



CISS 2015

# A Two-Layer Coalitional Game among Rational Cognitive Radio Users

This research was supported by the NSF grant CNS-1018447.

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# Introduction: Hardware Constrained CR

## Overlay Cognitive Radio (CR) Structure

- Secondary users (**SUs**) are required to **sense before accessing** the spectrum licensed to primary users (**PU**s).

## Hardware Constraints

- Each SU has **limited sensing capability**.
- Sensing outcomes are **prone to errors**:
  - Miss detection (**MD**) → Collision with PUs,
  - False alarm (**FA**) → Missed spectrum opportunities.
- No central control unit → **Distributed** CR access → **SU collision**.

## Approach: SU Cooperation

# Observations and Related Work

## Observations

- Traditionally, cooperative sensing is studied assuming
  - a **fixed** number of SUs & a **single channel**;
  - all participating SUs are **fully cooperative**.

} In practice, there are many possible channels!
- How to make **rational sensing & cooperation decisions**?
- How to **share** the detected spectrum resources **fairly**?

## Related Work:

- **Game theory** has been utilized recently for SU cooperation:
  - **cognitive access** is ignored in [1-3];
  - **sensing decision** is not studied in [4];
  - only [5] **jointly** considers **sensing & access** – but is **not fair**.

[1] B. Wang, et al., *IEEE Trans Commun.* 10

[2] W. Saad et al., *IEEE Trans. Veh. Technol.* 11

[3] W. Wang et al., *GLOBECOM* 10

[4] J. Rajasekharan et al., *Asilomar* 10

[5] X. Hao et al., *IEEE Trans. Wireless Commun.* 12

# Definition and Assumption

- **Set of all SUs**  $\mathcal{M} = \{1, \dots, M\}$ ; **Set of all channels**  $\mathcal{N} = \{1, \dots, N\}$ .
- **Top-layer coalition**  $\mathcal{C} = (S, n)$ : a set of SUs  $S$  sensing channel  $n$ ;  
 ➔ **Top-layer partition**  $\mathcal{P} = \{\mathcal{C}^1, \mathcal{C}^2, \dots, \mathcal{C}^N\}$ .
- **Bottom-layer coalition**  $\eta \subseteq S$ : a set of cooperating SUs;  
 ➔ **Bottom-layer partition**  $\rho$ .

$$\mathcal{M} = \{1, \dots, 8\}; \mathcal{N} = \{1, 2, 3\}$$

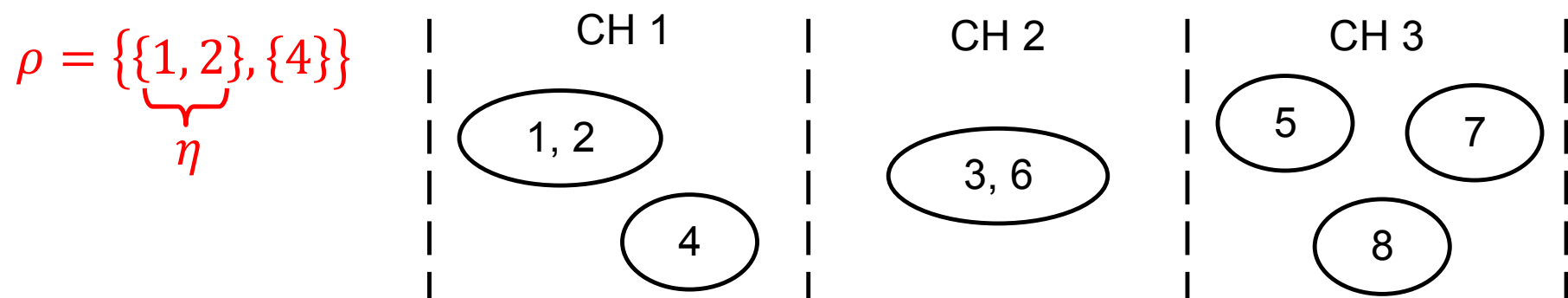
$$\mathcal{P} = \{(\underbrace{\{1, 2, 4\}}_{\mathcal{C}^1}, 1), (\underbrace{\{3, 6\}}_{\mathcal{C}^2}, 2), (\underbrace{\{5, 7, 8\}}_{\mathcal{C}^3}, 3)\}$$

