V.3

Objective function: $\max_{\lambda,\mu\geq 0} \min_{x,z,Y} \Big[\sum_i f_m^i (1+\alpha_i) Y_i + \sum_j f_d^j (1+\beta_j) Y_j$

$+ \sum_{ij} (v_m^i + \frac{3}{2000} \cdot d_{ij}^{MD} (1+r_{ij})) x_{ij} + \sum_{jc} O_{jc} (p^j + 9.75 + 3.5 \cdot \frac{d_{jc}^{DC}}{500})(1+\delta_j$

$) + \sum_{ij} \lambda (x_{ij} - u_{ij} Y_i) + \sum_{jc} \mu_{jc} (z_{jc} - v_{jc} Y_j) \Big]$

or Duality and Dual Simplex Algorithm.

Primal: $\min_{x,Y,z} f(x,z,Y)$ s.t $g_{ij}(x,Y) = x_{ij} - u_{ij} Y_i \leq 0$

$h_{ij}(Y,z) = z_{jc} - v_{jc} Y_j \leq 0.$

Lagrange function: $L(x,z,Y;\lambda,\mu) = f(x,z,Y) + \sum_{ij} \lambda_{ij} (x_{ij} - u_{ij} Y_i)$

$+ \sum_{jc} \mu_{jc} (z_{jc} - v_{jc} Y_j) \quad \forall \lambda, \mu \geq$

Dual: $G(\lambda,\mu) = \min_{x,z,Y} L(x,z,Y;\lambda,\mu)$

Dual problem: $\max_{\lambda,\mu\geq 0} G(\lambda,\mu).$

$\forall i, j \in I$ for all 6 Candidates

$\forall c \in C$ for all 30 Customer regions

Variable: Y_i, Y_j Binary

$x_{ij} \geq 0$ inflow variable from MFG $\rightarrow$ DC，辆/年

$z_{jc} \geq 0$ Outflow Variable from DC $\rightarrow$ C，辆/年

transportation: f_m^i, f_d^j, v_m^i, d_{ij}^{MD}, d_{jc}^{DC}，P为处理费。

Sensitivity/Robustness parameters: α_i, β_j, γ_{ij}, δ_{jc} (inflation)

Tight Upper Bound: $U_{ij} > 0$ is upper bound for x_{ij} (ex. 30 Customers)

$V_{jc} = D_c$ is a upper bound of z_{jc}

Multiplier: $\lambda_{ij} \geq 0$, $\mu_{jc} \geq 0$ a Outer layer dual parameters (shadow).

Constraints:

1. Demand Satisfaction.

$$\sum_{j \in I} z_{jc} = v_c , \quad \forall c \in C$$

2. Flow Balance

$$\sum_{c \in C} z_{jc} = \sum_{c \in C} x_{ij} , \quad \forall j \in I$$

3. Variable domains and integrality

$$x_{ij} \geq 0 , \quad z_{jc} \geq 0 , \quad l_i , \; l_j \in [0, 1]$$

x_{ij} is the annual flows in bottles shipped from MFG to DC

z_{jc} is the annual flow from DC to c, $\forall c \in C$

Continuous variable must be non-negative, location binary var are binary.

4. Discrete shipment rule

$$O_{jc} \in \mathbb{Z}_+ , \quad O_{jc} \geq \frac{Z_{jc}}{6}.$$

O_{jc}. the numbers of orders shipped from Dc to $c \in C$.
Since given the information that 6 bottles per order. the total number of bottles delivered have to be $6 \cdot O_{jc}$

5. Notes regarding Big-M Constraints (linking Variable).

Big-M linking Constraints in V-1: $X_{ij} \leq U_{ij} Y_i$, $Z_{jc} \leq V_{jc} Y_j$

$\forall U_{ij}, Z_{jc} \in M$

In this Version, I introduced langrange relaxation, the Constraints are Softened. It's no longer being Considered as hard Constraints in inner problem. We multiple λ_{ij}, μ_{jc} as stated below

$$\max_{\lambda, \mu \geq 0} \sum_{ij} \lambda_{ij} (X_{ij} - U_{ij} Y_i) + \sum_{jc} \mu_{jc} (Z_{jc} - V_{jc} Y_j)$$

the $\max \lambda \mu \geq 0$ penalises any violation of these Constraints making inner problem (min) easier to solve. with fewer Constraints separable Structure

A rough draft of Objective function:

Variable: stated in the earlier pages

Constraints: take Constraints 1→3 as Hard Constraints denote H. take linking Constraints in Lagrange to be relaxed denote LK..

