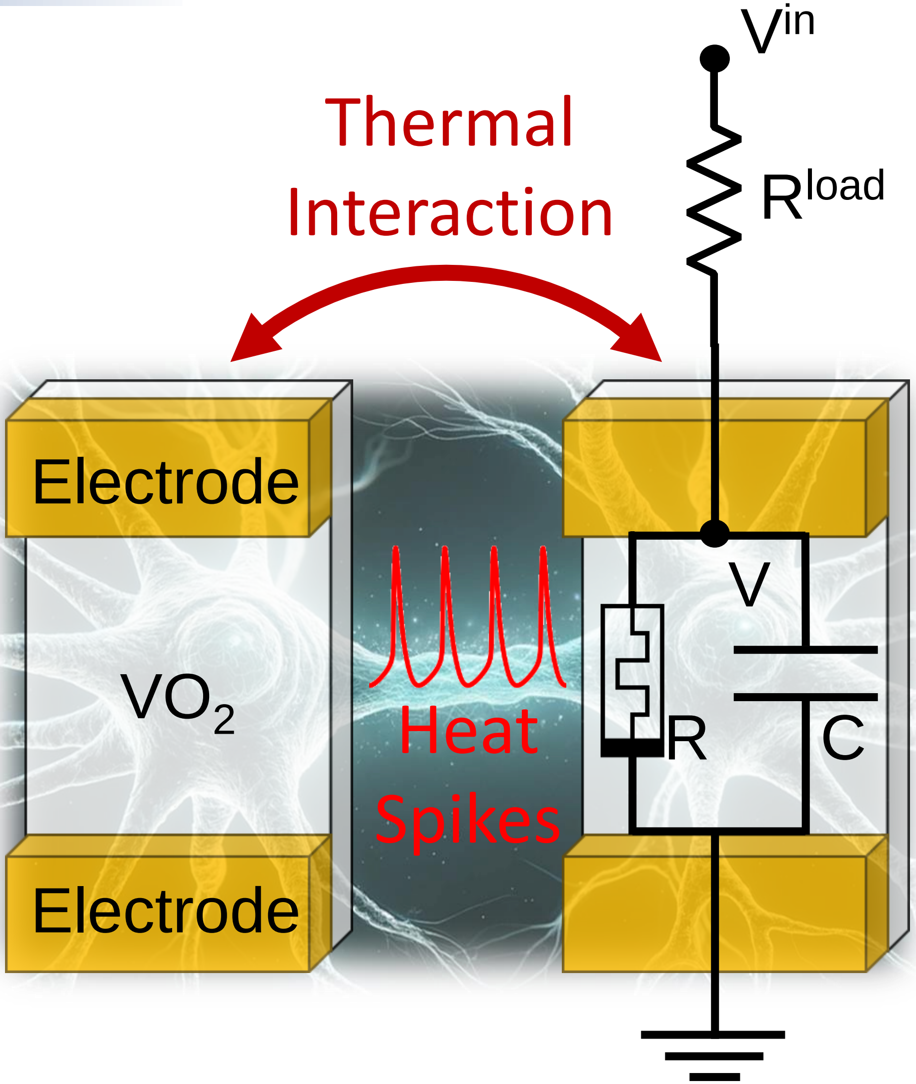
Slow memory dynamics

Thermal neuristor



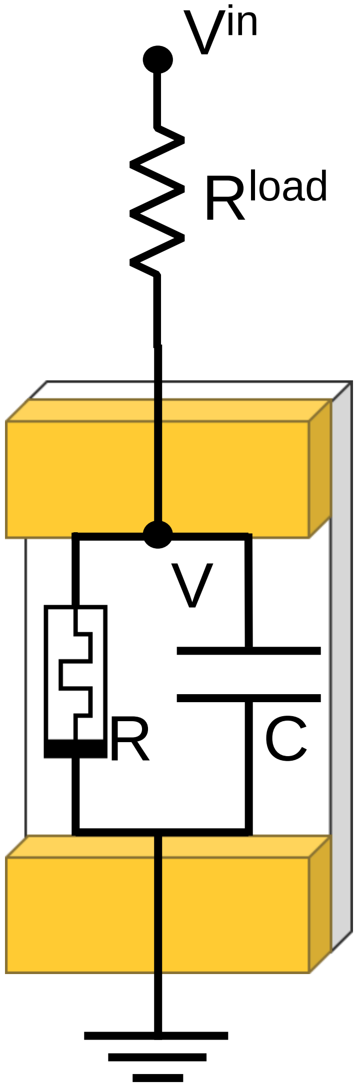


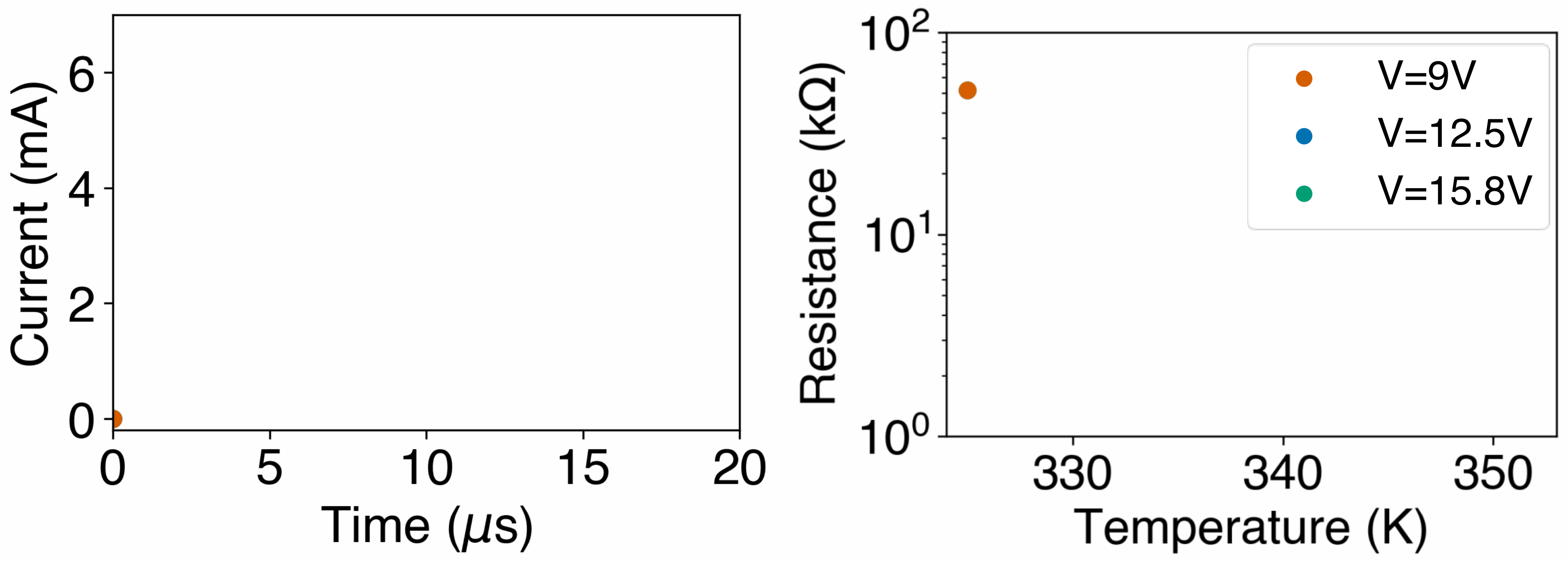
Substrate: Al_2O_3



Qiu, E., Zhang, Y. H., Di Ventra, M., & Schuller, I. K. (2023). Reconfigurable cascaded thermal neuristors for neuromorphic computing. *Advanced Materials*, 2306818.

Single neuristor characteristics





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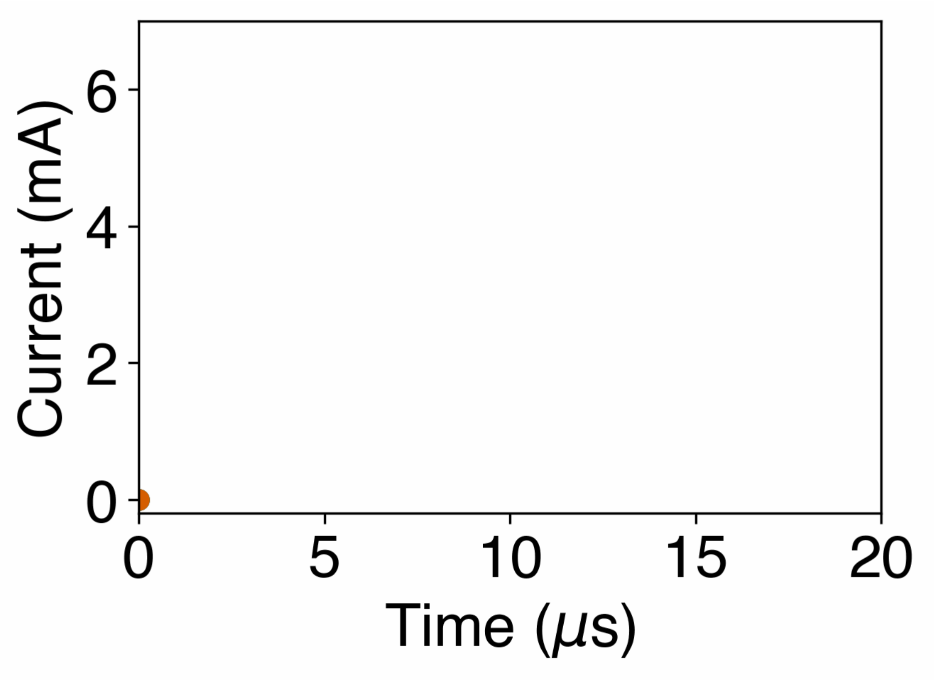
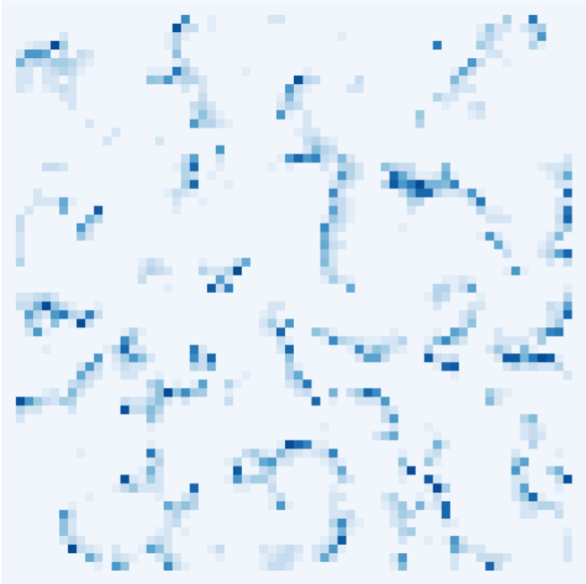
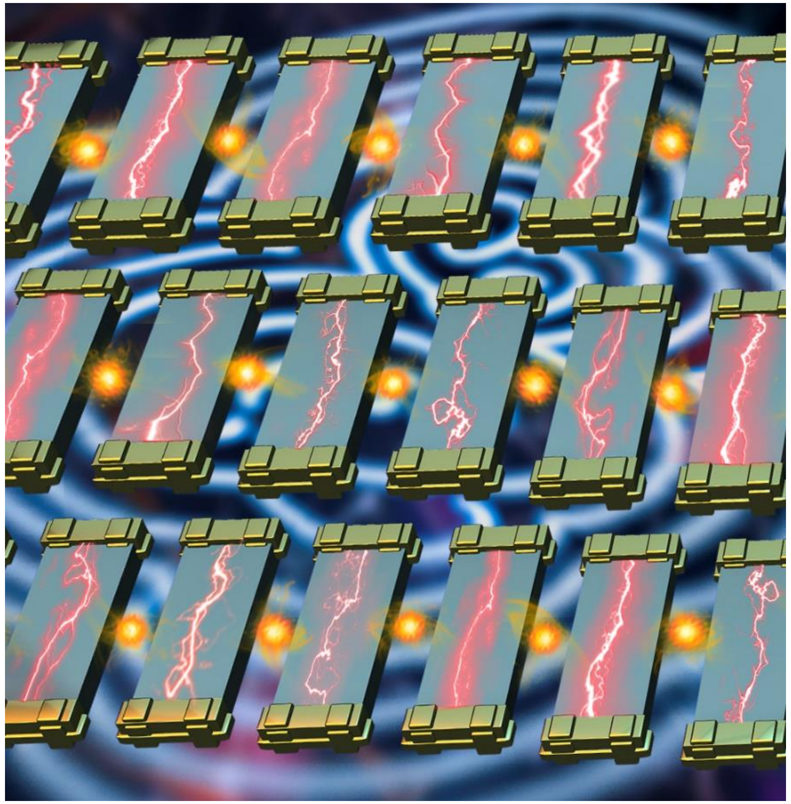
Thermal neuristor array

1 neuron

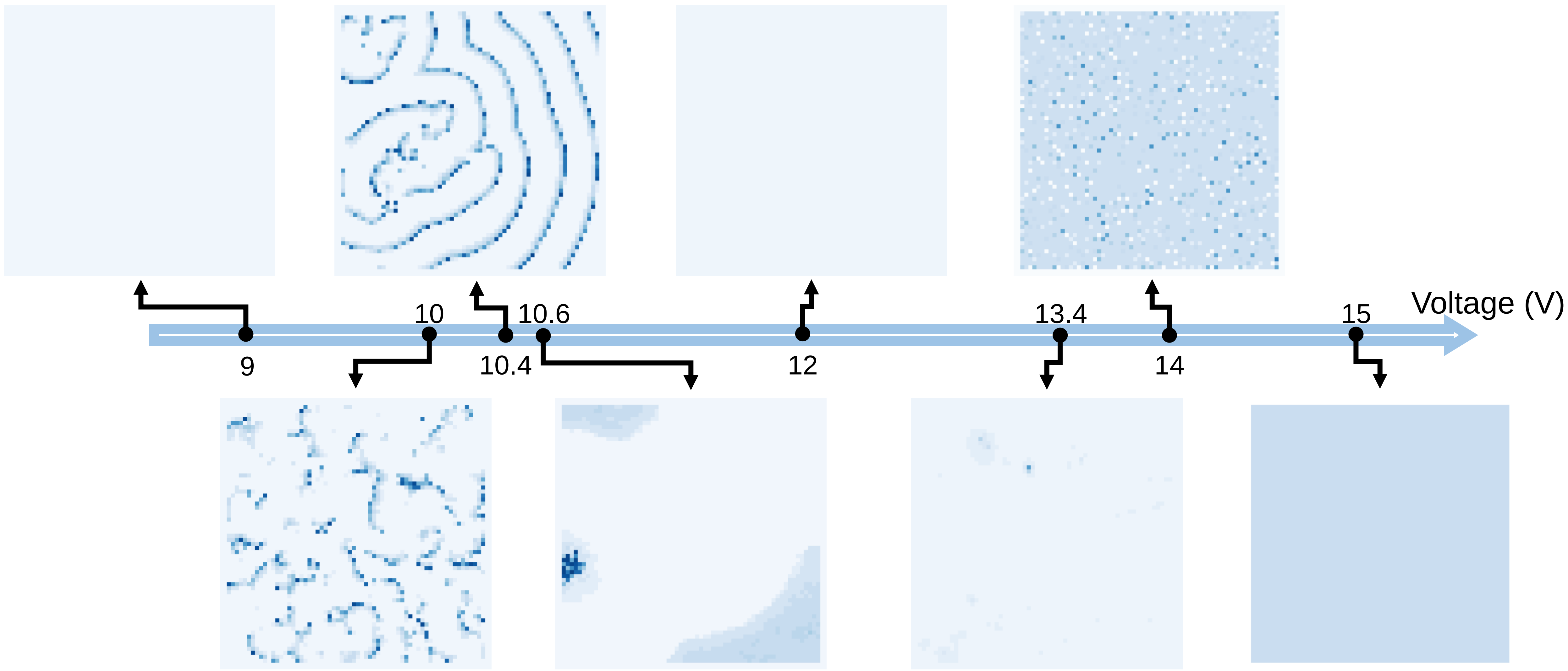


64x64
neurons



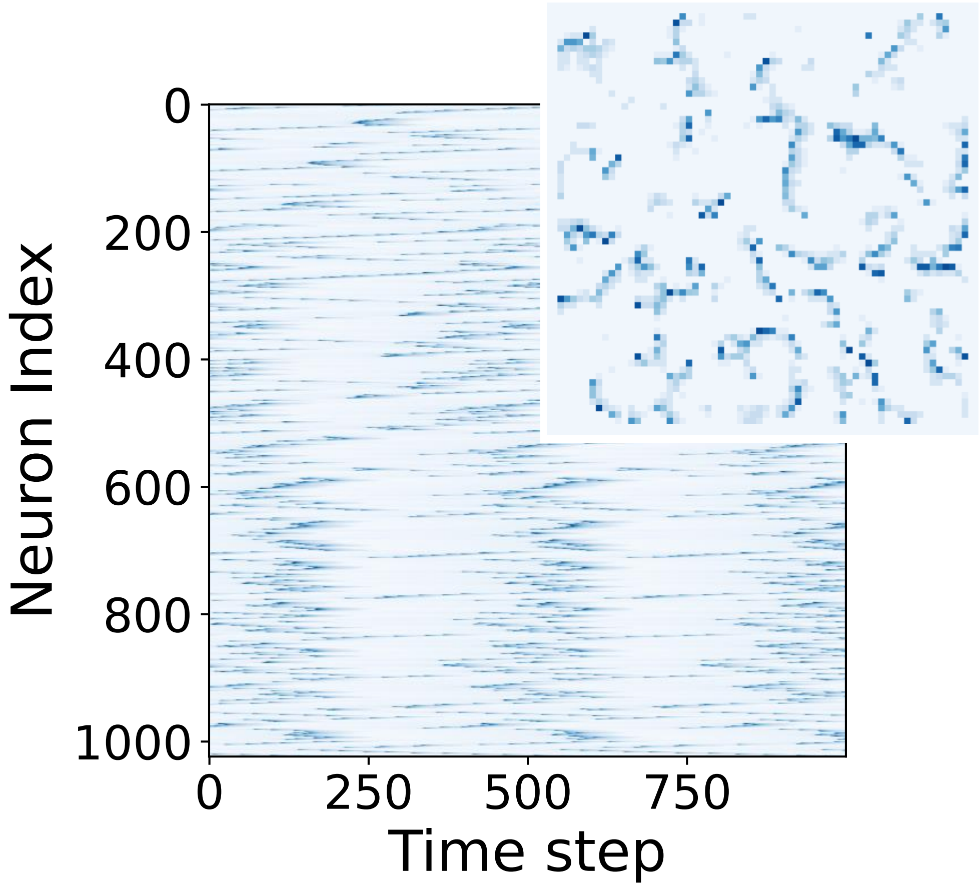


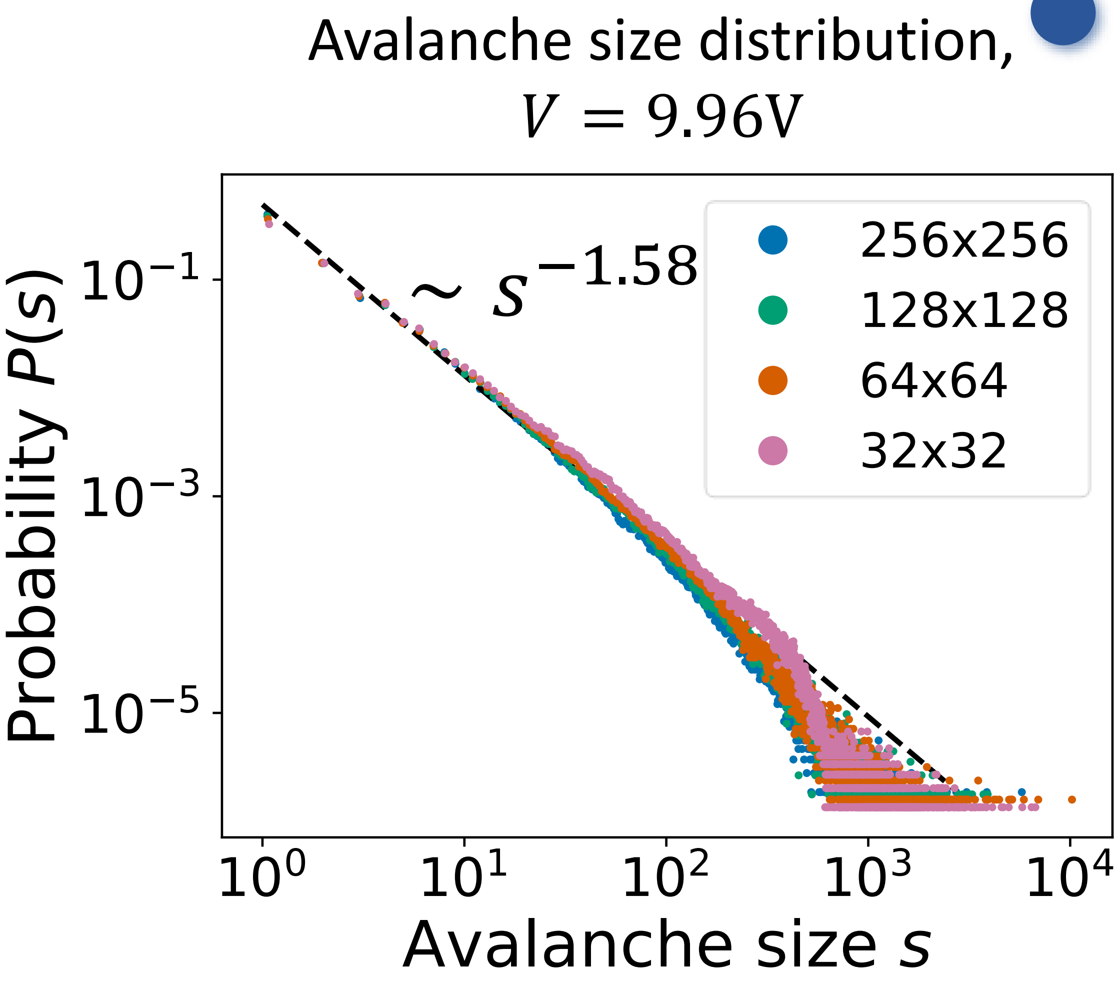
Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.



Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.

