# A Two-Layer Coalitional Game among Rational Cognitive Radio Users

Yuan Lu, Alexandra Duel-Hallen
Department of ECE, North Carolina State University, Raleigh, NC, 27606
{ylu8, sasha}@ncsu.edu

***Abstract*—In cognitive radio (CR) networks, secondary users (SUs) sense the spectrum to identify and possibly transmit over temporarily unoccupied channels that are licensed to primary users (PUs). However, when the received PU signal is weak, spectrum sensing by individual SUs becomes unreliable. To improve sensing accuracy, SUs can form disjoint coalitions and cooperate to discover idle time slots. These spectrum opportunities are then shared among the coalition members in a coordinated manner. It is proposed to decouple the coalition formation and the access (payoff) allocation problems by modeling these processes as a two-layer coalitional game. This game fosters cooperation by providing each SU with the access opportunities it deserves. Numerical results demonstrate that the proposed game outperforms previously investigated collaborative sensing and access approaches in terms of energy efficiency, throughput, and fairness.**

***Index Terms*—Coalitional game, cooperative sensing, cognitive radio (CR).**

## I. Introduction

Cooperative sensing exploits spatial diversity to improve sensing accuracy in cognitive radio (CR) systems. While most investigations assume a fixed number of fully cooperative secondary users (SUs) sensing a single channel, in practice, there are many possible channels for sensing and transmission. Thus, SUs need to choose both the portion of the spectrum to sense under the hardware constraints and their collaborators for sensing these bands.

Game theory has been utilized recently [1]–[5] to model and analyze the behavior, interactions, and decision-making of cooperative SUs. In [1], the best mixed strategy on the sensing probability of each SU is obtained under an evolutionary game framework. In [2], [3], SU coalition formation that balances the tradeoff between miss-detection and false-alarm probabilities is investigated. An information-theoretic metric that quantifies the contribution of each SU/coalition and allocates the available resources in a fair and stable manner is proposed in [4]. To the best of our knowledge, only [5] considers combined spectrum sensing and access where every coalition member obtains equal probability to access the sensed idle channel. However, the game in [5] ignores the fact that contributions of different SUs to their coalition might not be the same, and thus they should not be awarded equally. For example, an SU located closer to the primary user (PU) tends to obtain more accurate sensing results, and thus its contribution is greater than that of a more distant SU.

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TABLE I
Notation

| Notation | Explanation |
|---|---|
| $\mathcal{M}$ | Set of all SUs $\{1,\dots,M\}$. |
| $\mathcal{N}$ | Set of all channels $\{1,\dots,N\}$. |
| $(\mathcal{M},\succ_{\mathcal{M}})$ | Top-layer hedonic game with preference relation $\succ_m$ for each SU $m\in\mathcal{M}$. |
| $C=(S,n)$ | A top-layer coalition $C$ is a two-tuple, where $C(1)$ is a set of SUs $S\subseteq\mathcal{M}$ and $C(2)$ indicates the operating channel $n$. |
| $\mathcal{P}$ | A top-layer partition of $\mathcal{M}$ defines a set of disjoint top-layer coalitions $\mathcal{P}=\{C^1,C^2,\dots,C^N\}$ with cardinality $N$, where $\forall n\neq\tilde{n}$, $C^n(1)\cap C^{\tilde{n}}(1)=\phi$, $C^n(2)\neq C^{\tilde{n}}(2)$, $\cup_{n=1}^{N}C^n(1)=\mathcal{M}$ and $\cup_{n=1}^{N}\{C^n(2)\}=\mathcal{N}$. |
| $n_*^m$ | Sensing decision of SU $m$. |
| $\Pi(\mathcal{M},\mathcal{N})$ | Set of all $N^M$ possible top-layer partitions. |
| $x^{mC}$ | Top-layer utility for SU $m\in C(1)$ on channel $C(2)$. |
| $a^{mC}$ | SU $m$'s allocated payoff probability on channel $C(2)$ where $m\in C(1)$. |
| $R^{mn}$ | Transmission rate of SU pair $m$ on channel $n$. |
| $\gamma^{mn}$ | SU-to-SU signal-to-noise ratio (SNR) of SU pair $m$ on channel $n$. |
| $B^n$ | Bandwidth of channel $n$. |
| $\beta^n$ | Availability probability of channel $n$. |
| $(S,U^n)$ | Bottom-layer game among a set of SUs $S$ on channel $n$ with value function $U^n$. |
| $\eta$ | A bottom-layer coalition $\eta$ is a subset of SUs sensing the same channel, i.e., $\eta\subseteq C(1)$ for some $C\in\mathcal{P}$. |
| $\rho$ | A bottom-layer partition is a set of disjoint subsets of $C(1)$ with their union equal to $C(1)$ for some $C\in\mathcal{P}$. |
| $\rho\backslash\{\eta\}$ | Relative complement of $\{\eta\}$ in $\rho$, i.e., the set of all bottom-layer coalitions in $\rho$ except $\eta$. |
| $U^n(\eta;\rho)$ | Value for bottom-layer coalition $\eta$ on channel $n$ under bottom-layer partition $\rho$ where $\eta\in\rho$. |
| $U(\mathcal{P})$ | The total partition value $U(\mathcal{P})\triangleq\sum_{C\in\mathcal{P}}U^{C(2)}(C(1))$. |
| $P_{\text{MD}}^n(\eta)$, $P_{\text{FA}}^n(\eta)$ | Cooperative miss-detection and false-alarm probabilities of a bottom-layer coalition $\eta$ on channel $n$ regarding the PU presence. |
| $P_{\text{MD}}^n(\rho)$ | Integrated miss-detection probability on channel $n$ given a bottom-layer partition $\rho$. |
| $P_{\text{MD}}^{\text{Ch}}$ | Integrated miss-detection (or PU collision) probability constraint on each channel. |
| $P_{\text{MD}}^n(m)$ | Individual miss-detection constraint of SU $m$ on channel $n$. |
| $\tau^{mn}$ | Adaptive detection threshold of SU $m$ on channel $n$. |
| $\lambda^{mn}$ | PU-to-SU SNR per sample at SU $m$ on channel $n$. |
| $\nu$ | Number of collected samples for spectrum sensing. |

