Asset Allocation vs. Factor Allocation—*Can We Build a Unified Method?*

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Asset classes, such as equities and bonds, have constituted the primary building blocks for constructing comprehensive diversified portfolios for more than 60 years. Investors long held the view that different asset classes provide essential diversification benefits, but recent market downturns have seen an erosion of these diversification benefits. It has become conventional wisdom that "diversification disappears when you need it the most" (Page 2010).

Recent literature suggests that systematic risk factors, as opposed to asset classes, may provide better building blocks for optimal portfolio diversification (e.g., Clarke, de Silva, and Murdock 2005; Bender et al. 2010; and Page and Taborsky 2011). The notion is to identify risk and return factors and to build portfolios that can best capture these return premiums. For example, assets such as real estate and high yield embed a positive exposure to equity beta and interest rate duration, both important elements that determine the expected risk and return of a portfolio. A factor approach may target these common risk factors first and then identify the asset classes, such as real estate and high yield, that garner the optimal exposure to these factors considering the portfolio objectives.

This approach bridges the gap between the modern paradigm of asset pricing and investment practice. The factor-based approach transforms the asset allocation decision from the asset space to a smaller factor space, offering asset allocators the benefit of applying their ability to predict factor distributions and correlations. This benefit can come from an investor's insight into predicting factor returns or from the more comprehensible probability distribution of factor returns (Kritzman 2014). In our view, the factor-based approach also provides scalability of investment opportunities because a small set of factors can drive the returns of a large set of assets. The challenge, of course, is in identifying and generating forecasts of those factors that are as robust as traditional asset class forecasts, an important point made by critics of the factor-based approach (e.g., Idzorek and Kowara 2013).

The practice of factor-based asset allocation predates its recent ascent in popularity. In 1996, Bridgewater Associates launched its "all-weather" strategy, which is essentially a factor-based asset allocation framework. This approach allocates assets based on their exposures to economic growth and inflation and conditions the allocation based on economic regimes. Just two macro risk factors can be powerful building blocks in creating a hugely successful asset allocation strategy.

Liability driven investing (LDI) is another factor-based approach with a rich history, and it has proven quite popular with corporate defined benefit plans. In essence, an LDI approach segregates assets into one of two factors: a *hedging* factor portfolio and a *growth* factor portfolio. The hedging portfolio generally consists of long-duration bonds, designed to mimic the pension's liabilities, with the purpose of reducing the funding ratio volatility. Metrics such as duration and yield curve exposure are critical when constructing the hedge portfolio. The growth portfolio may consist of equities, high yield, real estate, or other growth assets, with the purpose of closing the funding ratio gap. Metrics such as equity beta and yield spread are important to monitor when constructing the growth portfolio.

Capitalizing on Bridgewater's early success, many asset managers have since developed their own factor-based asset allocation processes. They generally fall into one of two camps. The first camp relies on a set of cross-sectional asset pricing style factors from the stock anomaly literature, factors that are ideally uncorrelated with asset class premia. These style (or alternative risk premia) factors can be implemented as tilts in long-only portfolios or as "pure styles" in long–short portfolios. Factor portfolios from different asset classes can be combined to form a multi-asset-class portfolio. Asness et al. (2015) and Brightman and Shepherd (2016) present examples of these.

The second camp uses variables that capture macroeconomic states (e.g., economic growth and inflation) and constructs portfolios that have the desirable exposure to these macro factors. Optimization is usually used to obtain the portfolio with the least factor exposure deviation from the desired target. Macro factor exposures are estimated through time-series regression, sometimes coupled with discretionary judgment, to ensure that asset classes have intuitive exposures to the predetermined factor set. Blyth, Szigety, and Xia (2016) and Greenberg, Babu, and Ang (2016) present examples.

The framework we present here combines aspects of both camps. It is designed to be a comprehensive framework that can accommodate both style factors as well as macroeconomic states. We map asset pricing style factors to traditional asset classes. Additionally, our framework allows for discretionary judgment to ensure that asset classes have intuitive exposures to the predetermined factor set. Specifically, a unique aspect of this framework is to extend the existing factor-based asset allocation process to the derivation of asset expected returns and to integrate both quantitative and discretionary factor views. With asset expected returns that capture the desirable factor exposures, portfolio optimization can be used to generate optimal asset allocations for a wide range of investment objectives commonly faced by a multi-asset-class-solution provider.

The next section provides background on factor-based allocation models and how they differ from traditional asset allocation models. We then introduce a method for integrating the two in one framework so that both forecast-factor alphas and traditional alphas can be tied together. The third section walks through each of the steps of the framework using an empirical example to illustrate the stages.

FROM TRADITIONAL ASSET ALLOCATION TO FACTOR MODELS

Traditional asset allocation usually entails allocating among main asset classes (e.g., equities, commodities, fixed income, cash, and alternatives) and then allocating within asset classes to various subcomponents (e.g., US, UK/Europe, Japan, developed Asia ex Japan, and emerging markets within equities). Exhibit 1 shows an example of a typical asset allocation framework.

Investment plans often establish a strategic portfolio, based on long-term asset class forecasts of return and risk, taking account of their own investment objectives. Some investors take active positions away from the strategic portfolio based on shorter-term asset class forecasting models. Examples of these models include sector rotation models, country rotation models, and stand-alone signals using metrics such as liquidity, valuation, momentum/technical indicators, sentiment/risk indicators, and macroeconomic variables. Submodels can be either time-series or cross-sectional in nature, and in some cases the expected return of an asset class is a result of combining these time-series and cross-sectional models. For example, an alpha scaling process can be employed that converts cross-sectional asset Z-scores to returns and adds them to the time-series return predictions.

