Game-Theoretic Framework for Cooperative Sensing and Fair Spectrum Access in Multichannel Cognitive Radio Networks

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***Abstract*—In cognitive radio (CR) networks, secondary users (SUs) sense the spectrum to identify and possibly transmit over temporarily unoccupied channels licensed to primary users (PUs). When the received PU signal is weak, spectrum sensing by individual SUs becomes unreliable. To improve the sensing accuracy, SUs can cooperate to discover idle time slots. However, fair payoff (spectrum access) allocation has not been investigated for cooperative sensing in multichannel CR networks with heterogeneous PU signal quality at different SU sensors although such scenarios are prevalent in practice. In this paper, a two-layer coalitional game model is investigated, in which the coalition formation and the payoff allocation problems are decoupled: in the top-layer hedonic game, SUs form disjoint coalitions over potentially available channels while in the bottom-layer game, SUs negotiate over the payoff allocation within each coalition. The cooperatively detected spectrum opportunities are then shared among the coalition members in a coordinated manner according to their binding agreement on payoff allocation. The proposed game is decentralized and fosters cooperation by providing each SU with the transmission opportunities it deserves. Moreover, we propose a new physical-layer approach to distribute the network-level miss-detection (MD) constraints fairly among the interfering SUs for guaranteed PU protection. We demonstrate the performance advantages of the AND-rule combining of spectrum sensing results for heterogeneous SUs. Numerical results show that the proposed game outperforms previously investigated collaborative sensing and multichannel access approaches in terms of energy efficiency, throughput, SU fairness, and computational complexity. Finally, the effect of mobility on the computational complexity of the proposed game is discussed.**

***Index Terms*—Coalitional game, cooperative sensing and access, cognitive radio (CR).**

I. Introduction

Cooperative sensing exploits spatial diversity to improve sensing accuracy in cognitive radio (CR) systems [2]. While most investigations assume a fixed number of fully cooperative secondary users (SUs) with identical sensing capabilities monitoring a single channel, in practice, there are many possible channels for sensing and transmission, and the sensing accuracy varies over the spectrum and among the SUs. Under the hardware constraints, SUs need to choose both the channels to sense and their collaborators[1] for spectrum sensing.

TABLE I
Main Notation

Notation	Explanation
$\mathcal{M}$, $\mathcal{N}$	Set of all SUs and set of all channels.
$C = (S, n)$	A top-layer coalition C is a two-tuple, where $C(1)$ is a set of SUs $S \subseteq \mathcal{M}$ and $C(2)$ is the operating channel n.
$\mathcal{P}$	A top-layer partition of $\mathcal{M}$ defines a set of disjoint top-layer coalitions $\mathcal{P} = \{C^1, C^2, \ldots, C^N\}$ with cardinality N, where $\forall n \neq \tilde{n}$, $C^n(1) \cap C^{\tilde{n}}(1) = \phi$, $C^n(2) \neq C^{\tilde{n}}(2)$, $\cup_{n=1}^N C^n(1) = \mathcal{M}$ and $\cup_{n=1}^N \{C^n(2)\} = \mathcal{N}$.
x^{mC}	Top-layer utility, given by the expected data rate of SU $m \in C(1)$ operating on channel $C(2)$.
a^{mC}	Bottom-layer payoff for SU $m \in C(1)$ on channel $C(2)$, given by the probability that m transmits successfully.
R^{mn}	Transmission rate of SU pair m on channel n.
γ^{mn}	SU-to-SU signal-to-noise ratio (SNR) of SU pair m on channel n.
B^n, β^n	Bandwidth and availability probability of channel n.
(S, U^n)	Bottom-layer game among a set of SUs S on channel n with the value function U^n.
η	A bottom-layer coalition η is a subset of SUs sensing the same channel, i.e., $\eta \subseteq C(1)$ for some $C \in \mathcal{P}$.
ρ	A bottom-layer partition is a set of disjoint subsets of $C(1)$ with their union equal to $C(1)$ for some $C \in \mathcal{P}$.
$U^n(\eta; \rho)$	Value of bottom-layer coalition η, as given by η's overall successful transmission probability on channel n under the bottom-layer partition ρ where $\eta \in \rho$.
$P^n_{\text{MD}}(\eta)$, $P^n_{\text{FA}}(\eta)$	Cooperative miss-detection (MD) and false-alarm (FA) probabilities of a bottom-layer coalition η regarding the PU presence.
$P^{\text{Ch}}_{\text{MD}}$	Integrated MD (or PU collision) probability constraint on each channel.
$P^n_{\text{MD}}(m)$, $P^n_{\text{FA}}(m)$	Individual MD constraint and FA probability of SU m on channel n.
λ^{mn}	PU-to-SU SNR per sample at SU m on channel n.

Game theory has been utilized recently to model and analyze SU interactions in cooperative sensing [4]–[11] and opportunistic access [11]–[13], but, to the best of our knowledge, only the game in [11] takes into account transmission (spectrum access) opportunities when making channel sensing and collaboration decisions in a multichannel CR network. However, in the one-layer game of [11], the SUs sensing the same channel are forced to cooperate, and all coalition members have the same probability of transmission over the channel that has been sensed idle. Thus, the game in [11] is not suitable for the heterogeneous environment where the contributions of different SUs within a coalition can vary significantly. For example, an SU located closer to the primary user (PU) tends to obtain more accurate sensing results and,

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[1]We consider only cooperation among the SUs. The PU-SU cooperation [3] is out of the scope of this paper.

thus, its contribution to the cooperative sensing accuracy is greater than that of a more distant SU. Such heterogeneous scenarios are typical in wireless networks. To provide incentives for cooperation, the payoff of each player should match its contribution. In this paper, we propose a two-layer coalitional game where channel sensing decisions and access agreements are guided by a fair payoff allocation method among the SUs.

Moreover, to facilitate efficient spectrum sensing, we develop constraints on miss-detection (MD) rates of coalition members and provide novel insights into the performance of fusion rules of sensing results for heterogeneous networks under the constant detection rate (CDR) [14] constraints.

The *contributions* of this paper are:

- Development and analysis of a two-layer coalitional game that includes:
  (i) An efficient, stable, and distributed coalition formation algorithm across and within the channels.
  (ii) A contribution-based payoff allocation scheme to promote individual incentives for cooperation.
- Improved spectrum sensing approaches:
  (i) A fair distribution method of the integrated network-level primary collision probability constraint among the coalitions and member SUs sharing the same channel.
  (ii) Demonstration of performance advantages of the AND-rule combining for heterogeneous sensing environments under the CDR constraints.

While we have first proposed the two-layer game in [1], that paper did not contain several key proofs, practical validation, and complexity analysis of the coalitional game and did not address fusion rules for heterogeneous environments.

The rest of this paper is organized as follows. Table I summarizes significant notation. In section II, we introduce the system model and formulate the proposed two-layer game, which is then analyzed in Section III using coalitional game theory. Simulation results and comparison with [11] are presented in Section IV, and conclusions are drawn in Section V.

II. System Model and Two-Layer Game Formulation

We consider an overlay slotted[2] CR ad hoc network with multiple SU pairs and multiple channels under i.i.d. Bernoulli PU traffic. An SU can sense and access only one channel at each time slot due to the hardware constraints.

