- Common Criteria includes:
  - 1. Principle of Unbiased Estimation;
  - 2. Minimum Variance among unbiased estimators;
  - 3. Minimum Mean Square Error (MSE);
- bias( $\hat{\theta}$ ) =  $E(\hat{\theta}) \theta$
- $MSE(\hat{\theta}) = V(\hat{\theta}) + Bias(\hat{\theta})^2 = E(\hat{\theta} \theta)^2$
- The  $k^{th}$  population moment:  $E(X^k)$
- The  $k^{th}$  sample moment:  $\frac{1}{n} \sum_{i=1}^{n} X_i^k$

## Steps of Method of Moments:

**Step 1** Identify how many parameters we need to estimate. (Let's say m).

**Step 2** Find the first m population moments:  $E(X), E(X^2), \dots, E(X^m)$ 

**Step 3** Find the first m sample moments:  $\frac{1}{n} \sum_{i=1}^{n} X_i$ ,  $\frac{1}{n} \sum_{i=1}^{n} X_i^2$ , ...,  $\frac{1}{n} \sum_{i=1}^{n} X_i^m$ 

Step 4 Equalize each of the population moments to the corresponding sample moment.

$$E(X) = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$E(X^2) = \frac{1}{n} \sum_{i=1}^{n} X_i^2$$

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$$E(X^m) = \frac{1}{n} \sum_{i=1}^n X_i^m$$

**Step 5** The solutions for the above equations are the moment estimators for the parameters.

## Steps of Maximum Likelihood Estimation:

**Step 1**: Find the likelihood function  $L(\theta; x) = \prod_{i=1}^{n} f(X_i, \theta)$ .

**Step 2**: Find the natural logarithm of the likelihood function  $l(\theta) = l(\theta; x) = \log L(\theta; x)$ .

**Step 3**: Take a derivative of  $l(\theta)$  for each of the parameter. (If you have m parameters you need m derivatives).

**Step 4**: Equalize each of the derivative with 0.

**Step 5**: Solve the equations to find solutions. The solutions are the MLE estimators for the parameters.

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