(1) Cooperative sensing

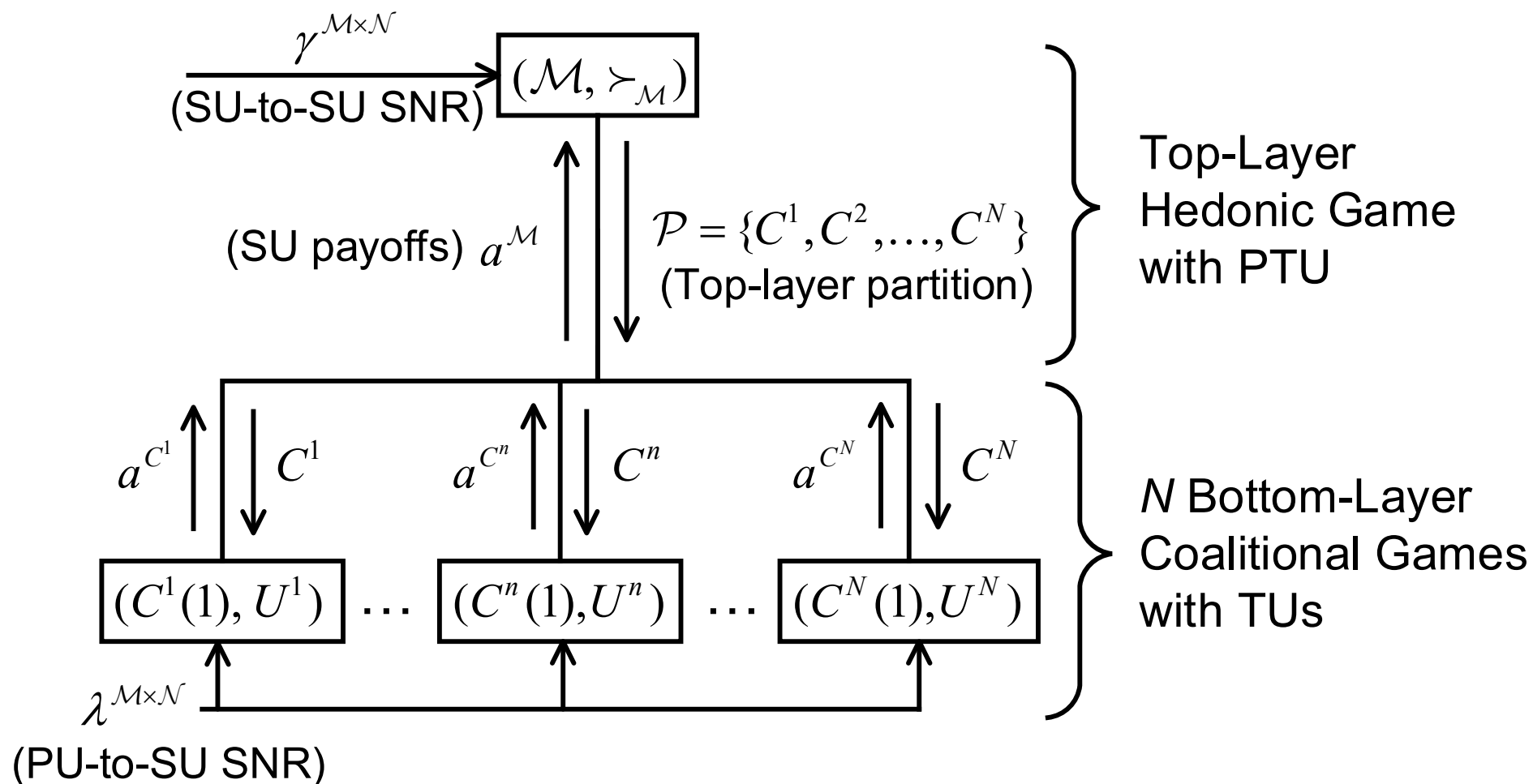
➔ Improved successful tx prob.

