Convolutional Networks

So far we have worked with deep fully-connected networks, using them to explore different optimization strategies and network architectures. Fully-connected networks are a good testbed for experimentation because they are very computationally efficient, but in practice all state-of-the-art results use convolutional networks instead.

First you will implement several layer types that are used in convolutional networks. You will then use these layers to train a convolutional network on the CIFAR-10 dataset.

```
# As usual, a bit of setup
import numpy as np
import matplotlib.pyplot as plt
from libs.classifiers.cnn import *
from libs.data_utils import get_CIFAR10_data
from libs.gradient_check import eval_numerical_gradient_array,
eval_numerical_gradient
from libs.layers import *
from libs.fast_layers import *
from libs.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of
plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-
modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) +
np.abs(y))))
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
for k, v in data.items():
  print('%s: ' % k, v.shape)
```

```
X_train:  (49000, 3, 32, 32)
y_train:  (49000,)
X_val:  (1000, 3, 32, 32)
y_val:  (1000,)
X_test:  (1000, 3, 32, 32)
y_test:  (1000,)
```

Convolution: Naive forward pass

The core of a convolutional network is the convolution operation. In the file `libs/layers.py`, implement the forward pass for the convolution layer in the function `conv_forward_naive`.

You don't have to worry too much about efficiency at this point; just write the code in whatever way you find most clear.

You can test your implementation by running the following:

```
x_shape = (2, 3, 4, 4)
w_shape = (3, 3, 4, 4)
x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
b = np.linspace(-0.1, 0.2, num=3)

conv_param = {'stride': 2, 'pad': 1}
out, _ = conv_forward_naive(x, w, b, conv_param)
correct_out = np.array([[[[-0.08759809, -0.10987781],
                           [-0.18387192, -0.2109216 ]],
                          [[ 0.21027089,  0.21661097],
                           [ 0.22847626,  0.23004637]],
                          [[ 0.50813986,  0.54309974],
                           [ 0.64082444,  0.67101435]]],
                         [[[-0.98053589, -1.03143541],
                           [-1.19128892, -1.24695841]],
                          [[ 0.69108355,  0.66880383],
                           [ 0.59480972,  0.56776003]],
                          [[ 2.36270298,  2.36904306],
                           [ 2.38090835,  2.38247847]]]])

# Compare your output to ours; difference should be around e-8
print('Testing conv_forward_naive')
print('difference: ', rel_error(out, correct_out))
```

```
Testing conv_forward_naive

---------------------------------------------------------------------------
-----
TypeError                                 Traceback (most recent call
last)
/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetwork
```

```
s.ipynb Cell 5' in <cell line: 24>()
     <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000004?line=21'>22</a> # Compare your output to ours; difference should be around e-8
     <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000004?line=22'>23</a> print('Testing conv_forward_naive')
---> <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000004?line=23'>24</a> print('difference: ', rel_error(out, correct_out))

/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb Cell 2' in rel_error(x, y)
     <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000001?line=20'>21</a> def rel_error(x, y):
     <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000001?line=21'>22</a>   """ returns relative error """
---> <a href='vscode-notebook-cell:/Users/yuanhawk/50.035-Computer-Vision/Lab2/week6/ConvolutionalNetworks.ipynb#ch0000001?line=22'>23</a>   return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))

TypeError: unsupported operand type(s) for -: 'NoneType' and 'float'
```