Avalanche size distribution





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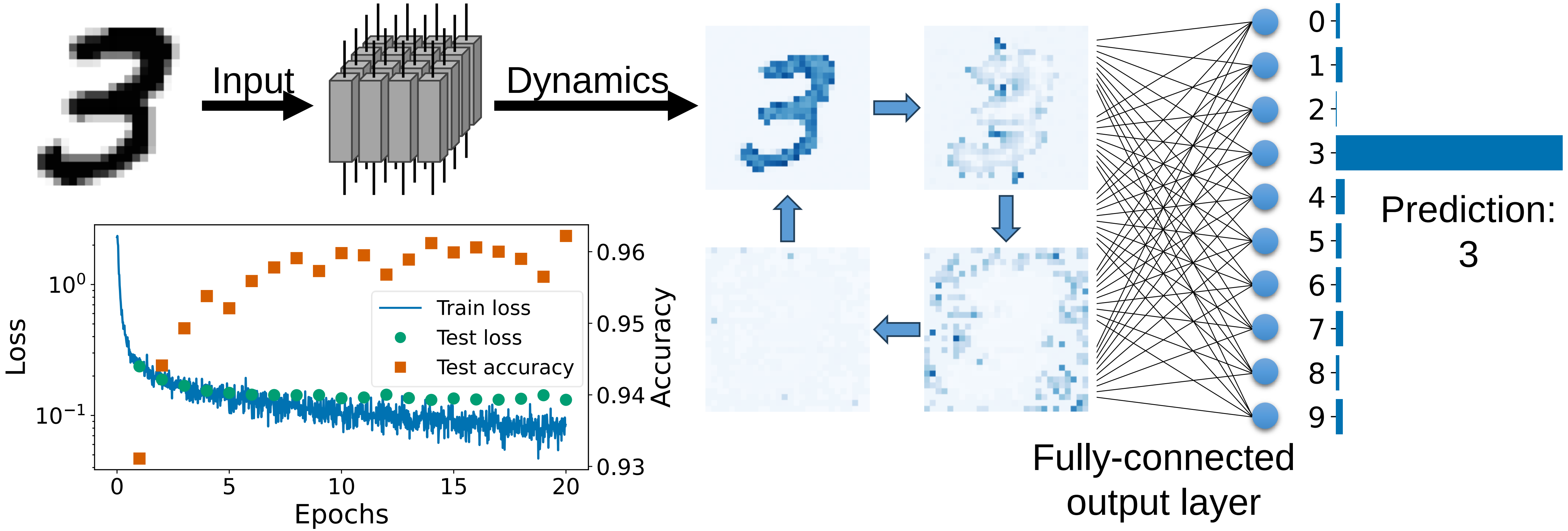
LRO for computing



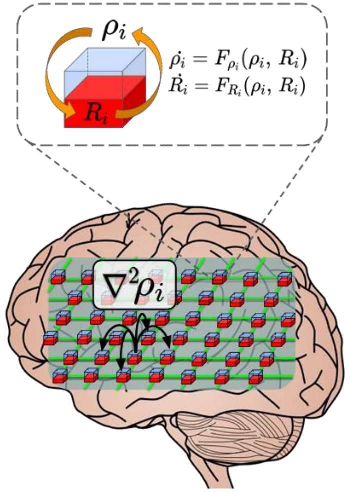
Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.

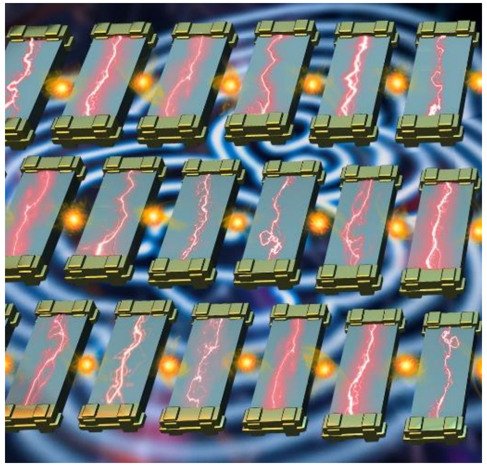
LRO for computing

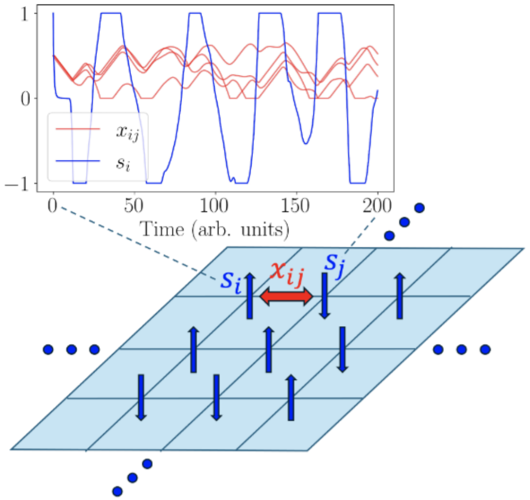
Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.

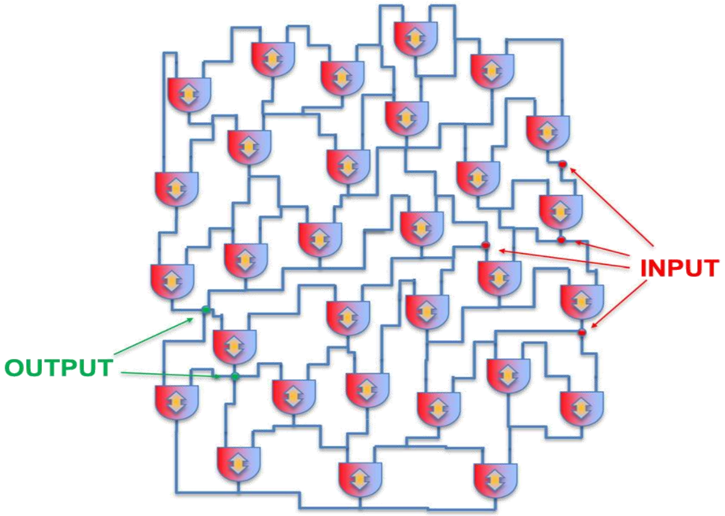
Memory-induced long-range order

- Memory, or time non-local interactions, induces spatial long-range order
- Tunable by adjusting the time scales
- An expansive LRO *phase* instead of a transition *point*
- The brain may not be critical



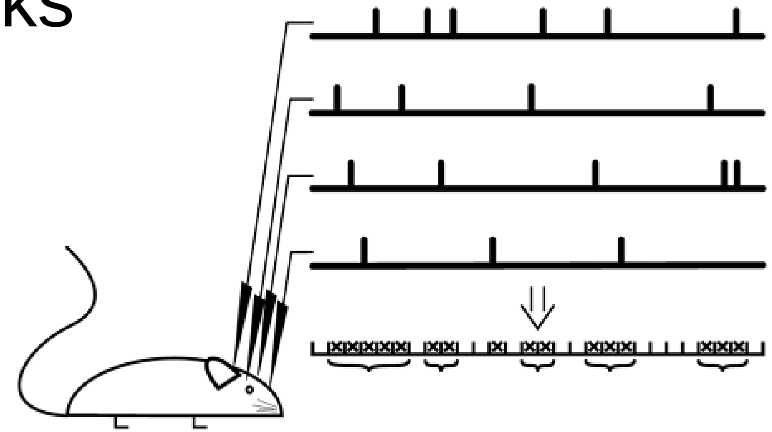


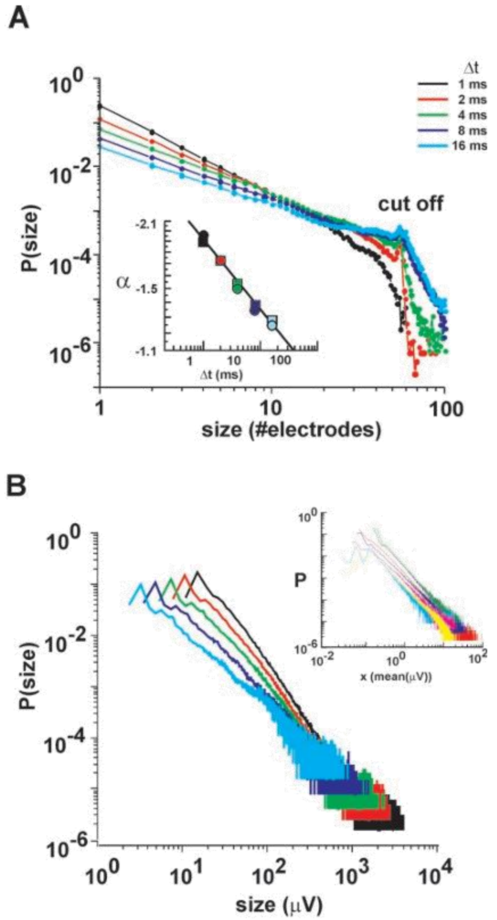




Why is LRO important?

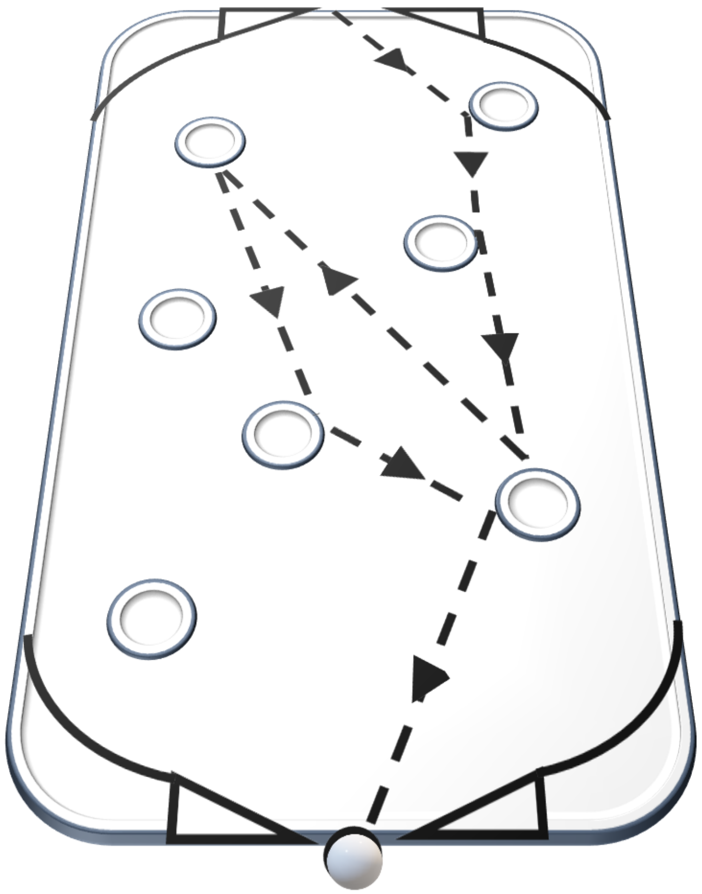
- Long-range order is observed in
  - Brain dynamics
  - Neural networks
  - MemComputing machines
- Beneficial for computing tasks





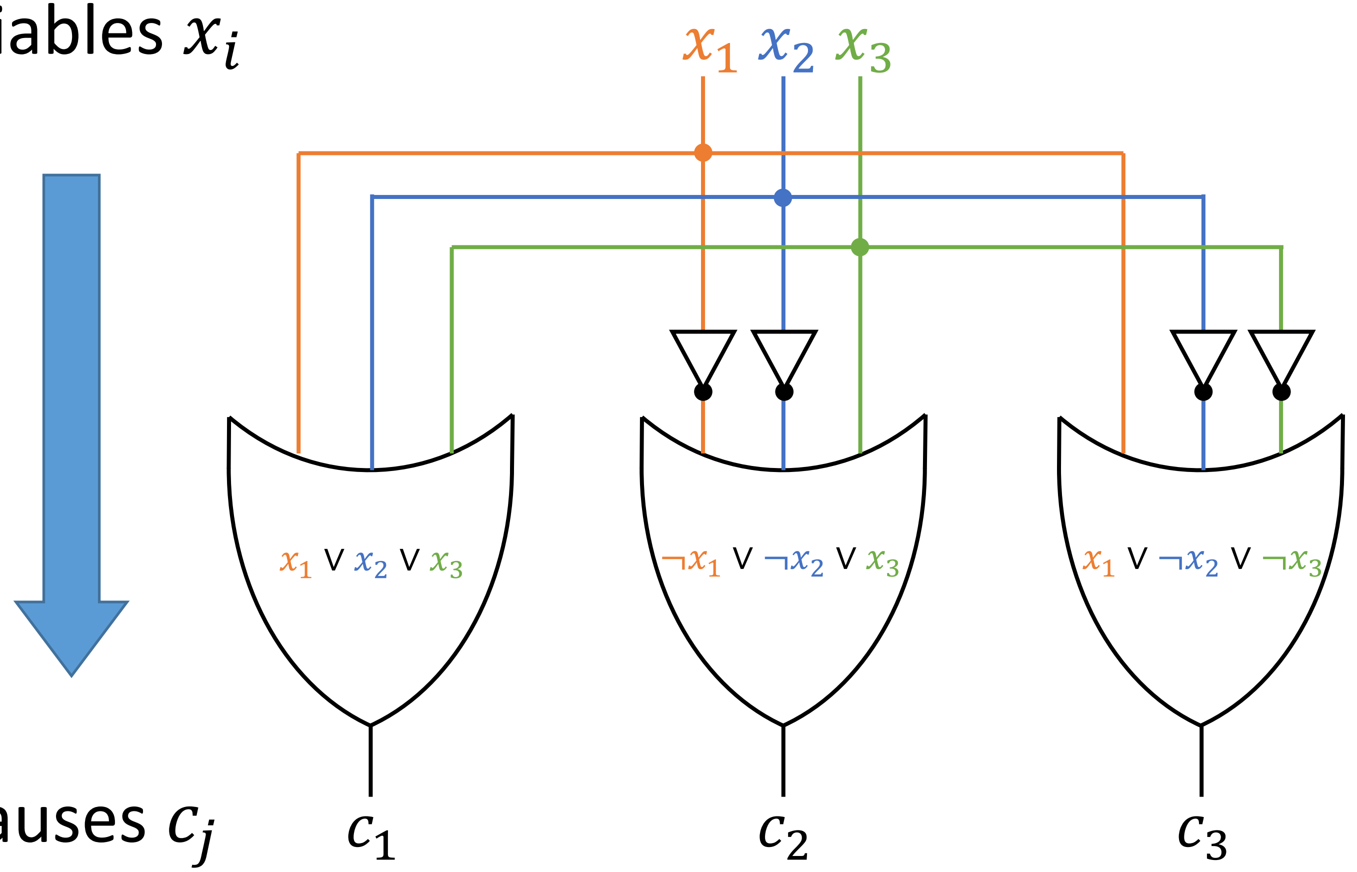
MemComputing: Computing with LRO

- Nonlinear dynamical system
- Equilibrium ↔ solution
- MILRO helps the system navigate towards the solution