On top of these models, discretionary views are often incorporated to arrive at a final portfolio. For instance, a fundamental-view portfolio can be generated by portfolio managers and researchers as long as they can express their ideas as trades. Black–Litterman-type models and their extensions are a popular way of combining prior qualitative views with quantitatively driven model outputs.

Exhibit 1

Traditional Asset Class Allocation Framework (Hypothetical Illustration)

Class		Strategic Asset Allocation (%)
Equity		**60**
	US Equity	30
	Europe Equity	15
	UK Equity	5
	Japan Equity	5
	Emerging Markets Equity	5
Fixed Income		**20**
	Global Investment Grade	15
	High Yield	5
Alternatives		**20**
	Hedge Funds	10
	Private Equity	10
Cash		**0**
	Total	**100**

Source: State Street Global Advisors (SSGA), for illustrative purposes only. Shown are nominal allocations.

How do we relate asset classes and factors to each other? Given that the existing literature has found that both macro and style factors can explain asset returns, we prefer to combine them in the same asset-pricing model and test whether they can be complementary. This is similar in spirit to the approaches taken by Clarke, de Silva, and Murdock (2005) and Bender et al. (2010). We collectively call macro and style factors *systemic factors*. We express asset returns as

$$R_t = B_t F_t + \epsilon_t \qquad (1)$$

where $t = 1, \ldots, T$; R_t is a vector of asset returns; F_t is a vector of macro and style factors; B_t is the sensitivity of asset returns to factors; and ϵ_t is a vector of residual returns and has zero expectation by design.

Note that, cross-sectionally, not all residuals are zero, even though the expected value is zero. Residual returns may exhibit specific dynamic patterns that can be predicted, such as seasonality among some agriculture and energy commodities. In addition, portfolio managers may have unfactorable insights that help to explain the residuals from certain asset classes that fall through the systemic factors. An example of this type of insight is a nonrecurring event, such as Brexit or an impeding country default, which may not be captured in the predefined factor model. In this case, the portfolio managers may understand that a forecasting model may prove ineffective given the outcome of these exogenous events.

Thus, we expand the residual ϵ_t as

$$\epsilon_{i,t} = \Gamma_{i,t} I_{i,t} + V_{i,t} + \eta_{i,t} \qquad (2)$$

where $I_{i,t}$ is a vector of within-asset-class idiosyncratic factors that capture locally predictable residual return patterns; $\Gamma_{i,t}$ is a vector of dynamic exposures to the idiosyncratic factors; $V_{i,t}$ is a vector of portfolio managers' views (allowing portfolio managers to summarize their ideas as discretionary views that cannot be captured otherwise); and $\eta_{i,t}$ is a vector of residuals after the these sources of return have been taken into account.

To understand how strategic versus tactical allocation fits in this framework, it can be helpful to rewrite Equations 1 and 2 such that we break out persistent long-term components within factors ($\overline{F}_t$) and transitory short-term components within factors (ΔF_t). These components capture return opportunities at different investment horizons. By grouping the short-term component of the systemic factors with Equation 2 and suppressing the subscript i, we rewrite Equation 1 as the sum of strategic and tactical components:

$$R_t = \underbrace{B_t \overline{F}_t}_{\text{Strategic}} + \underbrace{B_t\, \Delta F_t + \Gamma_t I_t + V_t + \eta_t}_{\text{Tactical}} \qquad (3)$$

A FACTOR-BASED MULTI-ASSET-CLASS ALPHA FRAMEWORK

The title of this section is a mouthful, but it encapsulates the framework's purpose and premise. That is, it is a framework that allows us to blend factor forecasts with traditional asset class views. Such portfolios have the potential to outperform more standard asset allocation schemes because we are incorporating information about both factor and asset class future expected returns. This is a useful and flexible approach that allows information about factors and asset classes to be combined in a systematic and consistent way.

With factor exposures and return forecasts, we use the asset pricing model in Equation 3 to directly forecast future asset returns:

$$E(R_{t+1}) = B_t E(\overline{F}_{t+1}) + B_t E(\Delta F_{t+1}) + \Gamma_t E(I_{t+1}) + E(V_{t+1}) \qquad (4)$$

Although Equation 4 can be applied directly to compute expected asset returns, we want to have more flexibility in combining all types of factors into one framework with more control over the factor composition in the return prediction. Therefore, we propose an extension of the alpha construction methodology by Jones, Lim, and Zangari (2007) to multi-asset-class scenarios. This methodology allows more features to control the exposure composite explicitly in the alpha computation.

Our cross-asset-class alpha model framework contains the following steps, which are later described in detail:

1. Select macroeconomic and style factors.
2. Estimate asset class factor exposures to these factors.
3. Construct factor-mimicking portfolios.
4. Forecast factor-mimicking portfolio returns.
5. Create an optimal factor portfolio.
6. Infer asset expected returns.
7. Construct investable strategy portfolios.

Intuitively, the framework can be described as follows. We use information about which factors are the primary drivers of return in a multi-asset-class world. We link said factors to asset classes and use information we have about the future path of factor returns alongside the information we have about the future path of asset class returns. We use both sources of information to build the optimal portfolio.

We highlight the fact that our framework differs from those of Blyth, Szigety, and Xia (2016) and Greenberg, Babu, and Ang (2016) by converting the asset allocation problem from the factor space back to the asset space with a set of consistently derived expected returns. These expected returns are consistent with the risk premiums earned by the *optimal* factor exposures. This set of expected returns can then be used to construct strategies that satisfy different investment objectives.

DETAILS OF THE SEVEN-STEP FRAMEWORK VIA AN EMPIRICAL EXAMPLE

In this section, we describe the steps in detail and investigate whether our methodology can produce reasonable empirical results. This proof-of-concept analysis uses 10 representative asset classes and follows the procedure outlined previously to compute asset expected returns for tactical and strategic allocation. For one set of tactical expected returns, we also illustrate how discretionary views can be incorporated with a set of live trades placed by portfolio managers at our firm.[1] We run backtests with these sets of expected returns with practical constraints and compute the strategy performance.