Aside: Image processing via convolutions

As fun way to both check your implementation and gain a better understanding of the type of operation that convolutional layers can perform, we will set up an input containing two images and manually set up filters that perform common image processing operations (grayscale conversion and edge detection). The convolution forward pass will apply these operations to each of the input images. We can then visualize the results as a sanity check.

```
from imageio import imread
from PIL import Image

kitten = imread('notebook_images/kitten.jpg')
puppy = imread('notebook_images/puppy.jpg')
# kitten is wide, and puppy is already square
d = kitten.shape[1] - kitten.shape[0]
kitten_cropped = kitten[:, d//2:-d//2, :]

img_size = 200   # Make this smaller if it runs too slow
resized_puppy = np.array(Image.fromarray(puppy).resize((img_size, img_size)))
resized_kitten = np.array(Image.fromarray(kitten_cropped).resize((img_size, img_size)))
```

```python
x = np.zeros((2, 3, img_size, img_size))
x[0, :, :, :] = resized_puppy.transpose((2, 0, 1))
x[1, :, :, :] = resized_kitten.transpose((2, 0, 1))

# Set up a convolutional weights holding 2 filters, each 3x3
w = np.zeros((2, 3, 3, 3))

# The first filter converts the image to grayscale.
# Set up the red, green, and blue channels of the filter.
w[0, 0, :, :] = [[0, 0, 0], [0, 0.3, 0], [0, 0, 0]]
w[0, 1, :, :] = [[0, 0, 0], [0, 0.6, 0], [0, 0, 0]]
w[0, 2, :, :] = [[0, 0, 0], [0, 0.1, 0], [0, 0, 0]]

# Second filter detects horizontal edges in the blue channel.
w[1, 2, :, :] = [[1, 2, 1], [0, 0, 0], [-1, -2, -1]]

# Vector of biases. We don't need any bias for the grayscale
# filter, but for the edge detection filter we want to add 128
# to each output so that nothing is negative.
b = np.array([0, 128])

# Compute the result of convolving each input in x with each filter in
w,
# offsetting by b, and storing the results in out.
out, _ = conv_forward_naive(x, w, b, {'stride': 1, 'pad': 1})

def imshow_no_ax(img, normalize=True):
    """ Tiny helper to show images as uint8 and remove axis labels """
    if normalize:
        img_max, img_min = np.max(img), np.min(img)
        img = 255.0 * (img - img_min) / (img_max - img_min)
    plt.imshow(img.astype('uint8'))
    plt.gca().axis('off')

# Show the original images and the results of the conv operation
plt.subplot(2, 3, 1)
imshow_no_ax(puppy, normalize=False)
plt.title('Original image')
plt.subplot(2, 3, 2)
imshow_no_ax(out[0, 0])
plt.title('Grayscale')
plt.subplot(2, 3, 3)
imshow_no_ax(out[0, 1])
plt.title('Edges')
plt.subplot(2, 3, 4)
imshow_no_ax(kitten_cropped, normalize=False)
plt.subplot(2, 3, 5)
imshow_no_ax(out[1, 0])
plt.subplot(2, 3, 6)
```

```
imshow_no_ax(out[1, 1])
plt.show()
```

Convolution: Naive backward pass

Implement the backward pass for the convolution operation in the function `conv_backward_naive` in the file `libs/layers.py`. Again, you don't need to worry too much about computational efficiency.

When you are done, run the following to check your backward pass with a numeric gradient check.

```
np.random.seed(231)
x = np.random.randn(4, 3, 5, 5)
w = np.random.randn(2, 3, 3, 3)
b = np.random.randn(2,)
dout = np.random.randn(4, 2, 5, 5)
conv_param = {'stride': 1, 'pad': 1}

dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_param)[0], b, dout)

out, cache = conv_forward_naive(x, w, b, conv_param)
dx, dw, db = conv_backward_naive(dout, cache)

# Your errors should be around e-8 or less.
print('Testing conv_backward_naive function')
print('dx error: ', rel_error(dx, dx_num))
print('dw error: ', rel_error(dw, dw_num))
print('db error: ', rel_error(db, db_num))
```

Max-Pooling: Naive forward

Implement the forward pass for the max-pooling operation in the function `max_pool_forward_naive` in the file `libs/layers.py`. Again, don't worry too much about computational efficiency.

