PERSPECTIVES

Factor-Based Asset Allocation vs. Asset-Class-Based Asset Allocation

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This article addresses the issue of the alleged superiority of risk-factor-based asset allocations over the more traditional asset-class-based asset allocation. The authors used both an idealized model, capable of precise mathematical treatment, and optimizations based on different periods of historical data to show that neither approach is inherently superior to the other. Although the authors appreciate the role of risk models in portfolio management, they urge caution with respect to unwarranted claims of their dominance.

For the majority of the last 60 or so years since the publication of Markowitz (1952), the primary building blocks that have been optimized to determine an "asset allocation" have been asset classes. More recently, a new type of potential building block has emerged: risk factors. Recent articles on risk-factor-based asset allocation imply that the approach is somehow inherently superior to asset allocation based on asset classes. If a fair apples-to-apples comparison is conducted, neither approach is superior. In the absence of superior information, simply reorganizing securities into different building-block clusters does not generate superior risk-adjusted returns. In a strategic asset allocation setting, superior returns come from a large, granular opportunity set, free from artificial constraints, coupled with superior skill at forecasting the long-term capital market assumptions of the opportunity set. In a tactical asset allocation setting, superior returns relative to the target come from superior skill at forecasting the short-term capital market assumptions. In this article, we examine whether asset allocation based on risk factors or asset allocation based on asset classes is inherently superior. We mathematically prove that neither approach is inherently superior and provide empirical examples to help illustrate the point in a more realistic setting. Our goal is to set the record straight regarding the two approaches,

but we emphasize that we are fans of risk models and advocates of many common risk factors. We applaud the innovation of using risk factors in the asset allocation process.

Finally, the information presented here should not be confused with another popular topic that has the word "risk" in its name: risk parity. Conceptually, risk parity approaches focus on achieving maximum diversification (however that is defined) as opposed to the more traditional approach of maximizing return per unit of risk. If you believe in risk parity, you can certainly apply a risk parity approach to an opportunity set of either risk factors or asset classes.

A Brief History of Risk Factors

We begin with a brief history of asset allocation, the needs that lead to risk-factor-based approaches, and examples of recent studies that imply that risk-factor-based asset allocation is inherently superior to asset allocation based on asset classes.

Harry Markowitz's work on mean–variance optimization in the 1950s created the crux of modern portfolio theory and the most widely accepted method for creating portfolios. As practitioners embraced mean–variance optimization throughout the 1970s, 1980s, and 1990s, the investment process developed into two distinct processes. First, because it is nearly impossible to run a single-stage optimization that includes all available individual securities, the individual securities are grouped into asset classes and the optimization is performed on the asset classes. This first stage is often referred to as the asset allocation, or beta, decision; presumably,

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the asset classes in question have an inherent, non-skill-based return premium in which the primary source of that premium is beta, or market risk. In the second part of the process, managers that specialize in security selection, typically within each of the individual asset classes, are hired to implement the target asset allocation. This second step is often referred to as the product, or alpha, decision.[1]

The asset allocation decision rarely involves more than 20 asset classes, whereas the problem faced by a fund manager building a portfolio of individual securities within a given asset class can involve thousands of individual securities. Mean–variance optimization requires the practitioner to estimate expected returns, standard deviations, and the correlations of each asset relative to all the other assets being optimized. To optimize thousands of individual securities, it is necessary, yet impossible, to estimate all the correlations using time-series data;[2] this problem led to the development of structural multifactor risk models.[3] These models attempt to identify a reasonable number of common factors that explain the returns of individual securities.

The Sharpe–Lintner–Mossin–Treynor capital asset pricing model, in which the "market" became *the* risk factor (at least for equities), is the super factor for most risk-factor models.[4] Stephen A. Ross, using his arbitrage pricing theory (APT), was the first to formalize the argument that multiple risk factors contribute to asset-class returns. Ross (1976) put forth a more general model and left the specific factors and robust theory for future research. That research was undertaken in due course by academia. Fama and French (1992) supplemented the market factor by identifying size and valuation factors that could not be explained by the market-risk-factor model. Jegadeesh and Titman (1993), together with Carhart (1997), added momentum to the lineup of generally recognized factors.[5] These original factors were motivated by a desire to explain the market and its anomalies rather than by the needs of those involved in portfolio building, who sooner or later adopted the factors. That step was facilitated by Grinold and Kahn (2000), who devoted parts of their standard investment textbook to building factor-based portfolios.

The next set of proposed factors came from a combination of two lines of thought: (1) an attempt to explain manager performance and (2) the realization that many seemingly innovative strategies can be cheaply implemented by systematic passive exposures. Thus, Fung and Hsieh (2004) proposed a seven-factor model to explain hedge fund returns.[6] Similarly, Jensen, Yechiely, and Rotenberg (2005) documented how many hedge fund return series can be reasonably approximated by purely passive

algorithmic strategies that mimic such strategies as momentum, distressed debt, and merger arbitrage. Then, there was "exotic beta," a Goldman Sachs concept that tried to categorize somewhat cheaply implementable exposures to the less traditional segments of the market, such as commodities.[7] Some of the latest factors *du jour* are volatility itself and relative liquidity.[8]

For the vast majority of this journey through factor development, the primary application of structural multifactor risk models was the construction of portfolios of individual securities. Only during the most recent 20 years or so have structural multifactor risk models been used to more closely monitor portfolio managers and perform detailed attribution analyses. Arriving at our current issue, in the last 10 years or so, asset managers steeped in the science of constructing security-level portfolios started using structural multifactor risk models to develop multi-asset-class portfolios. The problem is not this innovation. The problem is that authors who strongly suggest that risk-factor-based asset allocation is inherently superior are at best overstating their case and at worst confusing investors with a false value proposition.[9]