A. The Top-Layer Game

The top-layer game is played by all SUs in $\mathcal{M}=\{1,\ldots,M\}$ across all channels in $\mathcal{N}=\{1,\ldots,N\}$. The set of SUs S sensing and transmitting over the same channel n forms a top-layer coalition $C=(S,n)$[3], i.e., the sensing decision of each SU $m\in S$ is

$$n_*^m = n \Leftrightarrow \{m\in C(1)\ \&\ C(2)=n\}. \tag{1}$$

[2]Our results can be extended to continuous PU traffic by adjusting the primary collision constraints [13].

[3]Note that a top-layer coalition is a two-tuple specifying both the set of SUs S and the channel index n because the same set of SUs S can achieve different throughputs on different channels.

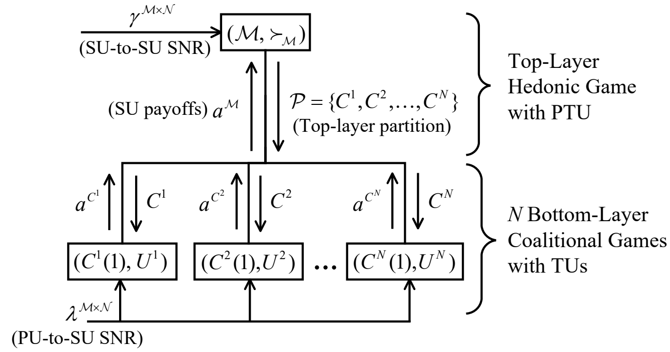

Fig. 1. Two-layer coalitional game.

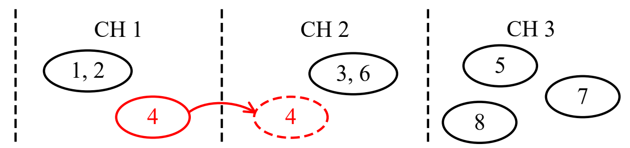

Fig. 2. An example of the two-layer coalition structure and SU movements with $\mathcal{M}=\{1,\ldots,8\}$ and $\mathcal{N}=\{1,2,3\}$. The top-layer partition is $\mathcal{P}=\{(\{1,2,4\},1),(\{3,6\},2),(\{5,7,8\},3)\}$ and the bottom-layer partitions are $\rho^1=\{\{1,2\},\{4\}\}$, $\rho^2=\{\{3,6\}\}$ and $\rho^3=\{\{5\},\{7\},\{8\}\}$. SU $m=4$ prefers to move from channel $n=1$ to $\tilde{n}=2$ if (3) is satisfied with $C^n=(\{1,2,4\},1)$, $C^{\tilde{n}}=(\{3,6\},2)$, $\tilde{C}^n=(\{1,2\},1)$ and $\tilde{C}^{\tilde{n}}=(\{3,6,4\},2)$.

Consequently, the set of all SUs $\mathcal{M}$ is partitioned into a total number of N disjoint top-layer coalitions on different channels, resulting in a top-layer partition $\mathcal{P}=\{C^1,C^2,\ldots,C^N\}$. Every possible top-layer coalition $C=(S,n)$ generates a partially transferable utility (PTU) measured by the expected data rate

$$x^{mC}=a^{mC}R^{mn} \tag{2}$$

for each member SU $m\in S$, where the successful transmission probability a^{mC} is a transferable utility (TU) [15] given by the allocated individual payoff provided by the bottom-layer game (cf. Section III-A), and the data rate of SU m on channel n given by $R^{mn}=B^n\log_2(1+\gamma^{mn})$ is a nontransferable utility (NTU) [15], which can vary greatly across different SU pairs due to their spatial separation. We define a preference relation that guides an SU when choosing a top-layer coalition. An SU m prefers to move from channel n to $\tilde{n}$ if the following preference relation is satisfied:

$$\tilde{C}^{\tilde{n}}\succ_m C^n \Leftrightarrow \begin{cases} x^{m\tilde{C}^{\tilde{n}}} > x^{mC^n} \\ \displaystyle\sum_{i\in\tilde{C}^n(1)} a^{i\tilde{C}^n}+\sum_{i\in\tilde{C}^{\tilde{n}}(1)} a^{i\tilde{C}^{\tilde{n}}} > \sum_{i\in C^n(1)} a^{iC^n}+\sum_{i\in C^{\tilde{n}}(1)} a^{iC^{\tilde{n}}} \end{cases} \tag{3}$$

where $\tilde{C}^n=(C^n(1)\backslash\{m\},n)$ and $\tilde{C}^{\tilde{n}}$ are the top-layer coalitions that form on channels n and $\tilde{n}$, respectively, if SU m makes the move, and C^n and $C^{\tilde{n}}=(\tilde{C}^{\tilde{n}}(1)\backslash\{m\},\tilde{n})$ are the existing top-layer coalitions on these channels. Thus, the top-layer coalition $\tilde{C}^{\tilde{n}}$ is preferable to C^n for SU m if (i) its expected data rate (2) improves and (ii) the combined successful transmission probabilities of all SUs on channels n and $\tilde{n}$ improve. For example, in Fig. 2, SU $m=4$ prefers to switch from channel 1 to channel 2 if this movement improves not only its individual throughput, but also the overall successful transmission probability of all SUs currently residing on these channels, i.e., SUs 1, 2, 3, 4 and 6. Using the preference relation (3), which combines the social and individual objectives, the top-layer game can be modeled as a hedonic game $(\mathcal{M}, \succ_{\mathcal{M}})$ [16]. Given $(\mathcal{M}, \mathcal{N}, a^{\mathcal{M}}, \gamma^{\mathcal{M}\times\mathcal{N}})$, the top-layer game generates a partition $\mathcal{P}$ as shown in Fig. 1. We will describe the partition formation algorithm of the top-layer game and prove its convergence to a Nash-stable partition [16] in Section III-B.

B. The Bottom-Layer Game

An output partition $\mathcal{P}$ of the top-layer game determines the set of SUs that can sense and transmit over each channel. The SUs sensing the same channel can further form disjoint bottom-layer coalitions, resulting in a bottom-layer partition ρ, as illustrated in Fig. 2. Within each bottom-layer coalition, the SUs exchange and combine their sensing results to improve the overall successful transmission probability of this coalition. The detected spectrum opportunities are then shared among the bottom-layer coalition members in a coordinated manner using a payoff allocation rule, which determines an SU's share of the slot for transmission and provides collision avoidance. We formulate N coalitional games (S, U^n) at the bottom layer, which are played on different channels, where $S = C(1)$ and $n = C(2)$ for some $C \in \mathcal{P}$, and U^n is a value function for all subsets of S. In Section III-A, we prove the efficiency [17] of the grand coalitions on all channels and discuss several payoff allocation rules for the bottom-layer games.

Since SUs coordinate channel access within each bottom-layer coalition, there is no need to use a medium access control (MAC) scheme. In contrast, MAC would be needed if multiple bottom-layer coalitions formed since several such coalitions could simultaneously detect a spectrum opportunity correctly and, therefore, would compete for access. As in [18], we consider the following two MAC options:

1) 0/X-model: All competing SUs fail to transmit successfully.
2) 1/X-model: All competing SUs have equal access probability, and the overhead cost is ignored. After gaining the right to access, the winning SU will transfer its transmission opportunity to its bottom-layer coalition's members as required by their binding agreement on payoff allocation (cf. Section III-A).

The performance of a chosen MAC option is impacted by the players' noncooperation cost, which measures how SU collisions and overhead affect the transferable successful transmission probability. Due to space limitations, we consider only the 0/X- and 1/X-models since they correspond to the maximum and minimum noncooperation cost, respectively. In practice, fair distributed MAC can be realized through random backoff and control message exchange at the cost of some control overhead and/or missed transmission opportunities. In Section III-A, we prove the formation of the grand coalition assuming both MAC models, and we expect this property to hold under the assumption of other MAC options with moderate noncooperation costs, eliminating the need for SU competition and, thus, for actual MAC utilization in the proposed game. However, MAC needs to be assumed hypothetically to determine the payoff allocation (cf. Section III-A).