Check your implementation by running the following:

```
x_shape = (2, 3, 4, 4)
x = np.linspace(-0.3, 0.4, num=np.prod(x_shape)).reshape(x_shape)
pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}

out, _ = max_pool_forward_naive(x, pool_param)
```

```python
correct_out = np.array([[[[-0.26315789, -0.24842105],
                          [-0.20421053, -0.18947368]],
                         [[-0.14526316, -0.13052632],
                          [-0.08631579, -0.07157895]],
                         [[-0.02736842, -0.01263158],
                          [ 0.03157895,  0.04631579]]],
                        [[[ 0.09052632,  0.10526316],
                          [ 0.14947368,  0.16421053]],
                         [[ 0.20842105,  0.22315789],
                          [ 0.26736842,  0.28210526]],
                         [[ 0.32631579,  0.34105263],
                          [ 0.38526316,  0.4       ]]]])

# Compare your output with ours. Difference should be on the order of
e-8.
print('Testing max_pool_forward_naive function:')
print('difference: ', rel_error(out, correct_out))
```

Max-Pooling: Naive backward

Implement the backward pass for the max-pooling operation in the function `max_pool_backward_naive` in the file `libs/layers.py`. You don't need to worry about computational efficiency.

Check your implementation with numeric gradient checking by running the following:

```python
np.random.seed(231)
x = np.random.randn(3, 2, 8, 8)
dout = np.random.randn(3, 2, 4, 4)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

dx_num = eval_numerical_gradient_array(lambda x:
max_pool_forward_naive(x, pool_param)[0], x, dout)

out, cache = max_pool_forward_naive(x, pool_param)
dx = max_pool_backward_naive(dout, cache)

# Your error should be on the order of e-12
print('Testing max_pool_backward_naive function:')
print('dx error: ', rel_error(dx, dx_num))
```

Fast layers

Making convolution and pooling layers fast can be challenging. To spare you the pain, we've provided fast implementations of the forward and backward passes for convolution and pooling layers in the file `libs/fast_layers.py`.

The fast convolution implementation depends on a Cython extension; to compile it you need to run the following from the `libs` directory:

```
python setup.py build_ext --inplace
```

The API for the fast versions of the convolution and pooling layers is exactly the same as the naive versions that you implemented above: the forward pass receives data, weights, and parameters and produces outputs and a cache object; the backward pass recieves upstream derivatives and the cache object and produces gradients with respect to the data and weights.

NOTE: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the following:

```
# Rel errors should be around e-9 or less
from libs.fast_layers import conv_forward_fast, conv_backward_fast
from time import time
np.random.seed(231)
x = np.random.randn(100, 3, 31, 31)
w = np.random.randn(25, 3, 3, 3)
b = np.random.randn(25,)
dout = np.random.randn(100, 25, 16, 16)
conv_param = {'stride': 2, 'pad': 1}

t0 = time()
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
t1 = time()
out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
t2 = time()

print('Testing conv_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('Difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
t1 = time()
dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting conv_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
```

```
print('dx difference: ', rel_error(dx_naive, dx_fast))
print('dw difference: ', rel_error(dw_naive, dw_fast))
print('db difference: ', rel_error(db_naive, db_fast))

# Relative errors should be close to 0.0
from libs.fast_layers import max_pool_forward_fast,
max_pool_backward_fast
np.random.seed(231)
x = np.random.randn(100, 3, 32, 32)
dout = np.random.randn(100, 3, 16, 16)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

t0 = time()
out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
t1 = time()
out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
t2 = time()

print('Testing pool_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive = max_pool_backward_naive(dout, cache_naive)
t1 = time()
dx_fast = max_pool_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting pool_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
```