Boolean Satisfiability

Variables x_i



Clauses c_j

$c_1 = x_1 \vee x_2 \vee x_3$

Find $\{x_i\}$,
such that all c_j
evaluates to TRUE

From logic to dynamics

discrete logic space

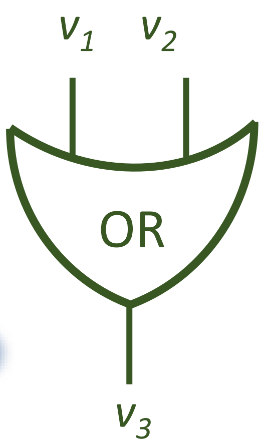
0 OR 0 = 0

1 OR 0 = 1

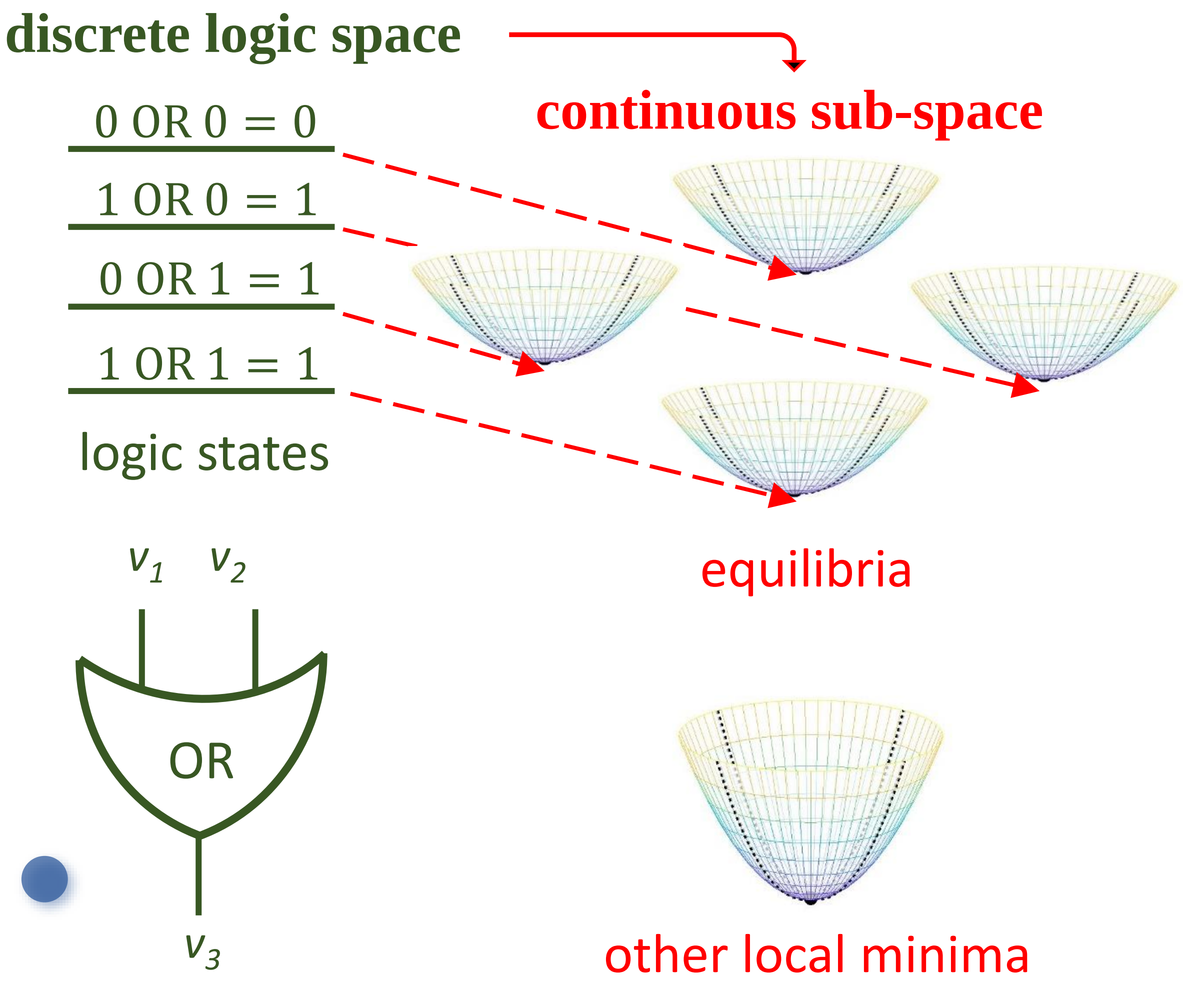
0 OR 1 = 1

1 OR 1 = 1

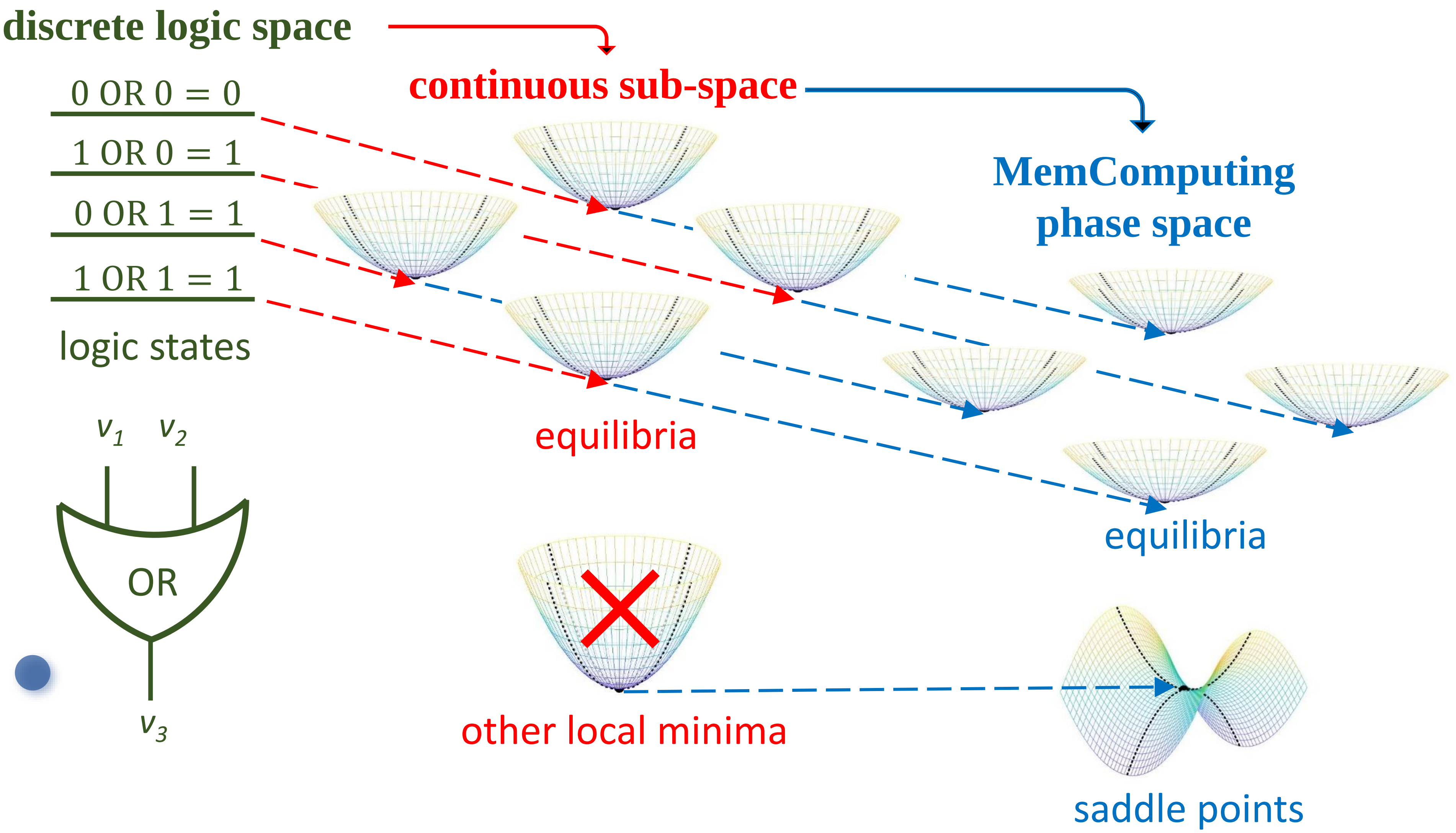
logic states



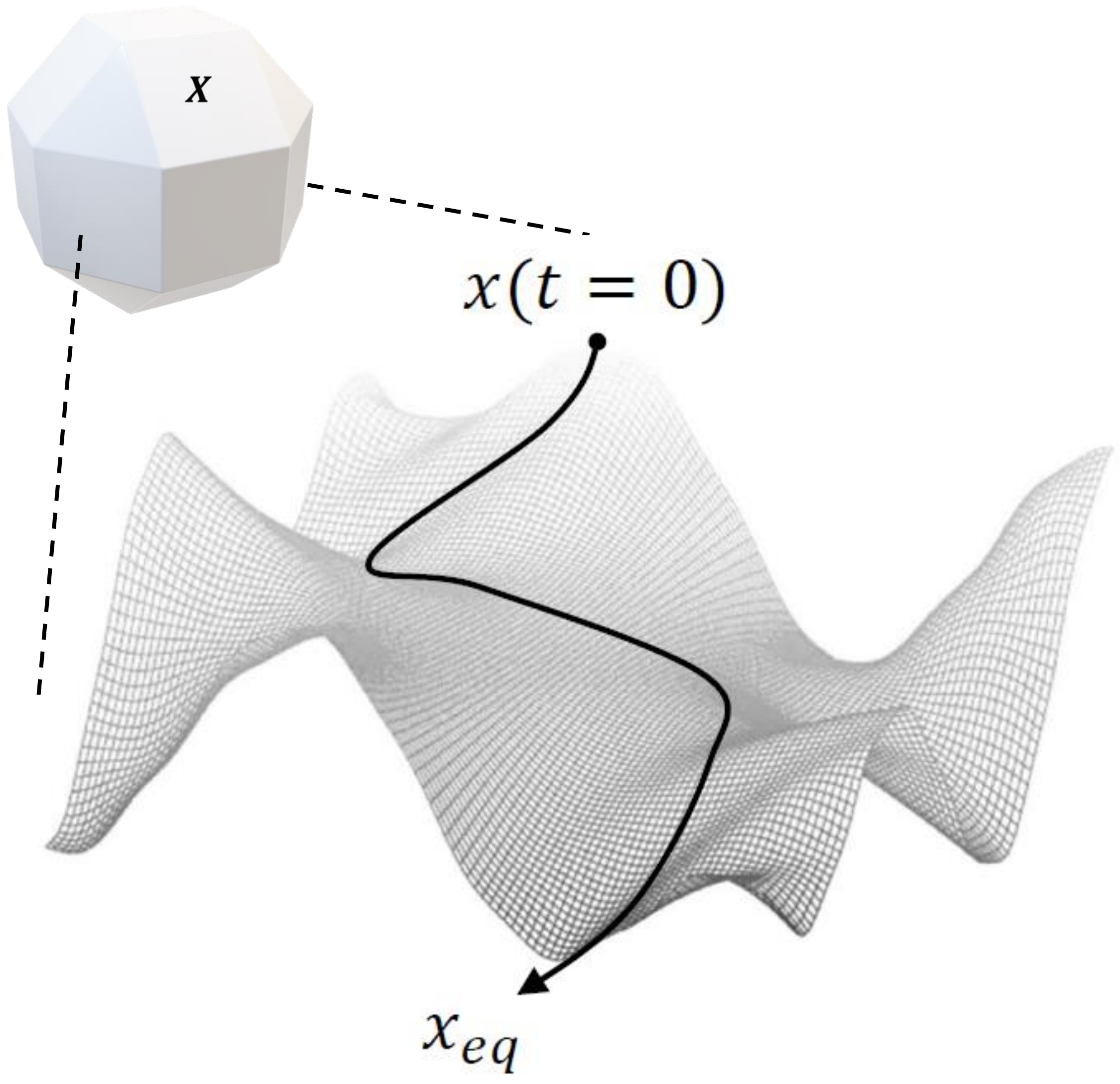
From logic to dynamics

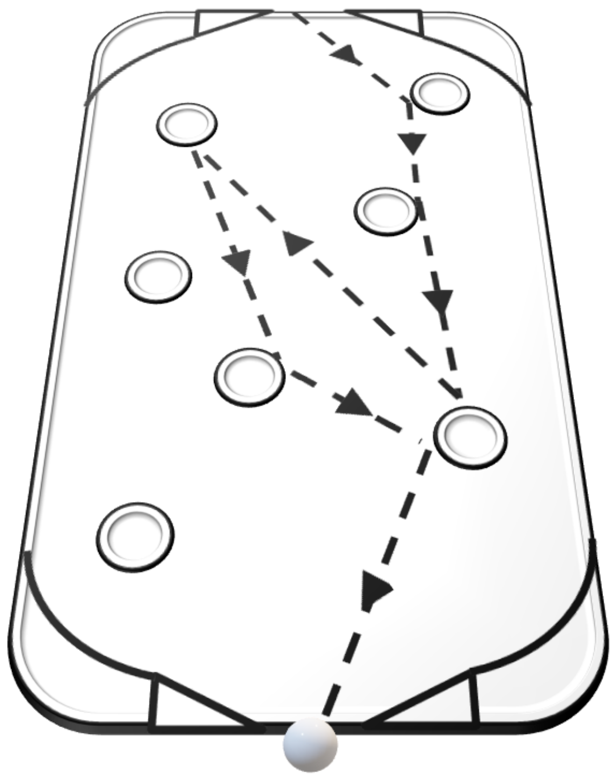


From logic to dynamics



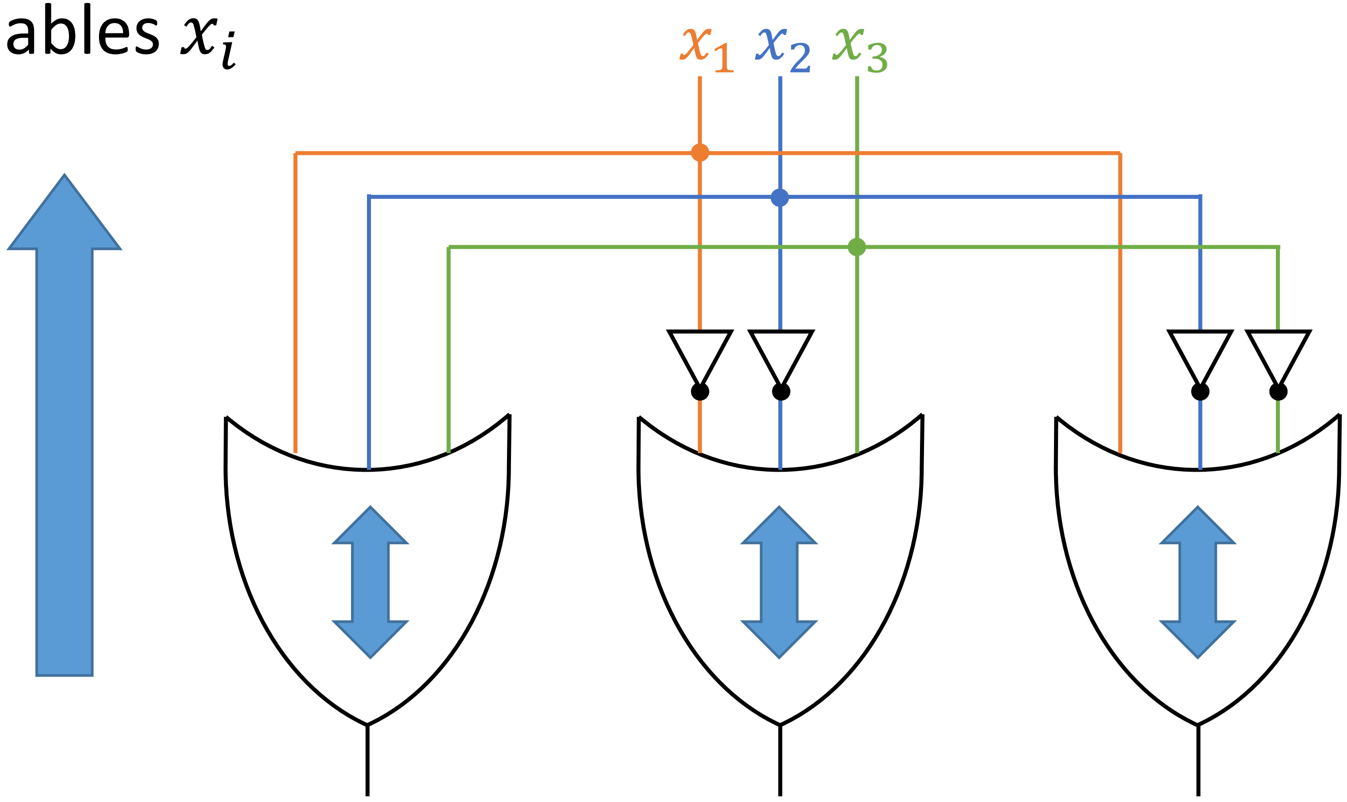
From logic to dynamics





Self-organizing logic gate

Variables x_i



Clauses c_j $c_1 = 1$ $c_2 = 1$ $c_3 = 1$

$c_1 = x_1 \vee x_2 \vee x_3$

From problem to solution

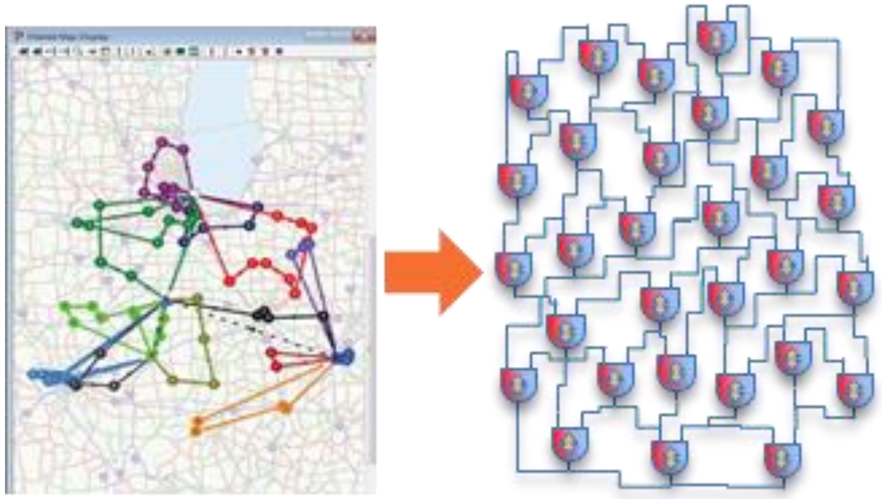
Problem

Boolean circuit representation

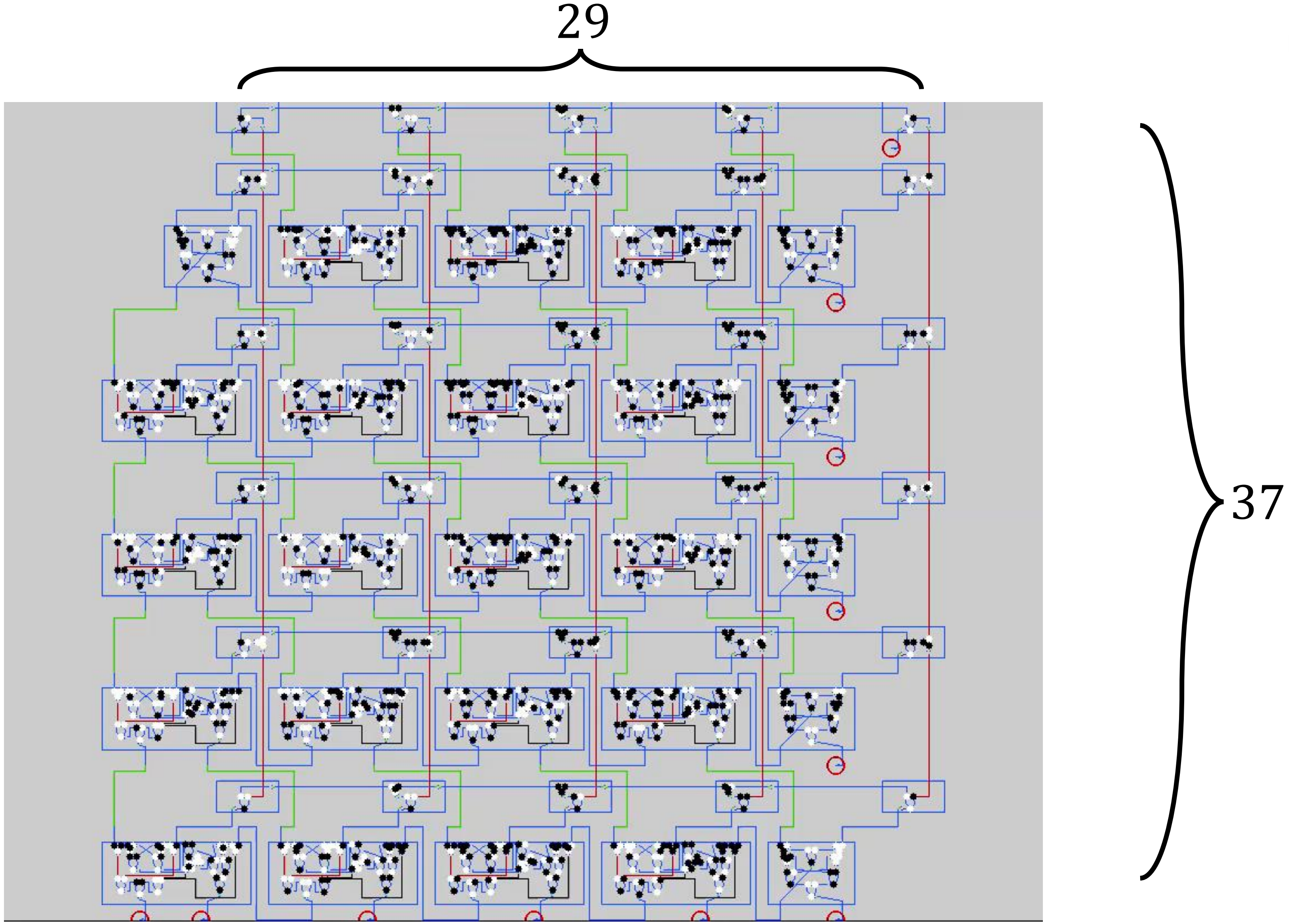
SOLG

Build / simulate dynamics

Solution

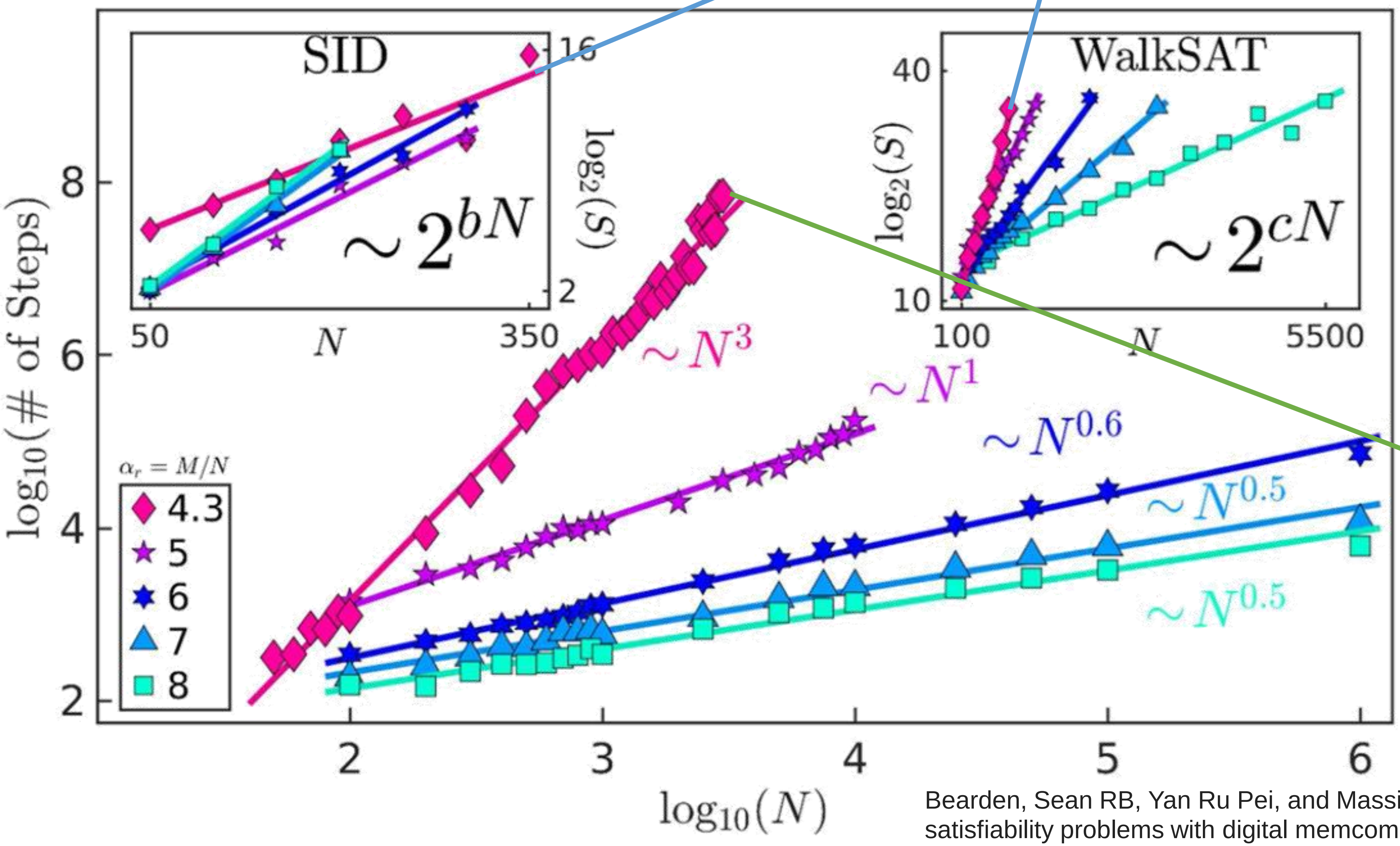


Example: $1073 = 29 \times 37$



Boolean Satisfiability

> age of the Universe (>10^{10} years)

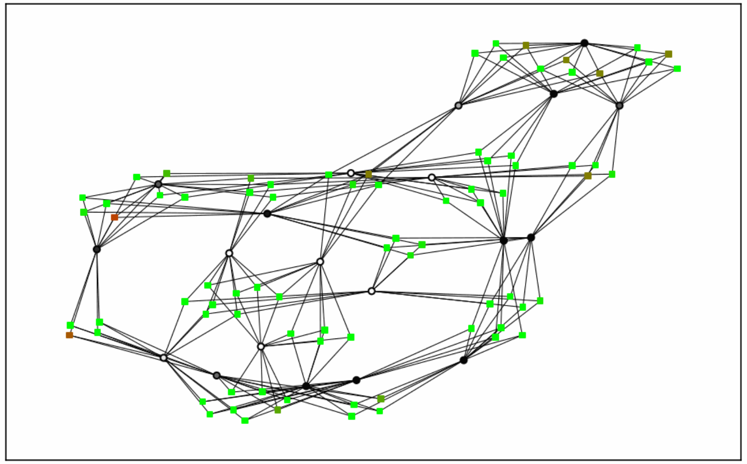


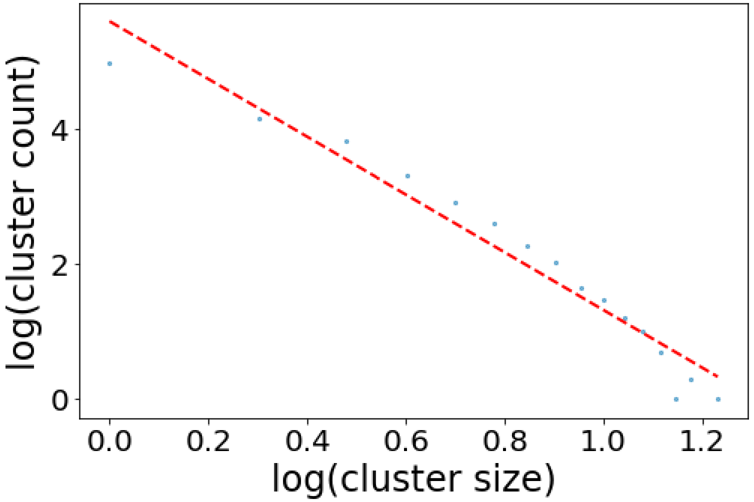
~ 15 hours

Bearden, Sean RB, Yan Ru Pei, and Massimiliano Di Ventra. "Efficient solution of Boolean satisfiability problems with digital memcomputing." *Scientific reports* 10.1 (2020): 1-8.

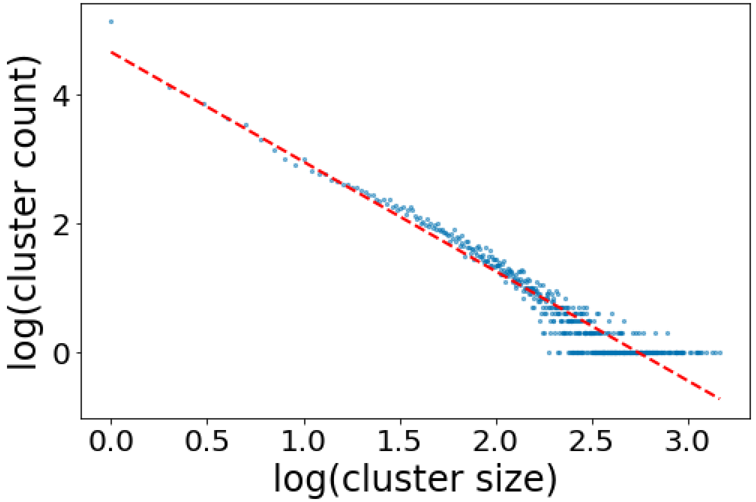
Why is LRO important?

- Stability ↔ Flexibility

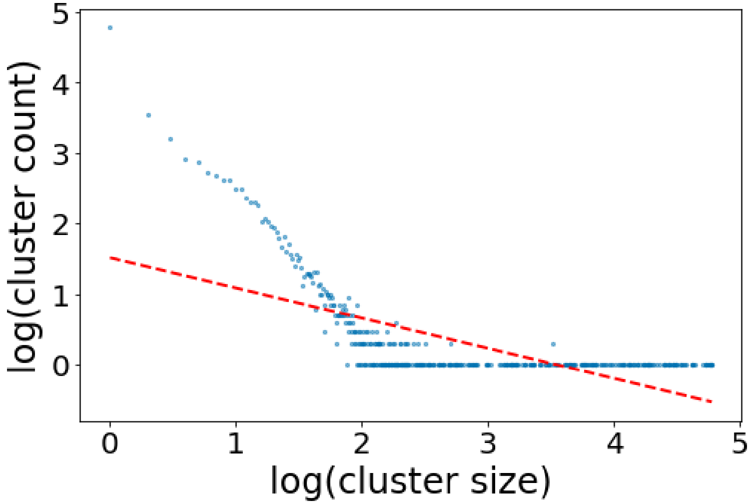




Small avalanches
Not enough correlation



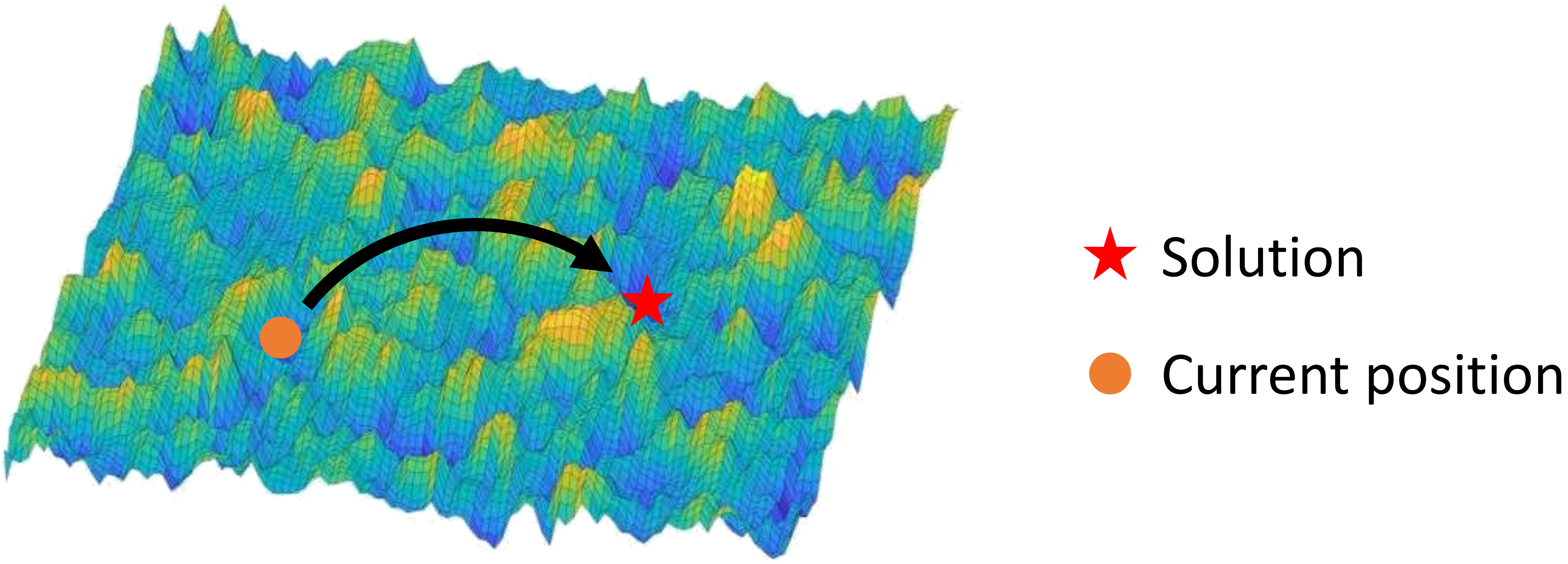
LRO
Ideal dynamics



Non-stopping avalanches
Chaotic dynamics

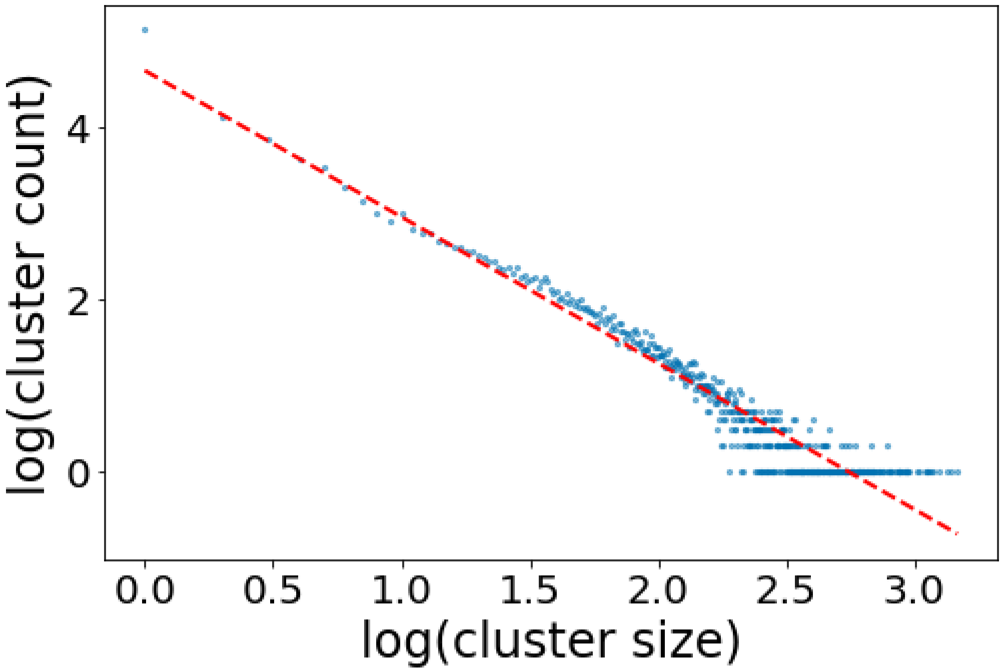
Why is LRO important?

