Modelling Price Dynamics, Optimal Portfolios, and Option Valuation for Crypto Assets

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Seven crypto assets:
bitcoin (BTC), ethereum (ETH), XRP (XRP), litecoin (LTC), bitcoin cash (BCH), EOS (EOS) and binance coin (BNB).
(covers July 25, 2017 to September 10, 2020)
In total 1143 days of return data, in-sample (estimation window) 360 days, and out-of sample (test window) 783 days.

Table: Mean monthly price volatility of four major Crypto assets and SPY prices in 08/1/2017-08/31/2020

	Bitcoin	Ethereum	Litecoin	Bitcoin Cash	SPY
Std($)	714.31	38.85	11.21	105.04	5
MDD(%)	18	22.78	25.01	28.25	4.19

1. The VaR and CVaR at α are defined as

$$\begin{aligned} \mathrm{VaR}_\alpha &= -\inf\{x \mid F(x) > \alpha, x \in \mathbb{R}\}, \\ \mathrm{CVaR}_\alpha &= -\mathbb{E}[x | x \le -\mathrm{VaR}_\alpha]. \end{aligned} \tag{1}$$

where $F(x) = Pr\{r \le x\}$ is the cdf.
2. The continuous ranked probability score (CRPS):

$$\mathrm{CRPS}(F, x) = \frac{1}{2}\mathbb{E}_F|X - X'| - \mathbb{E}_F|X - x|, \tag{2}$$

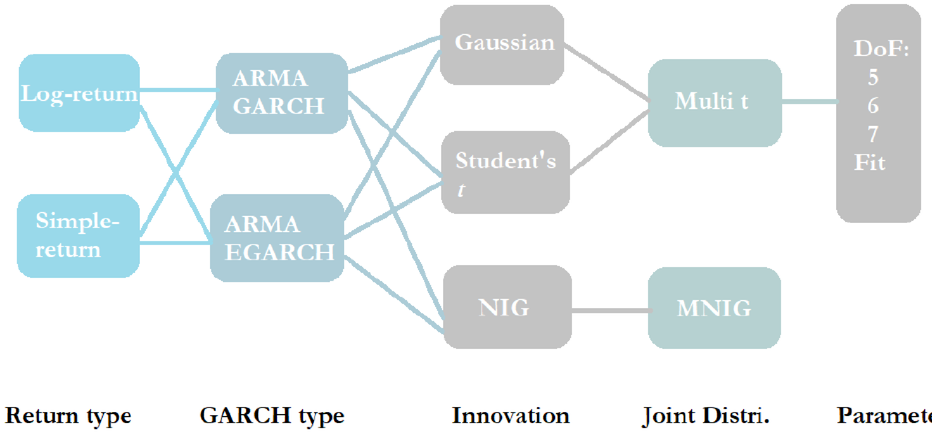
where X and X' are independent copies of a random variable with cumulative distribution function F and finite first moment.

Table: VaR backtesting results of ARMA(1,1)-GARCH(1,1) on log-return

Innovation	Gaussian				Student's t				NIG
Joint Distri.	Multivariate t				Multivariate t				MNIG
Variable	$\nu=5$	$\nu=6$	$\nu=7$	Fit ν	$\nu=5$	$\nu=6$	$\nu=7$	Fit ν	
Observations	783	783	783	783	783	783	783	783	783
					VaR95				
Failures	56	57	62	40	59	61	65	44	45
Expected Ratio[a]	39.15	39.15	39.15	39.15	39.15	39.15	39.15	39.15	39.15
Observed Ratio	1.43	1.46	1.58	1.02	1.51	1.56	1.66	1.12	1.15
Traffic Light[b]	yellow	yellow	yellow	green	yellow	yellow	red	green	green
Binomial Test[c]	reject	reject	reject	accept	reject	reject	reject	accept	accept
					VaR99				
Failures	25	26	25	11	26	29	30	12	16
Expected Ratio	7.83	7.83	7.83	7.83	7.83	7.83	7.83	7.83	7.83
Observed Ratio	3.19	3.32	3.19	1.40	3.32	3.70	3.83	1.53	2.04
Traffic Light	red	red	red	green	red	red	red	green	yellow
Binomial Test	reject	reject	reject	accept	reject	reject	reject	accept	reject

Table: CVaR backtesting results of ARMA(1,1)-GARCH(1,1) on log-return

Innovation	Gaussian				Student's t				NIG
Joint Distri.	Multivariate t				Multivariate t				MNIG
Variable	$\nu=5$	$\nu=6$	$\nu=7$	Fit ν	$\nu=5$	$\nu=6$	$\nu=7$	Fit ν	
Observations	783	783	783	783	783	783	783	783	783
					CVaR95				
Expected Severity[d]	1.44	1.40	1.38	1.68	1.44	1.40	1.38	1.69	1.48
Observed Severity	1.87	1.93	1.90	1.85	1.92	1.95	1.94	1.83	1.91
Traffic Light	red	red	red	green	red	red	red	green	red
					CVaR99				
Expected Severity	1.34	1.29	1.27	1.59	1.35	1.29	1.27	1.62	1.32
Observed Severity	1.60	1.68	1.78	1.56	1.64	1.66	1.71	1.52	1.70
Traffic Light	red	red	red	green	red	red	red	green	red