In this paper, we propose a two-layer coalitional game to study the cooperative sensing and coordinated access problems under limited individual sensing capabilities. The contributions of this paper are:

- An efficient, stable, and distributed coalition formation algorithm that improves the SU throughput.
- A fair payoff (access probability) allocation scheme to promote individual incentives for cooperation.
- A novel distributed threshold adaptation approach for cooperative sensing with guaranteed PU protection.

The rest of this paper is organized as follows. Table I summarizes significant notation. In section II, we introduce the system model and formulate the proposed two-layer game, which is then analyzed in Section III using coalitional game theory. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

## II. System Model and Game Formulation

We consider an overlay slotted CR ad hoc network with multiple SU pairs and multiple channels under i.i.d. Bernoulli PU traffic. An SU can sense and access only one channel at each time slot due to the hardware constraints. We formulate a two-layer game as illustrated in Fig. 1 and Fig. 2, where SUs make rational sensing decisions and can form coalitions for cooperative sensing and coordinated access.

### A. Top-Layer Game

The top-layer game is played by all SUs in $\mathcal{M}=\{1,\ldots,M\}$ across all channels in $\mathcal{N}=\{1,\ldots,N\}$. The set of SUs $S$ sensing and transmitting over the same channel $n$ forms a top-layer coalition $C=(S,n)$[1]. A top-layer partition of $\mathcal{M}$ is a set of $N$ two-tuples $\mathcal{P}=\{C^1,C^2,\ldots,C^N\}$, which determines SUs' sensing decisions:

$$n_*^m = n \Leftrightarrow \{m \in C(1) \;\&\; C(2)=n\} \tag{1}$$

for some $C\in\mathcal{P}$. Every possible top-layer coalition $C$ generates a partially transferable utility (PTU)

$$x^{mC} = a^{mC} R^{mC(2)} \tag{2}$$

for each coalition member $m\in C(1)$ where $a^{mC}$ is SU $m$'s allocated payoff probability on channel $C(2)$. The data rate $R^{mC(2)}=B^n \log_2(1+\gamma^{mC(2)})$ can vary greatly across different SU pairs due to their spatial separation, and thus is a non-transferable utility (NTU). The allocated payoff probability $a^{mC}$ is a transferable utility (TU) computed using the bottom-layer game in Section III-B. An SU $m$ prefers to move from channel $n$ to $\tilde{n}$ if the following preference relation is satisfied:

$$C_r^{\tilde{n}} \succ_m C^n \Leftrightarrow \begin{cases} x^{mC_r^{\tilde{n}}} > x^{mC^n} \\ \sum\limits_{i\in C_r^n(1)} a^{iC_r^n} + \sum\limits_{i\in C_r^{\tilde{n}}(1)} a^{iC_r^{\tilde{n}}} > \sum\limits_{i\in C^n(1)} a^{iC^n} + \sum\limits_{i\in C^{\tilde{n}}(1)} a^{iC^{\tilde{n}}} \end{cases} \tag{3}$$

where $C_r^n=(C^n(1)\backslash\{m\},n)$ and $C_r^{\tilde{n}}$ are the coalitions that form on channels $n$ and $\tilde{n}$, respectively, if the SU $m$ makes the move, and $C^n$ and $C^{\tilde{n}}=(C_r^{\tilde{n}}(1)\backslash\{m\},\tilde{n})$ are the existing coalitions on these channels. Thus, the top-layer coalition $C_r^{\tilde{n}}$ is preferable to $C^n$ for SU $m$ if (i) its individual utility (2) improves and (ii) the combined access probabilities of all SUs on channels $n$ and $\tilde{n}$ improve. Using the preference relation (3) which combines the social and individual objectives, the top-layer game can be modeled as a hedonic game $(\mathcal{M},\succ_{\mathcal{M}})$ [6]. Given $(\mathcal{M},\mathcal{N},a^{\mathcal{M}},\gamma^{\mathcal{M}\times\mathcal{N}})$, the top-layer game generates a partition $\mathcal{P}$ as shown in Fig. 1. We will describe the rules for switching coalitions in the top-layer game and will show that it converges to a Nash-stable [6] top-layer partition in Section III-C.

[1]Note that a top-layer coalition is a two-tuple specifying both the set of SUs $S$ and the channel index $n$ because the same set of SUs $S$ can achieve different throughputs on different channels.