(2) Coordinated access

➔ Reduced SU collisions



# Two-Layer Game Formulation

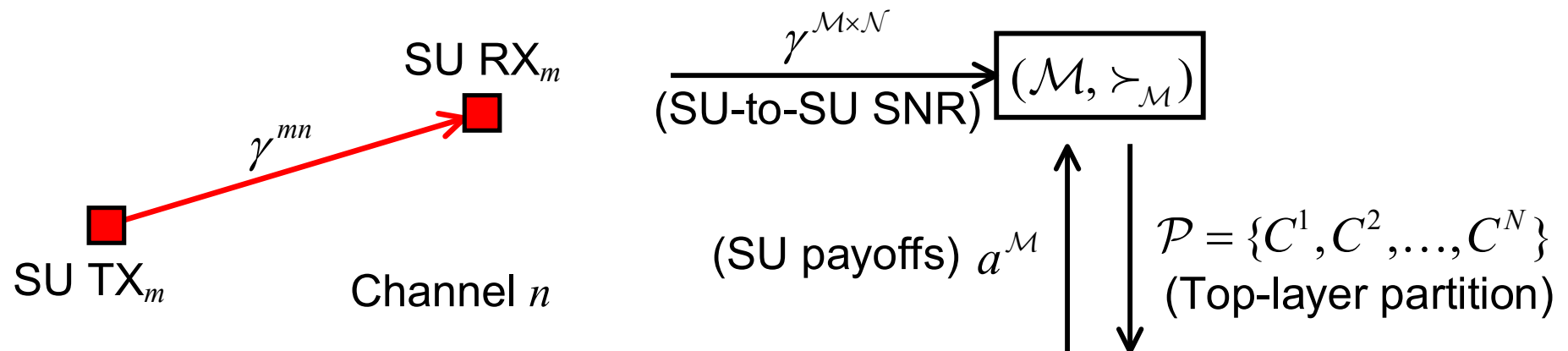


# Top-Layer Game Formulation

- Each SU  $m \in \mathcal{C}$  obtains a **partially transferable utility (PTU)** given by the **expected data rate**:

$$x^{m\mathcal{C}} = a^{m\mathcal{C}} R^{mn}$$

- measures the worth of top-layer coalition  $\mathcal{C}$  to SU  $m \in \mathcal{C}$  (1);
  - data rate  $R^{mn}$  is a non-transferable utility (**NTU**);
  - probability of successful transmission  $a^{m\mathcal{C}}$  is a transferable utility (**TU**) given by the payoff generated by the bottom-layer game.
- A **top-layer partition**  $\mathcal{P}$  determines SUs' **sensing decisions**.



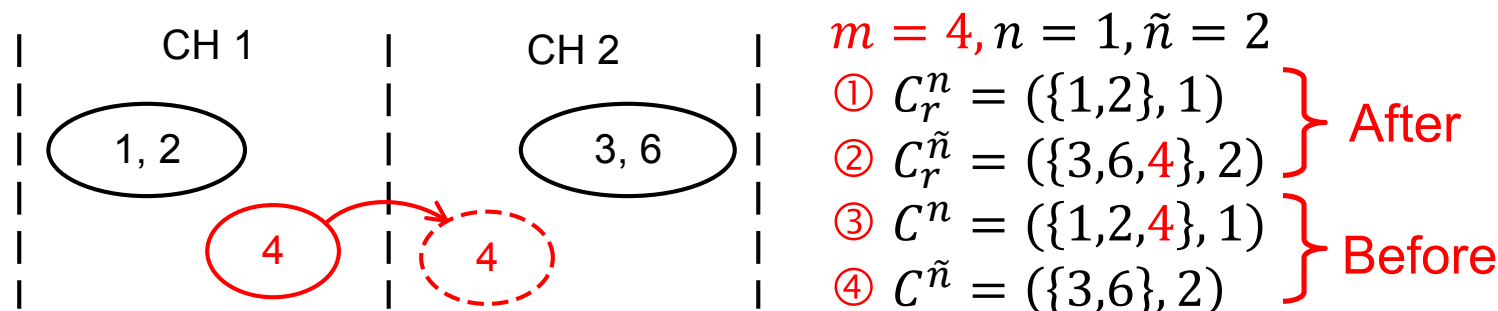
# Top-Layer Game Formulation

- An SU  $m$  **prefers** to move from channel  $n$  to  $\tilde{n}$  if

$$C_r^{\tilde{n}} \succ_m C^n \Leftrightarrow \begin{cases} x^{mC_r^{\tilde{n}}} > x^{mC^n} & \text{(i)} \\ \underbrace{\sum_{i \in C_r^n(1)} a^{iC_r^n} + \sum_{i \in C_r^{\tilde{n}}(1)} a^{iC_r^{\tilde{n}}}}_{\text{After the move}} > \underbrace{\sum_{i \in C^n(1)} a^{iC^n} + \sum_{i \in C^{\tilde{n}}(1)} a^{iC^{\tilde{n}}}}_{\text{Before the move}} & \text{(ii)} \end{cases}$$