The asset classes we employ are

1. Equities:
   a. US Large Cap (R1000)
   b. US Small Cap (R2000)
   c. US Real Estate Investment Trusts (REIT)
   d. Emerging Markets (EM)
2. Fixed Income:
   a. Lehman Aggregate (AGG)
   b. US High Yield (HYLD)
   c. Inflation-Linked Bonds (TIPS)
3. Commodities:
   a. Gold (GOLD)
   b. DJ Commodity Index (DJAIG)
4. Cash:
   a. US Cash (RF)

Exhibit 2 plots the cumulative returns of the 10 asset classes over the period January 2012 to December 2016.

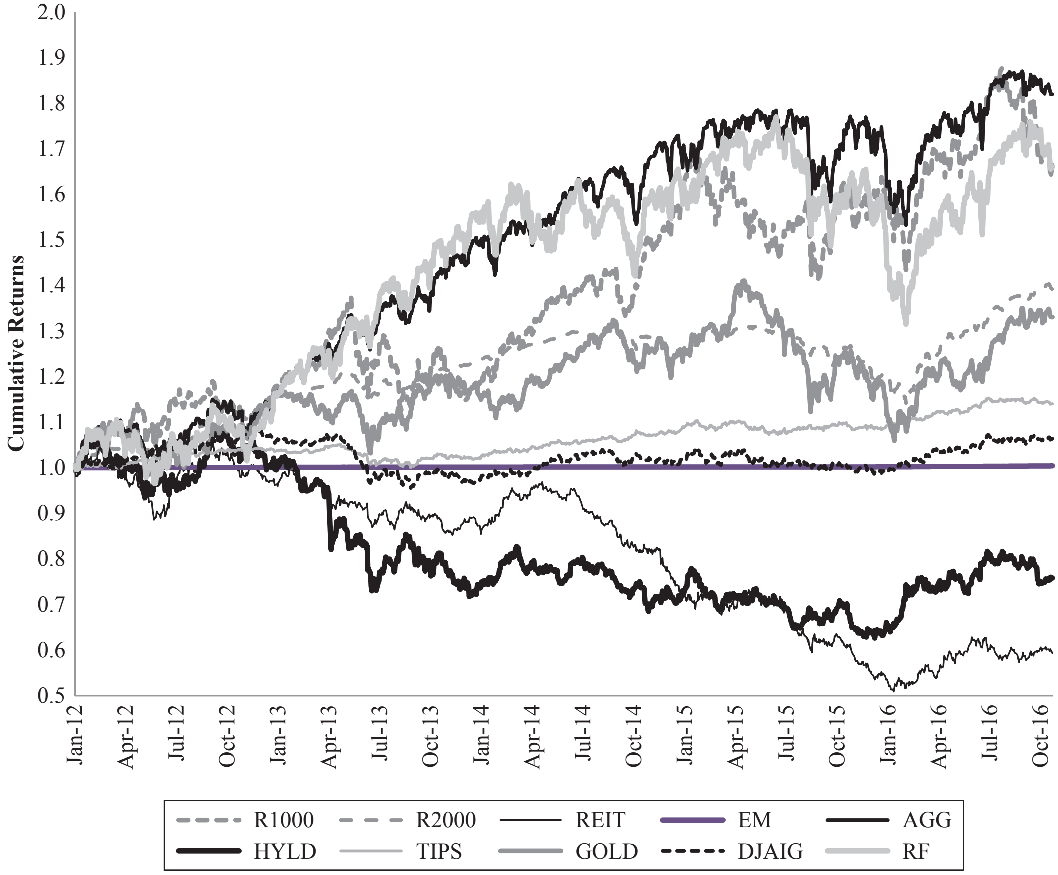
Step 1: Selecting State Variables and Style Factors

The factor-based framework starts by converting the asset allocation problem into a factor allocation problem. The first step is to select a robust set of factors that are both good at explaining the cross section of asset class returns and reasonably parsimonious.

Following Greenberg, Babu, and Ang (2016), we first list a set of macroeconomic factors. Next, we add style factors as done by Asness et al. (2015) and Brightman and Shepherd (2016) to the candidate list

[1] Specifically, every month, our portfolio managers trade a representative tactical asset allocation portfolio containing over two dozen asset classes at their discretion. We use their trades on the assets in our portfolio to form a zero-investment long–short portfolio that summarizes their discretionary views on these assets. We use trades over the period December 2011 to September 2016 for this illustration.

Exhibit 2
Asset Cumulative Returns (December 2011 to September 2016, Daily USD Returns)



Sources: SSGA, Bloomberg, and Datastream.

of factors. Exhibit 3 lists a full set of potential factors. We note that, to date, there is little research on combining macroeconomic and style factors. For example, which matters more and whether one camp subsumes or spans the other is still debated. For the idiosyncratic factors, these depend on the asset class. Exhibit 3 shows examples for macro factors, style factors, and idiosyncratic factors.

For the empirical illustration in this article, we choose the top three out of seven macro factors from Greenberg, Babu, and Ang (2016): economic growth (GRWTH), inflation (INFLTN), and real rates (REAL).[2] We add momentum (MMT) and volatility (VOL) from the set of four style factors used by Asness et al. (2015).[3] We select these style factors for ease of calculation and cross-asset-class consistency. We are not making a general statement as to which factors are optimal for inclusion; we limit our selection here for illustration purposes.

Step 2: Estimating Asset Class Factor Exposures

The transformation of assets to a factor is done through a factor-mimicking portfolio, which aggregates

[2] Note that the inflation factor is proxied by a time series that is positively correlated with deflation, as by Greenberg, Babu, and Ang (2016). Therefore, equity exposure to INFLTN is negative.