Recent papers that strongly suggest that risk-factor-based asset allocation is inherently superior to asset allocation based on asset classes include Page and Taborsky (2011) and Bender, Briand, Nielsen, and Stefek (2010). Positive messages from these papers with which we agree are that relaxing the long-only investment constraint and expanding one's opportunity set to include more potential exposures are often good things (although the latter point by itself is rather trivial). Unfortunately, most such papers use apples-to-oranges comparisons that lead all but the most careful reader to believe that risk-factor-based asset allocation is inherently superior to asset allocation based on asset classes; in fact, these papers offer neither real proof nor even a rigorous argument. The comparisons allegedly confirming the superiority of factor-based allocations are typically made between a relatively simple asset-class set and a risk-factor set that includes many more potential exposures, which is the sense in which we consider these to be apples-to-oranges comparisons. For example, Clarke, de Silva, and Murdock (2005), early proponents of risk-factor-based asset allocation, compared a five-exposure set (three asset classes and 2 factors) with a much more robust set of 14 factors that includes global equity and currency exposures, whereas Bender et al. (2010) compared a rather simplistic stock/bond portfolio with a 10-factor set that includes valuation, momentum, term and credit spreads, currency, and semi-active exposures to such strategies as convertible and merger arbitrage. Furthermore, many

of the presumed gains result from the fact that the comparisons involve different inherent constraints on the asset classes involved; setting a long-only portfolio next to a portfolio that may go long and short does not make for a meaningful comparison. The often-highlighted fact that the pairwise correlations among risk factors are lower than those among asset classes should not be confused with the superiority of one approach over the other.

Note that it would be impossible for the world as a whole to embrace risk-factor-based asset allocations. In a well-defined asset-class, or "grouping," scheme, all individual securities should be assigned to an asset class and the various asset classes should be mutually exclusive. This process results in what is known as a macro-consistent scheme, which enables all investors to hold the same portfolio should they so choose. With risk factors, individual securities often live in multiple factors, such as a bond that is part of both the duration and credit risk factors, and perhaps more importantly, it is impossible for all investors to hold the same portfolio because most factors require offsetting long and short positions, such as valuation (long value, short growth) or size (long small cap, short large cap). The entire world cannot simultaneously have a long position in the size premium, which would require everyone to short large caps.

Before we go into the details, let us briefly elaborate on what we mean by "risk factors" because the term can be understood in many different ways. Consistent with the current papers that proclaim risk-factor superiority, in this article, factors are essentially long–short portfolios of asset classes. We do not attempt to investigate the relative merits of APT or macro-based factors—such as industrial production, inflation expectations, consumption, and oil prices—for portfolio construction, nor do we try to assess the usefulness of models based on security characteristics, such as sector or industry.

Proof of Equality in an Idealized World

The mathematical proof is easiest in a simplified world in which the number of factors equals the number of assets, the asset-class returns are completely determined by the risk factors, and the risk factors are completely determined by the asset-class returns. In such a world, it is easy to demonstrate that the solutions to two *unconstrained* mean–variance optimizations—one in asset-class space and one in risk-factor space—are equivalent.

Above, we emphasized the word "unconstrained." It is important to realize that in practice, most optimizations impose a long-only constraint, which has different implications for opportunity sets of asset classes and opportunity sets of risk factors. By construction, many risk factors involve leverage (e.g., long small caps and short large caps are used to create the size factor); thus, a comparable "long-only" optimization of risk factors in which the optimizer is not allowed to short is, in fact, less constrained than a long-only optimization of asset classes. Ignoring potential legal and practical constraints associated with leverage extremes, most optimization constraints are actually arbitrary and reflect investor preferences. Intuitively, an investor who is willing to allocate to risk factors—which, by construction, require shorting—seemingly would not impose a long-only constraint on an asset-class optimization.

The realized returns, expected returns, and covariance matrices for both asset classes and risk factors are, respectively, denoted by

$$\mathbf{r}_a, \boldsymbol{\mu}_a, \boldsymbol{\Sigma}_a, \mathbf{r}_f, \boldsymbol{\mu}_f, \text{ and } \boldsymbol{\Sigma}_f,$$

where the subscripts a and f denote asset classes and risk factors, respectively.

In this simplified world, the vector of asset-class returns is a linear transformation of the factors based on matrix $\mathbf{L}$, where the asset class to risk factor exposure-mapping matrix $\mathbf{L}$ is square and invertible. In other words, we assume that in return space, asset classes can be fully expressed by factors and vice versa:

$$\mathbf{r}_a = \mathbf{L}\mathbf{r}_f, \tag{1a}$$

and

$$\mathbf{r}_f = \mathbf{L}^{-1}\mathbf{r}_a. \tag{1b}$$

The covariance matrix of the asset returns is calculated from the exposure-mapping matrix $\mathbf{L}$ coupled with the covariance matrix of risk factors, or alternatively, the covariance matrix of risk factors is calculated from the exposure-mapping matrix $\mathbf{L}$ coupled with the covariance matrix of asset classes:

$$\boldsymbol{\Sigma}_a = \mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}', \tag{2a}$$

and

$$\boldsymbol{\Sigma}_f = \mathbf{L}^{-1}\boldsymbol{\Sigma}_a(\mathbf{L}')^{-1}. \tag{2b}$$