Convolutional "sandwich" layers

Previously we introduced the concept of "sandwich" layers that combine multiple operations into commonly used patterns. In the file `libs/layer_utils.py` you will find sandwich layers that implement a few commonly used patterns for convolutional networks. Run the cells below to sanity check they're working.

```
from libs.layer_utils import conv_relu_pool_forward,
conv_relu_pool_backward
np.random.seed(231)
x = np.random.randn(2, 3, 16, 16)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
```

```
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
dx, dw, db = conv_relu_pool_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x:
conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w:
conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b:
conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], b, dout)

# Relative errors should be around e-8 or less
print('Testing conv_relu_pool')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
from libs.layer_utils import conv_relu_forward, conv_relu_backward
np.random.seed(231)
x = np.random.randn(2, 3, 8, 8)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_relu_forward(x, w, b, conv_param)
dx, dw, db = conv_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x,
w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x,
w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x,
w, b, conv_param)[0], b, dout)

# Relative errors should be around e-8 or less
print('Testing conv_relu:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

Three-layer ConvNet

Now that you have implemented all the necessary layers, we can put them together into a simple convolutional network.

Open the file `libs/classifiers/cnn.py` and complete the implementation of the `ThreeLayerConvNet` class. Remember you can use the fast/sandwich layers (already imported for you) in your implementation. Run the following cells to help you debug:

Sanity check loss

After you build a new network, one of the first things you should do is sanity check the loss. When we use the softmax loss, we expect the loss for random weights (and no regularization) to be about `log(C)` for `C` classes. When we add regularization the loss should go up slightly.

```
model = ThreeLayerConvNet()

N = 50
X = np.random.randn(N, 3, 32, 32)
y = np.random.randint(10, size=N)

loss, grads = model.loss(X, y)
print('Initial loss (no regularization): ', loss)

model.reg = 0.5
loss, grads = model.loss(X, y)
print('Initial loss (with regularization): ', loss)
```

Gradient check

After the loss looks reasonable, use numeric gradient checking to make sure that your backward pass is correct. When you use numeric gradient checking you should use a small amount of artifical data and a small number of neurons at each layer. Note: correct implementations may still have relative errors up to the order of e-2.

```
num_inputs = 2
input_dim = (3, 16, 16)
reg = 0.0
num_classes = 10
np.random.seed(231)
X = np.random.randn(num_inputs, *input_dim)
y = np.random.randint(num_classes, size=num_inputs)

model = ThreeLayerConvNet(num_filters=3, filter_size=3,
                          input_dim=input_dim, hidden_dim=7,
                          dtype=np.float64)
loss, grads = model.loss(X, y)
# Errors should be small, but correct implementations may have
# relative errors up to the order of e-2
for param_name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    param_grad_num = eval_numerical_gradient(f,
model.params[param_name], verbose=False, h=1e-6)
    e = rel_error(param_grad_num, grads[param_name])
```

```
    print('%s max relative error: %e' % (param_name,
rel_error(param_grad_num, grads[param_name])))
```

Overfit small data

A nice trick is to train your model with just a few training samples. You should be able to overfit small datasets, which will result in very high training accuracy and comparatively low validation accuracy.

```
np.random.seed(231)

num_train = 100
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

model = ThreeLayerConvNet(weight_scale=1e-2)

solver = Solver(model, small_data,
                num_epochs=15, batch_size=50,
                update_rule='sgd',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=1)
solver.train()
```

Plotting the loss, training accuracy, and validation accuracy should show clear overfitting:

```
plt.subplot(2, 1, 1)
plt.plot(solver.loss_history, 'o')
plt.xlabel('iteration')
plt.ylabel('loss')

plt.subplot(2, 1, 2)
plt.plot(solver.train_acc_history, '-o')
plt.plot(solver.val_acc_history, '-o')
plt.legend(['train', 'val'], loc='upper left')
plt.xlabel('epoch')
plt.ylabel('accuracy')
plt.show()
```

Train the net

By training the three-layer convolutional network for one epoch, you should achieve greater than 40% accuracy on the training set:

```
model = ThreeLayerConvNet(weight_scale=0.001, hidden_dim=500,
reg=0.001)

solver = Solver(model, data,
                num_epochs=1, batch_size=50,
                update_rule='sgd',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=20)
solver.train()
```

Visualize Filters

You can visualize the first-layer convolutional filters from the trained network by running the following:

```
from libs.vis_utils import visualize_grid

grid = visualize_grid(model.params['W1'].transpose(0, 2, 3, 1))
plt.imshow(grid.astype('uint8'))
plt.axis('off')
plt.gcf().set_size_inches(5, 5)
plt.show()
```