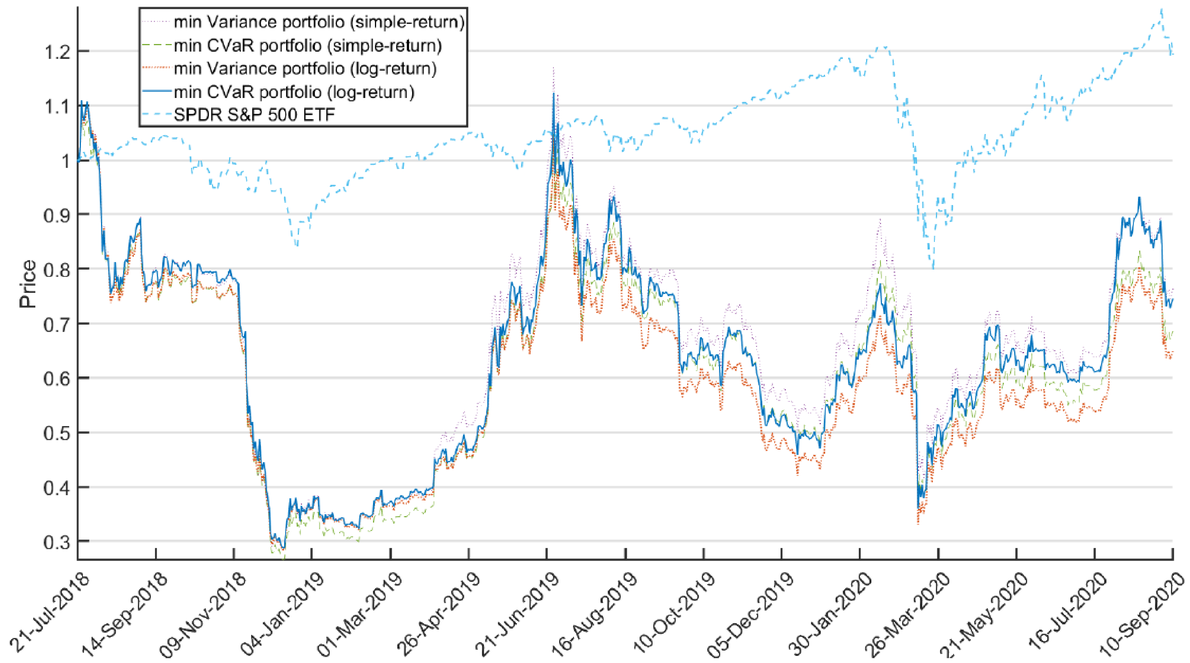


Table: Mean of CRPS based on different models

Innovation	Gaussian	Student's t
Joint Distri.	Multivariate t - fit ν	Multivariate t - fit ν
	Log-return	
ARMA(1,1)-GARCH(1,1)	2.16×10^{-4}	1.02×10^{-3}
ARMA(1,1)-EGARCH(1,1)	2.92×10^{-3}	1.49×10^{-4}
	Simple-return	
ARMA(1,1)-GARCH(1,1)	3.5×10^{-3}	5.9×10^{-3}
ARMA(1,1)-EGARCH(1,1)	2.94×10^{-3}	3.88×10^{-3}

1. $$\mathrm{MDD}(T) = sup_{t\in[0,T]}[sup_{s\in[0,t]}(S_s - S_t)] \tag{3}$$
2. $$\mathrm{Sharpe}(T) = \frac{\bar{R}(T) - R_f}{\sigma_P} \tag{4}$$
where $\bar{R}(T) = \frac{1}{T}\sum_{t=0}^{T}\tilde{r}_t$ is the mean portfolio return, R_f is the risk-free rate, and σ_P is the standard deviation of the portfolio's excess return within $[0, T]$.
3. Defined benchmark volatility as σ_M,
$$\mathrm{M2}(T) = \mathrm{Sharpe}(T)\sigma_M + R_f \tag{5}$$
4. We set $\alpha = \beta = 0.01$,
$$\mathrm{Rachev}_{\alpha,\beta}(T) = \frac{\mathrm{CVaR}_\beta(R_f - R_P)}{\mathrm{CVaR}_\alpha(R_P - R_f)} \tag{6}$$

Table: Mean of risk contribution of Crypto assets over out-of-sample data

Return Type	Log-return		Simple-return		
Portfolio	min Volatility	min CVaR	min Volatility	min CVaR	SPY
MDD	7.4×10^{-1}	7.4×10^{-1}	7.4×10^{-1}	7.6×10^{-1}	3.4×10^{-1}
Sharpe Ratio	-1.5×10^{-2}	-1.1×10^{-2}	8.7×10^{-3}	5.2×10^{-3}	1.6×10^{-2}
M2 Ratio	-1.8×10^{-4}	-1.2×10^{-4}	1.3×10^{-4}	8.7×10^{-5}	2.3×10^{-4}
Rachev Ratio	6.2×10^{-1}	8.1×10^{-1}	1.7	1.5	2.3

Note: The riskfree rate used in above table is the 10-year Treasury rate on 09/10/2020; And the dataset covers from 07/21/2018 to 09/10/2020. The SPY column is based on log-return. In additional, we provide the ratios of SPY based on simple-return here: Sharpe ratio is 2.2×10^{-2}, M2 ratio is 3.1×10^{-4}, and Rachev ratio is 2.9.

Table: Mean of risk contribution of Crypto assets

Return Type	Log-return		Simple-return	
Risk Neasure	$\mathrm{RC}^{\mathrm{Vol}}(\%)$	$\mathrm{RC}^{\mathrm{CVaR}}(\%)$	$\mathrm{RC}^{\mathrm{Vol}}(\%)$	$\mathrm{RC}^{\mathrm{CVaR}}(\%)$
BTC	10.5	6.6	10.41	4.8
ETH	14.2	15.1	14	16.1
XRP	13.3	19	13.5	22.9
LTC	14.5	17.2	14.6	19.6
BCH	17.1	24.9	17.5	27
EOS	17	15.4	17	9.6
BNB	13.1	1.9	13	0