Our initial goal is to prove the obvious; when risk factors perfectly explain the returns of asset classes, there is no inherent advantage from either approach. Stated differently, we can start with either risk factors or asset classes, derive an optimal portfolio, and move from one space to the other

with no gain or loss in efficiency. The solution to the unconstrained mean–variance maximization problem is as follows:

$$\mathbf{w} = \frac{1}{\theta} \Sigma^{-1} \mu, \tag{3}$$

where $\mathbf{w}$ represents the optimal weights and θ is the risk aversion coefficient. Notice that we have removed the subscripts because the solution can be calculated using either asset classes or risk factors. After calculating the optimal weights in one space, we can move to the other space using

$$\mathbf{w}_a = (\mathbf{L'})^{-1} \mathbf{w}_f \tag{4a}$$

and

$$\mathbf{w}_f = \mathbf{L'} \mathbf{w}_a. \tag{4b}$$

Using Equation 4a, to show that the return and risk of both portfolios are the same is a straightforward exercise. In Appendix A, we derive Equation 2a, demonstrate the link between Equations 3 and 4a, and verify that the returns and risks of asset-class- and risk-factor-based optimal portfolios are the same.

We admit that this example is somewhat artificial, but it mathematically demonstrates that there is no gain in efficiency from performing the *unconstrained* optimization in risk-factor space rather than in asset-class space. If there is a one-to-one mapping between asset-class and risk-factor returns, then there is no reason to expect that one set contains any less information than the other, as our calculations indeed confirm. This result is not new. Technically, it is known as the rotational indeterminacy of factors, which means that factors are determined up to a linear transformation. It is, nonetheless, a reminder that there is no obvious reason why risk factors

should produce a more efficient asset allocation. In fact, both asset classes and risk factors derive their returns from individual securities that, in turn, can be organized into both asset classes and risk factors. It is the somewhat arbitrary construction of the asset classes and risk factors, the selection of apples-to-oranges opportunity sets, and mean–variance optimization with different inherent constraints that can lead to differences—differences that should not be mistaken for inherent superiority of risk-factor-based asset allocation.

The Messy Real World

Having mathematically proven in an idealized setting that neither approach offers an inherent advantage, we turn to the far messier real world, for which an empirical example is necessary. Let us start by defining two opportunity sets that more or less cover the same underlying universe of individual securities; our set is U.S. centric because long-term data for the U.S. markets are readily available. We include cash as part of the factor opportunity set—even though it is not a risk factor—for reasons that will be obvious later (**Exhibit 1**).

The assumptions of our stylized example from the previous section clearly do not apply here because there is one less factor than there are asset classes; hence, there cannot be a one-to-one mapping between the two. Nevertheless, neither opportunity set would appear to have an inherent advantage obtained by including an additional uncorrelated asset class or risk factor that is unavailable to the other opportunity set.

In our asset-class opportunity set, we subdivide the universe of U.S. stocks along the dimensions of size and valuation. On the fixed-income

Exhibit 1. Opportunity Sets

Asset Classes	Proxy	Risk Factors	Proxy
A. Equity oriented			
U.S. large value	Russell 1000 Value Index	Market	Russell 3000 Index – Citigroup 3-month Treasury bills
U.S. large growth	Russell 1000 Growth Index	Size	Russell 2000 Index – Russell 1000 Index
U.S. small value	Russell 2000 Value Index	Valuation	Russell 3000 Value Index – Russell 3000 Growth Index
U.S. small growth	Russell 2000 Growth Index		
B. Fixed-income oriented			
U.S. mortgage backed		Mortgage spread	Barclays U.S. MBS – Barclays U.S. Treasury intermediate
U.S. Treasuries		Term spread (duration)	Barclays U.S. Treasury 20+ year – Citigroup 3-month Treasury bills
U.S. credit		Credit spread	Barclays U.S. credit – Barclays U.S. Treasury
C. Cash			
		Cash	Citigroup 3-month Treasury bills

side, we include the three biggest components of the U.S. investment-grade bond universe, which are Treasuries, mortgages, and corporate credit. In the risk-factor set, on the equity side, we include the equity premium (defined as the difference between the broad equity market and cash) and the size and valuation premiums (defined as the differences between small and large stocks and between value and growth stocks, respectively). On the fixed-income side, we include term (duration), mortgage, and credit spreads. The term spread is defined as the difference between a 20-year (or longer) Treasury and cash. The mortgage spread is defined as the difference between mortgages and intermediate-term Treasuries because the duration of the mortgage universe typically approximates that of intermediate Treasuries. Finally, the credit spread is equal to the difference between the returns of the credit and Treasury indices.

Because all the risk factors are derived series obtained by subtracting one series from another, they are zero-dollar investments. Interpreting what it means to allocate to a zero-dollar risk factor is tricky. Presumably, the allocation is self-financing because the negative (or short) position perfectly finances the positive (or long) position, with both positions summing to zero. Because the exposure nets out the return due to the risk-free rate for each factor, the way to think about it is that holding each factor is equivalent to holding a zero-dollar position in the factor itself plus a 100% position in cash. For example, a $100 portfolio that allocates 50% ($50) to the size premium and 50% ($50) to the valuation premium is equivalent to being $50

long in both the Russell 2000 Index and the Russell 3000 Value Index, $50 short in both the Russell 1000 Index and the Russell 3000 Growth Index, and $100 long in cash—which amounts to a $100 long position. Henceforth, we use this interpretation of factor exposures.