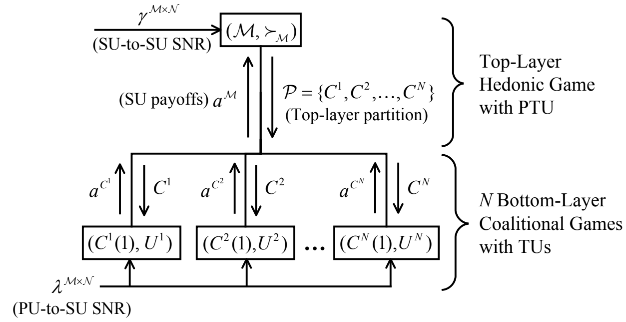


Fig. 1. Two-layer coalitional game.

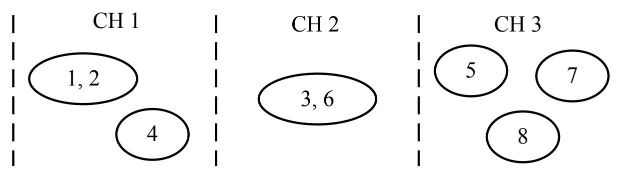


Fig. 2. An illustrative example of the two-layer coalition structure with the set of SUs $\mathcal{M}=\{1,\ldots,8\}$, the set of channels $\mathcal{N}=\{1,2,3\}$, the top-layer partition $\mathcal{P}=\{(\{1,2,4\},1),(\{3,6\},2),(\{5,7,8\},3)\}$, and the bottom-layer partitions $\rho^1=\{\{1,2\},\{4\}\}$, $\rho^2=\{\{3,6\}\}$ and $\rho^3=\{\{5\},\{7\},\{8\}\}$.

### B. Bottom-Layer Game

An output partition $\mathcal{P}$ from the top-layer game determines the set of SUs that can sense and transmit over each channel. The SUs sensing the same channel can further form disjoint bottom-layer coalitions for cooperative sensing, resulting in a bottom-layer partition $\rho$, as shown in Fig. 2. The detected spectrum opportunities are then shared among the bottom-layer coalition members. We formulate $N$ coalitional games $(S,U^n)$ that are played on different channels where $S=C(1)$ and $n=C(2)$ for some $C\in\mathcal{P}$, and $U^n$ is a value function for all subsets of $S$. Coalition formation and payoff allocation for these bottom-layer games are investigated in Sections III-A,B.

When multiple bottom-layer coalitions correctly detect a spectrum opportunity and compete for access, the successful access rate of each bottom-layer coalition depends on the medium access control (MAC) option at the physical layer. As in [7], we consider the following two collision models:

1) 0/X-model: All competing SUs fail to transmit successfully.
2) 1/X-model: All competing SUs gain equal probability for access. After gaining the right to access, the winning SU

may choose to transfer its access opportunity to other coalition members according to their binding agreement on payoff allocation (cf. Section III-B).

The value $U^n$ of each bottom-layer coalition $\eta \subseteq S$ is defined as the overall achievable access probability of $\eta$ on channel $n$. Label all coalitions in $\rho \backslash \{\eta\}$ as

$$\rho \backslash \{\eta\} = \{\xi_1, \xi_2, \ldots, \xi_{|\rho|-1}\} \tag{4}$$

and let us define a binary random vector of length $|\rho|-1$

$$\mathbf{X}_{\rho\backslash\{\eta\}} = (X_{\xi_1}, X_{\xi_2}, \ldots, X_{\xi_{|\rho|-1}}) \in \{0,1\}^{|\rho|-1} \tag{5}$$

where $X_{\xi_i} \in \{0,1\}$ is an indicator variable for the event that the coalition $\xi_i$ experiences a false alarm, i.e.,

$$\Pr(X_{\xi_i} = x_i) = \begin{cases} P^n_{\text{FA}}(\xi_i), & \text{if } x_i = 1 \\ 1 - P^n_{\text{FA}}(\xi_i), & \text{if } x_i = 0. \end{cases} \tag{6}$$

The coalition values for the two MAC models are given by

$$U^n_{\text{0/X}}(\eta; \rho) = \beta^n (1 - P^n_{\text{FA}}(\eta)) \cdot \prod_{i=1}^{|\rho|-1} P^n_{\text{FA}}(\xi_i) \tag{7}$$

and

$$U^n_{\text{1/X}}(\eta; \rho) = \beta^n (1 - P^n_{\text{FA}}(\eta)) \cdot \mathbb{E}\left[\frac{|\eta|}{|\eta| + J_{\rho\backslash\{\eta\}}\left(\mathbf{X}_{\rho\backslash\{\eta\}}\right)}\right] \tag{8}$$

$$= \beta^n (1 - P^n_{\text{FA}}(\eta)) \cdot \sum_{\mathbf{x} \in \{0,1\}^{|\rho|-1}} \left\{ \frac{|\eta| \prod_{i=1}^{|\rho|-1} \Pr(X_{\xi_i} = x_i)}{|\eta| + J_{\rho\backslash\{\eta\}}(\mathbf{x})} \right\}$$

where $\beta^n$ and $P^n_{\text{FA}}(\eta)$ are defined in Table I, and the number of competing SUs for cognitive access is given by

$$J_{\rho\backslash\{\eta\}}(\mathbf{x}) = \sum_{i=1}^{|\rho|-1} (1 - x_i)|\xi_i|. \tag{9}$$

Since the bottom-layer coalition value $U^n(\eta; \rho)$ depends on actions of its coalition members as well as on actions of SUs outside $\eta$, the bottom-layer coalitional game is in partition form [8]. We assume the overall access probability of a bottom-layer coalition is dividable and can be transferred among coalition members according to the allocated payoff probability $a^{mC}$ (cf. (2)), so that the bottom-layer game has transferable utility (TU)[2].