[3] For the purposes of this paper, we construct factors using basic definitions such as 12-month cumulative returns for momentum, and standard deviation of past 12-month daily returns for volatility. There are inevitable minor differences in design specifications but factor returns should be highly correlated with prior definitions.

Exhibit 3
Candidate Factors for Illustrative Example

Macro Factors	
Economic Growth	Risk associated with global economic growth *Broad-market equity index returns*
Real Rates	Risk of bearing exposure to real interest rate changes *Inflation-linked bond returns*
Inflation	Risk of bearing exposure to changes in nominal prices *Return of portfolio long nominal bonds, short inflation-linked bonds*
Credit	Risk of default or spread widening associated with financial distress *Return of portfolio long corporate bonds, short nominal bonds*
Emerging Markets	Risk that emerging sovereign governments will change capital market rules or general political risk in emerging markets *Basket of EM equity premiums, EM CDX, and EM FX*
Commodity	Risk associated with commodity markets *Weighted GSCI Commodity Index returns*
FX	Risk associated with exchange rate fluctuation *Trade-weighted dollar index*
Style Factors	
Value	Cheap assets tend to mean-revert *Asset-specific valuation spread portfolios*
Momentum	Asset prices trend tends to continue *Asset-specific momentum spread portfolios*
Carry	High-yielding assets earn higher return *Asset-specific yield spread portfolios*
Quality	Low-risk assets tend to outperform *Asset-specific volatility spread portfolios*
Idiosyncratic Factors	
Asset Flow	Large asset institutional flows generate sentiment *Aggregate fund flows by asset class*
Crowding	Crowding peaks lead crash *Within-asset-class pairwise correlation*
Statistical	Short-term statistical signals *Cointegrated spreads movement*

Source: SSGA, for illustrative purposes only.

assets with weights to capture the return premium of bearing the corresponding factor risk. To construct factor-mimicking portfolios, we estimate each asset class's factor exposures, ultimately allowing us to map exposures to weights.[4]

Generally, the exposures of asset classes to factors are relatively stable over a short horizon; therefore, we can use time-series regression to estimate the exposures. For longer windows, we can expect to see some changes in factor relationships. For example, equity and bond prices exhibited a positive correlation in the early 1980s, as disinflation boosted both markets. However, over much of the past 20 years bond prices have shown a negative monthly correlation to equities. For every asset in our model universe, we regress its total return (in USD) on the macro factor

[4] We can directly use factor exposures as asset weights to construct factor-mimicking portfolios, as suggested by Jones, Lim, and Zangari (2007), but we want to take into account the factor correlation among assets, so we need to scale exposures properly to reflect that consideration.

proxies in Exhibit 3.[5] There are two considerations in the exposure estimation. First, we want to maximize the explanatory power of the systematic component of asset class returns. Second, each asset class should only have meaningful exposures to relevant factors.[6] These regressions will provide us with the raw exposures on the macro factors. The raw exposures are standardized to have zero mean and unit standard deviation across all assets.

For the style factor exposures, we adopt the basic definitions from Asness et al. (2015). Because the style factor exposure calculation is different for each asset class, Asness et al. (2015) normalize (taking Z-scores) the raw exposure measures by asset class. For our needs, however, our goal is to create cross-asset-class factor-mimicking portfolios, so we need to compare factor exposures across asset classes. Therefore, we categorize style factors into two groups: one that relies on asset returns and one that includes asset-class-specific fundamental variables. For the return-based factors, we can directly compare factor exposure across asset classes, such as momentum and volatility. For factors that use asset-class-specific fundamental variables, such as value and carry, we compute factor exposure time-series percentiles or Z-scores, which are consistently comparable across asset classes.

Exhibit 4 shows the standardized exposures to the macro and style factors. Time-series regression using trailing 3 years of data is used to estimate asset exposure to the three macro factors. For the style factors, we compute trailing one-year cumulative returns as the exposure to momentum and trailing one-year asset return standard deviation as volatility. As an illustration,

Exhibit 4

Asset Exposures to Factors (short term, September 2016)

	GRWTH	INFLTN	REAL	MMT	VOL
R1000	1.14	–0.92	–1.18	0.63	0.34
R2000	1.43	–1.35	–1.26	0.63	1.20
REIT	0.62	0.06	0.53	–0.05	0.86
EM	0.52	–0.53	–0.59	0.40	0.17
AGG	–1.10	1.20	0.58	–0.50	–1.55
HYLD	–0.46	0.24	–0.48	1.08	–1.03
TIPS	–1.04	0.74	1.52	–0.28	–1.20
GOLD	–1.21	1.47	1.19	0.40	0.69
DJAIG	0.11	–0.92	–0.30	–2.32	0.52
RF	0	0	0	0	0

Source: SSGA, for illustrative purposes only.

Exhibit 4 shows an example of the factor exposure matrix for each of the assets in our investment universe.[7]

This framework can produce both tactical and strategic strategies. The difference in constructing tactical (TAA) versus strategic (SAA) allocations starts from the factor exposure estimation stage. The factor exposures are used to construct factor-mimicking portfolios. For TAA factor-mimicking portfolios, we use contemporaneous exposures to discover short-term factor exposure deviations from their equilibrium states. For SAA factor-mimicking portfolios, we use expanding window averages of short-term exposure estimates to capture the persistent relation between assets and factors.

Step 3: Constructing Factor-Mimicking Portfolios

Existing factor-based asset pricing theory links the underlying drivers of asset prices to a few factors. By exposing a portfolio to these factors in the present, assets earn risk premiums in the future; however, factors are not directly observable, nor are they investable. Thus, we create a portfolio of assets to mimic the return and risk exposure dynamics.

These factor-mimicking portfolios allow us to convert the asset allocation problem in the asset space into a factor allocation problem in the factor space. Factor-mimicking portfolios are the new building blocks.

