

MF921 Topics in Dynamic Asset Pricing

Week 5

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Black-Scholes (II): Dominance-Free Interval and Risk Neutral Pricing

A General Brownian Market Model

Given a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. $W(t) = (W_1(t), \dots, W_d(t))^T$, independent d -dimensional Brownian motion. The filtration $\mathcal{F}_t^W = \sigma(W(s) : 0 \leq s \leq t)$ which is complete and right-continuous.

A financial market \mathcal{M} with 1 bond and d stocks under a finite horizon $[0, T]$:

$$\begin{aligned} dS_0(t) &= r(t)S_0(t)dt, \quad S_0(0) = 1 \\ dS_i(t) &= S_i(t) \left(b_i(t) dt + \sum_{j=1}^d \sigma_{ij}(t) dW_j(t) \right), \quad \text{for } i \in 1, 2, \dots, d \end{aligned}$$

- $r(t)$: interest rate
- $b(t) = (b_1, \dots, b_d)$: appreciation rates
- $\sigma(t) = (\sigma_{ij}(t))$: volatility matrix