Next, we consider computation of the value function U^n for a coalition $\eta \subseteq S$ on channel n, defined as the overall successful transmission probability of η on this channel. Given a partition ρ on channel n, label all other bottom-layer coalitions in $\rho\backslash\{\eta\}$ on channel n as

$$\rho\backslash\{\eta\}=\{\xi_1, \xi_2, \ldots, \xi_{|\rho|-1}\} \tag{4}$$

and define a binary-valued random vector of length $|\rho|-1$

$$\mathbf{X}_{\rho\backslash\{\eta\}} = (X_{\xi_1}, X_{\xi_2}, \ldots, X_{\xi_{|\rho|-1}}) \in \{0,1\}^{|\rho|-1} \tag{5}$$

where $X_{\xi_i} \in \{0,1\}$ is an indicator variable for the event that the coalition ξ_i experiences a false alarm (FA), i.e.,

$$\Pr(X_{\xi_i} = x_i) = \begin{cases} P^n_{\mathrm{FA}}(\xi_i), & \text{if } x_i = 1 \\ 1 - P^n_{\mathrm{FA}}(\xi_i), & \text{if } x_i = 0. \end{cases} \tag{6}$$

The coalition values for the two MAC models can be expressed as:

$$U^n_{0/\mathrm{X}}(\eta;\rho) = \beta^n(1 - P^n_{\mathrm{FA}}(\eta)) \cdot \prod\nolimits_{i=1}^{|\rho|-1} P^n_{\mathrm{FA}}(\xi_i) \tag{7}$$

and

$$\begin{aligned} U^n_{1/\mathrm{X}}(\eta;\rho) &= \beta^n(1-P^n_{\mathrm{FA}}(\eta)) \cdot \mathbb{E}\left[\frac{|\eta|}{|\eta| + J_{\rho\backslash\{\eta\}}\left(\mathbf{X}_{\rho\backslash\{\eta\}}\right)}\right] \\ &= \beta^n(1-P^n_{\mathrm{FA}}(\eta)) \cdot \sum_{\mathbf{x}\in\{0,1\}^{|\rho|-1}} \left\{\frac{|\eta| \prod_{i=1}^{|\rho|-1} \Pr(X_{\xi_i}=x_i)}{|\eta| + J_{\rho\backslash\{\eta\}}(\mathbf{x})}\right\} \end{aligned} \tag{8}$$

where β^n and $P^n_{\mathrm{FA}}(\eta)$ are defined in Table I, and the number of competing SUs for cognitive access is given by

$$J_{\rho\backslash\{\eta\}}(\mathbf{x}) = \sum\nolimits_{i=1}^{|\rho|-1} (1 - x_i)|\xi_i|. \tag{9}$$

Since the bottom-layer coalition value $U^n(\eta;\rho)$ depends not only on the actions of the coalition members, but also on the actions of other SUs on channel n, this coalitional game is in partition form [15]. We assume the overall successful transmission probabilities (7) and (8) of a bottom-layer coalition are dividable and can be transferred among coalition members according to the allocated payoff probability a^{mC} (cf. (2)), so that the bottom-layer game has TU. To illustrate, consider the following example. Suppose the overall successful transmission probability of a two-SU bottom-layer coalition $\eta = \{1, 2\}$ on channel n is $U^n(\eta) = 0.8$, and both SUs agree on a payoff probability allocation of $a^{1n} = 0.2$ and $a^{2n} = 0.6$. Under the 0/X-model, the coalition η acquires a transmission opportunity when η discovers an idle time slot. On the other hand, under the 1/X-model, the coalition η acquires a transmission opportunity when η discovers an idle time slot, possibly competes with other coalitions, and finally wins a right to transmit. The acquired transmission opportunities are then shared between the two SUs according to their payoff allocation agreement, either in a probabilistic manner, with the conditional probabilities that SU 1 and SU 2 are allowed to transmit for the entire time slot given by $a^{1n}/(a^{1n}+a^{2n})=1/4$ and $a^{2n}/(a^{1n}+a^{2n})=3/4$, respectively, or in a time-division multiple access (TDMA) manner, for $1/4$

and $3/4$ of the time, respectively[4]. In either case, it is not hard to show that the successful transmission probability of SU 1 and SU 2 are $1/4 \times U^n(\eta) = 0.2$ and $3/4 \times U^n(\eta) = 0.6$, as desired.

C. Cooperative Sensing

Next, we present the cooperative sensing scheme at the physical layer. Consider a certain top-layer coalition $C = (S, n)$ and a bottom-layer partition ρ of S. The integrated MD probability on channel n is given by [1]

$$P^n_{\mathrm{MD}}(\rho) = 1 - \prod_{\eta\in\rho} \left(1 - P^n_{\mathrm{MD}}(\eta)\right). \tag{10}$$

To provide uniform protection to the PUs across the potentially available spectrum, ensure fairness among the SUs, and improve spectrum detection, we impose the following constraints:
Network-level constraint (C.1): $P^n_{\mathrm{MD}}(\rho) = P^{\mathrm{Ch}}_{\mathrm{MD}}$ (cf. Table I)
Coalition-level constraint (C.2): All equal-sized bottom-layer coalitions should maintain the same MD rate.
Node-level constraint (C.3): All SUs within a bottom-layer coalition must satisfy the same MD constraint.

Constraints (C.1) and (C.2) are satisfied by requiring:

$$P^n_{\mathrm{MD}}(\eta) = 1 - (1 - P^{\mathrm{Ch}}_{\mathrm{MD}})^{|\eta|/|S|} \tag{11}$$

for all $\eta \in \rho$.

Next, we evaluate performance of the AND- and the OR-combining rules for the proposed constrained spectrum sensing approach. The FA probabilities of these fusion rules are given by [14, eq. (10–17)][5]:

$$\begin{aligned} P^n_{\mathrm{FA,AND}}(\eta) &= \prod_{m\in\eta} P^n_{\mathrm{FA,AND}}(m) \qquad (12)\\ &= \prod_{m\in\eta} Q\left(\sqrt{2\lambda^{mn}+1}\phi + \sqrt{\nu}\lambda^{mn}\right)\\ P^n_{\mathrm{FA,OR}}(\eta) &= 1 - \prod_{m\in\eta}(1 - P^n_{\mathrm{FA,OR}}(m)) \qquad (13)\\ &= 1 - \prod_{m\in\eta}\left(1 - Q(\sqrt{2\lambda^{mn}+1}\tilde{\phi} + \sqrt{\nu}\lambda^{mn})\right) \end{aligned}$$

respectively, with (cf. (11) and (C.3))

$$\begin{aligned} \phi &\triangleq Q^{-1}\left(1 - P^n_{\mathrm{MD,AND}}(m)\right) = Q^{-1}\left((1 - P^n_{\mathrm{MD}}(\eta))^{\frac{1}{|\eta|}}\right)\\ &= Q^{-1}\left((1 - P^{\mathrm{Ch}}_{\mathrm{MD}})^{\frac{1}{|S|}}\right) \qquad (14)\\ \tilde{\phi} &\triangleq Q^{-1}\left(1 - P^n_{\mathrm{MD,OR}}(m)\right) = Q^{-1}\left(1 - (P^n_{\mathrm{MD}}(\eta))^{\frac{1}{|\eta|}}\right)\\ &= Q^{-1}\left(1 - \left(1 - (1 - P^{\mathrm{Ch}}_{\mathrm{MD}})^{|\eta|/|S|}\right)^{\frac{1}{|\eta|}}\right) \qquad (15) \end{aligned}$$