②      ③
①      ②
③      ④

- **expected data rate of SU  $m$**  improves (i);
  - **sum of the successful tx probabilities** on both channels increases (ii).
  - **preference relation  $\succ_m$**  combines **individual & social** objectives.
- Hedonic game**  $(\mathcal{M}, \succ_{\mathcal{M}})$  (A. Bogomolnaia & M. O. Jackson, *Game Econ. Behav.* 02)



# Bottom-Layer Game Formulation

- **$N$  coalitional games  $(S, U^n)$**  are played on different channels
  - a set of SUs  $S = C(1)$  on channel  $n = C(2)$  for some  $C \in \mathcal{P}$ .
- Medium access control (**MAC**) is needed when multiple bottom-layer coalitions **compete for detected spectrum opportunity**:
  - **0/X-model**: All competing SUs fail to transmit successfully.
  - **1/X-model**: All competing SUs gain equal probability for access.
- Define the **value  $U^n$**  of any **bottom-layer coalition  $\eta \subseteq S$**  as the **overall successful transmission probability** of  $\eta$  on channel  $n$ :

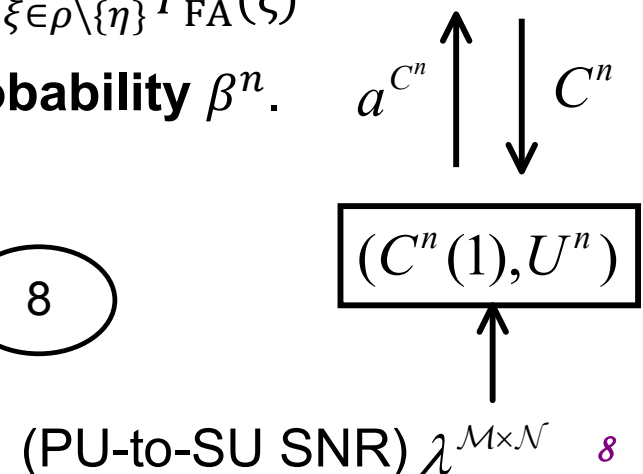
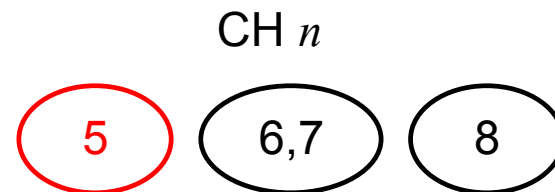
$$U_{0/X}^n(\eta; \rho) = \beta^n \cdot (1 - P_{\text{FA}}^n(\eta)) \cdot \prod_{\xi \in \rho \setminus \{\eta\}} P_{\text{FA}}^n(\xi)$$

- Bernoulli i.i.d. PU traffic with **availability probability  $\beta^n$** .

$$\eta = \{5\}$$

$$\rho = \{\{5\}, \{6,7\}, \{8\}\}$$

$$\rho \setminus \{\eta\} = \{\{6,7\}, \{8\}\}$$





# Bottom-Layer Game Formulation

- **Transmission opportunity** can be **transferred** within a bottom-layer coalition (if **all member SUs agree**):
  - **Coalition value**  $U^n(\eta) = \Pr[\text{some SU in } \eta \text{ transmits successfully}]$  is a transferable utility (**TU**);
  - **Allocated payoff probability**  $a^{mC} = \Pr[\text{SU } m \text{ transmits successfully}]$ ;
- How to **implement**?
  - If a slot is sensed idle  $\rightarrow$  SU  $m$  transmits with probability  $a^{mC} / U^n(\eta)$ ;
  - The resulting  $\Pr[\text{SU } m \text{ transmits successfully}] = a^{mC}$ .
- Example:  $U^n(\eta) = U^n(\{1,2\}) = 0.8$  for a 2-SU bottom-layer coalition.

