ECE410: Linear Control System Lab 2

Linear Control Systems Position Tracking for the Cart System

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1 Introduction

In this lab, we built the plant model of the pendulum cart system and designed two controllers to make the cart system track a square wave reference signal. In section 2, we design a controller that is based on state feedback stabilization. In section 3, we design a controller that uses output feedback stabilization and an observer to estimate state. In section 4, we discuss the optimal control value for output feedback stabilization implemented on the physical cart system. We control the cart via the motor voltage input and use the motor force and friction force to model cart position.

The mathematical expression of the pendulum cart is:

$$M\ddot{z} = F - B\dot{z}$$

For motor force, we will ignore the transient dynamics of the motor armature and simply assume that:

$$F = \alpha_1 u - \alpha_2 \dot{z}$$

After simplifying we have:

$$M\ddot{z} = \alpha_1 u - \alpha_2 \dot{z}$$

We define that:

$$x = [z\ \dot{z}]^T$$

We arrive at the plant model: $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-\alpha_2}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{\alpha_1}{M} \end{bmatrix}$

$$y = [\,1 \quad 0]x$$

Also, we keep in mind that some given parameters:

$$\alpha_1 = 1.7265\frac{N}{V}$$

$$\alpha_2 = 10.5\frac{Ns}{m}$$

$$M = 0.8211kg$$

2 State Feedback Stabilization

For State Feedback stabilization part, we define controller as:

$$u = Kx = [K_1 \; K_2]x = K_1 z + \; K_2 \dot{z}$$

$$\dot{x} = (\, A + BK)x$$

With the formula keep in mind, we calculate:

$$K \; = \; [-11.889, 1.326]$$

With the pole of $A + BK$ at {-5, -5}

Here we also define a desired equilibrium state $x_d = \begin{bmatrix} z_d \\ 0 \end{bmatrix}$

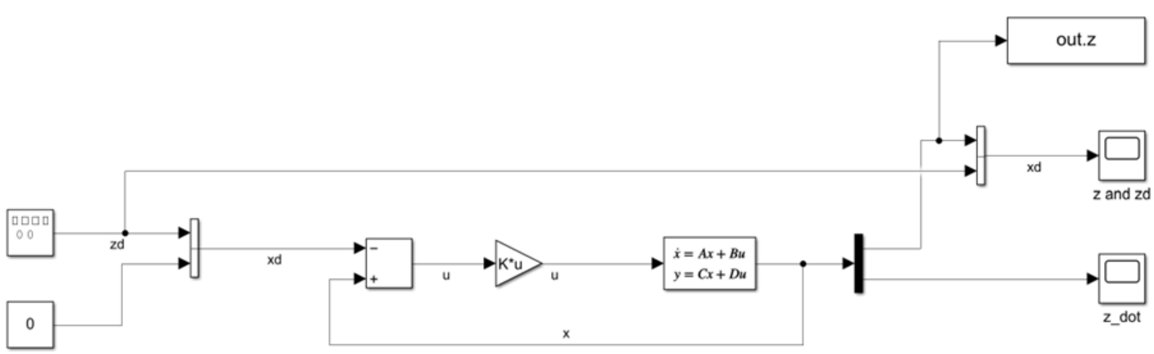
$$\dot{e} = Ax + Bu$$

$$\dot{e} = (A + BK)e$$

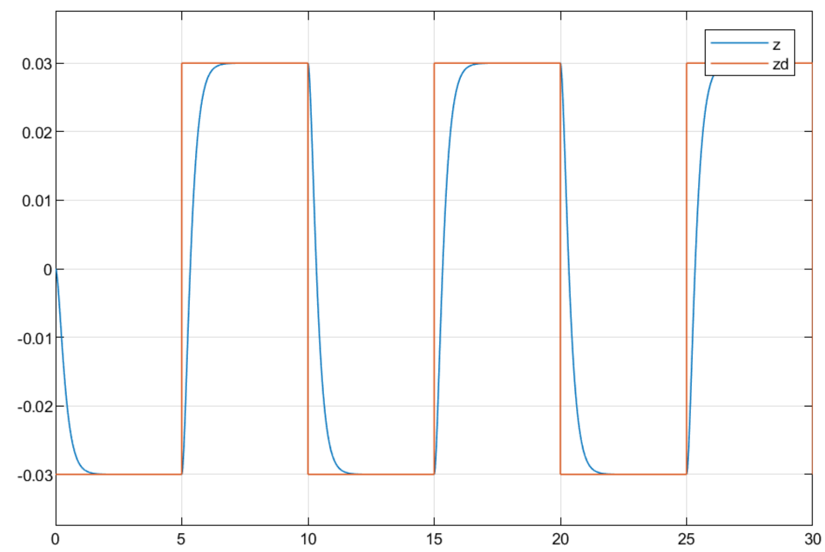
We finally get the formula:

$$u = K\begin{bmatrix} z - z_d(t) \\ \dot{z} \end{bmatrix}$$

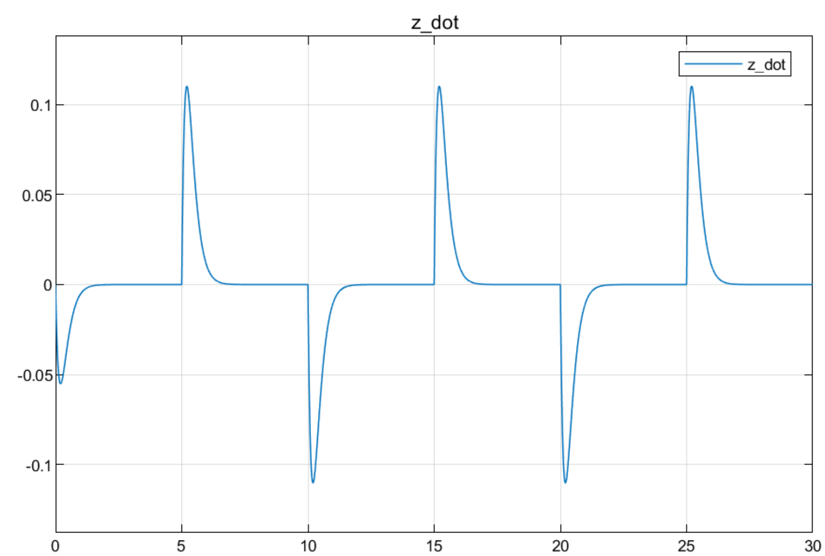
The Simulink model is:



The simulation plot for $z \; and \; z_d$:



The simulation plot for $\dot{z}$:



We use the cursor tool to measure the time for settling,

For eigenvalues at {-5, -5} $T_{s1} = 1.167 * 10^{-3}s$

For eigenvalues at {-2, -2} $T_{s2} = 2.905 * 10^{-3}s$

The ratio between them is $T_{s1}/T_{s2} = 0.401$

Which is the ratio we expect

We know that eigenvalues are responding for the settling time and eigenvalues far from origin will converge to target quickly, $\frac{p_5}{p2} = \frac{5}{2} = \frac{T_{s2}}{T_{s1}}$ which are also the ratio given in the handout

3 State Estimation and Output Feedback Control

Here we use an observer defined as:

$$\dot{\hat{x}} = A\hat{x} + Bu + L\,(y - C\hat{x})$$

We also know that:

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

After plug in all the equations, we get the final formula:

$$\dot{\hat{x}} = A\hat{x} + Bu + L\,(y - C\hat{x}) + Ly - BKx_d$$
$$u = K\,(\hat{x} - x_d(t))$$

We define:

$$\dot{\hat{x}} = A_{ctrl}\hat{x} + B_{ctrl}[\begin{matrix} y \\ x_d \end{matrix}]$$

$$u = C_{ctrl}\hat{x} + D_{ctrl}[\begin{matrix} y \\ x_d \end{matrix}]$$

For p_controller in [-2, -2] and p_observer = [-20, -20], we get:

$$[K1\ K2] = [-1.902\,, 4.179]$$

$$[L1\ L2] = [27.2\,, 52.01]$$

For the matrix about A_{ctrl} B_{ctrl} C_{ctrl} D_{ctrl}

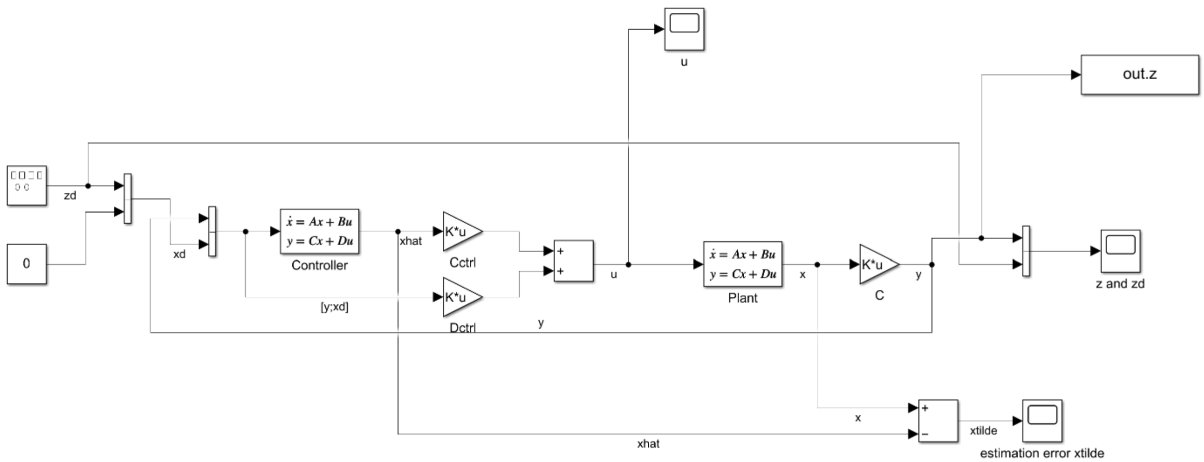
$$A_{ctrl} = \begin{bmatrix} -27.2 & 1 \\ -56 & -4 \end{bmatrix}$$

$$B_{ctrl} = \begin{bmatrix} 27.2 & 0 & 0 \\ 52.01 & 4 & -8.78 \end{bmatrix}$$

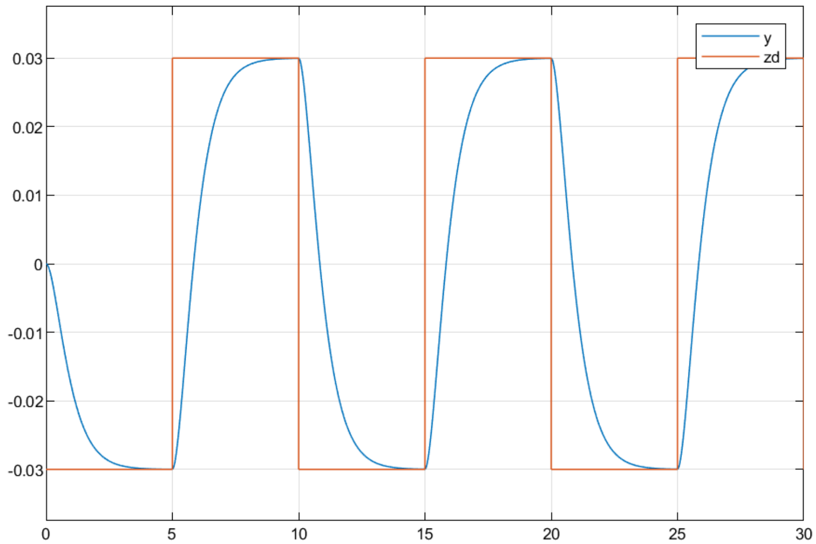
$$C_{ctrl} = [-1.902 \quad 4.179]$$

$$D_{ctrl} = [0 \quad 1.902 \quad -4.179]$$

The Simulink model for this controller is shown below:



The simulation plots from the three scopes are shown below:



y
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0.03
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0.01
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-0.01
-0.02
-0.03
0
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20
25
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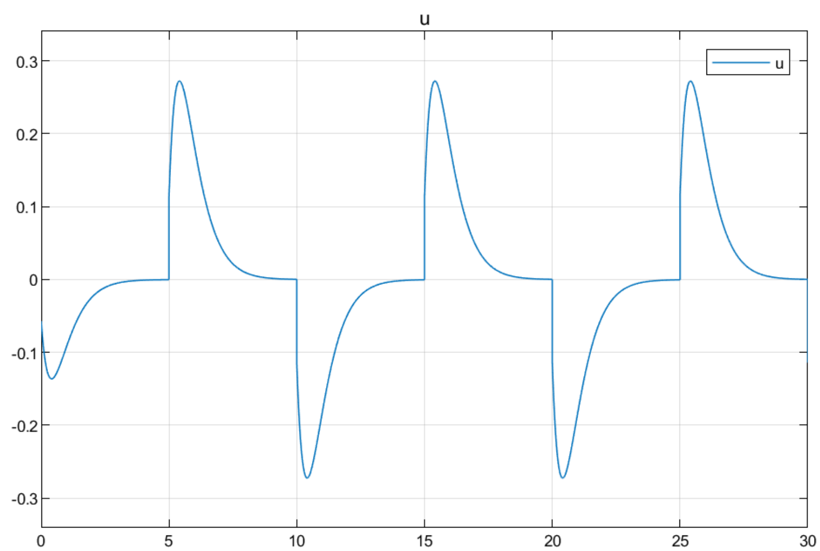

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0.3
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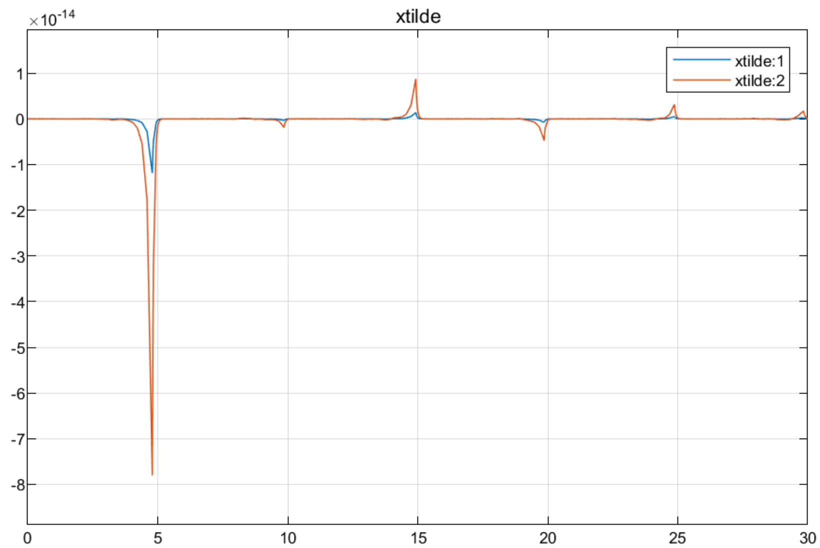
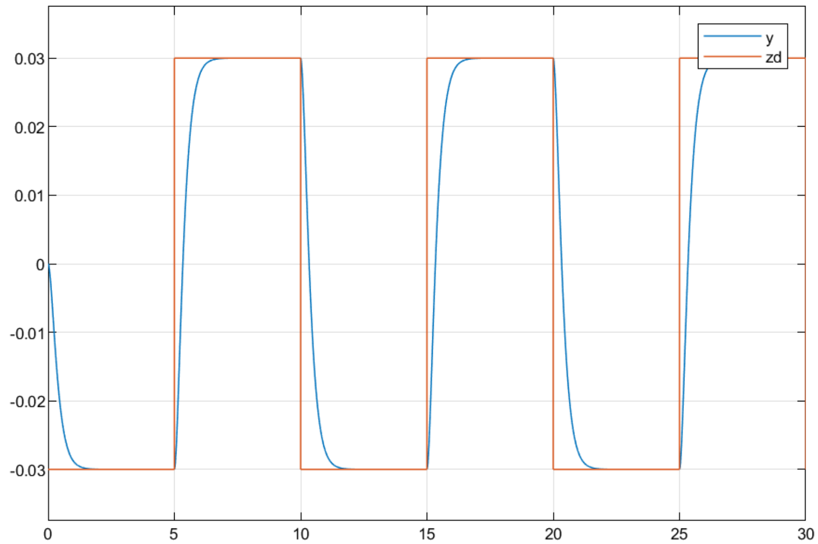