where $m \in \eta \in \rho$, ν is the number of collected samples for spectrum sensing, $Q(\cdot)$ is the Q-function [20, eq. (B.20)], and all other notation is defined in Table I. First, from (14,15), the individual MD constraint $P^n_{\mathrm{MD}}(m)$ depends on the size of the bottom layer coalition to which an SU belongs for the OR-rule, but not for the AND-rule. Thus, when using the AND-rule, SUs do not have to update the MD constraints when computing their values within different bottom-layer coalitions as long as the SU population $|S|$ on channel n does no change. Second, for both rules, as $|S|$ increases, each coalition η reduces its MD rate $P^n_{\mathrm{MD}}(\eta)$ in (11), leading to increased FA probability $P^n_{\mathrm{FA}}(\eta)$ in (12,13) and decreased coalition values in (7,8). Thus, large values of $|S|$ are penalized, balancing SU competition on all channels.

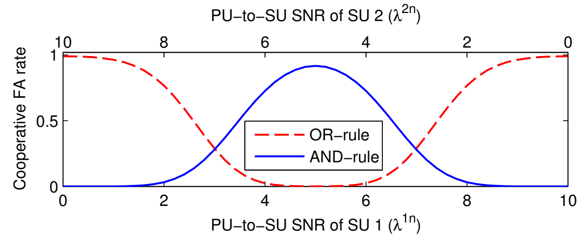

Fig. 3. Performance comparison of the $P^n_{\mathrm{FA}}(\eta)$ for the AND- and OR-rule under the proposed MD constraints; $\eta = \{1, 2\}$; $P^n_{\mathrm{MD}}(\eta) = 10^{-4}$; $\nu = 5$; $\bar{\lambda} = 0.5(\lambda^{1n} + \lambda^{2n}) = 5$ (7 dB).

Finally, our extensive simulation results using (12–15) show that the AND-rule is more suitable when cooperative SUs have heterogeneous sensing capabilities (i.e., different PU-to-SU SNRs) while the OR-rule provides better performance in homogeneous scenarios. Intuitively, if there is at least one SU $m \in \eta$ with favorable PU-to-SU SNR λ^{mn}, and thus with small FA probability $P^n_{\mathrm{FA,AND}}(m)$, the resulting $P^n_{\mathrm{FA,AND}}(\eta)$ is very small since $P^n_{\mathrm{FA,AND}}(\eta) \leq \min_{m\in\eta} P^n_{\mathrm{FA,AND}}(m)$ in (12). In contrast, for the OR-rule, if only one member SU has very poor sensing capacity, the entire bottom-layer coalition η suffers since $P^n_{\mathrm{FA,OR}}(\eta) \geq \max_{m\in\eta} P^n_{\mathrm{FA,OR}}(m)$ in (13). For example, in Fig. 3 we compare the integrated FA probabilities (12) and (13) for coalition $\eta = \{1, 2\}$ on channel n when the average PU-to-SU SNR of the two member SUs is fixed to $\bar{\lambda}$. We observe that the OR-rule and the AND-rule achieve their best performance for homogeneous and heterogeneous sensing capacities (PU signal strengths at the sensor) of the two SUs, respectively. The general case is explored below [21].

Proposition 1. Consider coalition η on channel n and a fixed bottom-layer-coalition-level MD probability constraint $P^n_{\mathrm{MD}}(\eta)$[6]. The FA probability of η for the AND-rule (12) (OR-rule (13)) is a quasiconcave (quasiconvex) function [22, Section 3.4], which achieves its global maximum (global minimum) when $\lambda^{mn} = \bar{\lambda}$, $\forall m \in \eta$, subject to an average PU-to-SU SNR constraint

$$\frac{1}{|\eta|}\sum_{m\in\eta} \lambda^{mn} = \bar{\lambda} \tag{16}$$

if $\forall m \in \eta$,

$$P^n_{\mathrm{MD}}(\eta) < (0.5)^{|\eta|} \tag{17}$$

$$\tilde{\phi} > -\frac{\sqrt{\nu}(2\lambda^{mn}+1)^{\frac{3}{2}}}{3\lambda^{mn}+2}. \tag{18}$$

[4]SU transmissions might be unsuccessful either due to SU collisions (under the 0/X-model) or MD of PU traffic (under both MAC models).

[5]As in [14], we assume additive white Gaussian noise (AWGN) PU-to-SU channels and thus time-invariant λ^{mn} values. In practice, these channels may be subject to fading and the ergodic sensing accuracy probabilities can be obtained by averaging over the fading distribution [5], [6], [19].

[6]Due to space limitations, we omit the proof, which shows the desired quasiconcavity and quasiconvexity by establishing the logconcavity of $P^n_{\mathrm{FA,AND}}(\eta)$ and $1 - P^n_{\mathrm{FA,OR}}(\eta)$, respectively [21].

Note that the constraints (17) and (18) are mild[7]. For example, they can be easily satisfied if: (i) $P^n_{\mathrm{MD}}(\eta) \geq 10^{-4}$ and $|\eta| \leq 13$ and (ii) ($\nu \geq 56$ for $\lambda^{mn} \geq 0$) or ($\nu \geq 13$ for $\lambda^{mn} \geq 1$) or ($\nu \geq 2$ for $\lambda^{mn} \geq 10$), i.e., when the MD constraint $P^n_{\mathrm{MD}}(\eta)$ is not extremely stringent, the coalition size $|\eta|$ is not too large, and the number of collected samples for spectrum sensing ν is sufficiently large[8].

Based on these observations, in the remainder of this paper we will assume the AND fusion rule to study a typical wireless CR network with a moderate number of cooperating SUs on each channel, which experience heterogeneous PU-to-SU channel conditions.

III. Two-Layer Coalitional Sensing and Access Game[9]

A. Grand Coalition Formation and Payoff Allocation in the Bottom-layer Game

Consider the bottom-layer game (S, U^n) on channel n.

Proposition 2. For the 0/X-model, the game $(S, U^n_{0/\mathrm{X}})$:

(i) reduces to a characteristic-form game, i.e., the value of a bottom-layer coalition η depends solely on the composition of η [15]: $U^n_{0/\mathrm{X}}(\eta;\rho) = U^n_{0/\mathrm{X}}(\eta;\tilde{\rho}) \triangleq U^n_{0/\mathrm{X}}(\eta)$, for any $\eta \subseteq S$ and any bottom-layer partitions ρ and $\tilde{\rho}$ of S, such that $\eta \in \rho$ and $\eta \in \tilde{\rho}$;

(ii) is superadditive [15], i.e., the SUs benefit from forming larger coalitions. Thus, for any two disjoint bottom-layer coalitions $\eta, \xi \subset S$, $U^n_{0/\mathrm{X}}(\eta \cup \xi) \geq U^n_{0/\mathrm{X}}(\eta) + U^n_{0/\mathrm{X}}(\xi)$;

(iii) from (ii), has efficient grand coalition [17], i.e., the grand coalition value is at least as large as the combined value of all coalitions in any other partition: $\forall \tilde{\rho} \neq \{S\}$, $U_{0/\mathrm{X}}(S) \geq \sum_{\eta\in\tilde{\rho}} U_{0/\mathrm{X}}(\eta)$.