	Allocated payoff prob.	Transmission prob. given a slot is sensed idle by $\eta$
<b>SU 1</b>	$a^1 = 0.2$	$a^1 / U(\eta) = 0.25$
<b>SU 2</b>	$a^2 = 0.6$	$a^2 / U(\eta) = 0.75$



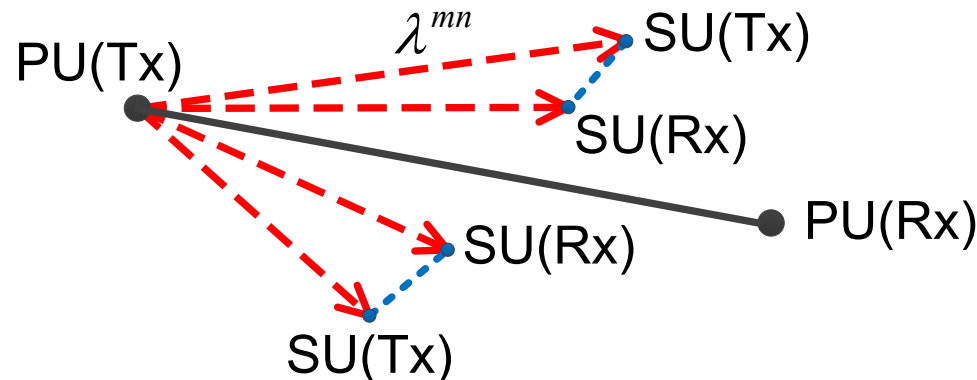
# Cooperative Sensing

- We regulate  $P_{\text{MD}}^n(m)$  and adjust  $\tau^{mn}$  to guarantee **PU protection**.
- **Individual** MD & FA probabilities for SU  $m$  on channel  $n$  are [1]:

$$P_{\text{MD}}^n(m) = 1 - Q\left((\tau^{mn}/2\nu - \lambda^{mn} - 1)\sqrt{\nu/(2\lambda^{mn} + 1)}\right)$$

$$P_{\text{FA}}^n(m) = Q((\tau^{mn}/2\nu - 1)\sqrt{\nu})$$

- $\tau^{mn}$  is the detection threshold and  $\nu$  is the number of samples;
- Adaptive threshold control:  $P_{\text{FA}}^n(m)$  decreases with PU-to-SU SNR  $\lambda^{mn}$ .
- Tight  $P_{\text{MD}}^n(m)$  constraint  $\rightarrow$  low  $\tau^{mn}$   $\rightarrow$  large  $P_{\text{FA}}^n(m)$ .

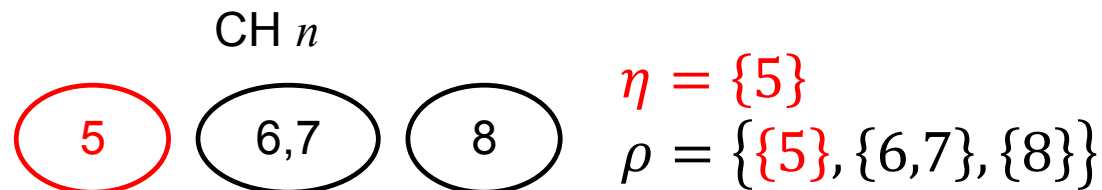


# Cooperative Sensing

- We assume the **AND-rule hard decision combining** [1].
- SUs in bottom-layer coalition  $\eta \in \rho$  **cooperate** to sense channel  $n$ :

$$P_{\text{MD}}^n(\eta) = 1 - \prod_{m \in \eta} (1 - P_{\text{MD}}^n(m)), \quad P_{\text{FA}}^n(\eta) = \prod_{m \in \eta} (P_{\text{FA}}^n(m)).$$

- **Integrated** MD probability is  $P_{\text{MD}}^n(\rho) = 1 - \prod_{\eta \in \rho} (1 - P_{\text{MD}}^n(\eta))$ .



- Require: (i)  $P_{\text{MD}}^n(\rho) = P_{\text{MD}}^{\text{Ch}}$  (MD constraint on each channel) (ii) Any 2 equal-sized bottom-layer coalitions maintain the same MD rate.

$$P_{\text{MD}}^n(m) = 1 - (1 - P_{\text{MD}}^n(\eta))^{\frac{1}{|\eta|}} = 1 - (1 - P_{\text{MD}}^{\text{Ch}})^{\frac{1}{|S|}}.$$

- Large SU population  $|S| \rightarrow$  tight  $P_{\text{MD}}^n(m)$  constraint  $\rightarrow$  increased  $P_{\text{FA}}^n(m)$   $\rightarrow$  reduced coalition value.

