期中补充题目

$$egin{aligned} &\lim_{x o +\infty} x^2 (n^{rac{1}{x}} - n^{rac{1}{x+1}}) \ &x^2 n^{rac{1}{x+1}} (n^{rac{1}{x} - rac{1}{x+1}} - 1) \ &= x^2 n^{rac{1}{x+1}} (e^{rac{1}{x(x+1)} \ln n} - 1) \ &\sim rac{x^2}{x(x+1)} n^{rac{1}{x+1}} \ln n o \ln n (n o + \infty) \end{aligned}$$

$$(rac{f(x_0+rac{1}{n})}{f(x_0)})^n, n o +\infty$$
 $a=e^{\ln a}$
 $e^{n\lnrac{f(x_0+rac{1}{n})}{f(x_0)}}$
 $e^{f(x_0+rac{1}{n})-f(x_0)}rac{1}{f(x_0)} - e^{rac{f'(x_0)}{f(x_0)}}$

注:也可以看作 $\ln f(x)$ 求导

设f在[a,b]上连续,在(a,b)上二阶可导,证明:存在 $\eta \in (a,b)$,有:

$$f(b) + f(a) - 2f(\frac{a+b}{2}) = \frac{(b-a)^2}{4}f''(\eta)$$

设
$$k = rac{f(b) + f(a) - 2f(rac{a+b}{2})}{rac{(b-a)^2}{4}}$$

$$\Rightarrow f(b)+f(a)-2f(\frac{a+b}{2})-\frac{(b-a)^2}{4}k=0$$

$$F(x) = f(x) + f(a) - 2f(rac{x+a}{2}) - rac{k(x-a)^2}{4}$$

$$F(a) = F(b) = 0 \Rightarrow F'(x_1) = 0$$

$$F'(x_1) = f'(x_1) - f'(\frac{x+a}{2}) - k\frac{x_1 - a}{2} = 0$$

$$\Rightarrow k = rac{f'(x_1) - f'(rac{x_1 + a}{2})}{rac{x_1 - a}{2}} = f''(\eta)(Lagrange)$$