Proposition 3. Under the 1/X-model, all bottom-layer partitions of S are equally efficient, i.e., for any two partitions ρ and $\tilde{\rho}$ of S, $\sum_{\eta\in\rho} U^n_{1/\mathrm{X}}(\eta;\rho) = \sum_{\eta\in\tilde{\rho}} U^n_{1/\mathrm{X}}(\eta;\tilde{\rho})$.

Proof. Under the 1/X-model, a spectrum opportunity on channel n is wasted if all bottom-layer coalitions make FAs on the PU presence. Thus, for any bottom-layer partition ρ of any S on any channel n (cf. (12))

$$\sum_{\eta\in\rho} U^n_{1/\mathrm{X}}(\eta;\rho) = 1 - \prod_{\eta\in\rho} P^n_{\mathrm{FA}}(\eta) = 1 - \prod_{m\in S} P^n_{\mathrm{FA}}(m) \quad (19)$$

which is independent of ρ. □

Proposition 4. Under the 1/X-model, the bottom-layer game $(S, U^n_{1/\mathrm{X}})$ exhibits nonpositive externalities [24], i.e., a merger between two coalitions cannot improve the values of other coalitions.

Proof. See Appendix A. □

[7] (17) implies that ϕ (14) and $\tilde{\phi}$ (15) are both negative since $|\eta| \geq 1$. (18) implies that $\tilde{\phi} > -\frac{3\sqrt{\nu}\sqrt{2\lambda^{mn}+1}}{2}$ since $\lambda^{mn} \geq 0$.

[8] In practice, low PU-to-SU SNR (e.g. ≤ 1) often requires ν on the order of hundreds to thousands and medium-to-high PU-to-SU SNR (e.g. ≥ 10) often requires less than 10 samples for good performance (See, e.g., [19], [23]).

[9] Proofs of Propositions 2 and 5–8 can be found in [1].

Proposition 5. The grand coalition always forms in the bottom-layer game for the 1/X and 0/X MAC models.

The proof of Proposition 5 is based on Propositions 2–4 and follows from the strong and weak efficiency of the grand coalition [15], [17], [24] for the 0/X and 1/X models, respectively [1]. Proposition 5 implies that all SUs sensing channel n cooperate, and MAC is not utilized. Moreover, it is feasible to divide the value of the grand coalition $U^n(S;\{S\})$ (hereafter referred to as $U^n(S)$ for brevity) while satisfying every member SU with its allocated payoff. Generally, a good payoff allocation rule ensures the stability of and the fairness within the grand coalition [25] by relating the actual individual payoff to the hypothetical payoff that an SU could have obtained by leaving the grand coalition and/or the marginal value it brings into the grand coalition. Therefore, despite the formation of the grand coalition, we need to compute the hypothetical values of smaller bottom-layer coalitions using (7) or (8) for the two collision models. Fair payoff allocation rules for traditional single-layer coalitional games are extensively studied in the literature assuming (hypothetical) breakdown of the grand coalition, e.g., the Shapley value [15], the Owen value [26], the nucleolus [15], the Nash bargaining solution (NBS) [27], [28], etc. In this paper, we employ the NBS due to its computational efficiency [27].

Proposition 6. For the characteristic-form (cf. Proposition 2) bottom-layer game $(S, U^n_{0/\mathrm{X}})$, NBS assigns a payoff (successful transmission probability) [27]

$$a^{mC}_{\mathrm{NBS},0/\mathrm{X}} = \frac{U^n_{0/\mathrm{X}}(S) - \sum_{i\in S} U^n_{0/\mathrm{X}}(\{i\})}{|S|} + U^n_{0/\mathrm{X}}(\{m\}) \quad (20)$$

$\forall m \in S$, where the disagreement point is $\left\{U^n_{0/\mathrm{X}}(\{m\})_{m\in S}\right\}$.

The m^{th} component of the disagreement point $U^n_{0/\mathrm{X}}(\{m\})$ represents SU m's minimal expected individual payoff and is given by the hypothetical successful transmission probability obtained if bargaining fails. Since SUs with stronger sensing capabilities have higher singleton values $U^n_{0/\mathrm{X}}(\{m\})$, the NBS (20) provides each SU with the allocated payoff commensurate with its sensing contribution, resulting in a fair payoff allocation.

When the 1/X-model is assumed, the selfish individual payoff $U^n_{1/\mathrm{X}}(\{m\};\rho)$ depends not only on that SU's sensing capability, but also on other SUs' reactions when a given SU does not cooperate. In this paper, we employ the *fine* NBS (*f*NBS) [28], which implies all SUs stand alone if bargaining is not successful, i.e., if the grand coalition breaks down.

Proposition 7. Under the 1/X model, the payoff of each SU $m \in S$ using *f*NBS is [28]

$$\begin{aligned} a^{mC}_{f\mathrm{NBS},1/\mathrm{X}} &= \frac{1}{|S|}\left[U^n_{1/\mathrm{X}}(\{S\}) - \sum_{i\in S} U^n_{1/\mathrm{X}}\left(\{i\};\{\{j\}_{j\in S}\}\right)\right] \\ &\quad + U^n_{1/\mathrm{X}}(\{m\};\{\{j\}_{j\in S}\}) \\ &= U^n_{1/\mathrm{X}}(\{m\};\{\{j\}_{j\in S}\}). \end{aligned} \quad (21)$$

Thus, the allocated payoff probability is simply given by the successful transmission probability that each SU could have obtained individually assuming all other SUs have also formed singletons. However, this does not imply that SUs should deviate from the grand coalition, because each SU is not guaranteed a payoff of $a^{mC}_{f\text{NBS},1/\text{X}}$ if it deviates. In fact, if an SU decides to remain isolated, its payoff is at risk due to the nonpositive externalities (Proposition 4) if other SUs collude. As a result, an SU m might end up with a much worse payoff than its singleton value $U^n_{1/\text{X}}(\{m\};\{\{j\}_{j\in S}\})$. Therefore, all SUs have an incentive to join the grand coalition. Moreover, we found that a *coarse* NBS (*c*NBS) [28] allocation where all other SUs form a coalition and try to reduce the hypothetical individual payoff of the deviating SU, results in similar throughput and energy consumption as that of the *f*NBS method in Proposition 7 [21].

Note that the allocated payoff probabilities (20) and (21) are *group-rational* [15]:

$$U^n(S) = \sum_{m\in S} a^{mC}. \tag{22}$$

Finally, the conditional transmission probability of an SU $m \in S$ given that a spectrum opportunity is detected successfully is set to $a^{mC}/\sum_{i\in S} a^{iC}$ (see Example at the end of Section II-B). From (22), the successful transmission probability for SU m sensing channel n is a^{mC}, $\forall m \in S$. The bottom-layer payoff allocation algorithm is summarized in Algorithm 1.

B. Coalition Formation at the Top Layer

In the top-layer game, SUs use (3) to determine if switching to another channel is advantageous. The proposed Algorithm 2 employs a distributed switching scheduling scheme to facilitate fast convergence to an SU network partition. At most one switch is allowed in each time slot. Initially, all SUs actively compete for the right to switch using an out-of-band control channel [29]. In every time slot, the winning SU m randomly chooses a potential new channel and notifies other SUs in $\mathcal{M}\backslash\{m\}$ about its decision by broadcasting (on the control channel) (i) a SWITCH signal if (3) holds, (ii) a HOLD signal if (3) fails and it still plans to continue searching, or (iii) a SLEEP signal when all switching opportunities have been exhausted unsuccessfully, i.e., the current channel is the most preferable for this SU. Competition for the right to switch continues until all SUs are asleep, indicating convergence of the partition formation process. Note that all SUs (including sleeping SUs) should cognitively monitor the environment changes, e.g., variation of the PU-to-SU SNRs, the PU/SU locations, the number of SUs, the number of channels, etc., and repeat the partition formation algorithm in Algorithm 2 when changes occur. Assuming that these parameters are fixed, the following Proposition holds:

Proposition 8. The proposed coalition formation algorithm (Algorithm 2) converges in at most N^M switches.