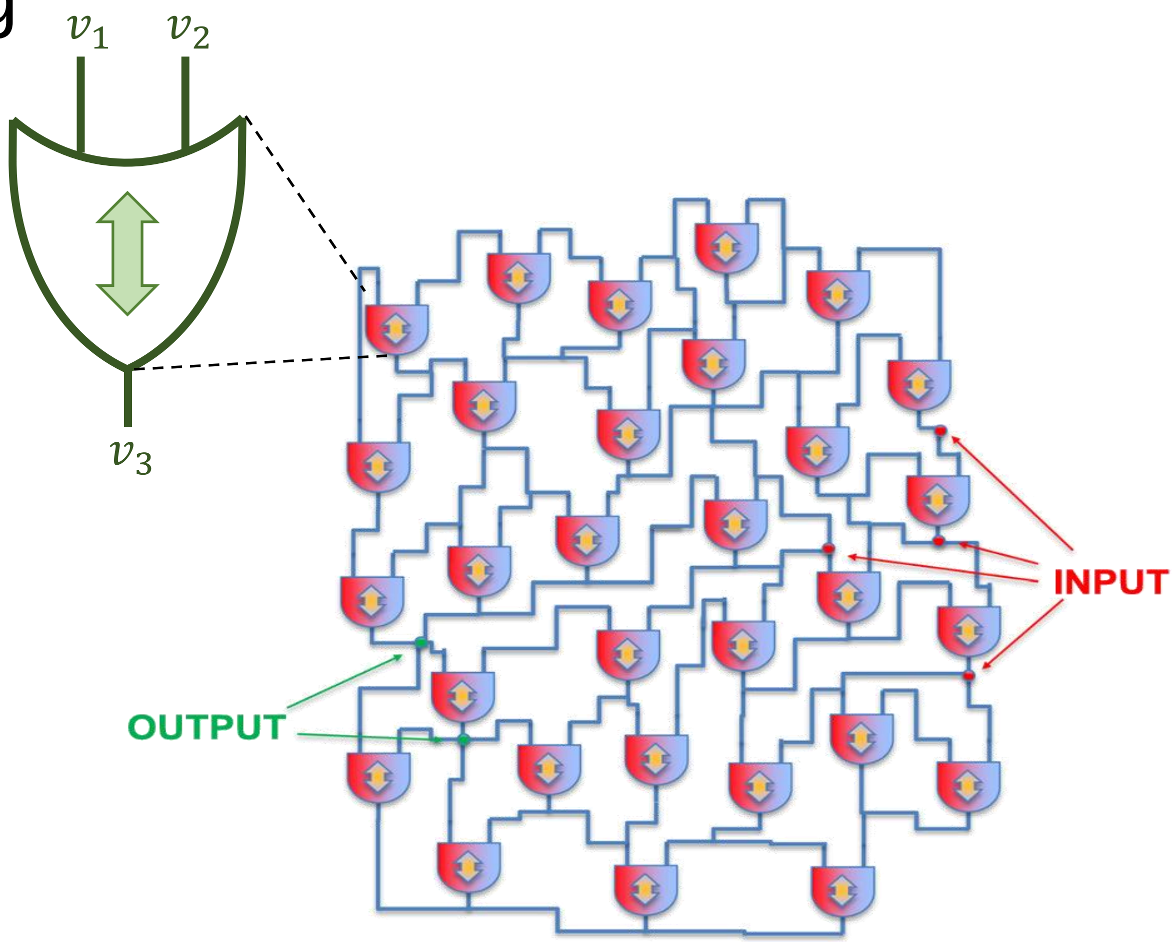
- Exploration of complex loss landscape



Why is LRO important?

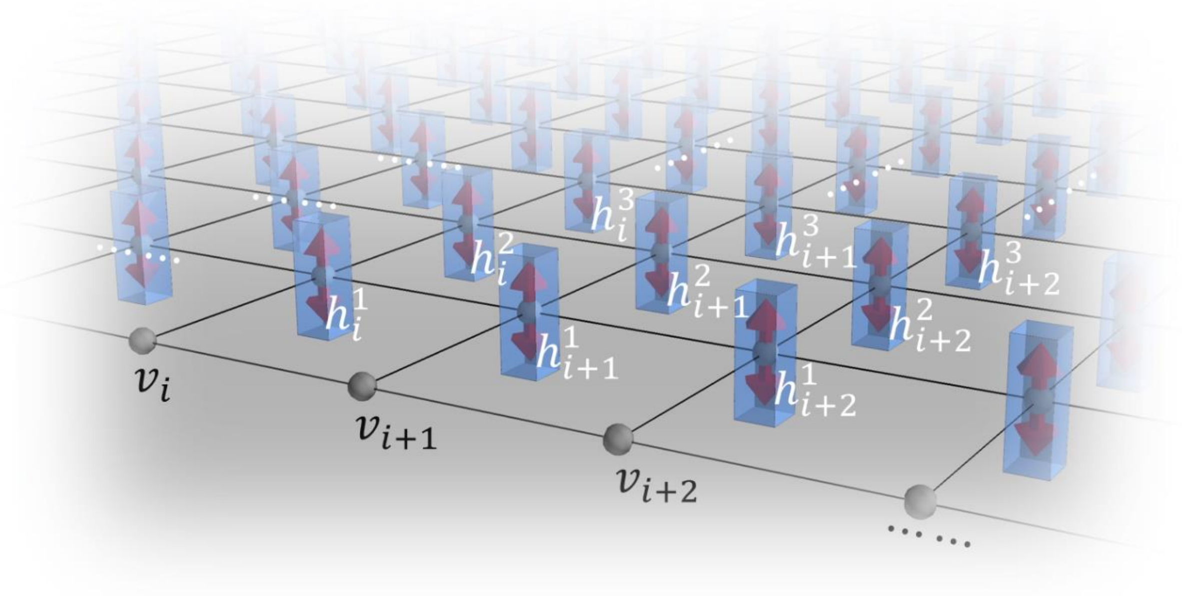
- Multi-scale, hierarchical processing





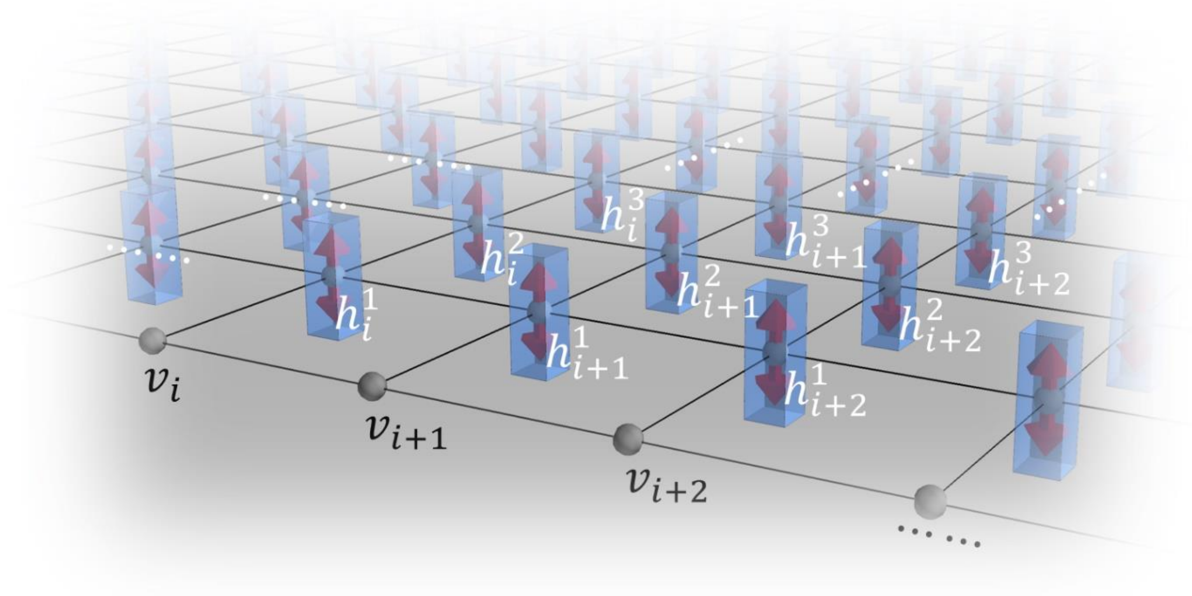
Future work

- Systematically analyze long-range order in neural networks
- MemComputing neural network: neural network entirely built with self-organizing logic gates



Future work

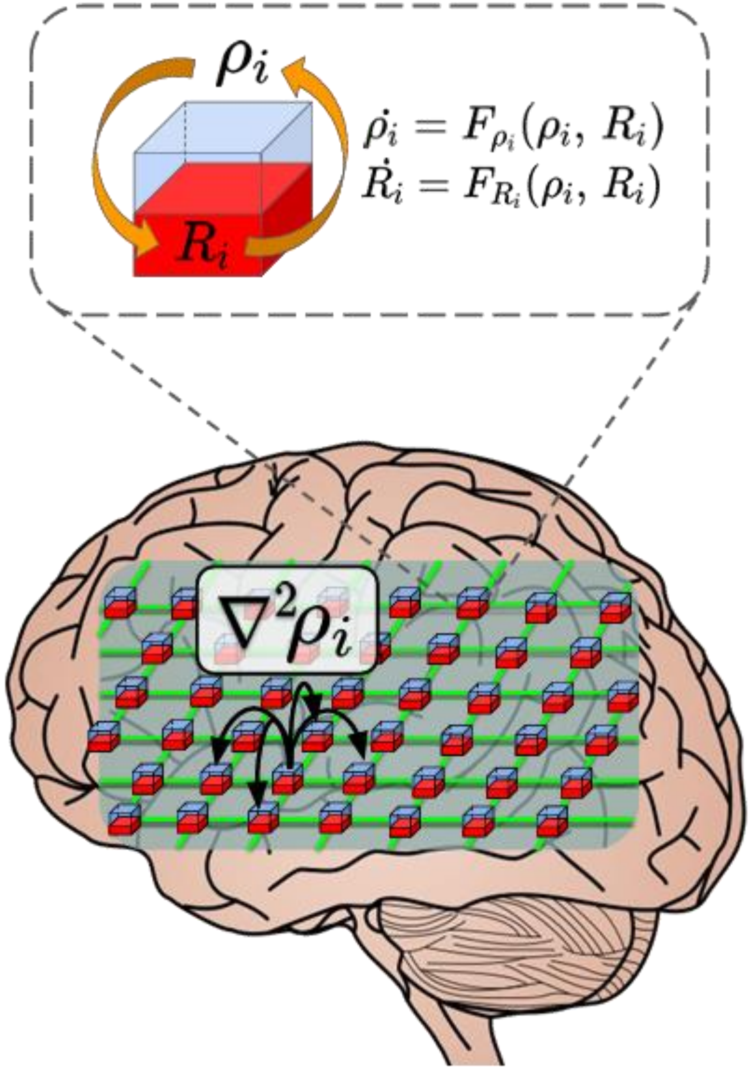
- Designing and optimizing dynamical systems with machine learning
- Optimized MemComputing dynamics for better performance



Thank you!

Yuanhang Zhang

MILRO in neural activity



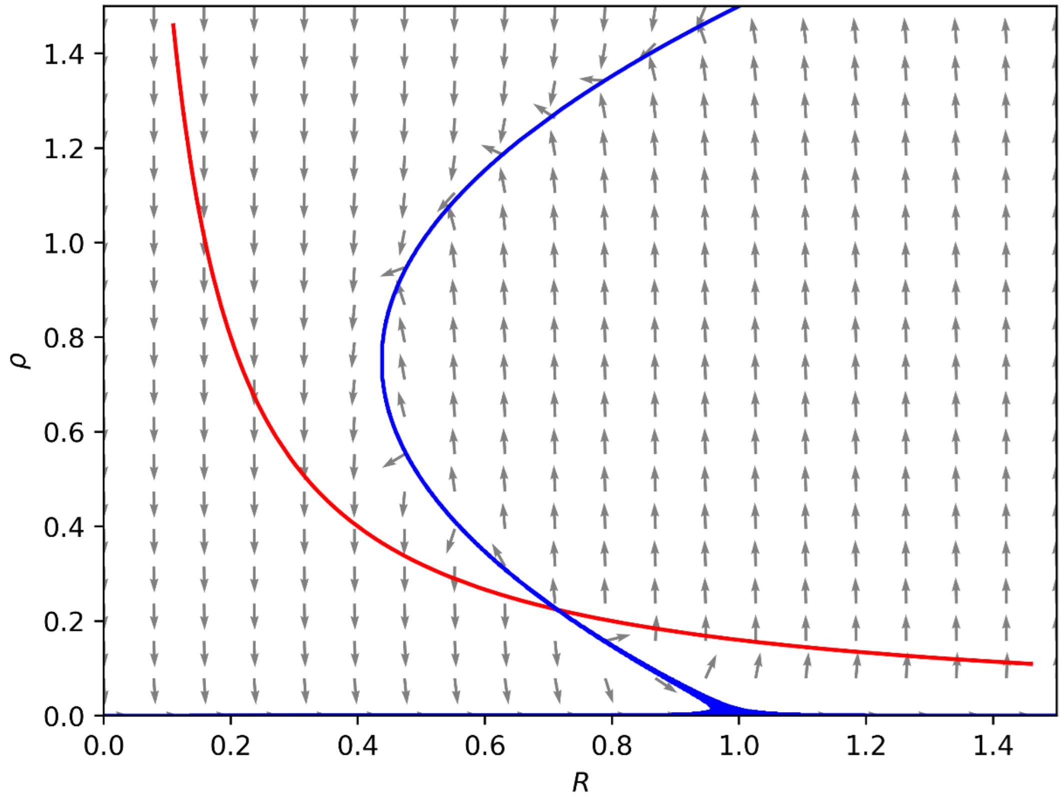
$$\dot{\rho}_i = (-a + R_i)\rho_i + b\rho_i^2 - c\rho_i^3 + h + D\nabla^2\rho_i + \sigma\eta_i$$

$$\dot{R}_i = \delta - \frac{1}{\tau_D}(R_i\rho_i + \sigma\eta_i)$$

Wilson-Cowan model,
slightly simplified

J. K.-C. Sun, C. S. Sipling, Y.-H. Zhang, M. Di Ventra. “Memory in neural activity: long-range order without criticality." *In preparation.*

MILRO in neural activity



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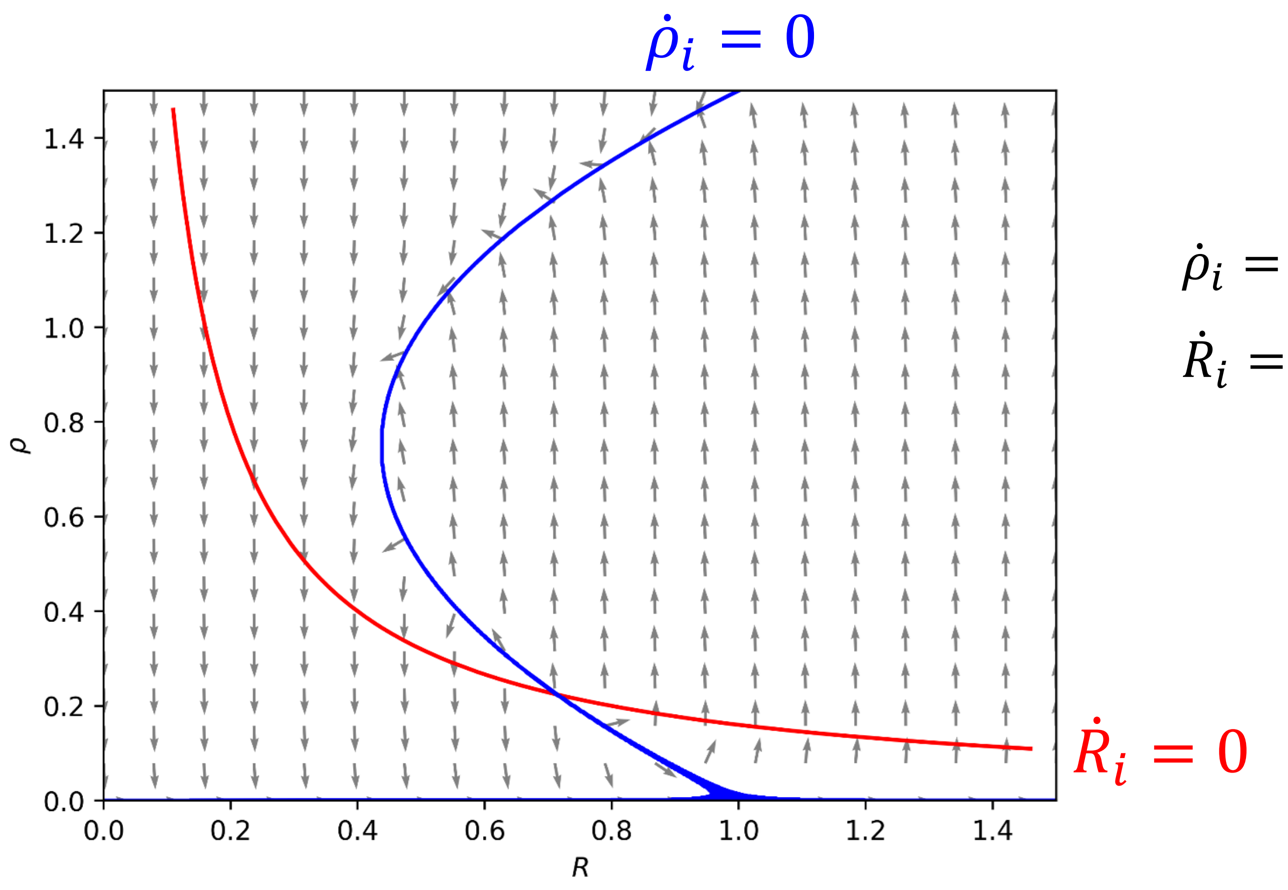

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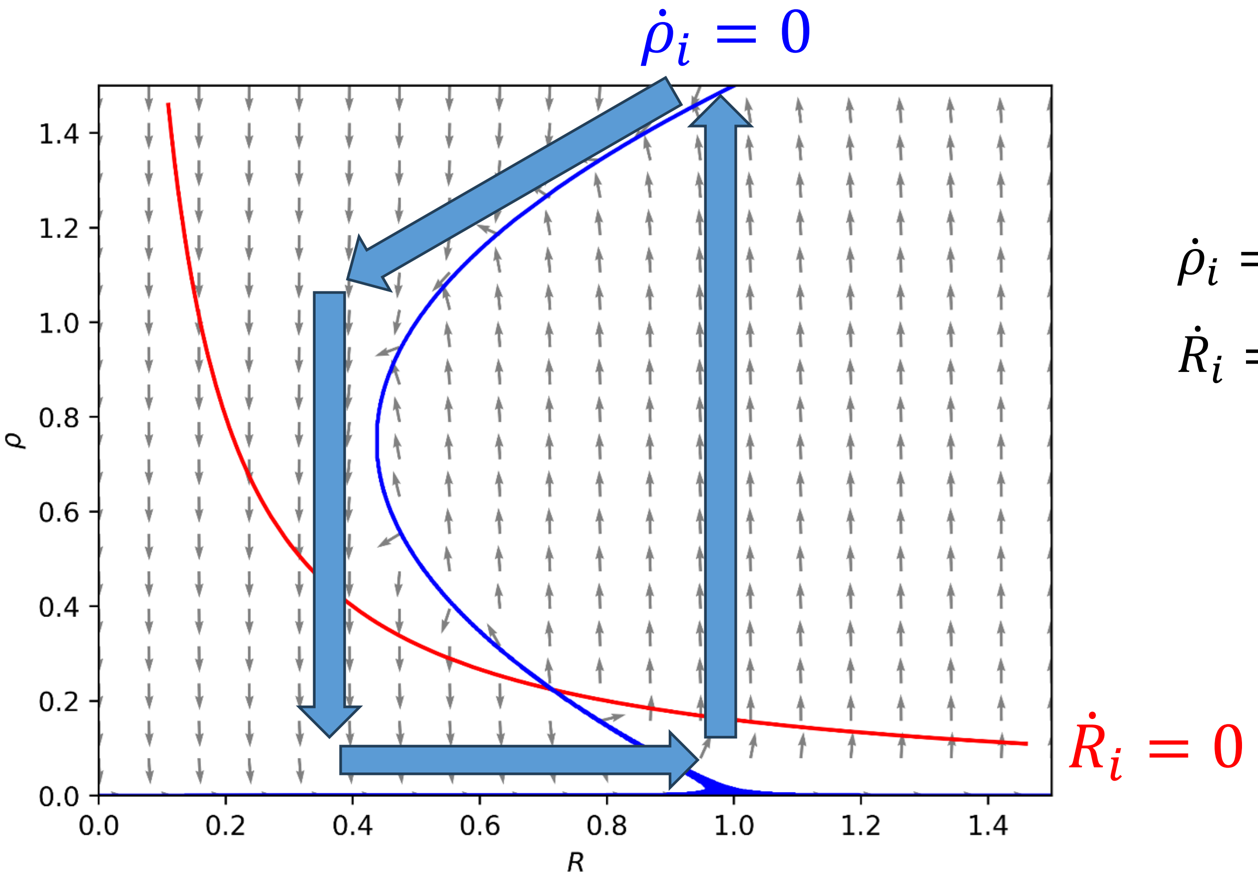