# Bottom-Layer: Coalition Formation

- **0/X-model:**  $(S, U_{0/X}^n)$  reduces to a **superadditive** coalitional game in **characteristic form** [1]:
  - The **value** of any bottom-layer coalition  $\eta$  is **independent of  $\rho \setminus \{\eta\}$** .
  - SUs obtain **larger coalition values** from **forming larger coalitions**.
- **1/X-model:**  $(S, U_{1/X}^n)$  exhibits **nonpositive externalities** and all bottom-layer partitions of  $S$  are **equally efficient**:
  - A **merger** between two coalitions **cannot benefit** the other coalitions.
  - For any partitions  $\rho$  and  $\tilde{\rho}$  of  $S$ ,  $\sum_{\eta \in \rho} U_{1/X}^n(\eta; \rho) = \sum_{\eta \in \tilde{\rho}} U_{1/X}^n(\eta; \tilde{\rho})$ .
- **Grand coalition  $\rho = \{S\}$  forms** for both 0/X & 1/X models [2,3].
  - All SUs in  $S$  are **willing to cooperate**.
  - **Successful transmission probability** for some SU on channel  $n$  = **grand coalition value**  $U^n(S)$ .

[1] W. Saad et al., *IEEE Signal Process. Mag.* 09

[2] E. Maskin, *Presidential Address to the Econometric Society* 03

[2] I. E. Hafilir, *Games Econ.Behav.* 07

# Bottom-Layer: Payoff Allocation

- How to **allocate** the **value**  $U^n(S)$  to every SU in  $S$ ?
  - **Individual payoff** that an SU **could have obtained** by leaving  $U^n(S)$  (**disagreement point**) → **Nash Bargaining Solution (NBS)** [1,2].
- 0/X-model:  $a_{\text{NBS},0/X}^{mC} = U_{0/X}^n(\{m\}) + \frac{1}{|S|} [U_{0/X}^n(S) - \sum_{i \in S} U_{0/X}^n(\{i\})]$ 
  - $\geq$  **hypothetical individual payoff** (guaranteed);
  - the 2nd term allocates the **surplus due to cooperation** equally to all SUs on channel  $n$ .
- 1/X-model:  $a_{f\text{NBS},1/X}^{mC} = U_{1/X}^n(\{m\}; \{\{j\}_{j \in S}\})$ 
  - = **hypothetical individual payoff** (assume other SUs are also **alone**);
  - $\nRightarrow$  SUs should **deviate from the grand coalition** → may end up with a **much worse payoff** if other SUs **collude**.

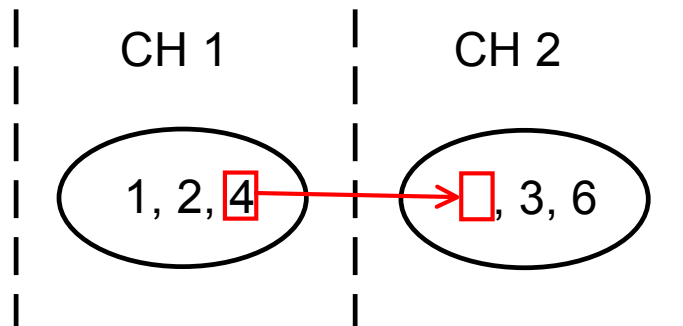
[1] K. Avrachenkov et al., *Networking* 11

[2] T. Kawamori & T. Miyakawa, *Osaka Univ. Econ. Work. Paper Series* 12

[3] W. Saad et al., *IEEE Signal Process. Mag.* 09

# Top-Layer: Coalition Formation

- SUs evolve to different top-layer coalitions in a **distributed** manner.
- An SU  $m$  **switches** to another channel if the newly formed coalition is **strictly preferable** ( $\succ_m$ ) to its current coalition.



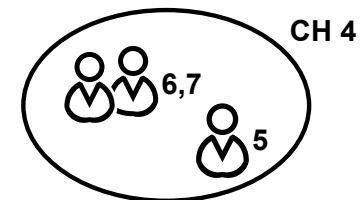
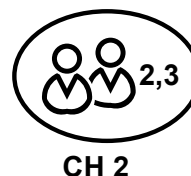
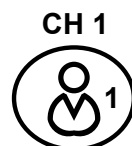
$$m = 4, n = 1, \tilde{n} = 2$$

Current coalition =  $(\{1, 2, 4\}, 1)$

New coalition =  $(\{4, 3, 6\}, 2)$

$(\{4, 3, 6\}, 2) \succ_4 (\{1, 2, 4\}, 1)$ ?