Variation of the number of SUs/PUs and mobility are explored in Section IV.

The computational complexity of the proposed game is dominated by the FA rate computations (line 1 in Algorithm 1). For SU m on channel n (cf. (12) and (14)), this rate is determined by λ^{mn} and SU population size $|C^n_t(1)|$. Each SU m on channel n maintains three values of its FA rate, corresponding to the current and potential SU populations sizes $|C^n_t(1)| \pm i$, $i = 0, 1$, and this information is exchanged over the control channel as needed. Thus, the initialization step requires $3M$ FA rate computations. When an SU explores whether to switch to channel $\tilde{n}$ using (3), it needs to compute its potential FA rate on that channel, corresponding to the updated SU population size $|C^{\tilde{n}}(1)| + 1$ and its PU-to-SU SNR $\lambda^{m\tilde{n}}$, resulting in one FA computation in each time slot. In those slots where an SU m actually switches to channel $\tilde{n}$, new coalitions $\tilde{C}^n$ and $\tilde{C}^{\tilde{n}}$ form (cf. (3)), and SU m needs to compute two additional FA rates corresponding to the SU population sizes $|\tilde{C}^{\tilde{n}}(1)| \pm 1$ while all other SUs in $\tilde{C}^n(1)$ and $\tilde{C}^{\tilde{n}}(1)$ have to compute only one additional FA rate each, corresponding to SU population sizes decreased and increased by 2 on their respective channels n and $\tilde{n}$. Since each FA rate computation takes $\mathcal{O}(1)$ time, the total computational complexity of the algorithm until convergence is

$$\mathcal{O}\Big(3M + T_{\text{Converge}} + \sum_{t\in\mathcal{T}_{\text{Switch}}} \big(|\tilde{C}^n_t(1)| + |\tilde{C}^{\tilde{n}}_t(1)| + 1\big)\Big) \tag{23}$$

where $\mathcal{O}(\cdot)$ is the big O notation [30], $\mathcal{T}_{\text{Switch}}$ is the set of time slots in which an SU switches in the interval $[1, T_{\text{Converge}}]$ and T_{Converge} denotes the convergence time, i.e., the time needed to execute the while loop from line 4 to 21 in Algorithm 2.

Algorithm 1 Bottom-layer payoff allocation

Input: $C{=}(S,n)$, $\{\lambda^{mn}{:}m{\in}S\}$, $P^{\text{Ch}}_{\text{MD}}$, ν, β^n
Output: $\{a^{mC}{:}m{\in}S\}$

1: $P^n_{\text{FA}}(m){=}Q\Big(\sqrt{2\lambda^{mn}{+}1}Q^{-1}((1-P^{\text{Ch}}_{\text{MD}})^{\frac{1}{|S|}}){+}\lambda^{mn}\sqrt{\nu}\Big)$
2: **if** 0/X-MAC **then** $\forall m{\in}S$:
3: $\quad U^n_{0/\text{X}}(S) = \beta^n(1 - \prod_{i\in S} P^n_{\text{FA}}(i))$
4: $\quad U^n_{0/\text{X}}(\{m\}){=}\beta^n(1 - P^n_{\text{FA}}(m)) \prod_{i\in S\backslash\{m\}} P^n_{\text{FA}}(i)$
5: $\quad$ Compute $a^{mC}_{\text{NBS},0/\text{X}}$ using (20)
6: **if** 1/X-MAC **then** $\forall m{\in}S$:
7: $\quad a^{mC}_{f\text{NBS},1/\text{X}}{=}U^n_{1/\text{X}}(\eta{=}\{m\};\rho{=}\{\{j\}_{j\in S}\})$ as in (8)

IV. Simulation Results

We assume the following simulation setup throughout this section unless otherwise noted: The sensing and slot durations are 5 ms and 100 ms, respectively. The SU sensing power P_S, SU transmission power P_{SU}, PU transmission power P_{PU}, and noise power P_N are 10 mW, 10 mW, 100 mW and 0.1 mW, respectively. All users are randomly placed in a square region of $100\,\text{m}\times100\,\text{m}$, and only path loss effects are considered with the pass loss exponent equal to 2 [11]. The PU-to-SU SNR is given by $\lambda^{mn} = P_{\text{PU}} d^{-2}_{mn}/P_N$, where d_{mn} is the distance between PU n and SU m. Similarly, the SU-to-SU SNR between two SUs m and m' is given by $\gamma^{mm'} = P_{\text{SU}} d^{-2}_{mm'}/P_N$, where $d_{mm'}$ is the SU distance.

In Fig. 4, we compare the proposed game (i,iii) with the game in [11] (ii,iv) for the two MAC models (cf. (7,8)). Initially there are $M = 10$ SUs and $N = 5$ PUs. At time slots 2000 and 4000, these parameters change to $(M = 10, N = 6)$

Algorithm 2 Distributed top-layer partition formation

Input: $\mathcal{M}, \mathcal{N}, \beta^{\mathcal{N}}, \gamma^{\mathcal{M}\times\mathcal{N}}, \lambda^{\mathcal{M}\times\mathcal{N}}$
Output: $\mathcal{P}$

Initialization: Each SU m randomly senses a channel n_*^m
$C^n(1) = \{m : n_*^m = n\}$ and $C^n(2) = n$, $\forall n \in \mathcal{N}$
$\mathcal{P} = \{C^1 \cdots C^N\}$ and Action = SWITCH
while $\mathcal{M}_{\text{Active}} \neq \emptyset$ **(at each time slot)**:
  if Action = SWITCH **then**: $\mathcal{M}_{\text{Active}} = \mathcal{M}$
  if Action $\neq$ HOLD **then**:
    SUs in $\mathcal{M}_{\text{Active}}$ contend for the right to switch
    SU $m \in C^n(1)$ wins and $\mathcal{N}_{\text{Candidate}} = \mathcal{N} \backslash \{n\}$
  m randomly chooses channel $\tilde{n} \in \mathcal{N}_{\text{Candidate}}$
  $\tilde{C}^n = (C^n(1) \backslash \{m\}, n)$, $\tilde{C}^{\tilde{n}} = (C^{\tilde{n}}(1) \cup \{m\}, \tilde{n})$
  m plays the bottom-layer game $(\tilde{C}^{\tilde{n}}, U^{\tilde{n}})$ in Fig. III-B
  if (3) holds **then**:
    $a^{iC^n} = a^{i\tilde{C}^n}$, $a^{jC^{\tilde{n}}} = a^{j\tilde{C}^{\tilde{n}}}$, $\forall i \in C^n(1)$, $j \in C^{\tilde{n}}(1)$
    $\mathcal{P} = \mathcal{P} \backslash \{C^n\} \backslash \{C^{\tilde{n}}\} \cup \{\tilde{C}^n\} \cup \{\tilde{C}^{\tilde{n}}\}$
    $C^n = \tilde{C}^n$, $C^{\tilde{n}} = \tilde{C}^{\tilde{n}}$, Action = SWITCH
  else:
    m stays on channel n and $\mathcal{N}_{\text{Candidate}} = \mathcal{N} \backslash \{\tilde{n}\}$
    if $\mathcal{N}_{\text{Candidate}} \neq \emptyset$ **then**: Action = HOLD
    else: $\mathcal{M}_{\text{Active}} = \mathcal{M} \backslash \{m\}$, Action = SLEEP
  if a slot is sensed idle by $C = (S, n)$ **then** $\forall m \in S$:
    SU m transmits with probability $a^{mC} / \sum_{i \in S} a^{iC}$