A thought experiment may be useful here as an analogy of how we think about our factors. Imagine that an exchange-traded fund (ETF) provider has created a set of ETFs that captures the factors we have just described. Ignoring fees and tracking errors for the sake of simplicity, the return of each ETF would equal the return of cash plus the return of the factor, or risk premium, in question. A portfolio of such ETFs would be subject to the same constraint that a typical long-only portfolio is subject to. In particular, this means that the sum of all the investments would add up to 100%, which, in turn—because of how the factors were constructed—means that the sum of all the long positions in the assets that make up the factors cannot exceed 100%. This is how we interpret a factor-based portfolio.

Table 1 shows annualized *total* return and *excess* return arithmetic means and standard deviations based on historical monthly data starting in January 1979 and ending in December 2011. Recall that the asset-class indices used to construct the risk-factor series are identified in Exhibit 1.

Table 2 shows the correlations. Note that, consistent with authors who proclaim the superiority of risk-factor allocations, the average pairwise correlation for the risk factors (without cash) is considerably lower (0.06) than that for the asset

Table 1. Historical Returns and Standard Deviations, January 1979–December 2011

Asset Class/Factor	Total Return		Excess Return	
	Arithmetic Mean	Standard Deviation	Arithmetic Mean	Standard Deviation
Barclays U.S. Treasury TR	8.59%*	6.12%*	2.96%	5.79%
Barclays U.S. credit TR	9.05*	7.87*	3.40	7.50
Barclays U.S. MBS TR	8.82*	7.40*	3.18	7.02
Russell 1000 Growth TR	12.22*	19.92*	6.42	18.97
Russell 1000 Value TR	13.04*	16.97*	7.20	16.15
Russell 2000 Growth TR	12.18*	26.47*	6.38	25.24
Russell 2000 Value TR	14.86*	20.40*	8.93	19.44
Market	12.70**	17.85**	6.88	17.00
Size	6.29**	11.02**	0.77	10.49
Valuation	6.44**	10.24**	0.91	9.68
Term spread (duration)	10.64**	13.44**	4.91	12.83
Credit spread	5.93**	3.96**	0.42	3.73
Mortgage spread	5.71**	3.97**	0.22	3.66
Citigroup 3-month Treasury bills	5.48	1.04	0.00	0.00

Note: TR stands for total return.
  *Historical capital market assumptions for the asset classes being optimized.
**Historical capital market assumptions for the risk factors being optimized.

Table 2. Historical Correlations

	Barclays U.S. Treasury TR	Barclays U.S. Credit TR	Barclays U.S. MBS TR	Russell 1000 Growth TR	Russell 1000 Value TR	Russell 2000 Growth TR	Russell 2000 Value TR	Barclays U.S. Treasury TR (excess)	Barclays U.S. Credit TR (excess)	Barclays U.S. MBS TR (excess)	Russell 1000 Growth TR (excess)	Russell 1000 Value TR (excess)	Russell 2000 Growth TR (excess)	Russell 2000 Value TR (excess)
Barclays U.S. Treasury TR	1.000*	0.863*	0.846*	0.072*	0.112*	–0.027*	0.030*	0.985	0.844	0.831	0.064	0.104	–0.033	0.023
Barclays U.S. credit TR	0.863*	1.000*	0.881*	0.271*	0.326*	0.188*	0.255*	0.860	0.991	0.877	0.268	0.323	0.186	0.252
Barclays U.S. MBS TR	0.846*	0.881*	1.000*	0.172*	0.210*	0.090*	0.133*	0.835	0.868	0.989	0.166	0.204	0.086	0.127
Russell 1000 Growth TR	0.072*	0.271*	0.172*	1.000*	0.843*	0.861*	0.755*	0.067	0.267	0.168	0.998	0.842	0.859	0.753
Russell 1000 Value TR	0.112*	0.326*	0.210*	0.843*	1.000*	0.738*	0.852*	0.105	0.319	0.205	0.840	0.998	0.736	0.849
Russell 2000 Growth TR	–0.027*	0.188*	0.090*	0.861*	0.738*	1.000*	0.878*	–0.027	0.188	0.090	0.861	0.739	0.999	0.878
Russell 2000 Value TR	0.030*	0.255*	0.133*	0.755*	0.852*	0.878*	1.000*	0.027	0.252	0.130	0.754	0.851	0.877	0.998
Barclays U.S. Treasury TR (excess)	0.985*	0.860*	0.835*	0.067*	0.105*	–0.027*	0.027*	1.000	0.865	0.845	0.070	0.108	–0.025	0.029
Barclays U.S. credit TR (excess)	0.844	0.991	0.866	0.267	0.319	0.188	0.252	0.865	1.000	0.881	0.271	0.325	0.191	0.256
Barclays U.S. MBS TR (excess)	0.831	0.877	0.989	0.168	0.205	0.090	0.130	0.845	0.881	1.000	0.171	0.208	0.092	0.133
Russell 1000 Growth TR (excess)	0.064	0.268	0.166	0.998	0.840	0.861	0.754	0.070	0.271	0.171	1.000	0.843	0.862	0.755
Russell 1000 Value TR (excess)	0.104	0.323	0.204	0.842	0.998	0.739	0.851	0.108	0.325	0.208	0.843	1.000	0.739	0.852
Russell 2000 Growth TR (excess)	–0.033	0.186	0.086	0.859	0.736	0.999	0.877	–0.025	0.191	0.092	0.862	0.739	1.000	0.879
Russell 2000 Value TR (excess)	0.023	0.252	0.127	0.753	0.849	0.878	0.998	0.029	0.256	0.133	0.755	0.852	0.879	1.000
Market	0.078	0.301	0.186	0.966	0.945	0.868	0.856	0.083	0.304	0.190	0.967	0.947	0.868	0.857
Size	–0.149	–0.038	–0.085	0.156	0.117	0.620	0.590	–0.142	–0.032	–0.079	0.158	0.120	0.621	0.592
Valuation	0.050	0.011	0.013	–0.545	–0.009	–0.469	–0.079	0.047	0.008	0.010	–0.546	–0.010	–0.469	–0.080
Term spread	0.919	0.785	0.730	0.059	0.088	–0.024	0.012	0.937	0.793	0.743	0.063	0.092	–0.021	0.016
Credit spread	0.159	0.636	0.430	0.420	0.467	0.410	0.453	0.176	0.647	0.444	0.425	0.473	0.413	0.458
Mortgage spread	0.031	0.310	0.559	0.211	0.220	0.211	0.202	0.034	0.311	0.563	0.212	0.221	0.211	0.202
Citigroup 3-month Treasury bills	0.128	0.054	0.100	0.029	0.046	0.000	0.020	–0.047	–0.082	–0.045	–0.027	–0.020	–0.042	–0.036
Market (plus cash)	0.086	0.305	0.192	0.967	0.947	0.868	0.857	0.080	0.299	0.187	0.965	0.946	0.866	0.855
Size (plus cash)	–0.137	–0.033	–0.076	0.159	0.122	0.619	0.591	–0.147	–0.040	–0.084	0.156	0.118	0.617	0.588
Valuation (plus cash)	0.063	0.017	0.023	–0.538	–0.004	–0.465	–0.076	0.042	0.000	0.005	–0.545	–0.012	–0.470	–0.083
Term spread (plus cash)	0.931	0.791	0.740	0.061	0.092	–0.024	0.014	0.936	0.788	0.741	0.061	0.091	–0.025	0.013
Credit spread (plus cash)	0.191	0.643	0.451	0.423	0.474	0.405	0.453	0.161	0.617	0.427	0.414	0.462	0.397	0.443
Mortgage spread (plus cash)	0.064	0.315	0.568	0.212	0.225	0.204	0.200	0.020	0.280	0.533	0.198	0.209	0.193	0.186

