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ECE 4960 2018sp – HW2

Programming environment: Eclipse (Java8)
Language: Java
Development platform: JDK
Operating system: MacOS Sierra 10.12.6

Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \quad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \quad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	**0.05000000000000093**	**0.049999967217445374**	**2.220446049250313E-16**	**2.2351741790771484E-8**
10^{-2}	**0.005000000000000003375**	**0.004999876022338867**	**8.881784197001252E-16**	**2.086162567138672E-7**
10^{-3}	**4.999999998487326E-4**	**5.012154579162598E-4**	**8.237854842718662E-14**	**2.0265579223632812E-6**
10^{-4}	**4.999999958599233E-5**	**1.6927719116210938E-5**	**3.8768988019910466E-13**	**2.0325183868408203E-5**
10^{-5}	**5.000006964905879E-6**	**1.3208389282237665E-4**	**1.000088900582341E-12**	**1.3208389282237665E-4**
10^{-6}	**4.999621834311796E-7**	**0.0016222000122071423**	**9.99866855977416E-13**	**0.0016222000122071423**
10^{-7}	**5.054390306291623E-8**	**0.031424522399902566**	**2.875544247160633E-11**	**0.031424522399902566**
10^{-8}	**6.07747192966755E-9**	**0.2549419403076173**	**3.3019136314038633E-9**	**0.2549419403076173**
10^{-9}	**8.274037077704577E-8**	**1.0**	**2.722921954578794E-8**	**1.0**
10^{-10}	**8.274037077704577E-8**	**1.0**	**8.274037077704577E-8**	**1.0**
10^{-11}	**8.274037077704577E-8**	**1.0**	**8.274037077704577E-8**	**1.0**
10^{-12}	**8.890058234078957E-5**	**1.0**	**3.338943110953174E-5**	**1.0**
10^{-13}	**7.992778373592246E-4**	**1.0**	**2.4416632504664637E-4**	**1.0**
10^{-14}	**7.992778373592246E-4**	**1.0**	**7.992778373592246E-4**	**1.0**
10^{-15}	**0.11022302462515654**	**1.0**	**0.05471187339389871**	**1.0**
10^{-16}	**1.0**	**1.0**	**0.44488848768742184**	**1.0**
10^{-17}	**1.0**	**1.0**	**1.0**	**1.0**
10^{-18}	**1.0**	**1.0**	**1.0**	**1.0**

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \quad E(h) = O(h) = \frac{1}{2}hf''(x) + O(h^2) \quad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \quad E(2h) = O(h) = \frac{1}{2}2hf''(x) + O(h^2) \quad (4)$$

$$f'(x) = \frac{-1}{2h}f(x+2h) - \frac{3}{2h}f(x) + \frac{2}{h}f(x+h) + O(h^2) \quad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \quad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \quad (7)$$

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
2^{-4}	**0.19140625**	**0.390625**	**0.0078125**	**2.0408163265306123**	**2.1176470588235294**

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2^{-5}	0.0947265625	0.19140625	0.001953125	2.020618556701031	2.0606060606060606
2^{-6}	0.047119140625	0.0947265625	4.8828125E-4	2.010362694300518	2.0307692307692307
2^{-7}	0.02349853515625	0.047119140625	1.220703125E-4	2.005194805194805	2.0155038759689923
2^{-8}	0.0117340087890625	0.02349853515625	3.0517578125E-5	2.0026007802340704	2.007782101167315
2^{-9}	0.005863189697265625	0.0117340087890625	7.62939453125E-6	2.0013012361743656	2.003898635477583
2^{-10}	0.0029306411743164062	0.005863189697265625	1.9073486328125E-6	2.0006508298080052	2.001951219512195
2^{-11}	0.0014650821685791016	0.0029306411743164062	4.76837158203125E-7	2.0003254678600486	2.0009760858955588
2^{-12}	7.324814796447754E-4	0.0014650821685791016	1.1920928955078125E-7	2.000162747172268	2.000488162069807
2^{-13}	3.6622583866119385E-4	7.324814796447754E-4	2.9802322387695312E-8	2.000081376897099	2.000244110826315
2^{-14}	1.8310919404029846E-4	3.6622583866119385E-4	7.450580596923828E-9	2.000040689276341	2.000122062862374
2^{-15}	9.155366569757462E-5	1.8310919404029846E-4	1.862645149230957E-9	2.000020344845125	2.0000610332936617
2^{-16}	4.5776600018143654E-5	9.155366569757462E-5	4.656612873077392E-10	2.0000101724743016	2.000030517112471
2^{-17}	2.2888241801410913E-5	4.5776600018143654E-5	1.1641532182693481E-10	2.000005086250086	2.000015258672648
2^{-18}	1.1444091796875E-5	2.2888241801410913E-5	0.0	2.000005086263021	2.000005086237151
2^{-19}	5.7220458984375E-6	1.1444091796875E-5	0.0	2.0	2.0000101725260415
2^{-20}	2.86102294921875E-6	5.7220458984375E-6	0.0	2.0	2.0
2^{-21}	1.430511474609375E-6	2.86102294921875E-6	0.0	2.0	2.0
2^{-22}	7.152557373046875E-7	1.430511474609375E-6	0.0	2.0	2.0
2^{-23}	3.5762786865234375E-7	7.152557373046875E-7	0.0	2.0	2.0
2^{-24}	1.7881393432617188E-7	3.5762786865234375E-7	0.0	2.0	2.0
2^{-25}	8.940696716308594E-8	1.7881393432617188E-7	0.0	2.0	2.0
2^{-26}	4.470348358154297E-8	8.940696716308594E-8	0.0	2.0	2.0
2^{-27}	2.9802322387695312E-8	4.470348358154297E-8	0.0	1.5	3.0
2^{-28}	0.0	2.9802322387695312E-8	0.0	Infinity	0.5
2^{-29}	0.0	0.0	0.0	NaN	Infinity
2^{-30}	0.0	0.0	0.0	NaN	NaN
2^{-31}	0.0	0.0	0.0	NaN	NaN
2^{-32}	0.0	0.0	0.0	NaN	NaN
2^{-33}	0.0	0.0	0.0	NaN	NaN
2^{-34}	0.0	0.0	0.0	NaN	NaN
2^{-35}	0.0	0.0	0.0	NaN	NaN
2^{-36}	0.0	0.0	0.0	NaN	NaN
2^{-37}	0.0	0.0	0.0	NaN	NaN
2^{-38}	0.0	0.0	0.0	NaN	NaN
2^{-39}	0.0	0.0	0.0	NaN	NaN
2^{-40}	0.0	0.0	0.0	NaN	NaN