and ($M = 14, N = 6$), respectively. Each PU uses one channel with $B^n = 10\,\text{MHz}$ exclusively. The channel availability probability is $\beta^n = 0.2$. For fair comparison, the AND-rule combining scheme under the constraints described in Section II is used for both the proposed two-layer game and the game in [11] assuming $\nu = 5$ and $P_{\text{MD}}^{\text{Ch}} = 0.01$. Note that [11] aims to maximize the energy efficiency while the objective of the proposed game is to maximize the expected data rate (2). Nevertheless, the proposed game achieves better average energy efficiency and throughput in Fig. 4(a,d), since in this game the SUs' payoffs increase as they sense closer PUs, resulting in improved sensing accuracy. Moreover, each SU is allocated the access opportunities it deserves and thus is provided with sufficient incentives to participate in the proposed game. In particular, as shown in Fig. 4(b,c), every SU has higher data rate when playing the proposed game than operating alone (i.e., all SUs are satisfied with their data rates). Despite the fact that the proposed game can have a greater number of dissatisfied SUs in terms of energy efficiency than the game in [11] under the 1/X-model (curves iii vs. iv in Fig. 4(e)), these SUs experience negligible energy efficiency loss (Fig. 4(f)), and the overall energy efficiency is significantly higher for the two-layer game (Fig. 4(d)). We also note that the changes in the numbers of PUs/SUs at time slots 2000 and 4000 do not cause significant disruptions to these games. Finally, while the overall system throughputs and energy consumption in Fig. 4(a,d) are very similar under both MAC models, individual SUs report a higher degree of satisfaction under the 0/X-model as shown in Fig. 4(b,c,e,f) since this model has the maximum noncooperation cost (Section II-B).

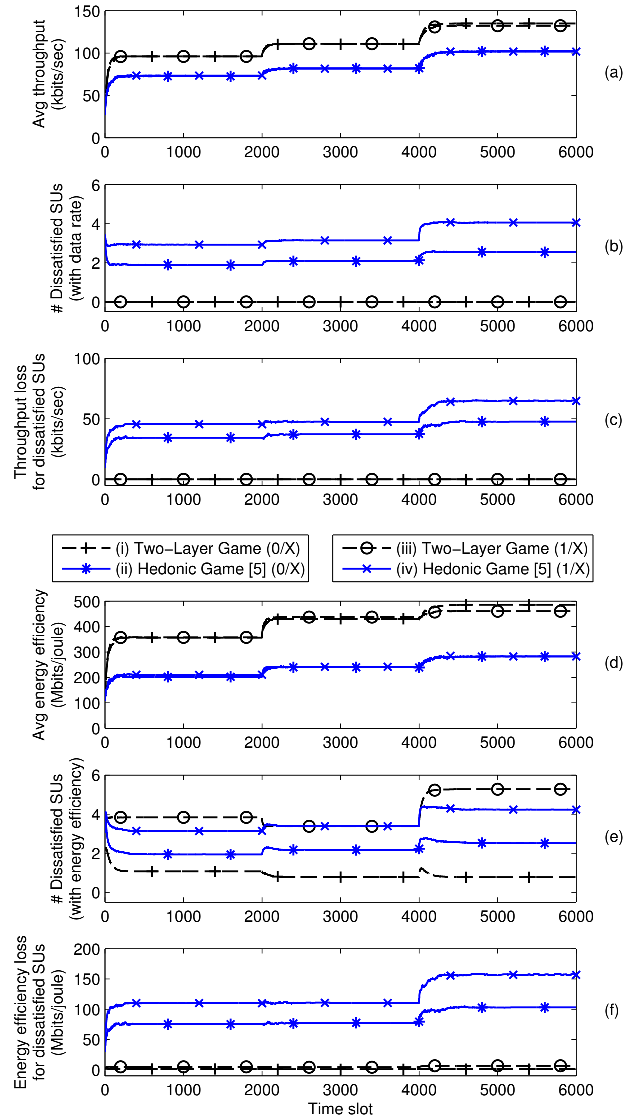

Fig. 4. Performance comparison of the proposed two-layer game and the hedonic game in [11].

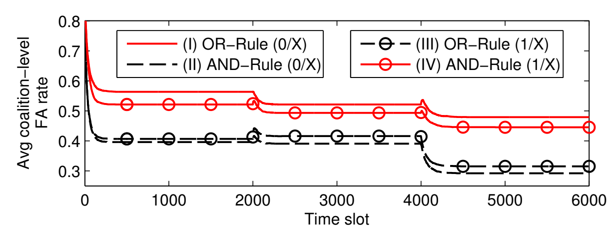

Fig. 5. Average coalition-level FA rate of the proposed hedonic game.

In Fig. 5 we validate the superiority of the AND-rule over the OR-rule for heterogeneous environments (see Section II-C) by comparing the average FA rates of the coalitions formed in the proposed game for the realistic scenario of Fig. 4. Not surprisingly, we observe significant sensing accuracy degradation using the OR-rule, which can be explained by diverse PU-to-SU SNRs among the coalition members.

Next, we compare the computational complexity (23) and convergence time in Fig. 6 for varying N values, assuming

$$U^n_{1/\mathrm{X}}(\eta;\rho) = \beta^n(1-P^n_{\mathrm{FA}}(\eta))\cdot\sum_{\tilde{\mathbf{x}}\in\{0,1\}^{|\rho|-3}}\left\{\sum_{(x_1,x_2)\in\{0,1\}^2}\frac{|\eta|\Pr(X_{\xi_1}=x_1)\Pr(X_{\xi_2}=x_2)\prod_{i=1}^{|\rho|-3}\Pr(X_{\xi_{i+2}}=\tilde{x}_i)}{|\eta|+J_{\{\xi_1,\xi_2\}}((x_1,x_2))+J_{\rho\backslash\{\eta\}\backslash\{\xi_1\}\backslash\{\xi_2\}}(\tilde{\mathbf{x}})}\right\} \tag{24}$$

$$U^n_{1/\mathrm{X}}(\eta;\tilde{\rho}) = \beta^n(1-P^n_{\mathrm{FA}}(\eta))\cdot\sum_{\tilde{\mathbf{x}}\in\{0,1\}^{|\rho|-3}}\left\{\sum_{x_1\in\{0,1\}}\frac{|\eta|\Pr(X_{\xi_1\cup\xi_2}=x_1)\prod_{i=1}^{|\rho|-3}\Pr(X_{\xi_{i+2}}=\tilde{x}_i)}{|\eta|+J_{\{\xi_1\cup\xi_2\}}((x_1))+J_{\tilde{\rho}\backslash\{\eta\}\backslash\{\xi_1\cup\xi_2\}}(\tilde{\mathbf{x}})}\right\} \tag{25}$$

$$U^n_{1/\mathrm{X}}(\eta;\rho) - U^n_{1/\mathrm{X}}(\eta;\tilde{\rho}) = \beta^n(1-P^n_{\mathrm{FA}}(\eta))\cdot\sum_{\tilde{\mathbf{x}}\in\{0,1\}^{|\rho|-3}}\left\{\prod_{i=1}^{|\rho|-3}\Pr(X_{\xi_{i+2}}=\tilde{x}_i)\cdot A(\tilde{\mathbf{x}})\right\} \tag{26}$$