*Historical capital market assumptions for the asset classes being optimized.
**Historical capital market assumptions for the risk factors being optimized.

Table 2. Historical Correlations (continued)

	Market	Size	Valuation	Term Spread	Credit Spread	Mortgage Spread	Citigroup 3-Month Treasury Bills	Market (plus cash)	Size (plus cash)	Valuation (plus cash)	Term Spread (plus cash)	Credit Spread (plus cash)	Mortgage Spread (plus cash)
Barclays U.S. Treasury TR	0.078	–0.149	0.050	0.919	0.159	0.031	0.128	0.086	–0.137	0.063	0.931	0.191	0.064
Barclays U.S. credit TR	0.301	–0.038	0.011	0.785	0.636	0.310	0.054	0.305	–0.033	0.017	0.791	0.643	0.315
Barclays U.S. MBS TR	0.186	–0.085	0.013	0.730	0.430	0.559	0.100	0.192	–0.076	0.023	0.740	0.451	0.568
Russell 1000 Growth TR	0.966	0.156	–0.545	0.059	0.420	0.211	0.029	0.967	0.159	–0.538	0.061	0.423	0.212
Russell 1000 Value TR	0.945	0.117	–0.009	0.088	0.467	0.220	0.046	0.947	0.122	–0.004	0.092	0.474	0.225
Russell 2000 Growth TR	0.868	0.620	–0.469	–0.024	0.410	0.211	0.000	0.868	0.619	–0.465	–0.024	0.405	0.204
Russell 2000 Value TR	0.856	0.590	–0.079	0.012	0.453	0.202	0.020	0.857	0.591	–0.076	0.014	0.453	0.200
Barclays U.S. Treasury TR (excess)	0.083	–0.142	0.047	0.937	0.176	0.034	–0.047	0.080	–0.147	0.042	0.936	0.161	0.020
Barclays U.S. credit TR (excess)	0.304	–0.032	0.008	0.793	0.647	0.311	–0.082	0.299	–0.040	0.000	0.788	0.617	0.280
Barclays U.S. MBS TR (excess)	0.190	–0.079	0.010	0.743	0.444	0.563	–0.045	0.187	–0.084	0.005	0.741	0.427	0.533
Russell 1000 Growth TR (excess)	0.967	0.158	–0.546	0.063	0.425	0.212	–0.027	0.965	0.156	–0.545	0.061	0.414	0.198
Russell 1000 Value TR (excess)	0.947	0.120	–0.010	0.092	0.473	0.221	–0.020	0.946	0.118	–0.012	0.091	0.462	0.209
Russell 2000 Growth TR (excess)	0.868	0.621	–0.469	–0.021	0.413	0.211	–0.042	0.866	0.617	–0.470	–0.025	0.397	0.193
Russell 2000 Value TR (excess)	0.857	0.592	–0.080	0.016	0.458	0.202	–0.036	0.855	0.588	–0.083	0.013	0.443	0.186
Market	1.000	0.205	–0.324	0.072	0.470	0.227	–0.026	0.998	0.203	–0.325	0.070	0.457	0.213
Size	0.205	1.000	–0.131	–0.131	0.153	0.072	–0.044	0.203	0.995	–0.135	–0.135	0.140	0.058
Valuation	–0.324	–0.131	1.000	0.034	–0.055	–0.054	0.020	–0.323	–0.129	0.995	0.035	–0.049	–0.047
Term spread	0.072	–0.131	0.034	1.000	0.130	–0.059	–0.069	0.068	–0.138	0.026	0.997	0.110	–0.076
Credit spread	0.470	0.153	–0.055	0.130	1.000	0.559	–0.089	0.464	0.145	–0.063	0.123	0.965	0.517
Mortgage spread	0.227	0.072	–0.054	–0.059	0.559	1.000	–0.012	0.226	0.070	–0.054	–0.061	0.549	0.965
Citigroup 3-month Treasury bills	–0.026	–0.044	0.020	–0.069	–0.089	–0.012	1.000	0.036	0.052	0.123	0.012	0.176	0.251
Market (plus cash)	0.998	0.203	–0.323	0.068	0.464	0.226	0.036	1.000**	0.206**	–0.317**	0.071**	0.468**	0.228**
Size (plus cash)	0.203	0.995	–0.129	–0.138	0.145	0.070	0.052	0.206**	1.000**	–0.123**	–0.134**	0.157**	0.082**
Valuation (plus cash)	–0.325	–0.135	0.995	0.026	–0.063	–0.054	0.123	–0.317**	–0.123**	1.000**	0.036**	–0.030**	–0.020**
Term spread (plus cash)	0.070	–0.135	0.035	0.997	0.123	–0.061	0.012	0.071**	–0.134**	0.036**	1.000**	0.125**	–0.055**
Credit spread (plus cash)	0.457	0.140	–0.049	0.110	0.965	0.549	0.176	0.468**	0.157**	–0.030**	0.125**	1.000**	0.577**
Mortgage spread (plus cash)	0.213	0.058	–0.047	–0.076	0.517	0.965	0.251	0.228**	0.082**	–0.020**	–0.055**	0.577**	1.000**