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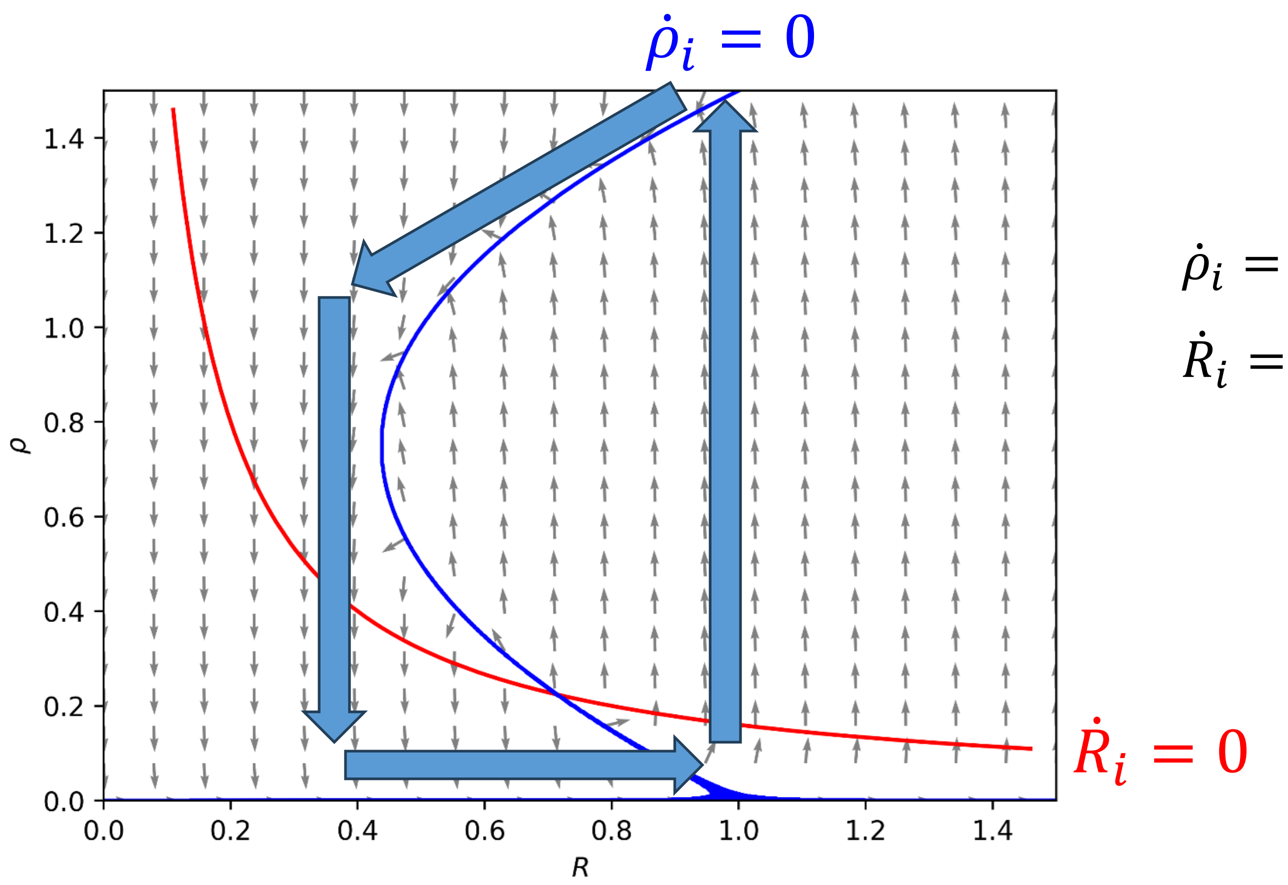

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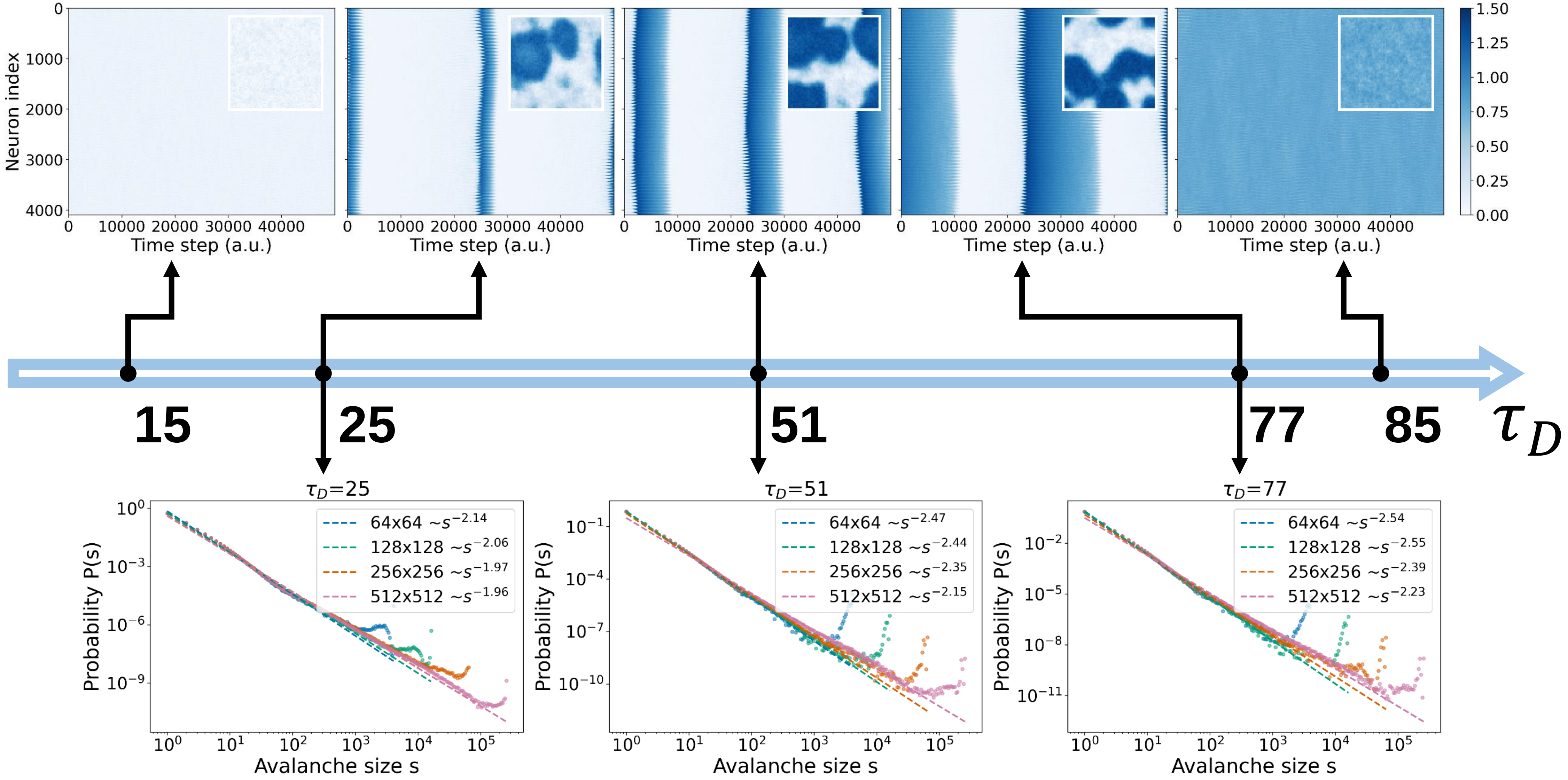
Time scale
to be tuned

J. K.-C. Sun, C. S. Sipling, Y.-H. Zhang, M. Di Ventra. “Memory in neural activity: long-range order without criticality." *In preparation.*

MILRO in neural activity



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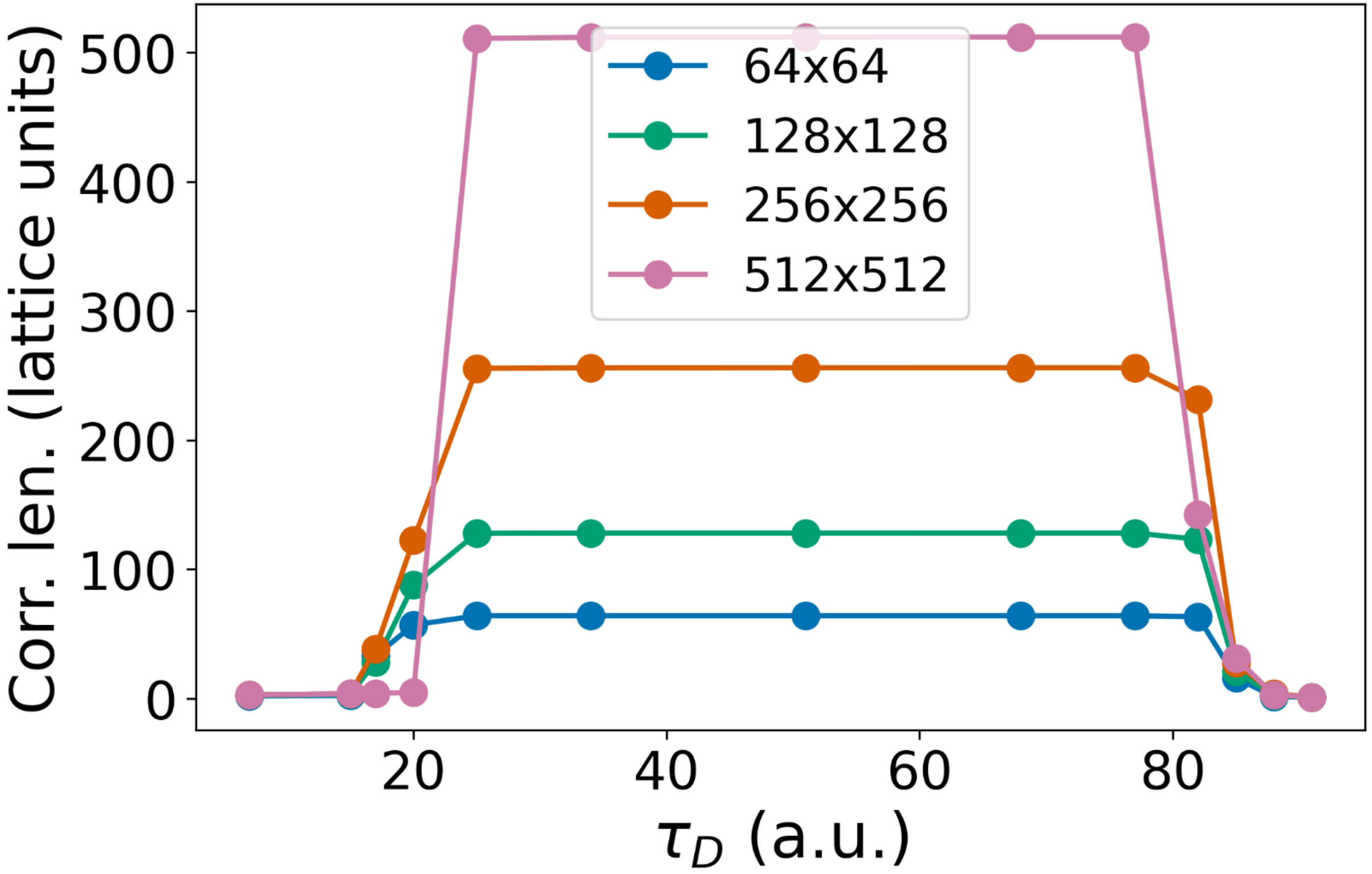
MILRO in neural activity



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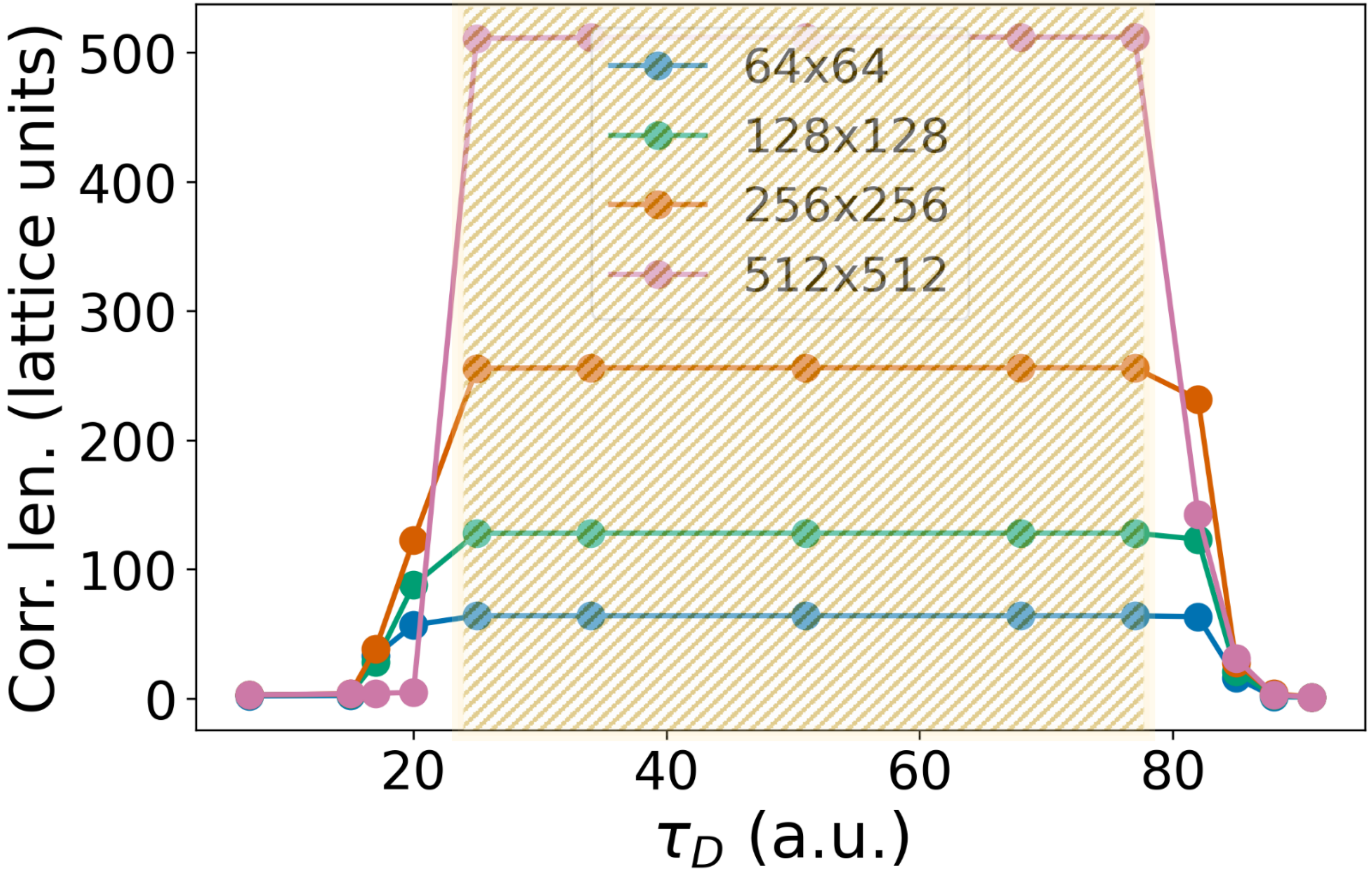
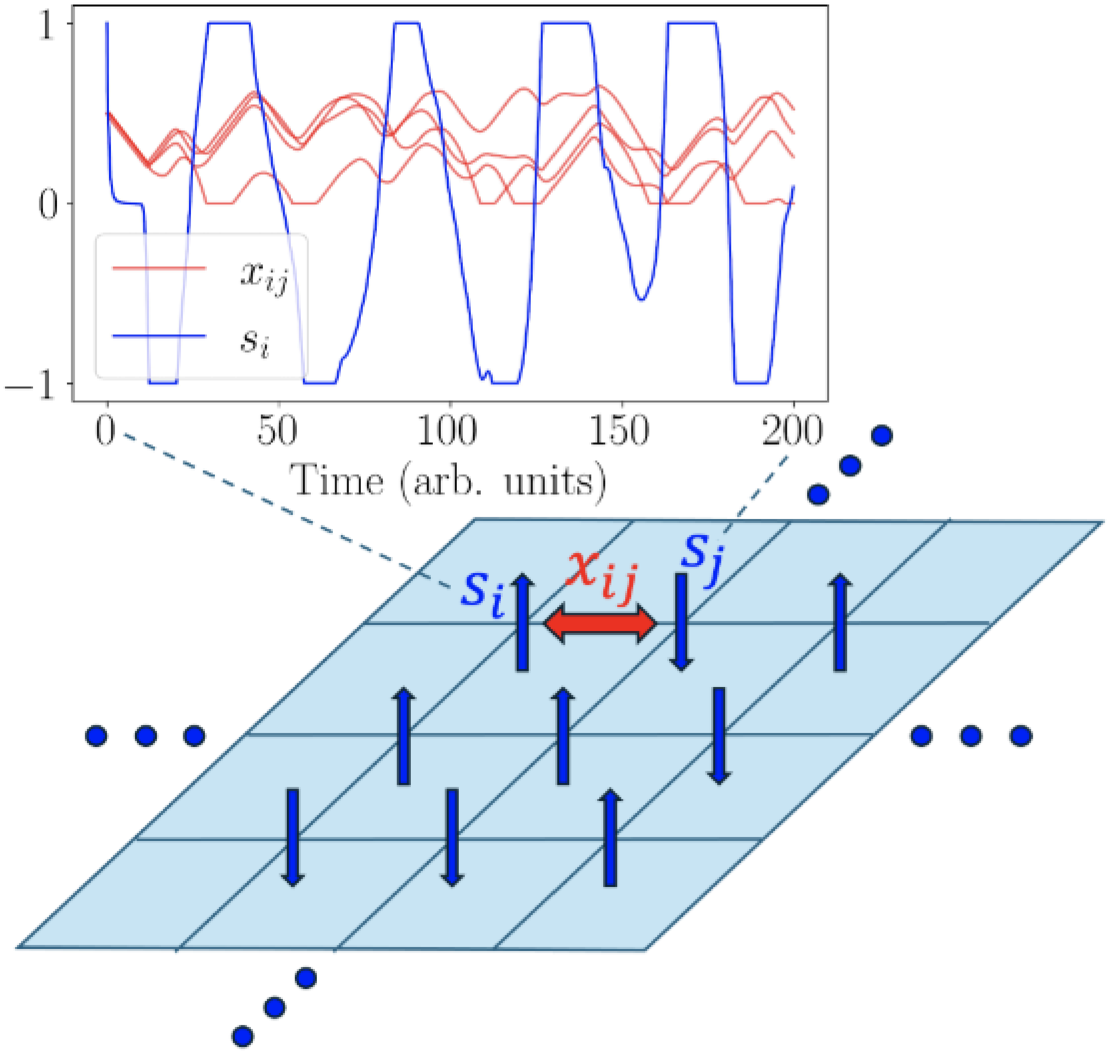
MILRO in neural activity



LRO
Phase!

J. K.-C. Sun, C. S. Sipling, Y.-H. Zhang, M. Di Ventra. “Memory in neural activity: long-range order without criticality." *In preparation.*

MILRO in dynamical systems

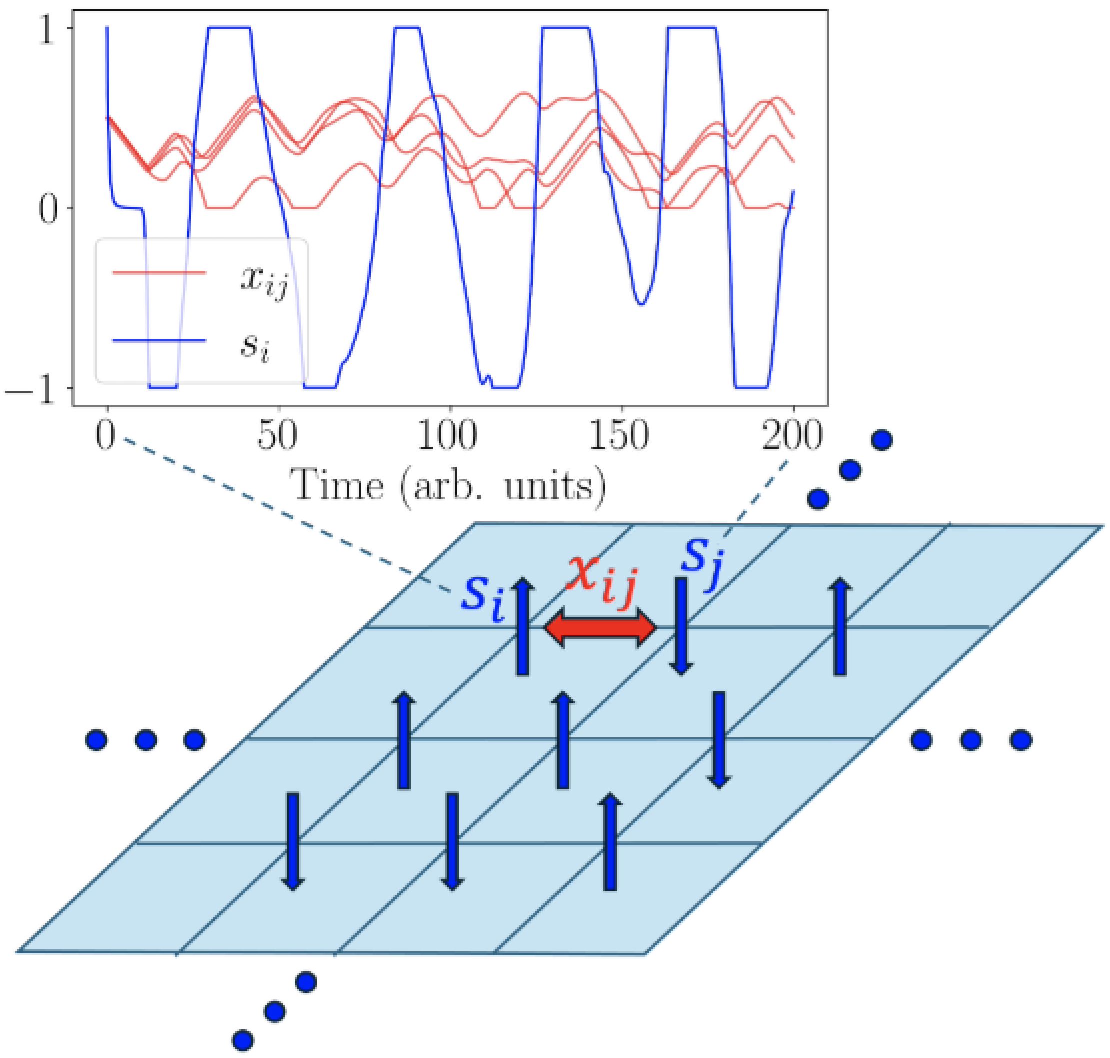


$$\dot{s}_i = \sum_{\langle ij \rangle} J_{ij} s_j - g \sum_{\langle ij \rangle} x_{ij} s_i$$

$$\dot{x}_{ij} = \gamma (C_{ij} - \delta)$$

Sipling, Chesson, and Massimiliano Di Ventra. "Memory-induced long-range order in dynamical systems." *arXiv preprint arXiv:2405.06834* (2024).