[5] We define the universe of securities used in model estimation as the estimation universe and the universe of securities that we used to generate return predictions as the model universe. For different portfolios, the securities, when they can be invested in, form the investment universe. The estimation universe is a subset of the model universe, but it can be identical to the model universe. The investment universe is usually a subset of the model universe.

[6] Other regression models can be used here as well. Greenberg, Babu, and Ang (2016) and Bass, Gladstone, and Ang (2017) used constrained stepwise regressions to determine the factors that best describe asset class returns. Blyth, Szigety, and Xia (2016) outlined a quasiquantitative process to determine factor exposures in which regression coefficients are modified according to economic priors. Furthermore, we can use Bayesian regression to produce posterior coefficient estimates that incorporate these economic priors.

[7] Note that our inflation proxy is deflation; therefore, equity assets and commodities have negative exposure to INFLTN.

Moreover, we can use returns from factor-mimicking portfolios to assess whether risk premiums in these factors can be captured with assets in the model universe.[8] These returns also provide sample data to study the probability distribution of factor returns, which helps us to make return predictions.

We have several options in constructing factor-mimicking portfolios:

- **Sorted portfolios:** The first approach is to sort all securities in our estimation universe into quintile portfolios by using an exposure metric and then to form a top-minus-bottom zero-investment spread portfolio. The factor-mimicking portfolios constructed in this way can be noisy but simple.
- **Optimization:** The second approach relies on an optimizer. The setup of the optimization problem is not unique. As an illustration, to create a factor-mimicking portfolio, the optimizer solves the following problem

$$\max_h h'\beta_i$$

$$s.t. h'\sum h = 1 \tag{5}$$

$$|h'\beta_i| \leq \tau, i \neq j$$

where h is a vector of weights of the factor-mimicking portfolio for macro factor i; Σ is the variance–covariance matrix of the estimation universe; and β_i is the exposure to macro factor i. The solver maximizes the portfolio's exposure to factor i but constrains exposures to other factors at tolerance τ.
- **Factor model:** The third approach adapts the portfolio approach of Jones, Lim, and Zangari (2007) and can be viewed as an extended Fama–MacBeth regression. Let's assume that we can express asset returns in the following factor model

$$R = FB + \varepsilon$$

where R is the vector of asset returns in the model universe, B is the exposure matrix of the assets in the model universe to the macro factors in Exhibit 1, and F is the return vector of the factor-mimicking portfolios to all macro factors.
With the Σ defined earlier, we can solve for F as a generalized least square coefficient:

$$\widehat{F} = \left[B'\sum^{-1}B\right]^{-1} B'\sum^{-1}R \tag{6}$$

This implies that the factor-mimicking portfolios P can be expressed as

$$P = \left[B'\sum^{-1}B\right]^{-1} B'\sum^{-1} \tag{7}$$

For this article, we use the *factor model* approach to create the factor-mimicking portfolios. For every factor in Exhibit 3, we compute its factor-mimicking portfolio following Equation 7. Factor-mimicking portfolios constructed in this way are unit exposure to a particular factor and zero exposures to the other factors. For each factor, we thus obtain a zero-investment unit factor exposure portfolio of assets. In addition, the discretionary view can also be expressed as a zero-investment portfolio.[9]

By repeating construction of the factor-mimicking portfolios in each period, we create a time series of portfolio returns. The cumulative returns of the tactical factor-mimicking portfolios are plotted in Exhibit 5, and strategic counterparts are plotted in Exhibit 6. The performance numbers of these factor-mimicking portfolios are summarized in Exhibits 7 and 8.

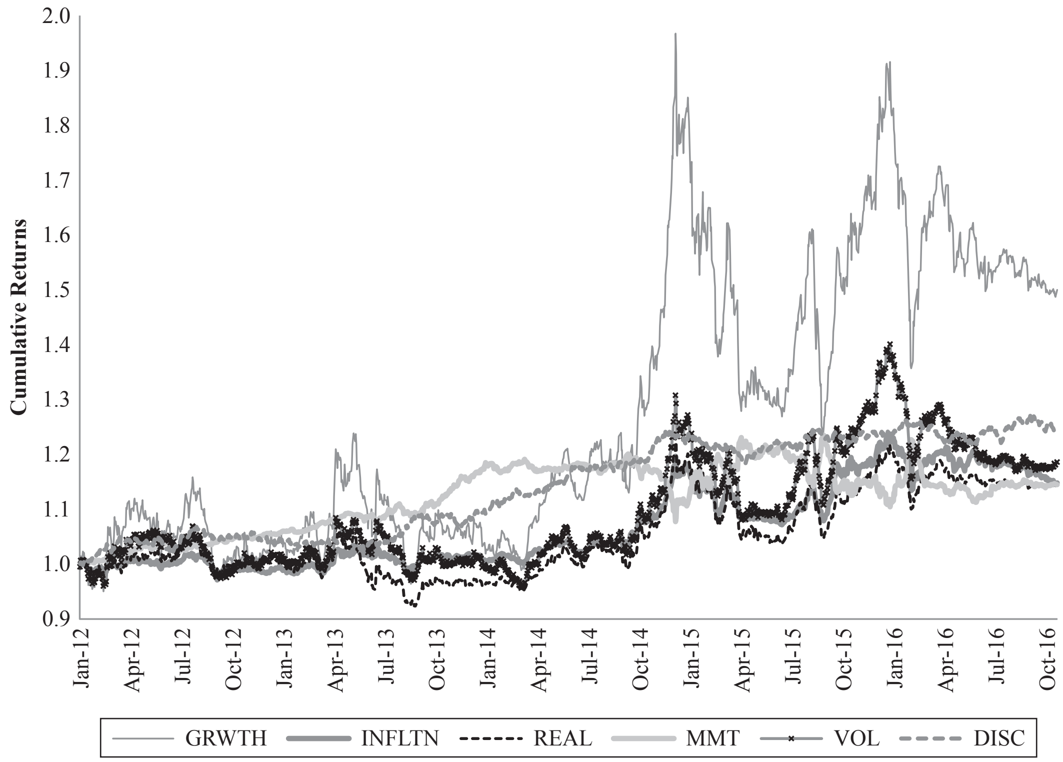
The performance of a factor-mimicking portfolio is crucial for us to understand whether (1) the factor has a premium; (2) such a premium can be replicated with the assets we can invest; and (3) the factor premium exists conditional on other factors. As for the discretionary-view portfolio, positive performance is a necessary condition. Exhibits 7 and 8 demonstrate the efficacy of the factor-mimicking portfolios in capturing factor premiums. Several observations are apparent from the data. First, all of the factors from both the tactical and strategic viewpoints show positive excess return. This suggests a positive return premium for bearing a