classes (0.38). This average correlation difference should be intuitive. The asset classes are all part of the "market" portfolio and, hence, carry with them an element of overall market beta. By construction, with the exception of the market risk factor, the risk factors do not share this common exposure to the market. This fact is directly related to a key point from the recent "importance of asset allocation literature"; most notably, Xiong, Ibbotson, Idzorek, and Chen (2010) emphasized that one must be extremely careful when comparing items that include the market factor with those that do not.

In our first optimization comparison, we constrain the weights of the asset classes and of the risk factors to be nonnegative and the allocations to sum to 1. The values from Table 1 that are marked with an asterisk are the historical capital market assumptions for the asset classes that are being optimized, and the values marked with two asterisks are the historical capital market assumptions for the risk factors that are being optimized. Recall from our zero-dollar investment discussion that when a dollar is invested in the risk factor, it is really being invested in Treasury bills plus the zero-dollar (self-financing) risk factor. In Tables 1 and 2, cash (T-bills) is not marked with an asterisk, but it is included in both opportunity sets.

As we proceed with our first optimization comparison, let us pause here and reflect on what we are actually comparing. From a mathematical point of view, in the asset-class space, our feasible set for weights is a seven-dimensional subset (because of the one linear constraint of unity) of the eight-dimensional asset-class space, subject to the constraint of the weights being positive. The risk-factor space is somewhat more complicated. But for our purposes, what matters is that the feasible set for risk-factor weights, which are a combination of some asset classes being offset by other asset classes, is a different subset (fewer dimensions because of more constraints) of the eight-dimensional asset-class space.[10]

The intuition we want to focus on is that although the risk-factor weights' feasible set has fewer dimensions than its asset-class counterpart, which suggests that it may be slightly handicapped, by construction, the risk factors include underlying asset-class exposures of –100%—something that is not allowed in our asset-class set. From this perspective, the "long-only" constraint imposed during the optimization of the risk-factor opportunity set may, in fact, be less constrained and the "long-only" label for this comparison may be a bit of a misnomer. *A priori*, one cannot say which opportunity set coupled with its respective explicit and implicit constraints will produce more efficient asset allocations; there is nothing obvious about it, and it could go either way, depending on the returns and covariances of the opportunity set in question. It is in this sense that we claimed at the beginning of this section that the superiority (or otherwise) of a risk-factor-based allocation is an empirical question.

Figure 1 shows the two *historical* efficient frontiers obtained from the mean–variance optimizations described above.[11] In Figure 1, the empirical question is thus answered for this set of risk factors and this particular time period. In contrast to the current risk-factor papers that highlight much shorter time periods in which equities have generally performed poorly, the efficient frontier based on asset classes dominates the efficient frontier based on risk factors.

Figure 1. Long-Only Constraints: Asset Classes vs. Classic Risk Factors, January 1979–December 2011
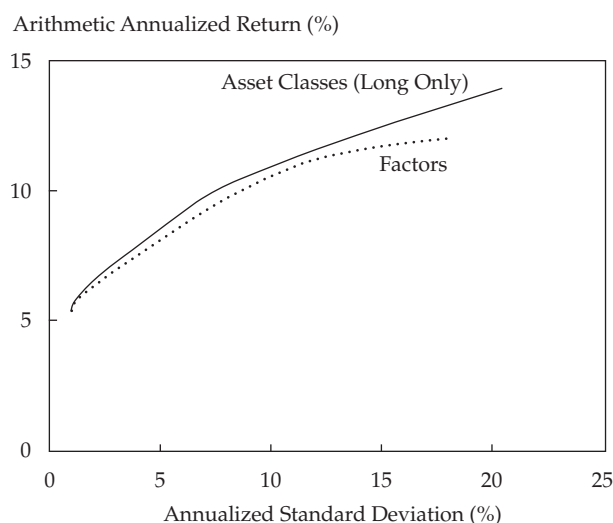

This result may look like a decisive score for asset-class-based asset allocation, but that conclusion is wrong. We reran this optimization comparison using a variety of *historical* capital market assumptions based on various shorter time periods; sometimes, asset classes worked better, and in some cases, risk factors worked better. By cherry picking a particular historical time period, almost any desired result can be found. We have omitted these results, except for the somewhat interesting results obtained from optimizations based on capital market assumptions from the most recent 10 years of data (January 2002–December 2011). In this example, neither asset classes nor risk factors consistently dominated along the whole risk spectrum.