MILRO in dynamical systems



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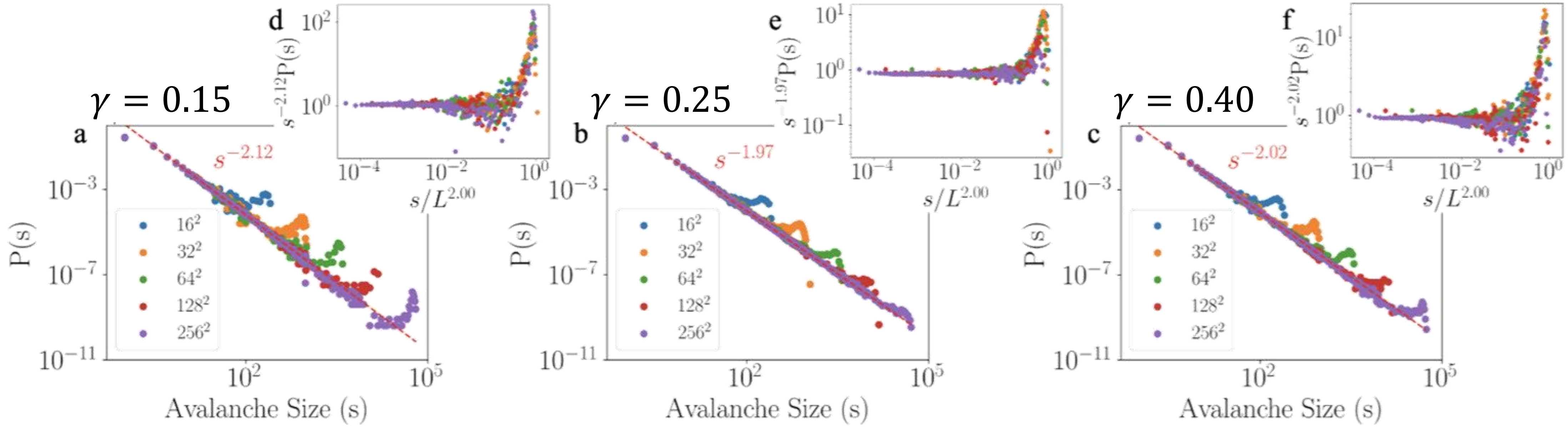
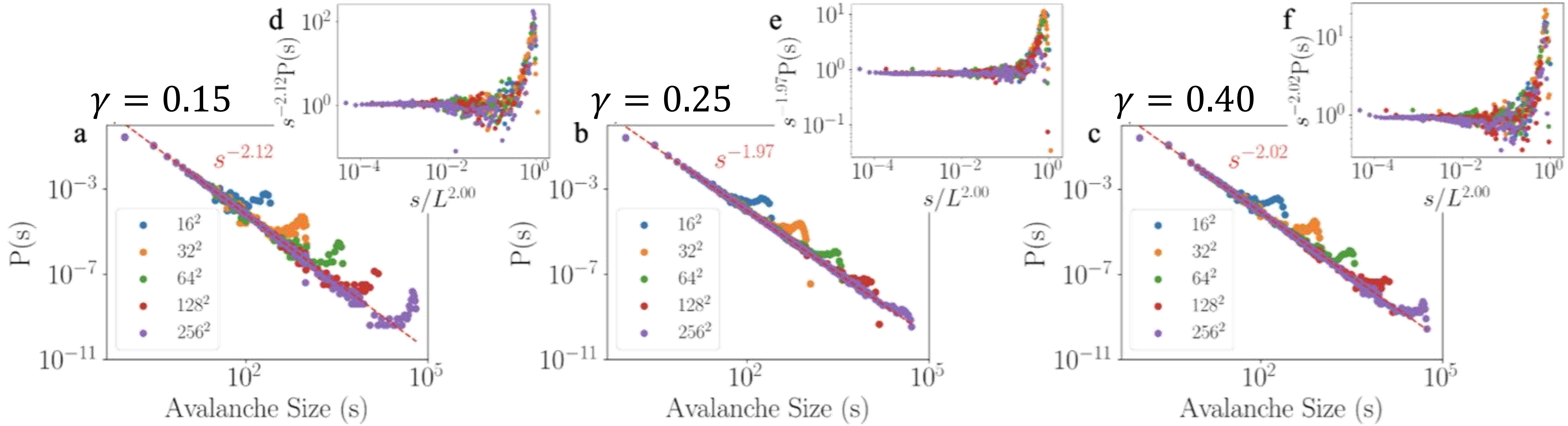
Time scale
to be tuned

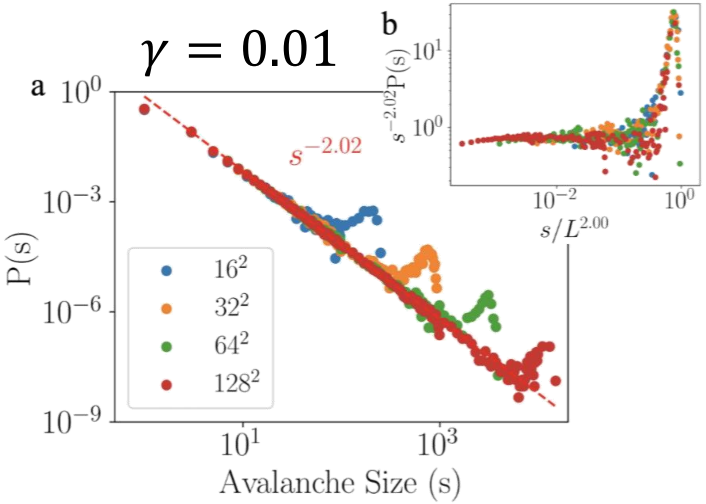
Sipling, Chesson, and Massimiliano Di Ventra. "Memory-induced long-range order in dynamical systems." *arXiv preprint arXiv:2405.06834* (2024).

MILRO in dynamical systems

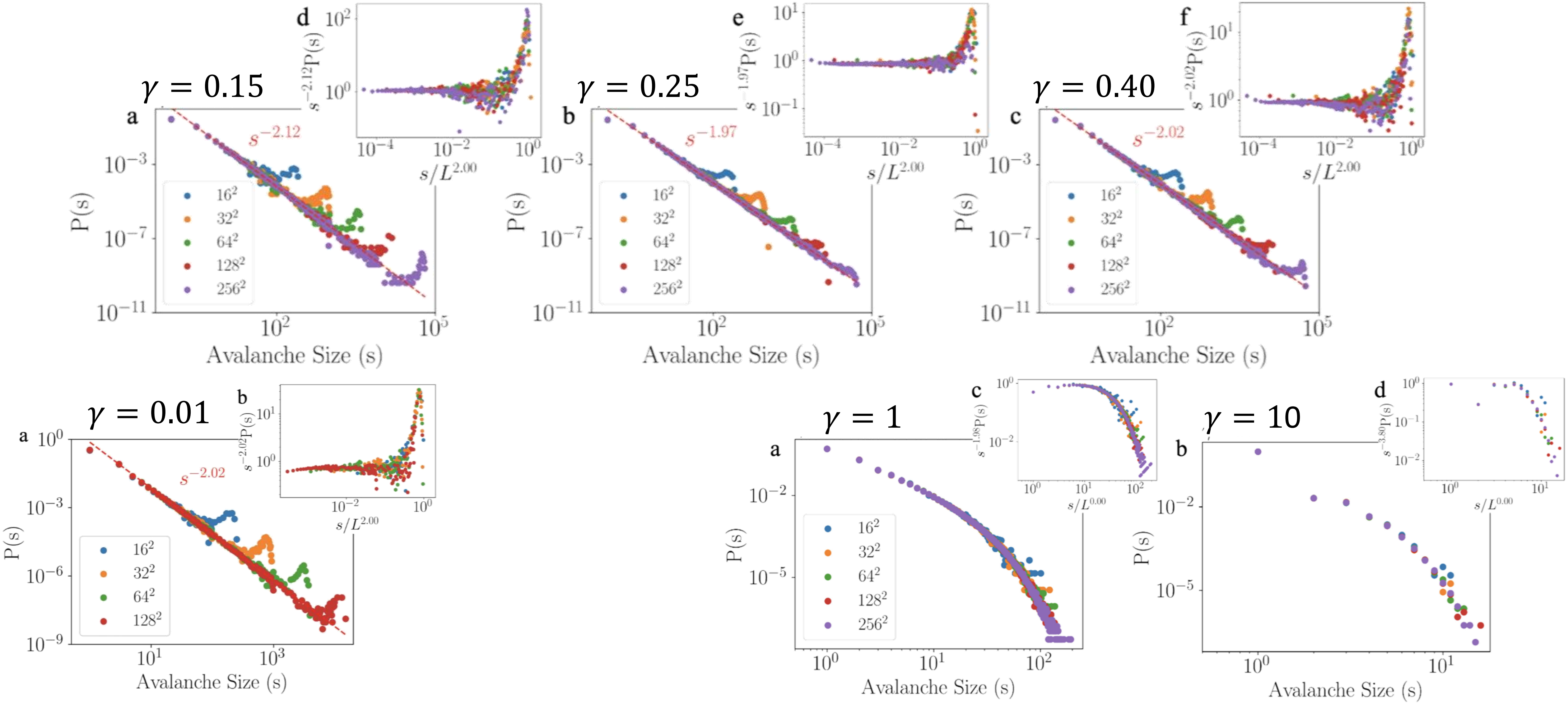


MILRO in dynamical systems

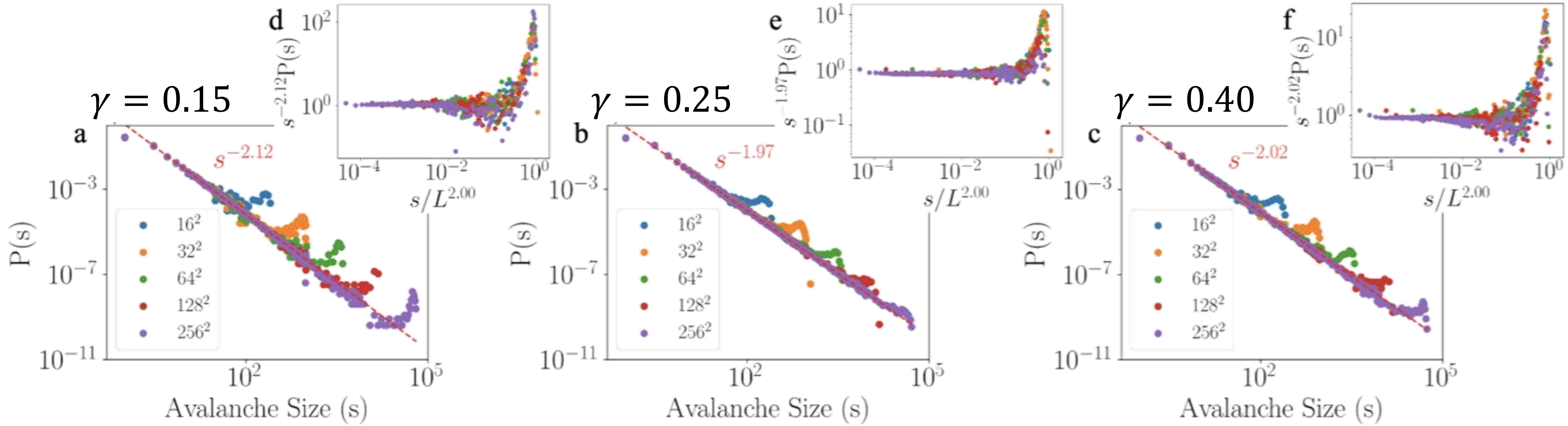


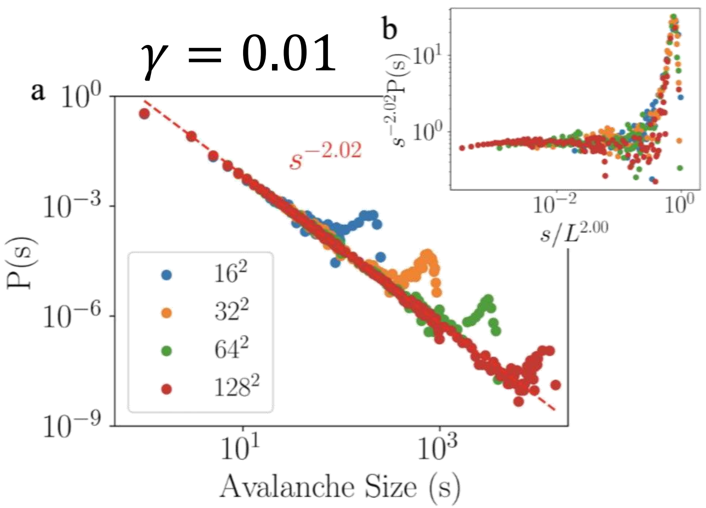


MILRO in dynamical systems

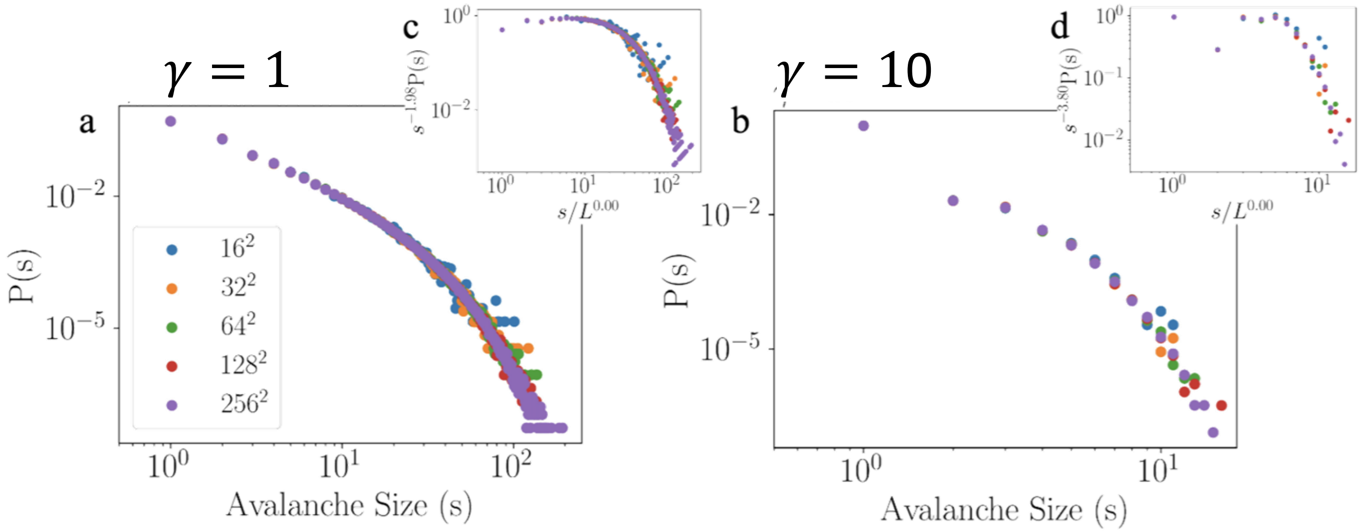


MILRO in dynamical systems



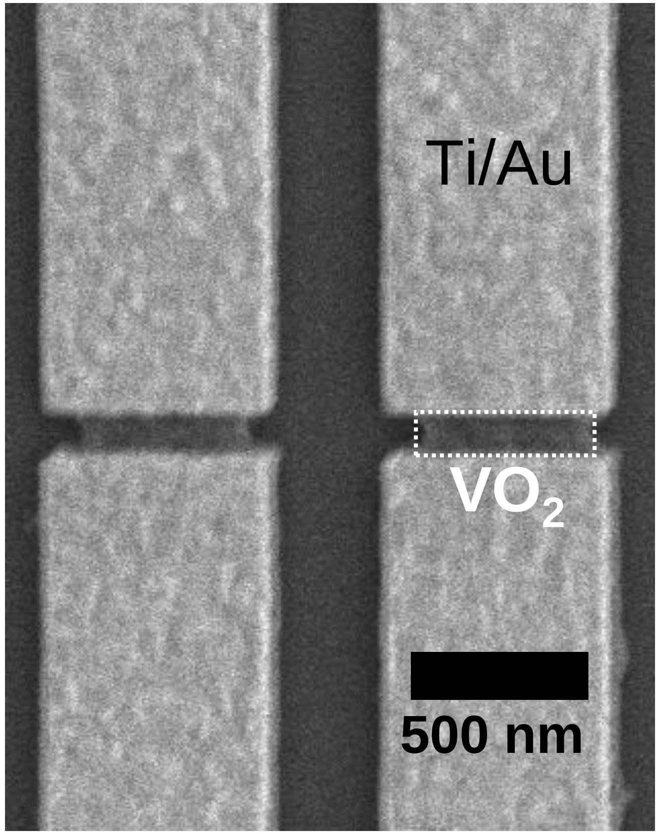


LRO
Phase!

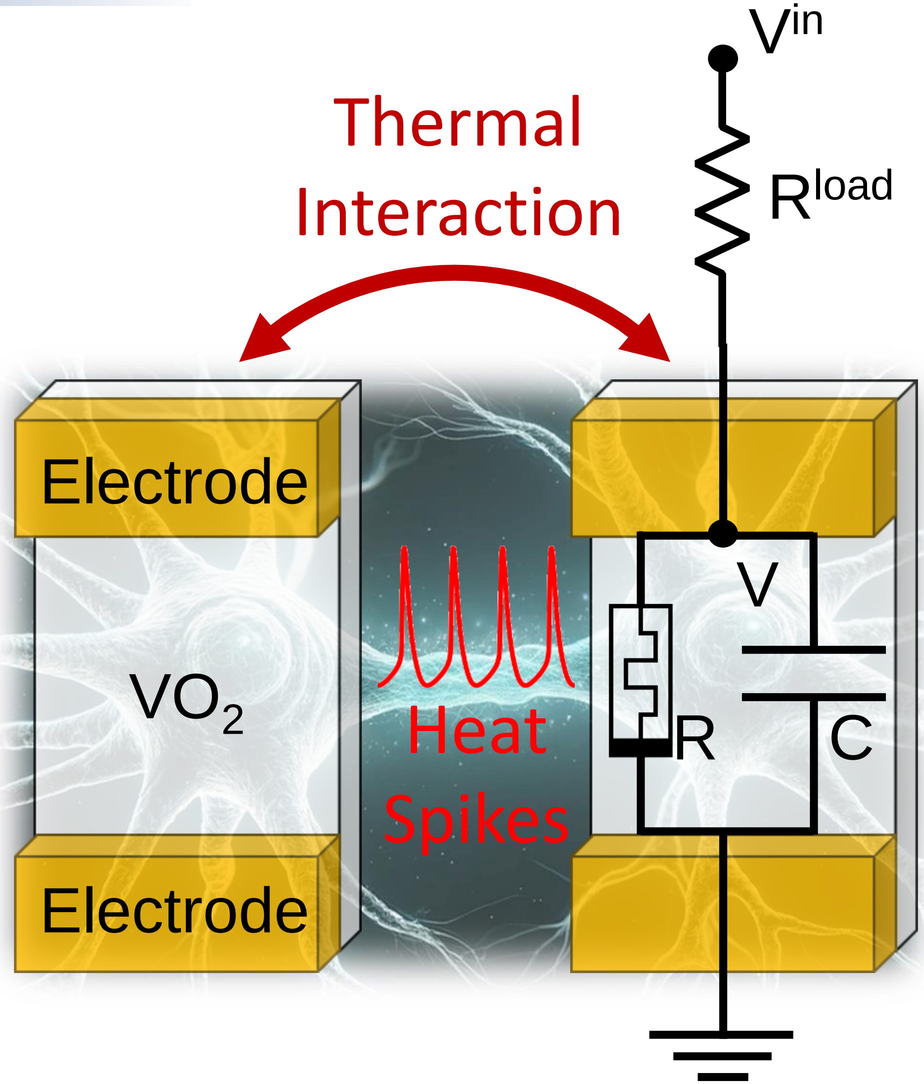


MILRO in neuromorphic systems

- The thermal neuristor: VO_2-based artificial spiking neuron

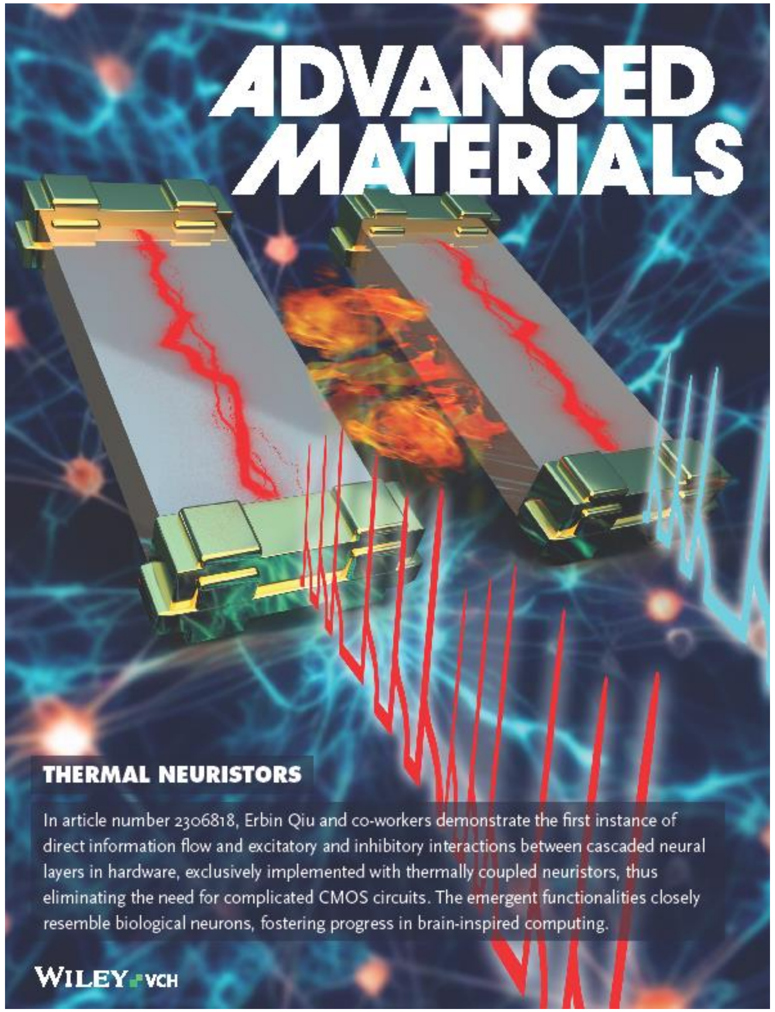


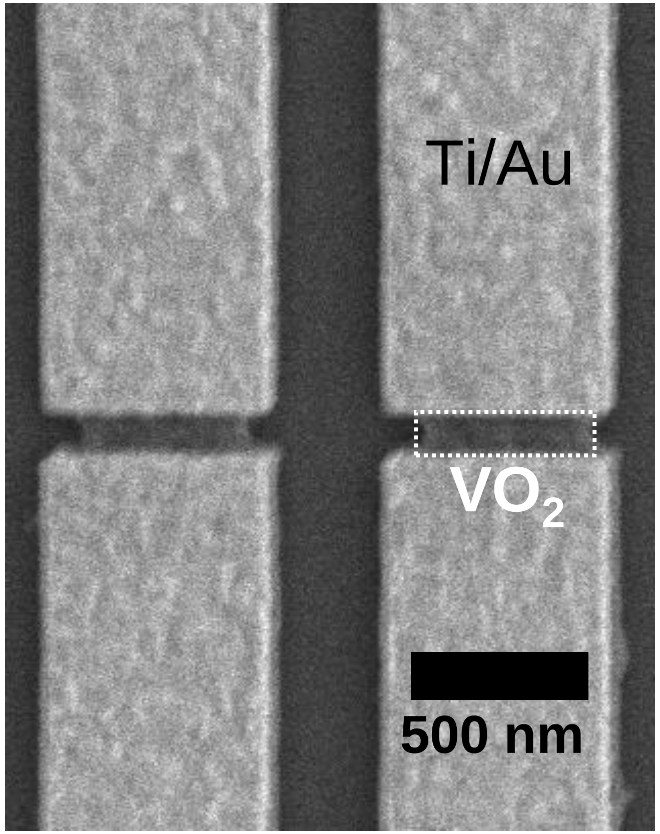
Substrate: Al_2O_3



Qiu, E., Zhang, Y. H., Di Ventra, M., & Schuller, I. K. (2023). Reconfigurable cascaded thermal neuristors for neuromorphic computing. *Advanced Materials*, 2306818.