Pf: the following Consists Several points.

Firstly, define the Lagrange and dual function.

$$L(X,Z,Y;\lambda,\mu) =$$

$$\sum_{i\in I} f_m^i(1+\alpha_i)Y_i + \sum_{j\in I} f_d^j(1+\beta_j)Y_j + \sum_{i,j}\left(V_m^i + \frac{3}{2000} d_{ij}^{mD}(1+\gamma_{ij})\right)X_{ij} + \sum_{jc}\frac{Z_{jc}}{6}\left(p^j+9.75 + 3.5\frac{d_{jc}^{DC}}{500}\right)(1+\delta_{jc}) + \sum_{ij}\lambda_{ij}(X_{ij}-U_{ij}Y_i) + \sum_{jc}\mu_{jc}(Z_{jc}-V_{jc}Y_j)$$

denote L

Dual (Lagrange) function: $g(\lambda,\mu) = \inf_{(X,Z,Y)\ \text{s.t.}\ H} L \qquad \forall \lambda,\mu \geq 0$

denote g

Dual problem: $\max_{\lambda\geq 0,\mu\geq 0} g(\lambda,\mu)$ denote D.

Secondly, the weak duality (general) situation.

Assume $(\bar{x}, \bar{z}, \bar{Y})$ a feasible for both H and LK.

then. $L(\bar{x}, \bar{z}, \bar{Y}; \lambda, \mu) = Cost(\bar{x}, \bar{z}, \bar{Y}) + \sum_{ij} \lambda_{ij}(\bar{x}_{ij} - u_{ij}\bar{Y}_i)$

$$+ \sum_{j,c} \mu_{jc}(\bar{z}_{jc} - v_{jc}\bar{Y}_j)$$

$$\le Cost(\bar{x}, \bar{z}, \bar{Y})$$

Since $\forall \lambda, \mu \ge 0$, then

$$g(\lambda, \mu) \le L(\bar{x}, \bar{z}, \bar{Y}; \lambda, \mu) \le Cost(\bar{x}, \bar{z}, \bar{Y})$$

taking the optimal vale $p^* = \min\{Cost(x, z, Y) : H, Lc\}$

then $g(\lambda, \mu) \le p^* = \min_{H, LK} Cost \quad \forall \lambda, \mu \ge 0$

$\longrightarrow \max g(\lambda, \mu) \le p^*$ weak duality

Thirdly, strong duality and complementary slackness

$\longrightarrow$ Linear Programming Relaxation.

if $Y_i, Y_j \in [0, 1]$ and the feasible region satisfies the

Slater Condition, then the problem is convex and strong duality

$$\min_{H, LK} Cost = \max_{\lambda, \mu \ge 0} g(\lambda, \mu) = L(X^*, Z^*, Y^*; \lambda^*, \mu^*)$$

As well as with the Complementary-slackness Conditions

$$\lambda_{ij}^{*}(x_{ij}^{*}-u_{ij}Y_i^{*})=0, \quad \mu_{jc}^{*}(Z_{jc}^{*}-u_{ij}Y_j^{*})=0.$$

denote cs

Hence, the saddle point $(x^{*}, z^{*}, Y^{*}; \lambda^{*}, \mu^{*})$ satisfy both primal and dual optimality

Fourthly, Structure of inner min function (λ, μ fixed)