An asset-based approach would have produced more efficient portfolios for lower risk levels, but the opposite is true for riskier portfolios, as shown in **Figure 2**.

Figure 2.   Long-Only Constraints: Asset Classes vs. Classic Risk Factors, January 2002–December 2011

Arithmetic Annualized Return (%)

Annualized Standard Deviation (%)
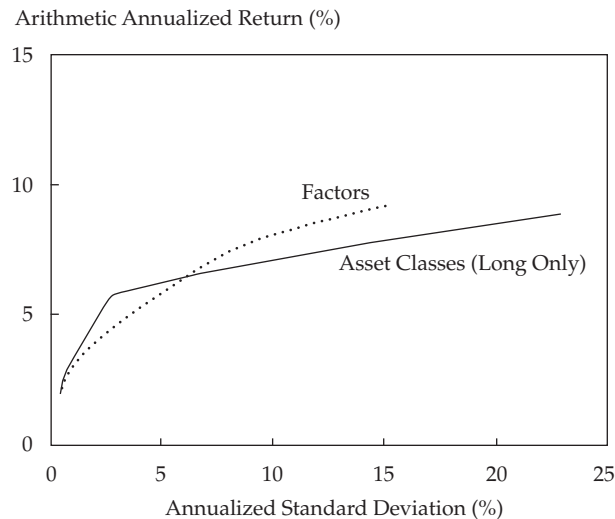
We hope we have clearly illustrated that there is nothing obvious about the superiority of asset allocation based on risk factors. Whether one uses historical data, as we have here, or future capital market assumptions, the realized or expected efficiency needs to be checked empirically (by which we mean the numbers must be run) to determine which approach may be more appropriate. Neither is simply superior.

Of course, practical implementation is a whole different story. Even if one were able to convince himself that for a given level of risk, a risk-factor-like portfolio is superior, very few institutional investors would be willing to take on the extreme positions implied by most risk factors. For example, let us take a simple risk factor—size, which we constructed earlier by shorting large-cap stocks (Russell 1000) and going long on small-cap stocks (Russell 2000). In the U.S. equity component of a strategic asset allocation, a policy portfolio that was allocated equally between large cap and small cap would be viewed as dramatically small-cap over-weighted from the typical market-capitalization viewpoint. In fact, it is almost impossible to imagine a policy portfolio with a U.S. equity split of 0% large cap and 100% small cap, let alone –100% large cap and 100% small cap, which is implicit in the typical size factor construction. Furthermore, why

stop here? Could we not create an even bigger or better size premium by going short 10,000% large cap and long 10,000% small cap?

Moving to another simple risk factor, valuation, which we constructed by shorting growth stocks (Russell 3000 Growth) and going long value stocks (Russell 3000 Value), we have observed policy portfolios that strongly favor value stocks but none that completely ignore growth stocks. Investing in the valuation risk factor would suggest an extremely high level of conviction in the value premium and a level of conviction that we have never observed in a traditional asset-class policy portfolio. Our point is that investors considering the use of a factor-based approach need to be aware of the rather extreme positions embedded in the factors. Finally, one can certainly construct an asset allocation using asset classes that is designed to capture many of the risk factors.

Conclusion

We mathematically proved that in an idealized world in which risk-factor returns are completely explained by asset-class returns and vice versa, neither approach is inherently superior to the other. Next, in a series of real-world optimizations based on historical data, we demonstrated that either approach may be superior over a given time period but it really comes down to the specifics of the comparison and of the composition of the factors. In particular, in some cases, the apparent superiority of the risk factors is a simple result of the fact that the risk factors are, in a guise, a set of asset classes with the long-only constraint removed. If one truly creates an even playing field, there is no gain in efficiency, which is hardly surprising. If risk factors could be approximated by asset classes, it would be quite odd if that risk-factor approximation contained more useful information than the totality of information contained in the asset classes that are used to approximate the risk factors themselves.

There are a couple practical issues associated with risk-factor-based asset allocation. First, remember that it would be impossible for the world to completely embrace risk-factor-based asset allocation because it implies a weighting scheme that is not macro-consistent. This observation puts the risk-factor-based asset allocation on the level of a strategy rather than a theoretically consistent model. A strategy that is not feasible for all may nonetheless be feasible for some, and just as any strategy, it may or may not result in a more efficient portfolio for a given time period. Second, the largest hurdle to adopting a risk-factor-based asset allocation is that most institutional investors are

simply uncomfortable with the extreme positions involving leverage that are implied by risk factors.

Finally, there is nothing wrong with risk-factor-based asset allocation, but it is not a magic bullet that will automatically lead to asset allocations that will dominate asset allocations based on asset classes.

Helpful comments were provided by Wai Lee and Philip Straehl.

This article qualifies for 0.5 CE credit.