### C. Cooperative Sensing

Next, we present the cooperative sensing scheme at the physical layer. Consider a certain top-layer coalition $C = (S, n)$ and a bottom-layer partition $\rho$ of $S$. The individual miss-detection and false-alarm probabilities for each SU $m \in S$ can be approximated as [9]

$$P^n_{\text{MD}}(m) = 1 - Q\Big((\tau^{mn}/2\nu - \lambda^{mn} - 1)\sqrt{\nu/(2\lambda^{mn} + 1)}\Big) \tag{10}$$

and

$$P^n_{\text{FA}}(m) = Q\left((\tau^{mn}/2\nu - 1)\sqrt{\nu}\right) \tag{11}$$

respectively, where $\lambda^{mn}$, $\tau^{mn}$ and $\nu$ are defined in Table I, and $Q(\cdot)$ is the Q-function [10, eq. (B.20)]. Since miss detection of the PU presence leads to undesired primary collisions, we impose constraints on the $P^n_{\text{MD}}(m)$ values. In the proposed threshold adaptation method, the individual detection threshold $\tau^{mn}$ is obtained by inverting (10), resulting in the individual false-alarm probability $P^n_{\text{FA}}(m)$ in (11). It is not hard to show that the resulting $P^n_{\text{FA}}(m)$ increases with the PU-to-SU SNR $\lambda^{mn}$, indicating stronger sensing capability of closer SUs to the PUs. When the SUs in the bottom-layer coalition $\eta \in \rho$ collaborate to sense channel $n$, the miss-detection and false-alarm probabilities of the final sensing result are [9]

$$P^n_{\text{MD}}(\eta) = 1 - \prod_{m \in \eta} (1 - P^n_{\text{MD}}(m)) \tag{12}$$

and

$$P^n_{\text{FA}}(\eta) = \prod_{m \in \eta} (P^n_{\text{FA}}(m)) \tag{13}$$

respectively, where we assume the AND-rule hard decision combining [9] with the proposed adaptive threshold control scheme due to its simplicity and good performance in heterogeneous environment [11] while the results can be extended to more general models. Finally, the PU transmission is interrupted if one or more bottom-layer coalitions in $\rho$ fail to detect its presence, so the integrated miss-detection probability on channel $n$ is

$$P^n_{\text{MD}}(\rho) = 1 - \prod_{\eta \in \rho} (1 - P^n_{\text{MD}}(\eta)). \tag{14}$$

Next, we impose the following conditions:

**Condition (C.1)**: $P^n_{\text{MD}}(\rho) = P^{\text{Ch}}_{\text{MD}}$, where $P^{\text{Ch}}_{\text{MD}}$ is the integrated miss-detection probability constraint on each channel.

**Condition (C.2)**: Any two equal-sized bottom-layer coalitions should maintain the same miss-detection probability.

Note that (C.1) ensures a certain level of protection to the PUs while (C.2) promotes fairness among the SUs. Conditions (C.1) and (C.2) are easily satisfied by setting (cf. (12) & (14))

$$P^n_{\text{MD}}(m) = 1 - (1 - P^n_{\text{MD}}(\eta))^{\frac{1}{|\eta|}} = 1 - (1 - P^{\text{Ch}}_{\text{MD}})^{\frac{1}{|S|}} \tag{15}$$

where $m \in \eta \in \rho$. As SU population $|S|$ on channel $n$ grows, each SU $m$ reduces its threshold $\tau^{mn}$ to satisfy (15), leading to increased false alarm probability $P^n_{\text{FA}}(m)$ (cf. (10)-(11)) and decreased coalition values in (7) or (8). Thus, large $|S|$ values are penalized, relieving SU competition on all channels.

## III. Two-Layer Coalitional Sensing and Access Game

### A. Formation of the Grand Coalition at the Bottom Layer

Let us focus on the bottom-layer game $(S, U^n)$ on channel $n$ where $S = C(1)$ and $n = C(2)$ for some $C \in \mathcal{P}$. To classify this game, we formulate the following propositions[3].

**Proposition 1.** *Under the 0/X-model, the bottom-layer game $(S, U^n_{0/X})$ reduces to a characteristic-form game, i.e., the value of a bottom-layer coalition $\eta$ depends solely on the composition of $\eta$ [8]: $U^n_{0/X}(\eta; \rho) = U^n_{0/X}(\eta; \tilde{\rho})$, $\forall \eta \subseteq S$ and any bottom-layer partitions $\rho$ and $\tilde{\rho}$ of $S$ such that $\eta \in \rho$ and $\eta \in \tilde{\rho}$.*

[2] Consider for example, the overall access probability of a two-SU bottom-layer coalition is $U(\{1,2\}) = 0.8$, and the allocated payoff probabilities are $a^1 = 0.2$ and $a^2 = 0.6$. Given that a time slot is sensed idle by the two SUs (cooperatively), SU 1 and SU 2 can access with probabilities $a^1/(a^1 + a^2) = 1/4$ and $a^2/(a^1 + a^2) = 3/4$, respectively.