[8] The returns earned by factor-mimicking portfolios help us to test whether there are risk premiums among these assets. If there are no returns, these factors may serve as control factors, which does not mean they are not useful.

[9] Although it is hard to adjust the exposure level, the discretionary portfolio can be normalized to have a predetermined level of risk or, with some return assumption, to have a certain level of risk-adjusted return.

E X H I B I T 5

Tactical Asset Allocation Factor-Mimicking Portfolio Returns (December 2011 to September 2016, Daily USD Returns)



Source: SSGA, for illustrative purposes only.

systematic risk, particularly for the macro factors. The style factors also show a positive return premium, and these results are consistent with prior studies.

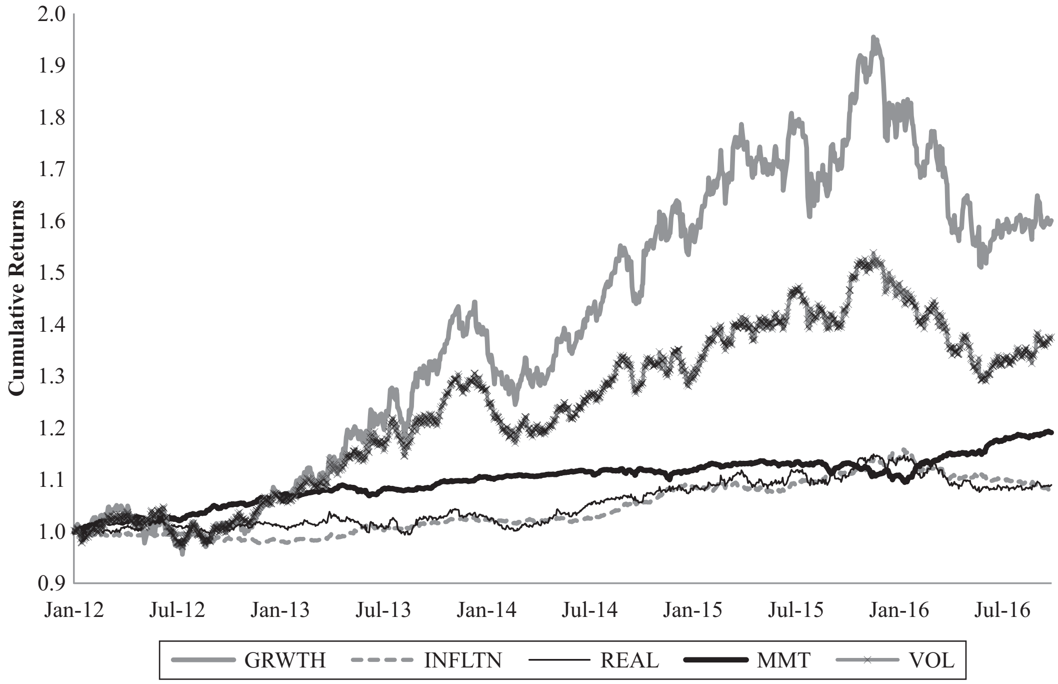
In addition, we see considerably more volatility in the performance of the tactical factors when compared to the strategic factors. This is most apparent when comparing the cumulative return of the growth factors of Exhibits 5 and 7. This difference is explained by the differences in the construction method. Whereas the strategic portfolios use an expanding window and averages short term exposures, the tactical portfolios are focused on short-term contemporaneous exposures, thereby leading to less stability in the formation of the portfolios.

Step 4: Forecasting Factor-Mimicking Portfolio Returns

With factor-mimicking portfolios as the new building blocks, we next create an optimal factor portfolio. As an illustration, our investment objective is to maximize the risk–return trade-off. To do so, we need to forecast factor-mimicking portfolio returns. By iterating the factor-mimicking portfolio calculation in Equation 7 for every time period and multiplying the portfolio weights with asset returns, we can generate return series for these factor-mimicking portfolios. The return data can help us to understand the distribution property of the factor-mimicking portfolio returns and make predictions.

- **Time-series forecast:** We can fit the factor-mimicking portfolio returns with time-series models. In the simplest way, we can use historical average factor portfolio returns as future return forecasts.
- **Economic factor-based forecast:** Because factor-mimicking portfolios are essentially proxies for factor premiums that can be influenced by macroeconomic

EXHIBIT 6
Strategic Asset Allocation Factor-Mimicking Portfolio Returns (December 2011 to September 2016, Daily USD Returns)



Source: SSGA, for illustrative purposes only.

EXHIBIT 7
Tactical Asset Allocation Factor-Mimicking Portfolio Performance Summary (September 2016, Daily USD Returns)

	GRWTH	INFLTN	REAL	MMT	VOL	DISC
Ann. Return	8.8%	2.9%	2.8%	2.8%	3.6%	4.6%
Ann. Std. Dev.	24.8%	6.5%	9.8%	6.5%	12.0%	4.7%
Sharpe Ratio	0.35	0.45	0.29	0.44	0.30	1.00

Source: SSGA, for illustrative purposes only.