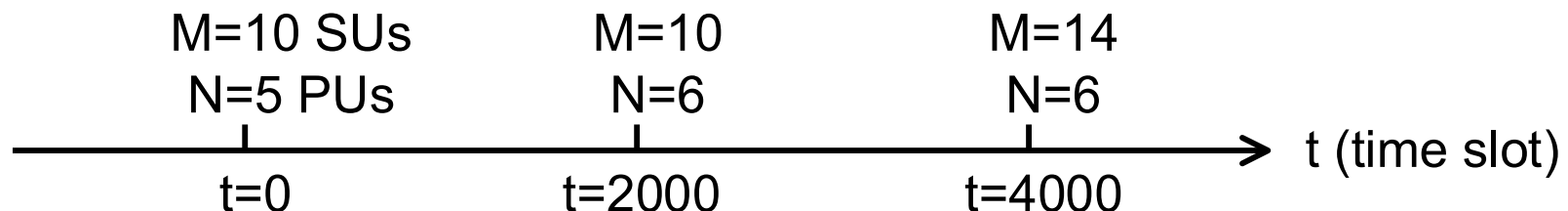
- We prove **convergence to a final top-layer partition**  $\mathcal{P}^*$ :
  - **Overall successful tx prob.** of the CR network **increases** in each transition;
  - SUs **cannot revisit** the same top-layer partition;
  - Only a **finite number** of  $N^M$  possible **top-layer partitions**.



# Simulation Setup

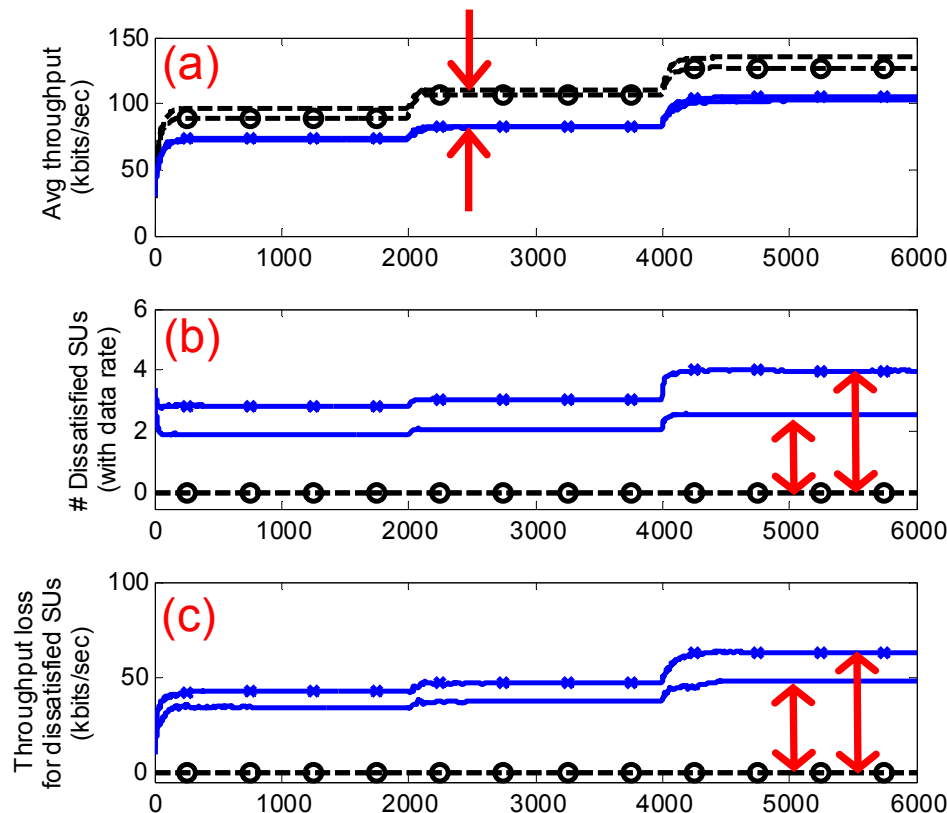
- Only consider the pass loss effects with the path loss exponent = 2.
- All users randomly placed in a square region of 100m × 100m.
- Each PU uses one channel with bandwidth = 10 MHz exclusively.

Parameter	Value
Sensing/Slot duration	5 ms/100ms
Sensing/Noise power	10mW/0.1mW
PU/SU transmission power	100mW/10mW
Number of samples $\nu$	5
Channel availability probability $\beta^n$	0.2



# Simulation Result: Throughput

- **Better network throughput** under both 0/X and 1/X models
- **All SUs are satisfied** with their individual throughputs.