Thermal neuristor





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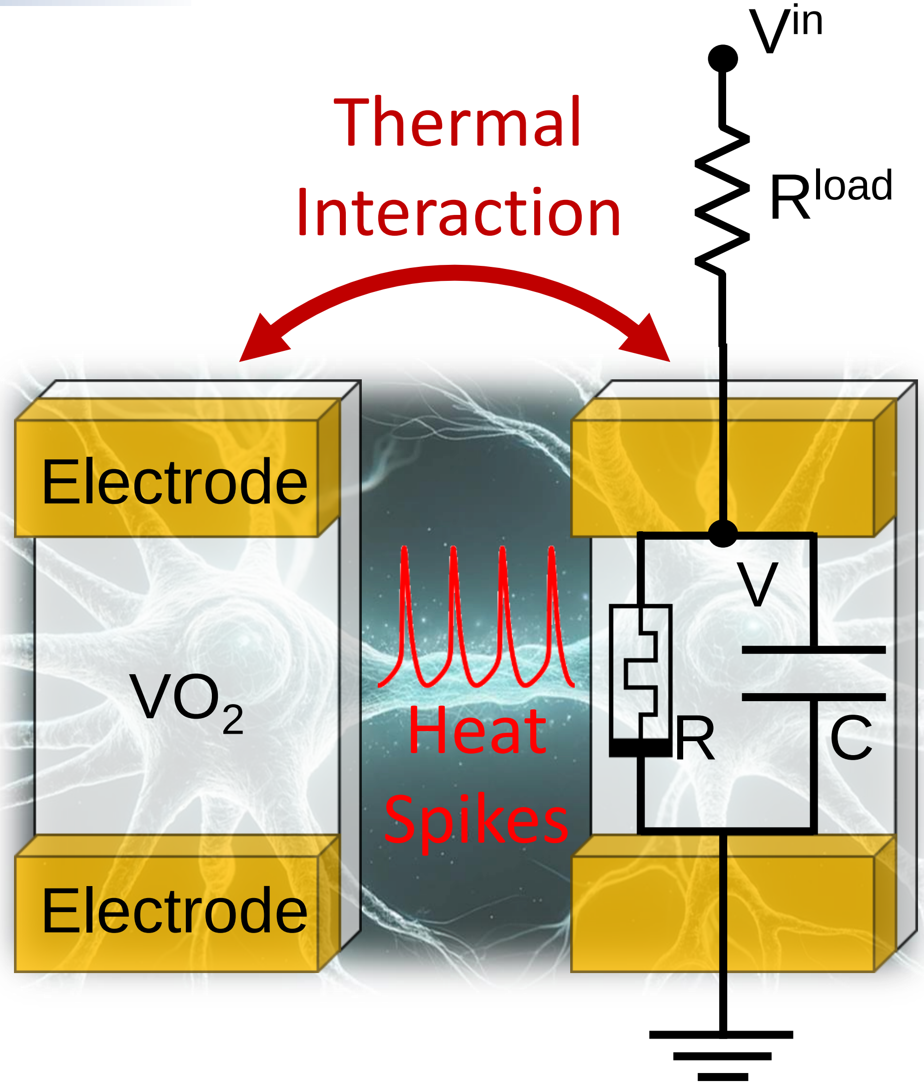
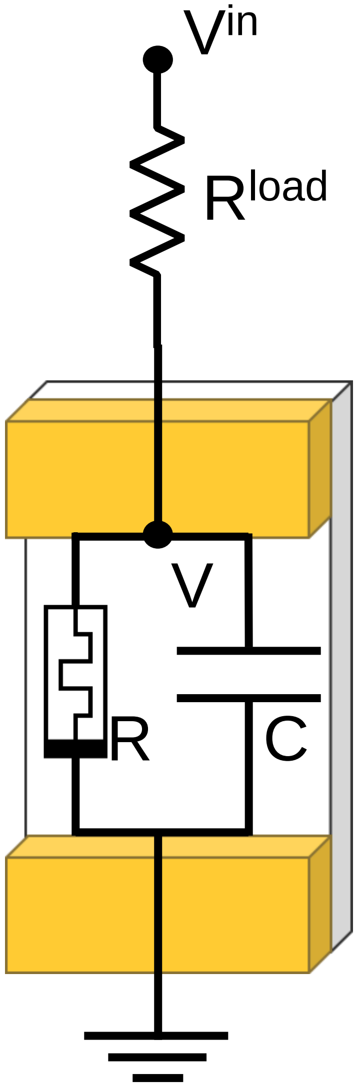
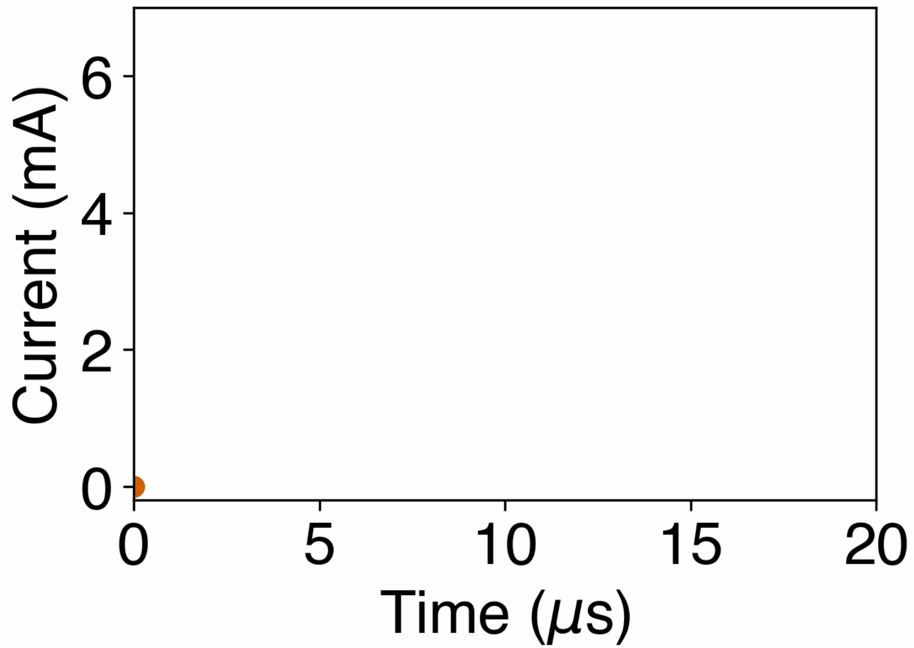
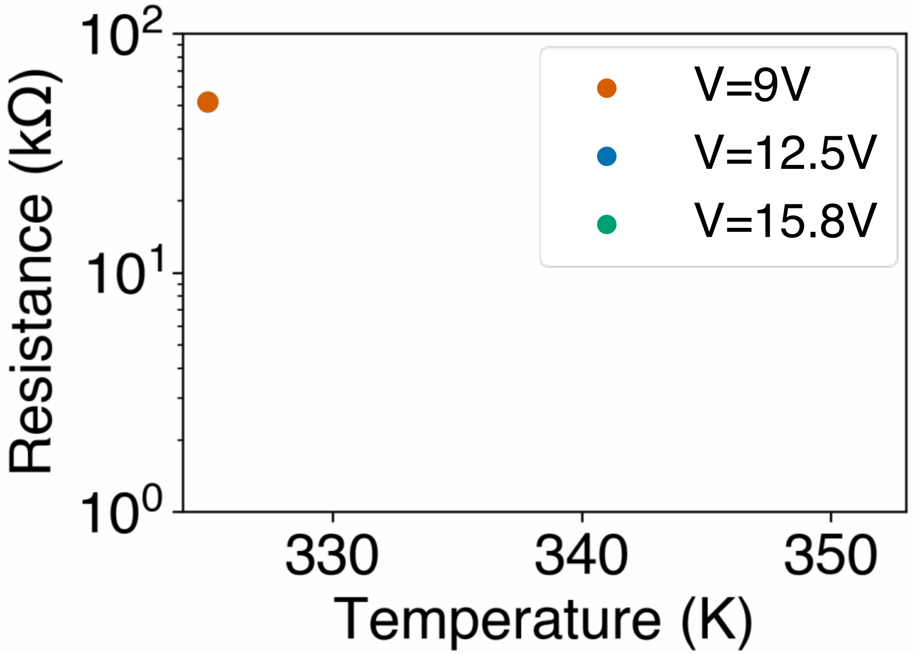


Qiu, E., Zhang, Y. H., Di Ventra, M., & Schuller, I. K. (2023). Reconfigurable cascaded thermal neuristors for neuromorphic computing. *Advanced Materials*, 2306818.

Single neuristor characteristics







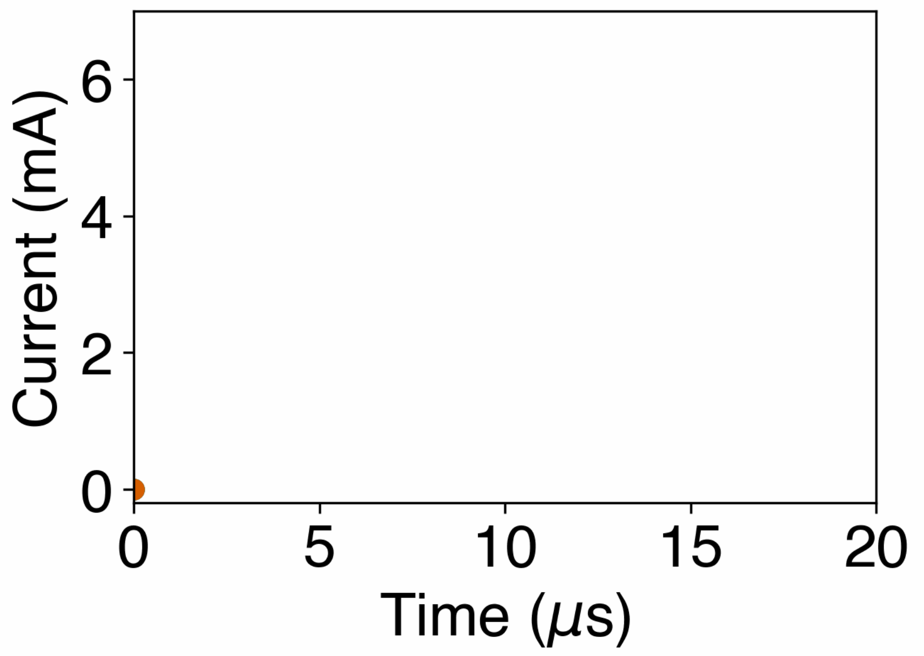
$$C\dot{V}_i = \frac{V_i^{in}}{R_i^{load}} - V_i\left(\frac{1}{R_i} + \frac{1}{R_i^{load}}\right)$$

$$C_{th}\dot{T}_i = \frac{V_i^2}{R_i} - S_e(T_i - T_0)$$

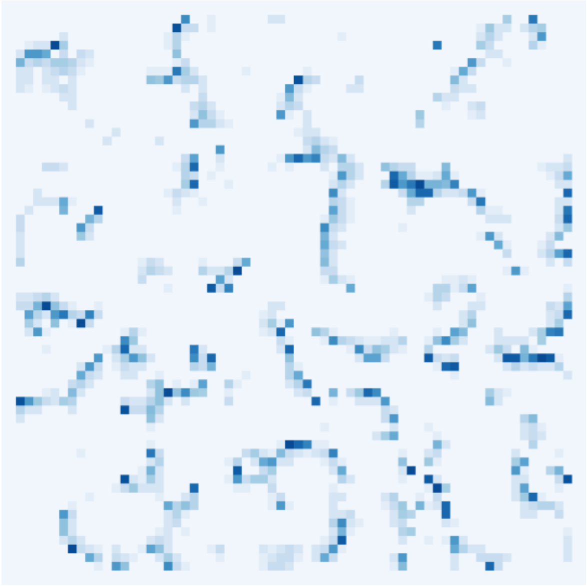
Qiu, E., Zhang, Y. H., Di Ventra, M., & Schuller, I. K. (2023). Reconfigurable cascaded thermal neuristors for neuromorphic computing. *Advanced Materials*, 2306818.

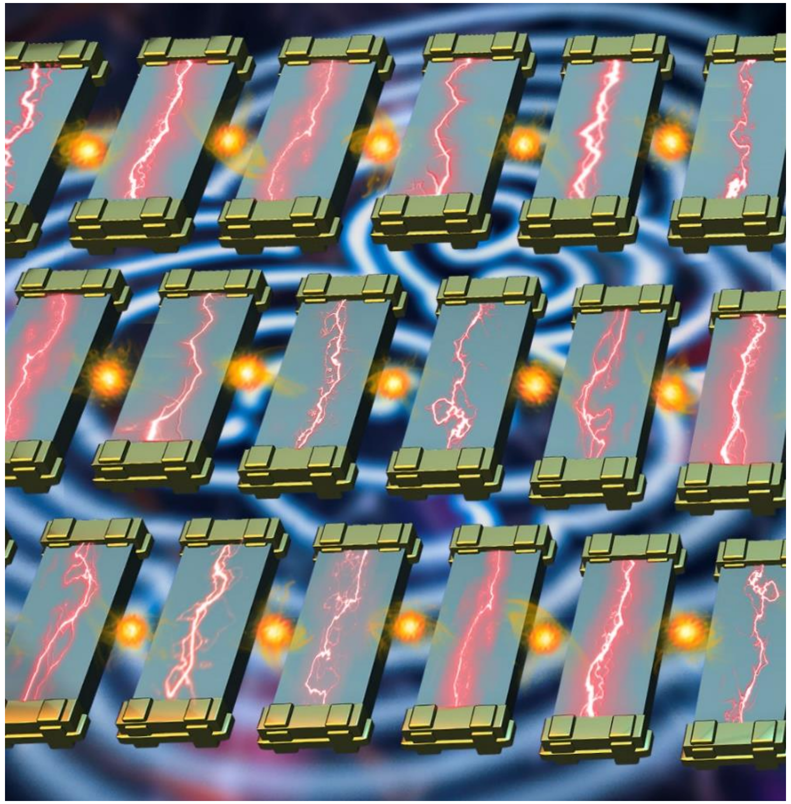
Thermal neuristor array

1 neuron

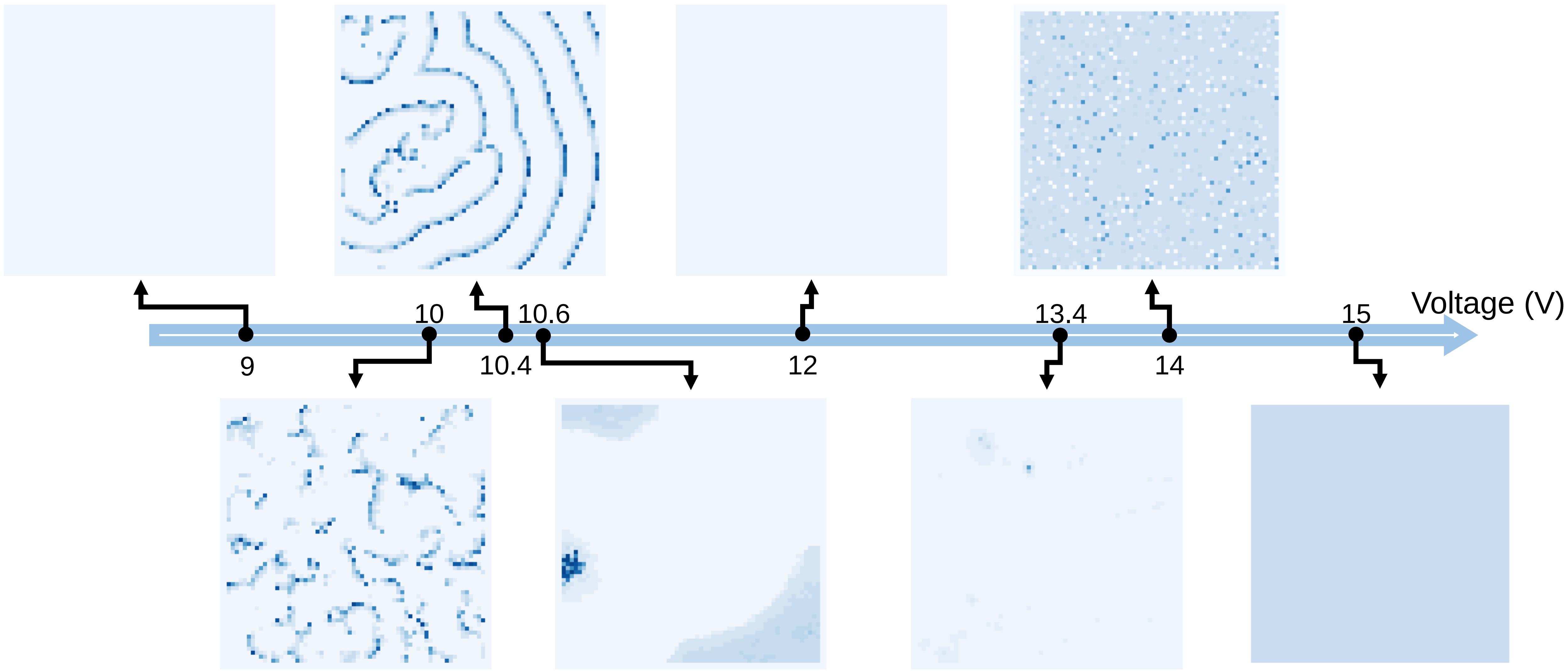


64x64
neurons





Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.



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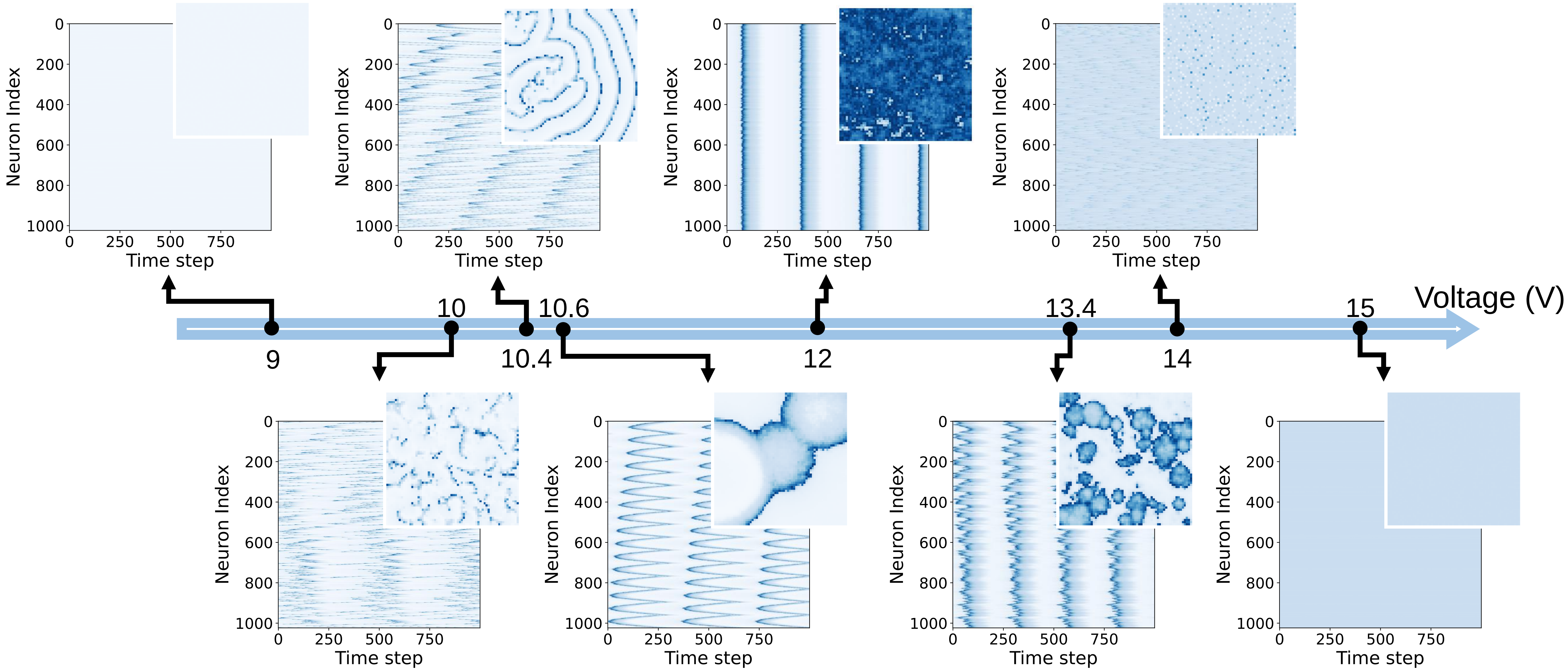
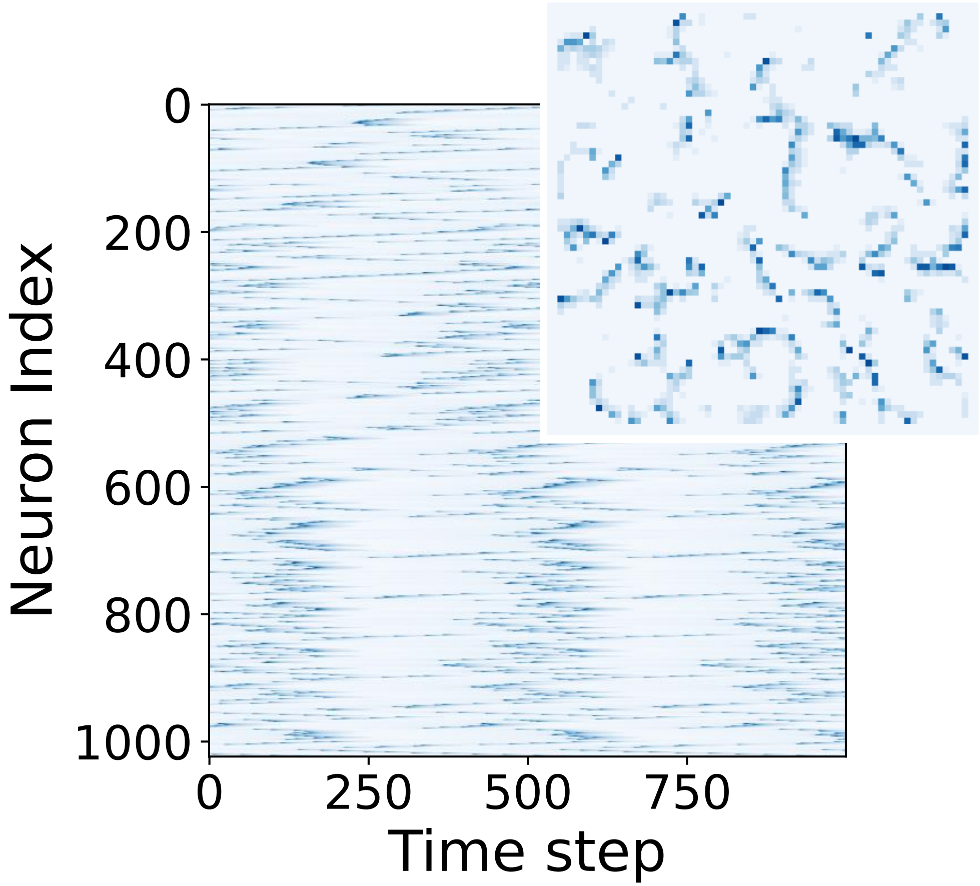
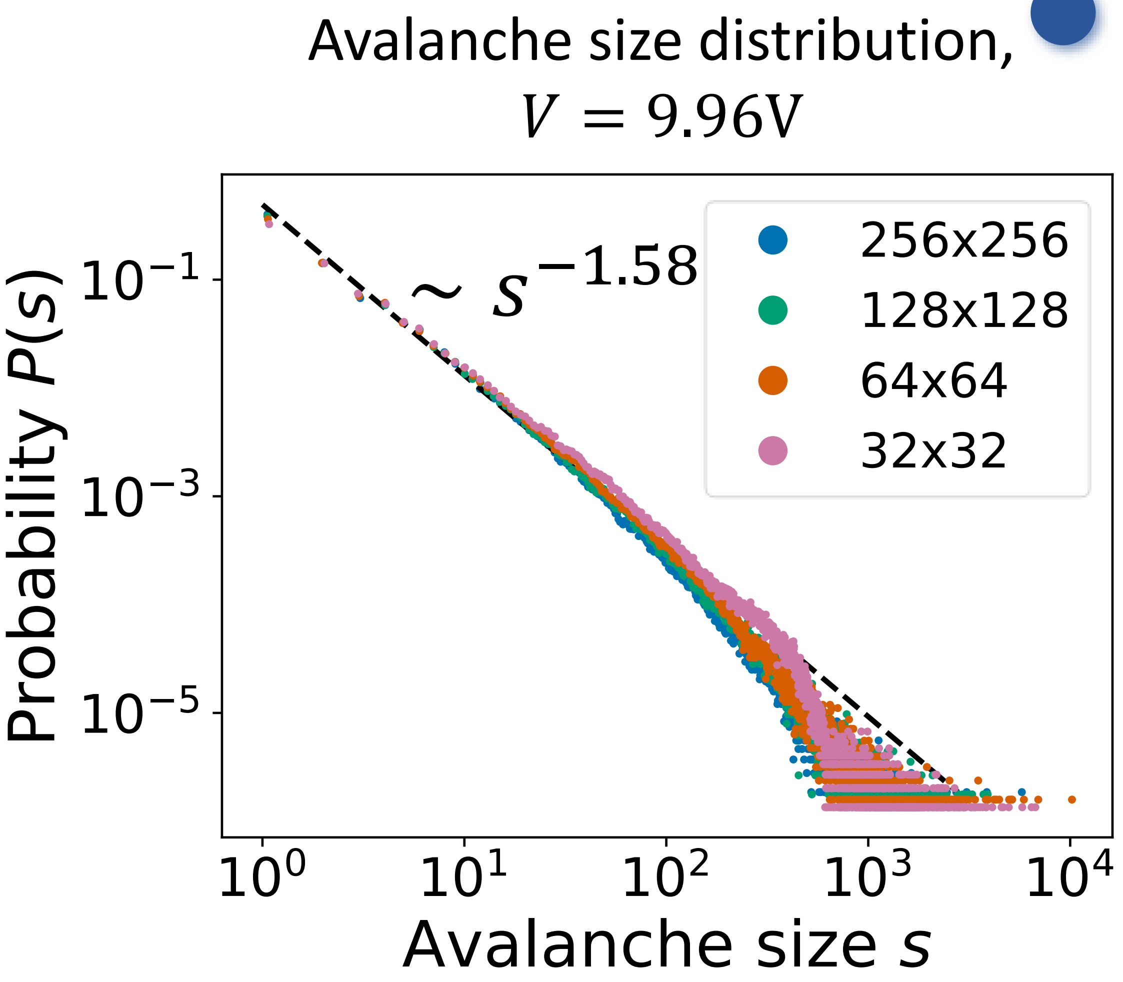