EXHIBIT 8
Strategic Asset Allocation Factor-Mimicking Portfolio Performance Summary (September 2016, Daily USD Returns)

	GRWTH	INFLTN	REAL	MMT	VOL
Ann. Return	10.3%	1.7%	1.8%	3.7%	6.8%
Ann. Std. Dev.	13.2%	2.5%	4.5%	1.9%	9.2%
Sharpe Ratio	0.78	0.66	0.40	2.00	0.74

Source: SSGA, for illustrative purposes only.

Exhibit 9

Factor-Mimicking and Discretionary Portfolio Returns and Covariance Matrix (December 2011 to September 2016, Daily USD Returns)

		Covariance Matrix					
	Avg. Ann. Ret.	GRWTH	INFLTN	REAL	MMT	VOL	DISC
GRWTH	7.7%	0.07	0.018	0.024	–0.016	0.032	–0.001
INFLTN	2.3%	0.018	0.005	0.006	–0.005	0.008	0
REAL	3.3%	0.024	0.006	0.009	–0.006	0.011	0
MMT	0.4%	–0.016	–0.005	–0.006	0.006	–0.007	0
VOL	2.1%	0.032	0.008	0.011	–0.007	0.017	0.001
DISC	1.5%	–0.001	0	0	0	0.001	0.002

Source: SSGA, for illustrative purposes only.

factors, we can further build factor models to forecast factor-mimicking portfolio returns.

For the purposes of this study, we adopt the simplest approach to forecast factor-mimicking portfolio returns, which is to use historical averages. Once again, to differentiate long-term versus short-term forecasts, we use trailing one-year factor-mimicking portfolio returns as the short-term forecast and expanding window averages as the long-term forecast. We also compute the covariance matrix with similar history considerations. Exhibit 9 provides an example of these estimates.

Step 5: Constructing the Optimal Factor Portfolio

The optimal factor portfolio expresses the *optimal* mix of factor exposures by reweighting factor-mimicking portfolios to meet some objective, such as a return target or expected risk level. The discretionary view portfolio can also be weighted along with the other factor portfolios.[10]

Three common objectives are as follows:

- **Minimum risk:** We select a set of weights on the factor-mimicking portfolios that minimizes the factor portfolio volatility. This can be achieved by solving the following optimization problem

$$\min_{\lambda} \lambda'\Omega\lambda$$

$$s.t. \quad \lambda'\iota = 1 \tag{8}$$

$$\lambda > 0$$

where ι is a vector of 1s, λ is a vector of factor weights, and Ω is the variance–covariance matrix of factor-mimicking portfolio returns. We put constraints on the factor weights so that they are positive and sum to 1.[11] This can be the setup we use to construct a strategic asset allocation.

- **Targeted factor exposure:** Portfolio exposures to factors can be controlled in a way that is consistent with top-down factor exposure decisions. This is similar in spirit to the approaches used by Blyth, Szigety, and Xia (2016) and Greenberg, Babu, and Ang (2016).
- **Maximum risk-adjusted returns:** The optimal factor weights solve the following optimization problem

$$\max_{\lambda} \lambda'\mathcal{Q} - \frac{1}{2}\lambda'\Omega\lambda \tag{9}$$

subject to a set of desirable constraints. $\mathcal{Q}$ is the expected factor-mimicking portfolio return vector. We leave out the specific constraints because we can have different prescriptions here based on different applications. For example, we

[10] In this subsection, without loss of generality, we do not differentiate quant factors versus discretionary views in the optimal factor portfolio construction.

[11] We assume all factor premiums are positive.

Exhibit 10
Optimal Factor Portfolio Weights (September 2016)

	TAA QUANT	TAA DISC
GRWTH	15%	–19%
INFLTN	46%	6%
REAL	42%	85%
MMT	50%	126%
VOL	–53%	–153%
DISC		55%

Source: SSGA, for illustrative purposes only.

can set them in the same way as in Equation 8 or let the optimal factor portfolio be an efficient zero-investment portfolio as done by Jones, Lim, and Zangari (2007).

For this study, we adopt the third approach; in every period, we maximize the risk-adjusted returns as in Equation 9 with the return and risk assumptions shown in Exhibit 9. Although we believe that assets earn positive factor premiums over the long run, we want to exploit short-term dislocations through tactical positioning. Therefore, we do not constrain the factor weights in the tactical optimal factor portfolio. For the strategic optimal factor portfolio, weights are constrained to be non-negative. In both cases, all weights must sum to 1. An example of the tactical optimal factor portfolio is reported in the TAA QUANT column of Exhibit 10. The TAA DISC column includes the discretionary view portfolio along with the factors.

Step 6: Inferring Expected Returns for Asset Classes

The optimal factor portfolio can be expressed as the product of the optimal factor weights λ and the factor-mimicking portfolio asset weights P. This portfolio represents an ideal portfolio held by a representative investor in a frictionless economy with expected asset returns α. Defining the variance–covariance matrix of all assets as Σ, the relation between ω_{OFP} and α can be described by the following equations:

$$\begin{aligned}\omega_{OFP} &= P'\lambda \\ &= \Sigma^{-1}\alpha\end{aligned} \tag{10}$$

Exhibit 11
Optimal Factor Portfolio and Implied Alpha (September 2016)

	Weights	Alpha
R1000	6.0%	4.7%
R2000	–0.9%	4.0%
REIT	0.5%	8.0%
EM	2.1%	2.7%
AGG	51.1%	1.7%
HYLD	–9.1%	–0.5%
TIPS	–18.2%	1.5%
GOLD	–1.2%	–0.4%
DJAIG	0.1%	–2.2%
RF	–30.4%	0.0%

Source: SSGA, for illustrative purposes only.