We then repeat the simulation plots with p_feedback = [-5; -5] p_observer = [-20; -20]



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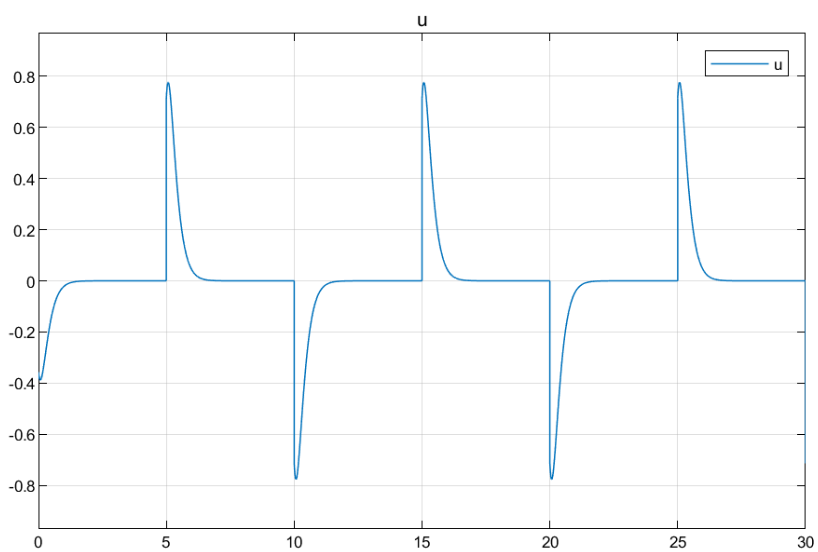