[3] Some proofs in this section are omitted due to space limitation and can be found in [11].

*Proof.* For any $\eta \subseteq S$ and any bottom-layer partition $\rho$ of $S$ such that $\eta \in \rho$, the value of $\eta$ is given by (cf. (7) & (13))

$$U^n_{\text{0/X}}(\eta;\rho) = \beta^n(1-P^n_{\text{FA}}(\eta)) \cdot \prod_{m\in S\setminus\eta} P^n_{\text{FA}}(m) \triangleq U^n_{\text{0/X}}(\eta) \quad (16)$$

which does not depend on $\rho$. $\square$

**Proposition 2.** *Under the 0/X-model, the bottom-layer game $(S, U^n_{0/X})$ is superadditive* [8]*, i.e., the SUs benefit from forming larger coalitions. Thus, for any two disjoint bottom-layer coalitions $\eta,\xi \subset S$, $U^n_{0/X}(\eta\cup\xi) \geq U^n_{0/X}(\eta)+U^n_{0/X}(\xi)$.*

*Proof.* Substituting (13) into (7) and simplifying, we obtain

$$\begin{aligned} &U^n_{\text{0/X}}(\eta\cup\xi) - U^n_{\text{0/X}}(\eta) - U^n_{\text{0/X}}(\xi) \quad (17)\\ &\quad = \beta^n\Big(1-\prod_{m\in\eta}P^n_{\text{FA}}(m)\Big)\Big(1-\prod_{m\in\xi}P^n_{\text{FA}}(m)\Big)\prod_{m\in S\setminus\eta\setminus\xi}P^n_{\text{FA}}(m)\end{aligned}$$

which is nonnegative. $\square$

**Proposition 3.** *Under the 1/X-model, all bottom-layer partitions of $S$ are equally efficient, i.e., for any two partitions $\rho$ and $\tilde\rho$ of $S$, $\sum_{\eta\in\rho} U^n_{1/X}(\eta;\rho) = \sum_{\eta\in\tilde\rho} U^n_{1/X}(\eta;\tilde\rho)$.*

**Definition 1.** *A partition-form game $(S, U)$ exhibits nonpositive externalities if a merger between two coalitions cannot improve the values of other coalitions* [12]*. Formally, if for any disjoint coalitions $\eta,\xi,\zeta \subset S$ and any partition $\rho$ of $S$ such that $\eta,\xi,\zeta\in\rho$, the coalition value*

$$U(\eta;\rho) \geq U(\eta;\rho\setminus\{\xi\}\setminus\{\zeta\}\cup\{\xi\cup\zeta\}) \quad (18)$$

*we say the game $(S, U)$ has nonpositive externalities.*

**Proposition 4.** *Under the 1/X-model, the bottom-layer game $(S, U^n_{1/X})$ exhibits nonpositive externalities.*

**Definition 2.** *In a partition-form game $(S, U)$, the grand coalition $\rho=\{S\}$ is efficient* [13] *if $\forall\tilde\rho\neq\rho$, the coalition value*

$$U(S;\rho) \geq \sum_{\eta\in\tilde\rho} U(\eta;\tilde\rho). \quad (19)$$

**Proposition 5.** *The grand coalition always forms under the 1/X and 0/X MAC models.*

*Proof.* If $\rho=\{S\}$ is efficient, the grand coalition always forms [8] [13]. We proved in Propositions 1 and 2 that the bottom-layer game $(S, U^n_{\text{0/X}})$ is a superadditive characteristic-form game. Thus the grand coalition $\rho=\{S\}$ is efficient and should always form under the 0/X-model.

Moreover, Propositions 3 implies the (weak) efficiency of the grand coalition under the 1/X-model, i.e., $\forall\tilde\rho\neq\{S\}$, $U^n_{\text{1/X}}(S;\rho)=\sum_{\eta\in\tilde\rho} U^n_{\text{1/X}}(\eta;\tilde\rho)$. From [12], [13], and Proposition 4, the grand coalition also forms under the 1/X-model. $\square$

### *B. Payoff Allocation at the Bottom Layer*

Next, we consider the allocation rule for the bottom-layer value of the grand coalition $U^n(S;\{S\})$ (hereafter referred to as $U^n(S)$ for brevity) where $S=C(1)$ and $n=C(2)$ for some $C\in\mathcal{P}$. Generally, a good payoff allocation rule relates the actual individual payoff to the hypothetical payoff that an SU could have obtained by leaving the grand coalition and/or the marginal value it brings into the grand coalition. Therefore, despite the formation of the grand coalition, we need to compute the hypothetical values of smaller bottom-layer coalitions using (7) or (8) for the two collision models. Both the Shapley value [8] and the Nash bargaining solution (NBS) [14], [15] can be utilized for payoff allocation, but we consider only NBS due to its computational efficiency.

First, consider the characteristic-form (cf. Proposition 1) bottom-layer game $(S, U^n_{\text{0/X}})$ under the 0/X-model.