The implied expected returns then can be computed as

$$\alpha = \Sigma P'\lambda \tag{11}$$

We convert factor weights in Exhibit 10 to asset weights using Equation 10. With Equation 11, we can convert portfolio weights to alpha. Both the weights and the alpha scores are shown in Exhibit 11. The implied alphas are also annualized, using the risk model.

Note that we can apply a portfolio formed from fundamental views (described earlier in the article) to this step as well. We could do this by expanding P and Ω to reflect the fundamental portfolio's historical asset returns, volatility, and correlation with other factor views.

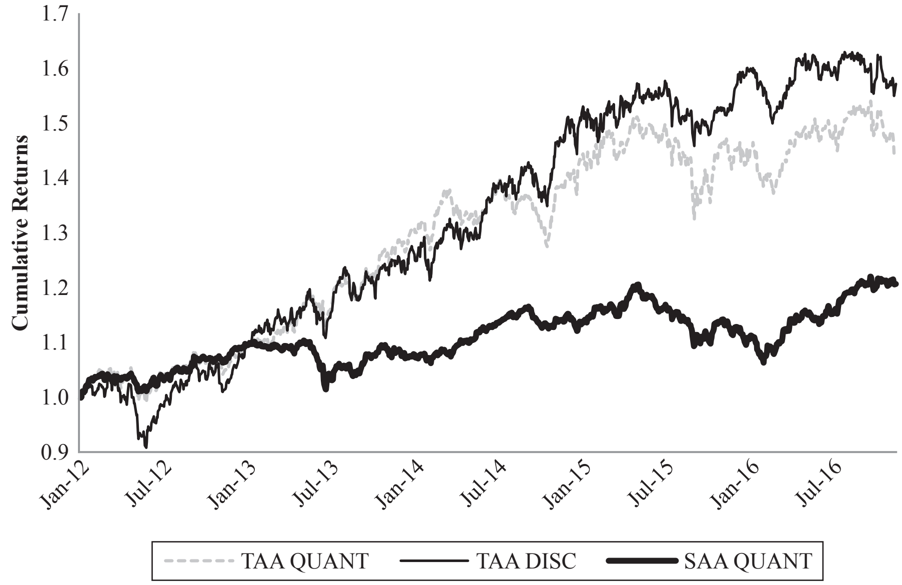
Step 7: Constructing the Final Optimal Portfolio

In the final step, the inferred asset class expected returns in the previous step are used to construct the final portfolio. Different objective functions can again be used, depending on the investment objective. Here, we maximize risk-adjusted return in keeping with how we constructed the optimal factor portfolio. The final portfolio is long-only; no leverage is used.

We use the process described earlier to produce three sets of alphas from December 2011 to September 2016. The three sets of alphas are SAA QUANT (uses

E X H I B I T 1 2

SAA Factor-Mimicking Portfolio Returns (December 2011 to September 2016, daily USD returns)



Source: SSGA, for illustrative purposes only.

long-term factor exposures and tighter constrained factor exposures), TAA QUANT (uses short-term factor exposures and more relaxed constraints), and TAA DISC (augments TAA QUANT with a discretionary view). At every month-end, we run a rebalance by maximizing the risk-adjusted return and constraining asset weights between 1% and 35% and summed to 1. For simplicity, we rebalance from cash every month. This also allows us to see the average turnover each strategy would incur.

Exhibit 12 plots the cumulative returns of these strategies over the period of December 2011 to September 2016. Although this example is meant to be illustrative and the time period short, all three strategies have produced reasonable risk-adjusted returns, with Sharpe ratios ranging from 0.82 to 0.94. As shown in Exhibit 13, the annualized one-way turnover of the strategic strategy is very low, at 23%. Furthermore, the tactical strategies still generate reasonable information ratios (relative to the strategic allocation), ranging from 0.52 to 0.75.

CONCLUSION

Our proposed multi-asset-allocation framework borrows from the existing factor-based asset allocation work that has emerged in recent years and extends it to a fully integrated factor-asset-class framework. Specifically, our approach extends the factor-based asset allocation framework of Blyth, Szigety, and Xia (2016) and Greenberg, Babu, and Ang (2016) with the alpha construction methodology of Jones, Lim, and Zangari (2007) to integrate factor allocation and asset class return prediction. Our innovations can be summarized as follows:

- We transform the asset allocation problem into a factor allocation problem in a way that allows information/forecasts of factors *and* asset classes to

E X H I B I T 1 3

SAA and TAA Strategy Performance (December 2011 to September 2016, daily USD returns)

	SAA QUANT	TAA QUANT	TAA DISC
Ann. Return	4%	8%	10%
Ann. Standard Dev.	5%	9%	10%
Sharpe Ratio	0.82	0.87	0.94
Turnover	23%	229%	240%
Excess Return		4%	6%
Tracking Error		8%	8%
Information Ratio		0.52	0.75

Source: SSGA, for illustrative purposes only.

be used in conjunction; the framework for translating factor forecasts into asset class forecasts is straightforward and intuitive.

- By using long-term and short-term exposures, the framework can identify strategic and tactical opportunities from systematic factors.
- Because systematic factors only partially characterize asset returns at short horizons, we add idiosyncratic factors; this hybrid structure allows room to leverage existing asset-class-specific insights omitted by the systematic risk factors.
- Discretionary views can be blended in the alpha estimates; the blending process provides a systematic way to express discretionary views by considering the risk and reward of the discretionary views conditioned on the quantifiable factors.
- The process produces consistent expected returns for all assets in the model universe, allowing different investment strategies to be created in a flexible manner.

The approach marries the insights of factor allocation with traditional asset allocation. With the caveat that information for both asset classes and factor forecasts is equally robust, this framework can result in optimal allocations that blend insights from both paradigms.

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