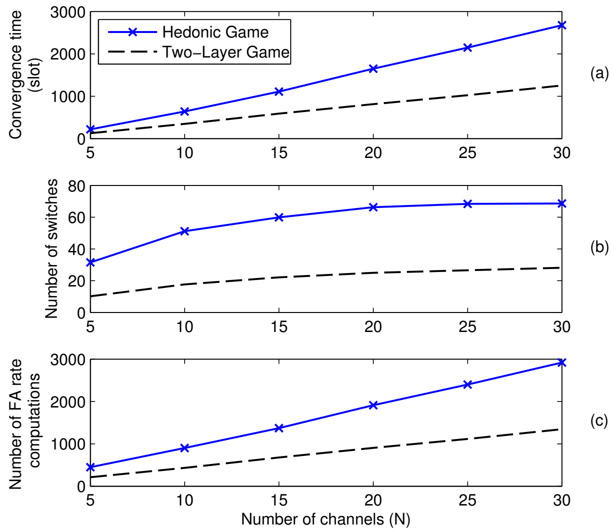

Fig. 6. Computational complexity vs. N; $M = 10$ SUs; 0/X-model: (a) convergence time T_{Converge}; (b) number of switches $|\mathcal{T}_{\text{Switch}}|$; (c) number of FA rate computations (23).

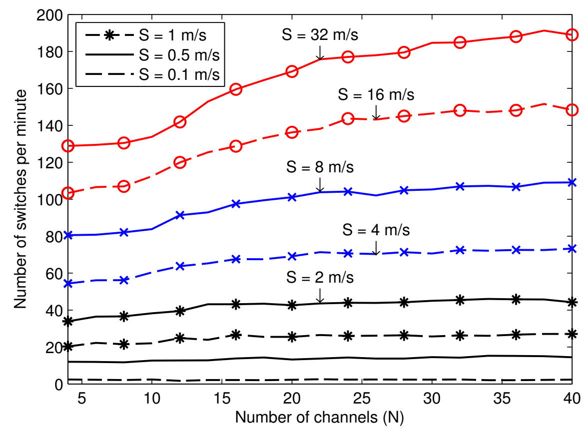

Fig. 7. Switch frequency vs. number of channels (N) for various traveling speeds (S); 0/X-model; $M = 10$ SUs.

stable SU/PU population and the 0/X MAC model. Note that when an SU determines whether it should switch to another channel, it needs the knowledge of the updated individual FA rates on the current and new channel for all involved SUs in both the proposed game and the game in [11]. Thus, the complexity analysis in Section III-B applies to both games. We notice a significant reduction in convergence time and computational complexity when the proposed game is played, due primarily to provision for the social utility improvement in the preference relation (3).

Finally, we study the effect of mobility on the performance of the proposed game. We assume that the nodes initially achieve a Nash-stable SU network partition and then move in randomly chosen independent directions at a constant speed S following the random direction model [31]. When a node reaches the simulation region boundary, it moves in a new randomly chosen direction in the next time slot. Fig. 7 illustrates the *switch frequency* given by the number of channel switches per minute in the network [5]. We observe that doubling of the mobile speed S results in 10 to 40 additional channel switches per minute. However, for moderate speeds, the switch frequency is insensitive to the numbers of channels in the network. Frequent channel switching increases $|\mathcal{T}_{\text{Switch}}|$ in (23) and, thus, results in much larger computational complexity. Moreover, the overhead grows with the mobile speed since the need for message exchange increases. While the proposed game adjusts well to sudden changes in the network settings (see Fig. 4) and node positions [21], continuous mobility, as in Fig. 7, prevents convergence and increases complexity and overhead. To resolve SU competition and provide high throughput in congested mobile CR networks, in [32], [33] we have investigated opportunistic sensing strategies that rely on multichannel diversity by adapting the reward of each SU pair to local channel state information prior to sensing.

V. Conclusion

The proposed two-layer game provides a comprehensive coalitional game-theoretical framework for cooperative sensing and access in multichannel multi-SU CR networks. Each SU is provided with the transmission opportunities it deserves and, thus, with sufficient incentives to participate in the proposed game. We also present a new cooperative sensing approach under MD constraints for guaranteed PU protection in the heterogeneous environment.

Appendix A
Proof of Proposition 4

Without loss of generality, we assume $\xi = \xi_1$ and $\zeta = \xi_2$ are the bottom-layer coalitions to be merged and denote the resulted bottom-layer partition as $\tilde{\rho} = \{\xi\cup\zeta\}\cup(\rho\backslash\{\xi\}\backslash\{\zeta\})$ with $|\tilde{\rho}| = |\rho|-1$. Expanding the bottom-layer coalition value $U^n_{1/\mathrm{X}}(\eta;\rho)$ in (8) over all possible values of (X_{ξ_1}, X_{ξ_2}) in $\{0,1\}^2$ yields (24) at the top of page 8 (cf. (5)). Similarly, the bottom-layer coalition value $U^n_{1/\mathrm{X}}(\eta;\tilde{\rho})$ after the merger is derived in (25) by expanding over all possible $X_{\xi_1\cup\xi_2}$ values. Noting that $\rho\backslash\{\eta\}\backslash\{\xi_1\}\backslash\{\xi_2\} = \tilde{\rho}\backslash\{\eta\}\backslash\{\xi_1\cup\xi_2\}$ and $P^n_{\mathrm{FA}}(\xi_1\cup\xi_1) = P^n_{\mathrm{FA}}(\xi_1)P^n_{\mathrm{FA}}(\xi_2)$ (cf. (12)), we subtract (25) from (24) and simplify to obtain (26) where (cf. (4)-(9))

$$\begin{aligned} A(\tilde{\mathbf{x}}) = \Big(\frac{1}{b_1(\tilde{\mathbf{x}})} - \frac{1}{b_{12}(\tilde{\mathbf{x}})}\Big)|\eta| P^n_{\mathrm{FA}}(\xi_2)\Big(1 - P^n_{\mathrm{FA}}(\xi_1)\Big) \quad (27)\\ + \Big(\frac{1}{b_2(\tilde{\mathbf{x}})} - \frac{1}{b_{12}(\tilde{\mathbf{x}})}\Big)|\eta| P^n_{\mathrm{FA}}(\xi_1)\Big(1 - P^n_{\mathrm{FA}}(\xi_2)\Big) \end{aligned}$$

with

$$\begin{aligned} b_1(\tilde{\mathbf{x}}) &= |\eta| + |\xi_1| + \sum\nolimits_{i=1}^{|\rho|-3} (1-\tilde{x}_i)|\xi_{i+2}|, \\ b_2(\tilde{\mathbf{x}}) &= |\eta| + |\xi_2| + \sum\nolimits_{i=1}^{|\rho|-3} (1-\tilde{x}_i)|\xi_{i+2}|, \qquad (28) \\ b_{12}(\tilde{\mathbf{x}}) &= |\eta| + |\xi_1| + |\xi_2| + \sum\nolimits_{i=1}^{|\rho|-3} (1-\tilde{x}_i)|\xi_{i+2}|. \end{aligned}$$

It is easy to show that $A(\tilde{\mathbf{x}}) > 0$ in (27), and thus the expression in (26) must be positive.

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