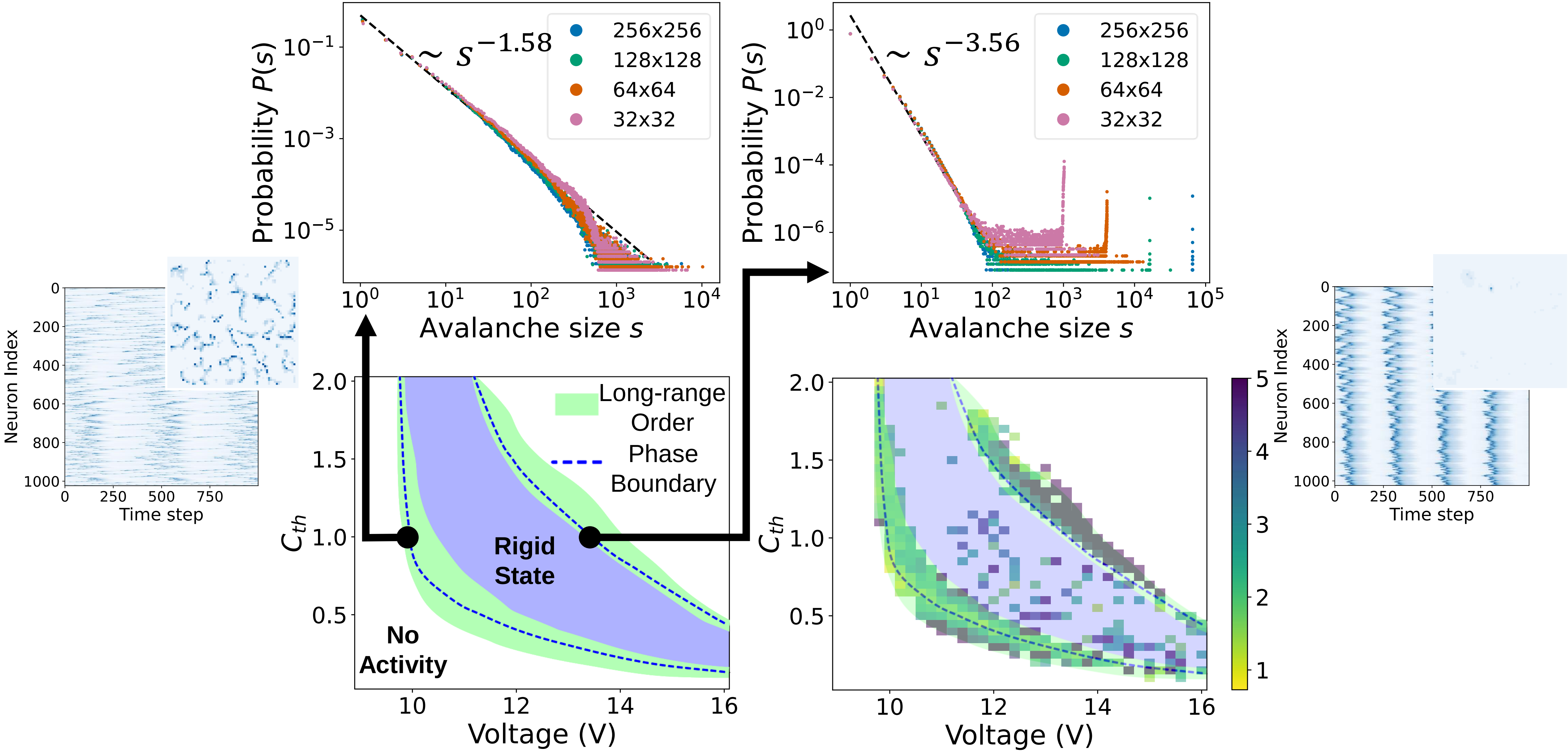
Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.

Avalanche size distribution





Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.



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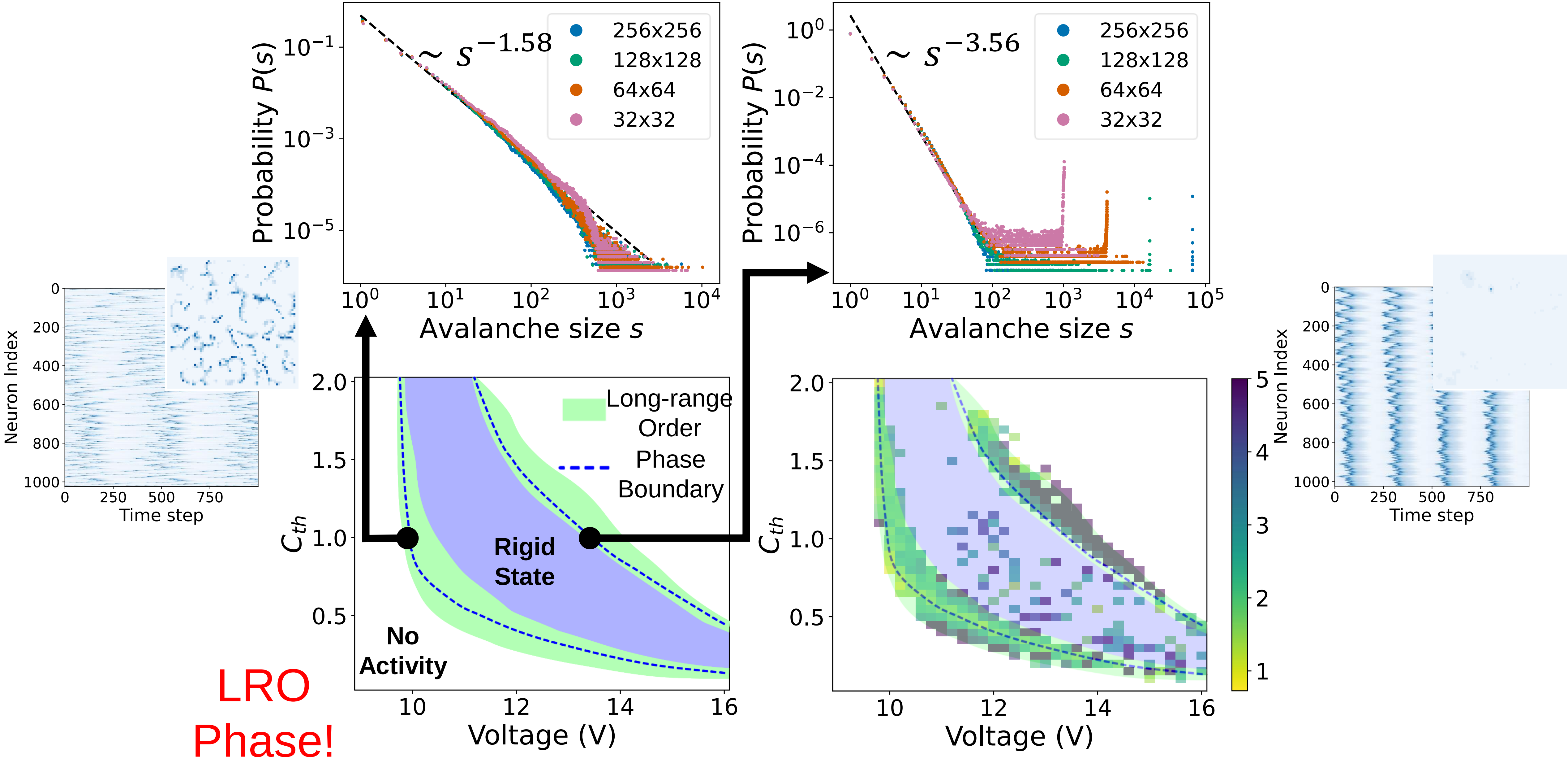
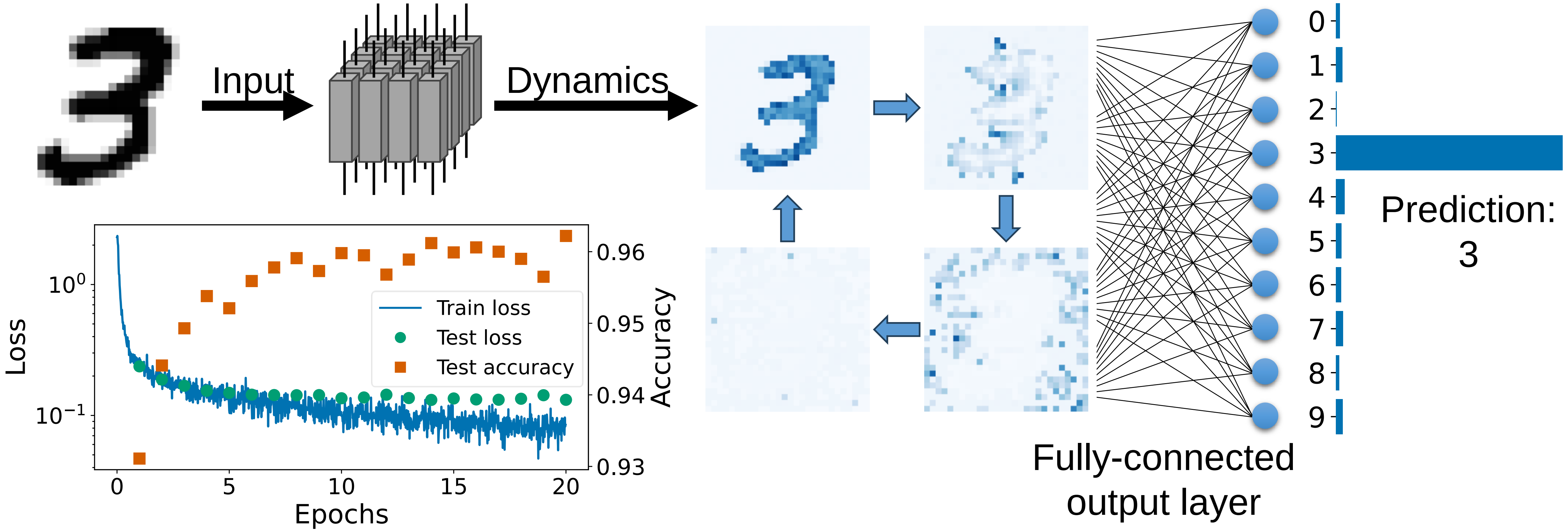


LRO
Phase!

Zhang, Y. H., Sipling, C., Qiu, E., Schuller, I. K., & Di Ventra, M. (2024). Collective dynamics and long-range order in thermal neuristor networks. *Nature Communications*, *15*(1), 6986.

LRO for computing



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LRO for computing

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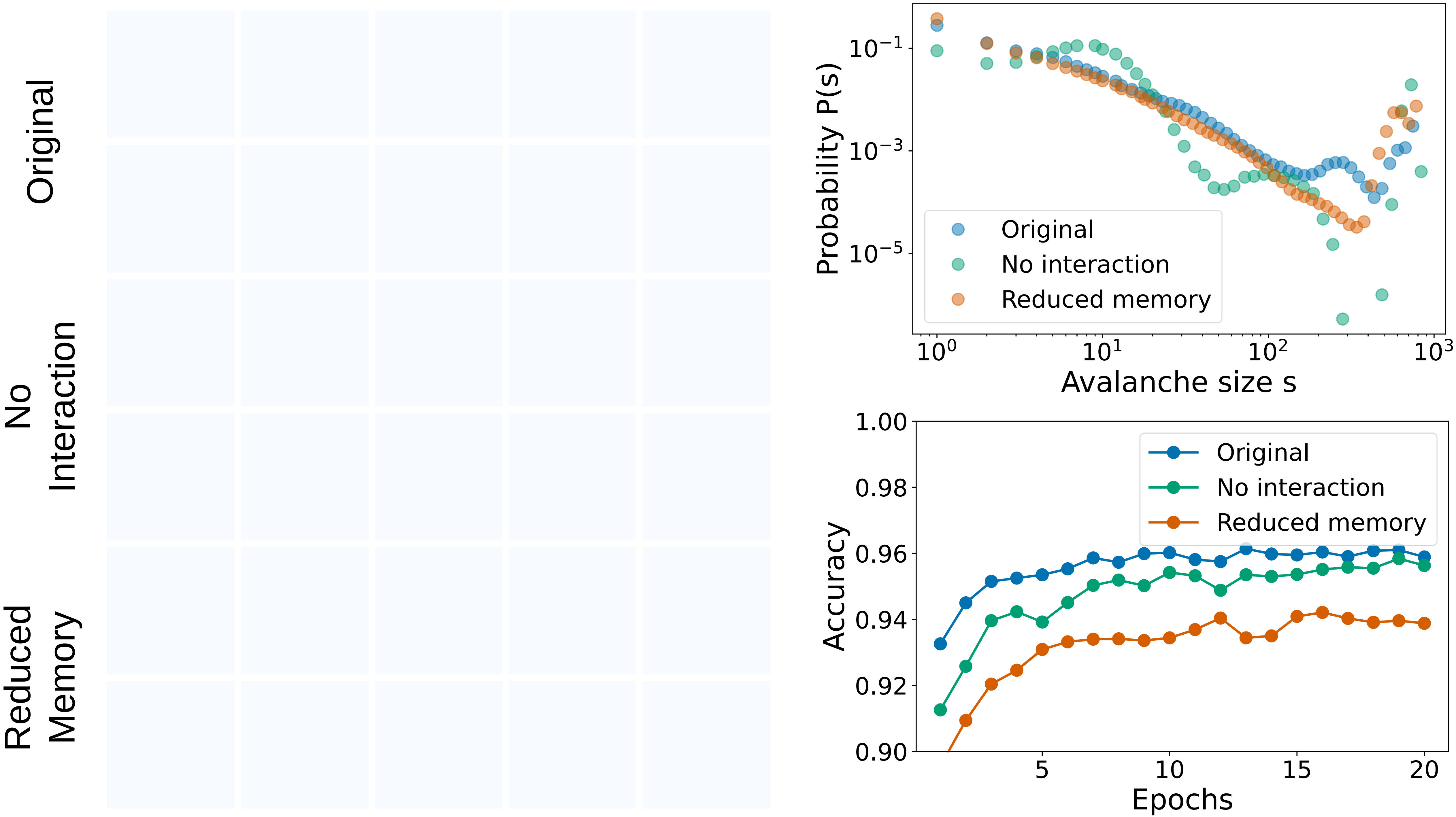
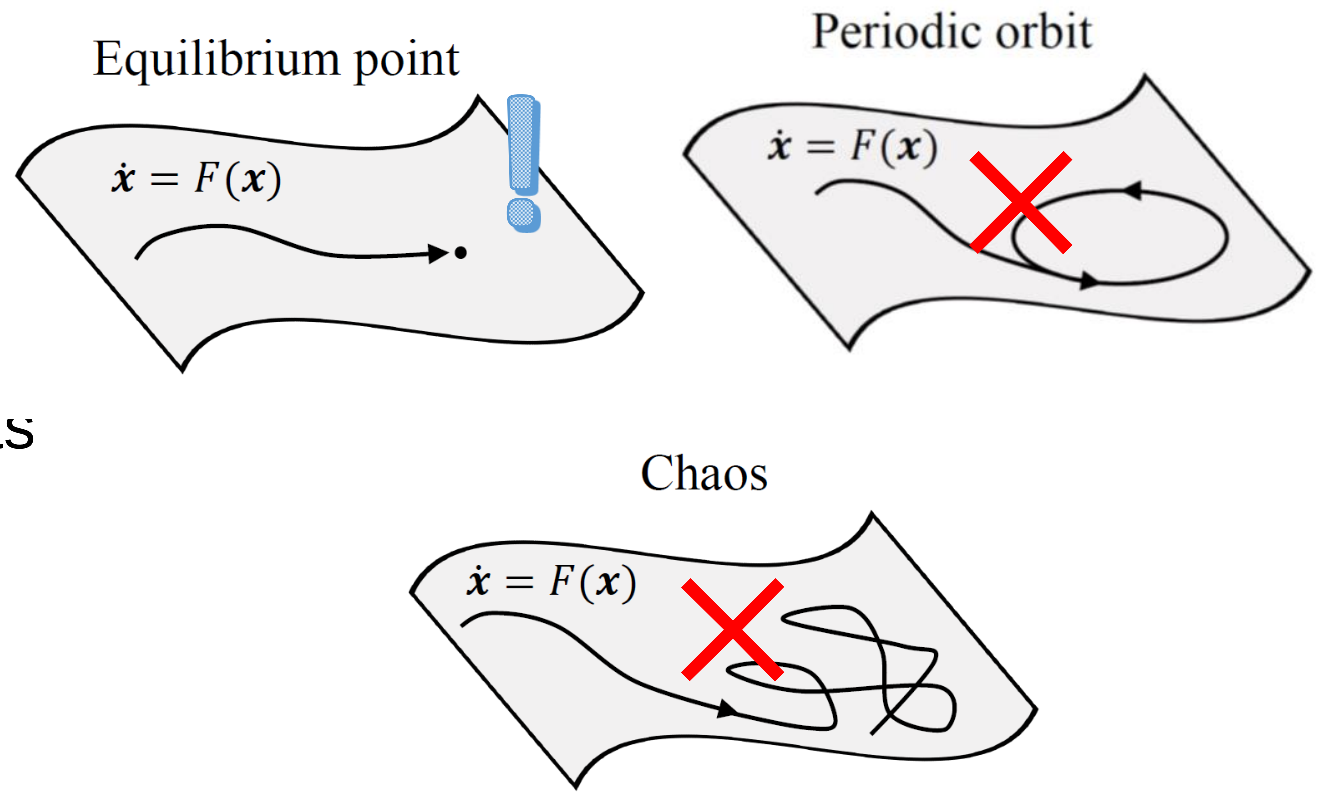

Original
No Interaction
Reduced Memory
Probability P(s)
10^{-1}
10^{-3}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}
Avalanche size s
Original
No interaction
Reduced memory
Accuracy
1.00
0.98
0.96
0.94
0.92
0.90
5
10
15
20
Epochs
Original
No interaction
Reduced memory

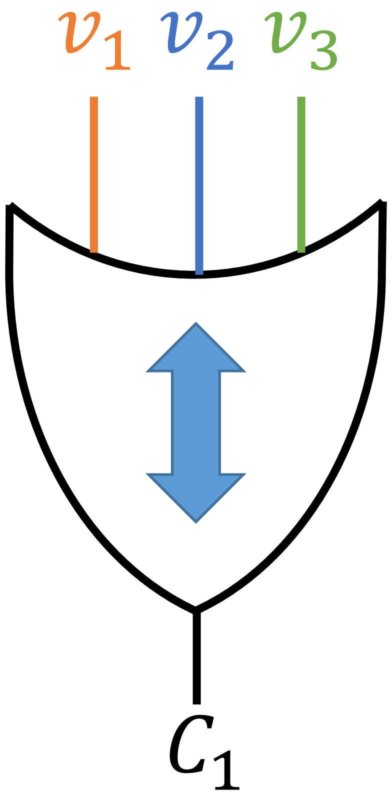
Phase space engineering

- Requirements:
  - Dissipative
  - Equilibrium ↔ Solution
  - No local minima
  - No (quasi-)periodic orbits
  - No chaos



M. Di Ventra, *MemComputing: Fundamentals and Applications*. Oxford University Press, 2022

Self-organizing logic gate



$$\dot{v}_n = \sum_m x_{l,m} x_{s,m} G_{n,m}(v_n, v_j, v_k) + (1 + \zeta x_{l,m})(1 - x_{s,m}) R_{n,m}(v_n, v_j, v_k)$$

$$\dot{x}_{s,m} = \beta(x_{s,m} + \epsilon)(C_m(v_i, v_j, v_k) - \gamma)$$

$$\dot{x}_{l,m} = \alpha(C_m(v_i, v_j, v_k) - \delta)$$

$$G_{n,m}(v_n, v_j, v_k) = \frac{1}{2} q_{n,m} \min[(1 - q_{j,m} v_j), (1 - q_{k,m} v_k]$$

$$R_{n,m}(v_n, v_j, v_k) = \begin{cases} \frac{1}{2}(q_{n,m} - v_n), C_m(v_i, v_j, v_k) = \frac{1}{2}(1 - q_{n,m} v_n) \\ 0, \text{otherwise} \end{cases}$$

Bearden, Sean RB, Yan Ru Pei, and Massimiliano Di Ventra. "Efficient solution of Boolean satisfiability problems with digital memcomputing." *Scientific reports* 10.1 (2020): 1-8.

MemComputing for Machine Learning

- How to find a better path?