**Proposition 6.** *Under the 0/X-model, NBS assigns a payoff (access probability)* [14]

$$a^{mC}_{NBS,0/X} = \frac{1}{|S|}\Big[U^n_{0/X}(S) - \sum_{i\in S} U^n_{0/X}(\{i\})\Big] + U^n_{0/X}(\{m\}) \quad (20)$$

*$\forall m\in S$, where the disagreement point is $\{U^n_{0/X}(\{m\})_{m\in S}\}$.*

The disagreement point represents the set of hypothetical individual payoffs that SUs expect to receive if they do not engage in bargaining, i.e. do not cooperate. Note that the first term in (20) is the same for all SUs in $S$, and the $U^n_{\text{0/X}}(\{m\})$ term is the singleton value of each SU $m$, which is independent of how the grand coalition breaks down because the game $(S, U^n_{\text{0/X}})$ is in the characteristic form. Using the superadditivity property in Proposition 2, it is not hard to show that the allocated payoff probability $a^{mC}_{\text{NBS,0/X}}$ is always larger than the hypothetical payoff $U^n_{\text{0/X}}(\{m\})$ that SU $m$ could have obtained by forming the singleton bottom-layer coalition $\{m\}$. Since SUs with stronger sensing capability obtain higher singleton values $U^n_{\text{0/X}}(\{m\})$, the NBS (20) provides each SU with the allocated payoff probability that is commensurate to its sensing contribution. Thus, all SUs should agree unanimously on the NBS allocation $a^{mC}_{\text{NBS,0/X}}$.

We now shift our focus to the partition-form bottom-layer game under the 1/X-model where the hypothetical individual access probability does not only depend on the individual SU's sensing capability, but is also related to how other SUs react when this SU does not cooperate. In this paper, we only consider the *fine* NBS (*f*NBS) [15] assuming all SUs stand alone if bargaining is not successful and plan to investigate other possibilities in our future work.

**Proposition 7.** *When the disagreement point is $\{U^n_{1/X}(\{m\};\{\{j\}_{j\in S}\})\}_{m\in S}$ (fNBS), the payoff to each SU $m\in S$ is* [15]

$$\begin{aligned} a^{mC}_{fNBS,1/X} &= \frac{1}{|S|}\Big[U^n_{1/X}(\{S\}) - \sum_{i\in S} U^n_{1/X}(\{i\};\{\{j\}_{j\in S}\})\Big] \\ &\qquad + U^n_{1/X}(\{m\};\{\{j\}_{j\in S}\}) \\ &= U^n_{1/X}(\{m\};\{\{j\}_{j\in S}\}) \quad (21)\end{aligned}$$

*under the 1/X-model.*

*Proof.* The first equation is from [15], and the second equation follows from Proposition 3, i.e., $U^n_{\text{1/X}}(\{S\})=\sum_{i\in S} U^n_{\text{1/X}}(\{i\};\{\{j\}_{j\in S}\})$. $\square$

Thus, the allocated payoff probability is simply given by the access probability that each SU could have obtained individually assuming all other SUs also formed singletons.

**Algorithm 1** Bottom-layer payoff allocation algorithm

**Input:** $C=(S,n)$, $\{\lambda^{mn}:m\in S\}$, $P^{\text{Ch}}_{\text{MD}}$, $\nu$, $\beta^n$
**Output:** $\{a^{mC}:m\in S\}$
$P^n_{\text{FA}}(m)=Q\left(\sqrt{2\lambda^{mn}+1}Q^{-1}((1-P^{\text{Ch}}_{\text{MD}})^{\frac{1}{|S|}})+\lambda^{mn}\sqrt{\nu}\right)$
**if** 0/X-MAC **then** $\forall m\in S$
  $U^n_{\text{0/X}}(S)=\beta^n(1-\prod_{i\in S}P^n_{\text{FA}}(i))$
  $U^n_{\text{0/X}}(\{m\})=\beta^n(1-P^n_{\text{FA}}(m))\prod_{i\in S\setminus\{m\}}P^n_{\text{FA}}(i)$
  Compute $a^{mC}_{\text{NBS,0/X}}$ using (20)
**end if**
**if** 1/X-MAC **then** $\forall m\in S$
  Compute $a^{mC}_{f\text{NBS,1/X}}=U^n_{\text{1/X}}(\eta=\{m\};\rho=\{\{j\}_{j\in S}\})$ by (8)
**end if**
**if** a slot is sensed idle **then** $\forall m\in S$
  SU $m$ transmits with probability $a^{mC}/\sum_{i\in S}a^{iC}$
**end if**

Fig. 3. Payoff allocation for the bottom-layer game $(S,U^n)$.

However, this does not imply that SUs should deviate from the grand coalition, because each SU is not guaranteed a payoff of $a^{mC}_{f\text{NBS,1/X}}$ if it deviates. In fact, if an SU decides to remain isolated, its payoff is at risk due to the nonpositive externalities if other SUs collude. As a result, an SU $m$ might end up with a much worse payoff than its singleton value $U^n_{\text{1/X}}(\{m\};\{\{j\}_{j\in S}\})$. Therefore, all SUs have an incentive to join the grand coalition.