Appendix A.

Following is the derivation of Equation 2a, which demonstrates that the asset-class covariance matrix can be derived from the risk-factor covariance matrix:

$$
\begin{aligned}
\boldsymbol{\Sigma}_a &= E\left[(\mathbf{r}_a - \boldsymbol{\mu}_a)(\mathbf{r}_a - \boldsymbol{\mu}_a)'\right] \\
&= E\left\{\left[\mathbf{L}(\mathbf{r}_f - \boldsymbol{\mu}_f)\right]\left[\mathbf{L}(\mathbf{r}_f - \boldsymbol{\mu}_f)\right]'\right\} \\
&= E\left\{\left[\mathbf{L}(\mathbf{r}_f - \boldsymbol{\mu}_f)\right]\left[(\mathbf{r}_f - \boldsymbol{\mu}_f)'\mathbf{L}'\right]\right\} \\
&= \mathbf{L}E\left[(\mathbf{r}_f - \boldsymbol{\mu}_f)(\mathbf{r}_f - \boldsymbol{\mu}_f)'\right]\mathbf{L}' \\
&= \mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}'.
\end{aligned}
$$

Next, we demonstrate the link between Equations 3 and 4a and that weights in either asset-class space or risk-factor space can be transformed into equivalent weights in the other space:

$$
\begin{aligned}
\mathbf{w}_a &= \frac{1}{\theta}\boldsymbol{\Sigma}_a^{-1}\boldsymbol{\mu}_a = \frac{1}{\theta}\left(\mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}'\right)^{-1}\mathbf{L}\boldsymbol{\mu}_f \\
&= \frac{1}{\theta}\left(\mathbf{L}'\right)^{-1}\boldsymbol{\Sigma}_f^{-1}\mathbf{L}^{-1}\mathbf{L}\boldsymbol{\mu}_f \\
&= \frac{1}{\theta}\left(\mathbf{L}'\right)^{-1}\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\mu}_f \\
&= \left(\mathbf{L}'\right)^{-1}\mathbf{w}_f.
\end{aligned}
$$

Also, we can verify that returns of the asset-class-based and factor-based optimal portfolios are the same:

$$
\begin{aligned}
\mathbf{w'}_a\mathbf{r}_a &= \left[\left(\mathbf{L}'\right)^{-1}\mathbf{w}_f\right]'\mathbf{L}\mathbf{r}_f \\
&= \mathbf{w'}_f\left[\left(\mathbf{L}'\right)^{-1}\right]'\mathbf{L}\mathbf{r}_f \\
&= \mathbf{w'}_f\left(\mathbf{L''}\right)^{-1}\mathbf{L}\mathbf{r}_f \\
&= \mathbf{w'}_f\left(\mathbf{L}\right)^{-1}\mathbf{L}\mathbf{r}_f \\
&= \mathbf{w'}_f\mathbf{r}_f.
\end{aligned}
$$

Finally, we show that the risks of the asset-class-based and factor-based optimal portfolios are the same:

$$
\begin{aligned}
\mathbf{w'}_a\boldsymbol{\Sigma}_a\mathbf{w}_a &= \left[\left(\mathbf{L}'\right)^{-1}\mathbf{w}_f\right]'\left(\mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}'\right)\left[\left(\mathbf{L}'\right)^{-1}\mathbf{w}_f\right] \\
&= \mathbf{w'}_f\left[\left(\mathbf{L}'\right)^{-1}\right]'\mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}'\left(\mathbf{L}'\right)^{-1}\mathbf{w}_f \\
&= \mathbf{w'}_f\left[\left(\mathbf{L''}\right)^{-1}\right]\mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}'\left(\mathbf{L}'\right)^{-1}\mathbf{w}_f \\
&= \mathbf{w'}_f\left[\left(\mathbf{L}\right)^{-1}\mathbf{L}\right]\boldsymbol{\Sigma}_f\left[\mathbf{L}'\left(\mathbf{L}'\right)^{-1}\right]\mathbf{w}_f \\
&= \mathbf{w'}_f\boldsymbol{\Sigma}_f\mathbf{w}_f.
\end{aligned}
$$

Notes

1. Waring and Siegel (2003) provided an excellent overview of this two-phase process.
2. The number of correlations that must be estimated is $N(N-1)/2$, where N equals the number of securities. Thus, for a portfolio of 1,000 securities, 495,000 correlations must be estimated. Using time series requires a minimum of 1,001 observations, which corresponds to roughly 4 years of daily return data, 83 years of monthly return data, or 250 years of quarterly return data.
3. Common examples of companies that offer structural multifactor risk models are MSCI (Barra) and Northfield Information Services.
4. See Sharpe (1964); Lintner (1965); Mossin (1966); and Treynor (1961, 1962).
5. Cliff Asness also contributed significantly to the development of momentum as a factor.
6. The factors were equity market, small-cap premium, change in 10-year U.S. Treasury and credit spreads, and three trend-following strategies for currencies, bonds, and commodities.
7. See, for example, Litterman (2005).
8. See Ibbotson, Chen, Kim, and Hu (2013).
9. A more appropriate value proposition is to claim superior skill at forecasting the return of risk factors, although the investing public is rightfully skeptical of this more traditional value proposition claim.
10. Note that, for practical purposes, the asset classes that are used in the construction of the factors can be expressed as combinations of the asset classes from the asset-class space; thus, for example, the Russell 3000 can be expressed as the weighted sum of the four Russell indices from the asset-class space.
11. All the optimizations are performed in total return space, with the cash return included.

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