Start with defining the terms in L

$$L=\sum_i \underbrace{\left(f_m^i(1+\alpha_i)-\sum_j \lambda_{ij}u_{ij}\right)}_{\phi_i(\lambda)}Y_i+\sum_j \underbrace{\left(f_d^j(1+\beta_j)-\sum_c \mu_{jc}v_{jc}\right)}_{\psi_j(\mu)}Y_j$$

$$+\sum_{ij}\underbrace{\left(v_m^i+\frac{3}{2000}d_{ij}^{md}(1+\tau_{ij})+\lambda_{ij}\right)}_{c_{ij}^{\sim x}(\lambda)}x_{ij}$$

$$+\sum_{jc}\underbrace{\left(f_d\left(p^j+9.75+3.5\frac{d_{jc}^{dc}}{500}\right)(1+\delta_{jc})+\mu_{jc}\right)}_{c_{jc}^{\sim z}(\mu)}z_{jc}$$

denote Lsp

$$L=\min \sum_i \phi_i(\lambda)Y_i+\sum_j \psi_j(\mu)Y_j+\sum_{ij}c_{ij}^{\sim x}(\lambda)x_{ij}+\sum_{jc}c_{jc}^{\sim z}(\mu)z_{jc}$$

denote inn.

WTS A): Each α, z, Y appears independent in H

then. $Y_i^*(\lambda) = \arg\min_{y \in \{0,1\}} \phi_i(\lambda)$ $\quad y = \begin{cases} 1 & \phi_i(\lambda) < 0 \\ 0 & \phi_i(\lambda) > 0 \\ \text{either } 0/1 & \phi_i(\lambda) = 0. \end{cases}$

implying $Y_i^*(\lambda) = 1\{\sum_j \lambda_{ij} u_{ij} > f_m^i(1+\alpha_i)\}$

denote Y rule for MFG

Works same for Y rule for DC as following

$$Y_j^*(\mu) = 1\{\sum_c \mu_{jc} v_{jc} > f_d^i(1+\beta_j)\}$$

means: $Y_j^* = 1$ when $\overrightarrow{\text{satisfied}}$.

B). inner minimization over X, z

$\min \sum_{ij} \tilde{c}_{ij}^{x}(\lambda) X_{ij} + \sum_{jc} \tilde{c}_{jc}^{z}(\mu) z_{jc}$ denote flow.

This is a minimum-cost flow subproblem.

optimal flows occurs on arcs with minimal reduced cost.

In the fifth step; the subgradient of the dual function.

If (x^*, z^*, Y^*) is optimal for fixed (λ, μ)

then the subgradient of the dual function is:

$$\partial_{\lambda_{ij}} g(\lambda, \mu) \ni x_{ij}^* - U_{ij} Y_i^* ,$$

$$\partial_{\mu_{jc}} g(\lambda, \mu) \ni x_{jc}^* - V_{jc} Y_j^*$$

then, the dual ascent update or primal-dual iteration is

$$\lambda_{ij}^{k+1} = [\lambda_{ij}^k + \gamma_k (x_{ij}^k - U_{ij} Y_i)]_+$$

$$\mu_{jc}^{k+1} = [\mu_{jc}^k + \gamma_k (x_{jc}^k - U_{jc} Y_j)]_+$$

With the step size $\gamma_k > 0$, for convex problems, $g(\lambda^k, \mu^k)$ is monotonically non-decreasing and converges to dual optimum as $k \to \infty$.

then the problem satisfied the KKT condition if:

if Y is relaxed to [0,1] the primal is

min Cost (X, Z, Y) s.t. H, Lf

also it's strong duality. A pair $(X^*, Z^*, Y^*; \lambda^*, \mu^*)$

is the optimal for ∀ solutions if

1. primal feasibility: all H, Lf holds

2. Dual feasibility: $\lambda^*, \mu^* \geq 0$

3. Complementary slackness:

$$\lambda^* (X_{ij}^* - U_{ij} Y_i^*) = 0, \quad \mu_{jc} (X_{jc}^* - V_{jc} Y_j^*) = 0 \qquad \text{CS}$$

4. Stationary: the gradient of Lagrangian

w.r.t cont variables vanishes which is equivalent to

reduced cost optimality in our case

then we are free to get the optimal solutions $\square$

My framework / workflow:

Farkas lemma → weak duality → strong duality → KKT condition → primal-dual convergence