Note that the allocated payoff probabilities (20) and (21) are *group-rational* [8]:

$$U^n(S)=\sum_{m\in S}a^{mC}. \tag{22}$$

Finally, the conditional access probability of an SU $m\in S$ given that a spectrum opportunity is detected successfully is set to $a^{mC}/\sum_{i\in S}a^{iC}$ (see footnote 2 on page 3 for an example). Due to the absence of SU competition in the grand coalition, the probability that the set of SUs $S$ discovers an idle slot is equal to the coalition value $U^n(S)$. Thus, a marginal access probability of $a^{mC}$ results $\forall m\in S$. The bottom-layer payoff allocation algorithm is summarized in Fig. 3 where the $P^n_{\text{FA}}(m)$ value is obtained by substituting (15) into (10)-(11).

### C. Coalition Formation at the Top Layer

The proposed CR network lacks a central entity that collects global SU information, computes all possible top-layer coalition values, and provides an optimized top-layer partition. Thus, the SUs need to organize themselves into different top-layer coalitions in the top-layer game in a distributed manner. Suppose the current partition is $\mathcal{P}=\{C^1,C^2,\ldots,C^N\}$ with $C^n(2)=n$. An SU $m$ attempts to deviate from its current top-layer coalition $C^n$ on channel $n$ to join another coalition $C^{\tilde n}$ on channel $\tilde n$ if the newly formed coalition $C^{\tilde n}_r$ is strictly preferable (cf. (3)). The proposed coalition formation algorithm is summarized in Fig. 4 (cf. Table I for notation). The algorithm will result in a Nash-stable [6] final partition $\mathcal{P}^*$. We will investigate distributed decision rules for stopping the algorithm when it converges in our future work.

**Algorithm 2** Top-layer partition formation algorithm

**Input:** $\mathcal{M},\mathcal{N},\beta^{\mathcal{N}},\gamma^{\mathcal{M}\times\mathcal{N}},\lambda^{\mathcal{M}\times\mathcal{N}}$
**Output:** $\mathcal{P}$
**Initialization**: Each SU $m$ randomly senses a channel $n^m_*$
$\mathcal{P}:=\{C^1\cdots C^N\}$ with $C^n(1)=\{m:n^m_*=n\}$ & $C^n(2)=n$
$i=1$
**while** $i\leq$MAX **do**
  Randomly choose an SU $m\in C^n(1)$
  $m$ randomly chooses another coalition $C^{\tilde n}$
  $C^n_r:=(C^n(1)\setminus\{m\},n)$
  $C^{\tilde n}_r:=(C^{\tilde n}(1)\cup\{m\},\tilde n)$
  $\tilde{\mathcal{P}}:=\mathcal{P}\setminus\{C^n\}\setminus\{C^{\tilde n}\}\cup\{C^n_r\}\cup\{C^{\tilde n}_r\}$
  $m$ requests the $\lambda^{\tilde m\tilde n}$ information from all $\tilde m\in C^{\tilde n}(1)$
  $m$ plays the bottom-layer game $(C^{\tilde n}_r,U^{\tilde n})$ in Fig. 3
  $m$ updates its PTU $x^{mC^{\tilde n}_r}:=a^{mC^{\tilde n}_r}R^{m\tilde n}$
  **if** (3) holds **then** SU $m$ switches to channel $\tilde n$
    $\mathcal{P}:=\tilde{\mathcal{P}}$, $C^n:=C^n_r$, $C^{\tilde n}:=C^{\tilde n}_r$ & $i:=i+1$
    $a^{iC^n}:=a^{iC^n_r}$ & $a^{jC^{\tilde n}}:=a^{jC^{\tilde n}_r}$ $\forall i\in C^n(1),j\in C^{\tilde n}(1)$
  **else** $m$ stays on channel $n$
  **end if**
**end while**

Fig. 4. Distributed partition formation for the top-layer game $(\mathcal{M},\succ_{\mathcal{M}})$.

**Proposition 8.** *The proposed coalition formation algorithm in Fig. 4 converges for sufficiently large MAX$\leq N^M$.*

*Proof.* When SU $m$ switches from channel $n$ to $\tilde n$, only the payoffs of the SUs on these two channels are affected. Thus

$$\begin{aligned}U(\tilde{\mathcal{P}})-U(\mathcal{P})&=\sum_{C\in\tilde{\mathcal{P}}}U^{C(2)}(C(1))-\sum_{C\in\mathcal{P}}U^{C(2)}(C(1)) \\ &=U^n(C^n_r(1))+U^{\tilde n}(C^{\tilde n}_r(1))-U^n(C^n(1))-U^{\tilde n}(C^{\tilde n}(1)) \\ &=\sum_{i\in C^n_r(1)}a^{iC^n_r}+\sum_{i\in C^{\tilde n}_r}a^{iC^{\tilde n}_r}-\sum_{i\in C^n(1)}a^{iC^n}-\sum_{i\in C^{\tilde n}(1)}a^{iC^{\tilde n}}>0\end{aligned} \tag{23}$$

where $\mathcal{P}$ and $\tilde{\mathcal{P}}$ are the current and new top-layer partitions, respectively, and $C^n$, $C^{\tilde n}$, $C^n_r$ & $C^{\tilde n}_r$ are defined as in (3). The third equation of (23) follows from the *group rationality* property in (22), and the last inequality is from (3). Since the total partition value $U(\mathcal{P})$ strictly increases as the coalition formation process evolves, SUs cannot visit the same top-layer coalition in $\Pi(\mathcal{M},\mathcal{N})$ twice. Moreover, $U(\mathcal{P})$ can take on only a finite number $|\Pi(\mathcal{M},\mathcal{N})|=N^M$ of values. Thus, the SUs must converge to a top-layer partition $\mathcal{P}^*$ in MAX$\leq N^M$ transitions. Finally, note that the Nash-stable partition $\mathcal{P}^*$ is not unique due to the short-sightedness [16] of the SUs' actions. □

