

### Math/Stat319 Homework 4

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1. Let  $X \sim \text{Binomial}(n, p)$ . In order to estimate population proportion of "success"  $p$ , the natural estimator of  $p$  is the sample proportion of success  $\hat{p}_1 = \frac{X}{n}$ . Given  $E(X) = np$  and  $V(X) = npq$ ,

1. Is  $\hat{p}_1$  an unbiased estimator for  $p$  ? Justify your answer.
2. What is the variance of  $\hat{p}_1$  ?
3. What is the mean square error (MSE) of  $\hat{p}_1$  for estimating  $p$  ?

Now consider another estimator  $\hat{p}_2 = \frac{X+2}{n+4}$ .

1. Is  $\hat{p}_2$  an unbiased estimator for  $p$  ? Justify your answer.
2. What is the variance of  $\hat{p}_2$  ?
3. What is the mean square error (MSE) of  $\hat{p}_2$  for estimating  $p$  ?

[*Comment:*] This one is a book example. Please check the book also to see how to compare the two MSEs.

2. Let  $X_1, \dots, X_n$  be random sample from  $N(\mu, \sigma^2)$ . The sample you collect is  $X_1 = 4, X_2 = 7, X_3 = 5, X_4 = 8, X_5 = 6$ . The probability of density function is  $f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ .
1. If both  $\mu$  and  $\sigma^2$  are unknown, please (1) find the point estimator for  $\mu$  and  $\sigma^2$  by method of moment. (2) AND use the sample you collect to calculate the estimates.
  2. If both  $\mu$  and  $\sigma^2$  are unknown, please (1) find the point estimator for  $\mu$  and  $\sigma^2$  by maximum likelihood estimation. (2) AND use the sample you collect to calculate the estimates.
  3. If  $\mu$  is known to be 2 and we want to estimate  $\sigma^2$ , (1) what is the maximum likelihood estimator for  $\sigma^2$ ? (2) AND use the sample you collect to calculate the estimate. (3) What is the maximum likelihood estimate for  $\sigma$ ?