(a) Average network throughput

(b) # Dissatisfied SUs (with throughput)

(c) Individual throughput loss (for dissatisfied SUs)

[5] X. Hao et al., *IEEE Trans. Wireless Commun.* 12

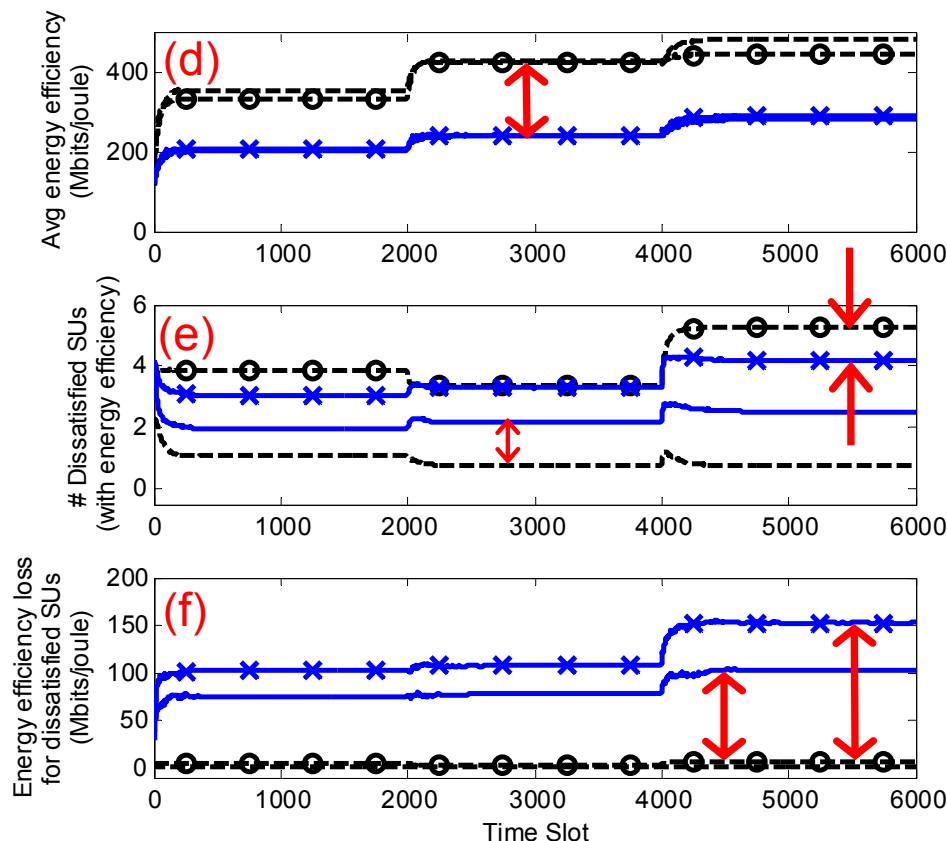
--- (i) Two-Layer Game (0/X)  
 — (ii) Hedonic Game [5] (0/X)

---○--- (iii) Two-Layer Game (1/X)  
 —×— (iv) Hedonic Game [5] (1/X)



# Simulation Result: Energy Efficiency

- **One exception:** more dissatisfied SUs under the 1/X-model (d).
  - **Negligible** energy efficiency **loss** for these individuals in (e).
  - **Significantly improved** overall energy efficiency (f).



(d) Average energy efficiency of the CR network

(e) # Dissatisfied SUs (with energy efficiency)

(f) Individual energy efficiency loss for dissatisfied SUs

[5] X. Hao et al., *IEEE Trans. Wireless Commun.* 12

--- (i) Two-Layer Game (0/X)  
 — (ii) Hedonic Game [5] (0/X)

---○--- (iii) Two-Layer Game (1/X)  
 —×— (iv) Hedonic Game [5] (1/X)

# Conclusion

- A **comprehensive** two-layer coalitional game framework for **SU cooperation** in **multichannel multi-SU** CR networks.
- An **efficient**, **stable**, and **distributed** coalition formation algorithm that improves the SU throughput.
- A **fair** payoff allocation scheme to promote **individual incentives** for cooperation.
- A novel **distributed threshold adaptation** approach for cooperative sensing with **guaranteed PU protection**.

**Thank you!**