## IV. Simulation Results

In Fig. 5, we compare the proposed game (i,iii) with the game in [5] (ii,iv) for the two MAC models (cf. (7)-(8)). The sensing and slot durations are 5 ms and 100 ms, respectively. The SU sensing power, SU transmission power, PU transmission power, and noise power are 10 mW, 10 mW, 100 mW and 0.1 mW, respectively. All users are randomly placed in a square region of $100\,m\times100\,m$, and only path loss

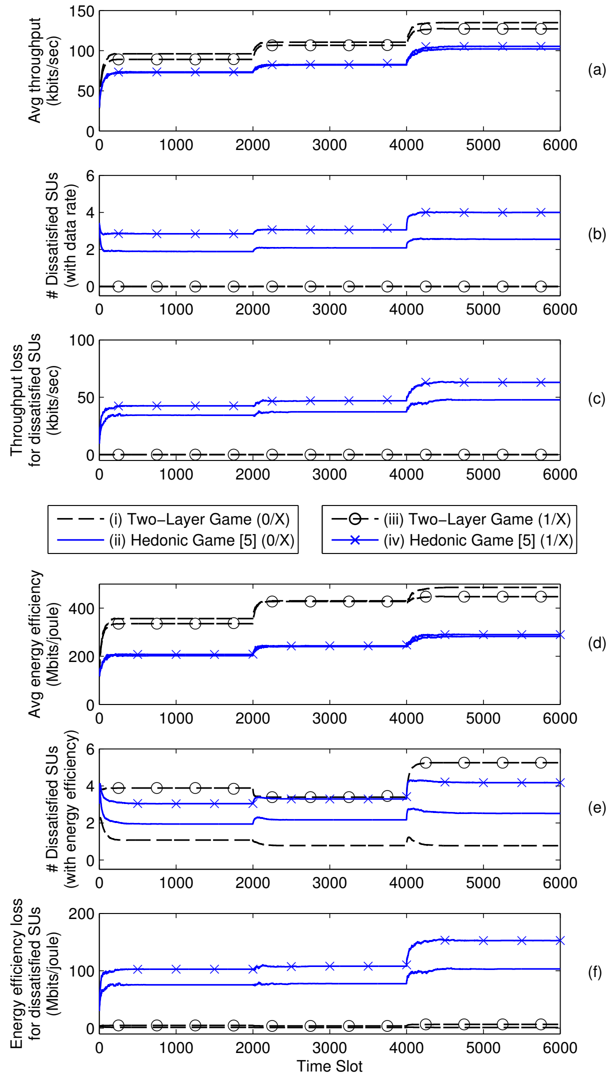


Fig. 5. Performance comparison of the proposed two-layer game and the hedonic game in [5].

effects are considered with the pass loss exponent equal to 2 [5]. Initially there are $M{=}10$ SUs and $N{=}5$ PUs. At time slots 2000 and 4000, these parameters change to $(M{=}10, N{=}6)$ and $(M{=}14, N{=}6)$, respectively. Each PU uses one channel with $B^n{=}10\,$MHz exclusively. For the sake of fair comparison, the AND-rule combining scheme with adaptive threshold described in Section II is used for both our proposed game and the game in [5] assuming $\nu{=}5$ and $P_{\mathrm{MD}}^{\mathrm{Ch}}{=}0.01$. Note that [5] aims to maximize the energy efficiency while the proposed two-layer game aims to maximize the expected data rate (2). Nevertheless, the proposed game achieves better average energy efficiency and throughput in Fig. 5(a,d), since in this game the SUs' payoffs increase as they sense closer PUs, resulting in improved sensing accuracy. Moreover, each SU is allocated the access opportunities it deserves and thus is provided with sufficient incentive to participate in the proposed game. In particular, as shown in Fig. 5(b,c), every SU has higher data rate when playing the proposed game than operating alone (i.e., all SUs are satisfied with their data rates). Finally, we observe from Fig. 5(e) that the proposed game can have a greater number of dissatisfied SUs in terms of energy efficiency (curves iii vs. iv) when the proposed game is played vs. the game in [5] under the 1/X-model. However, these SUs experience negligible energy efficiency loss as shown in Fig. 5(f) while the overall energy efficiency improves significantly when the proposed two-layer game is implemented as illustrated in Fig. 5(d).

## V. Conclusion

The proposed two-layer game provides a comprehensive coalitional game-theoretical framework for cooperative spectrum sensing and access in multichannel multi-SU CR networks. Each SU is provided with the spectrum access opportunities it deserves, and thus with sufficient incentives to participate in the proposed game. We also present a new cooperative sensing approach with adaptive threshold control for guaranteed PU protection.

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