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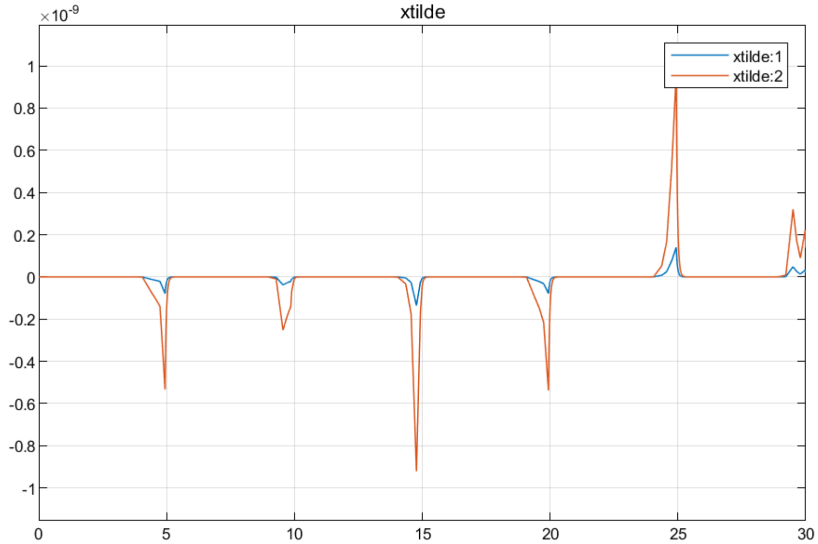
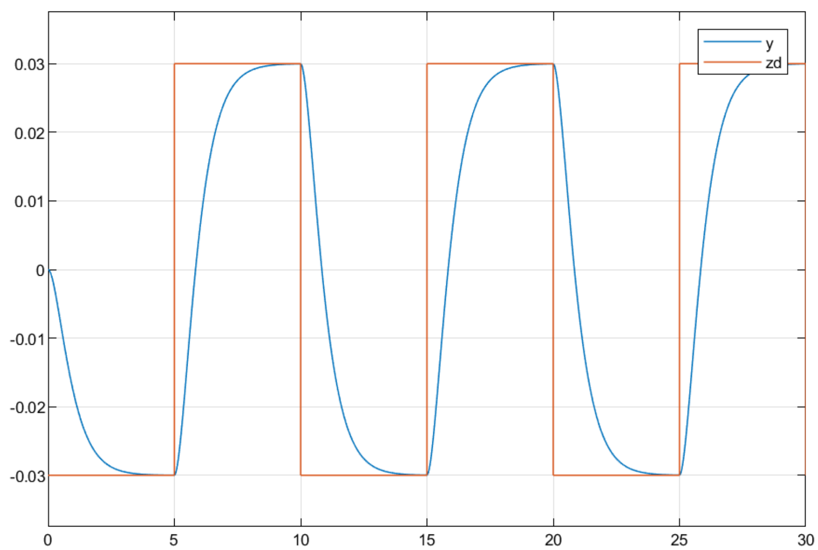
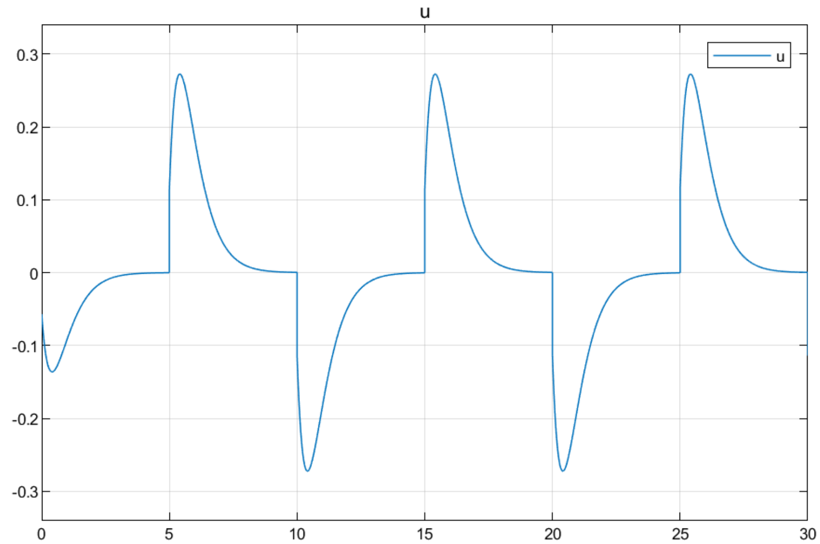
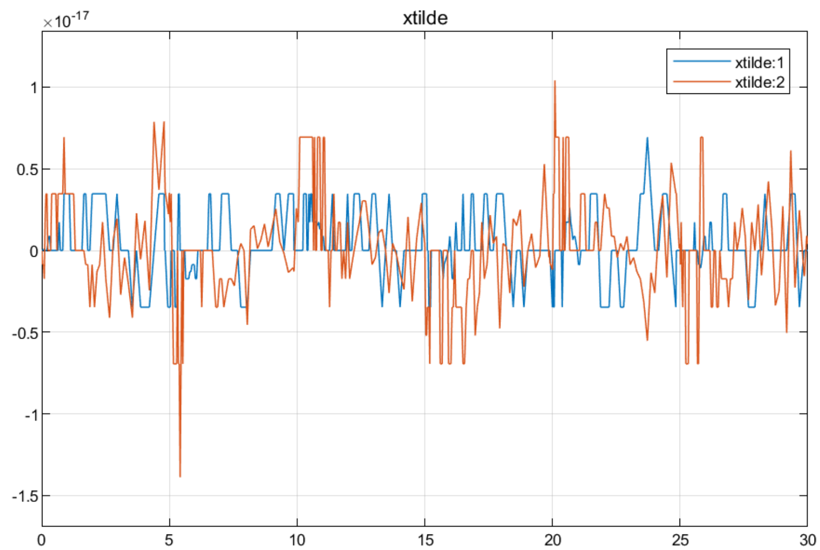


Through the plots above, we noticed that after we changed p_feedback from [-2; -2] to [-5; -5], z follows more closely to zd; thus, the tracking error decreased. Furthermore, the peak value of the motor voltage magnitude u, increased from 0.3 to 0.8. This is expected as the motor needs more power to follow the reference signal more closely.

Then, we repeat with p_feedback = [-2, -2] p_observer = [-10 -10]







Through the plots above, we noticed that after we changed p_feedback from [-20; -20] to [-10; -10], we find out that state estimation error is smaller magnitude than previous.

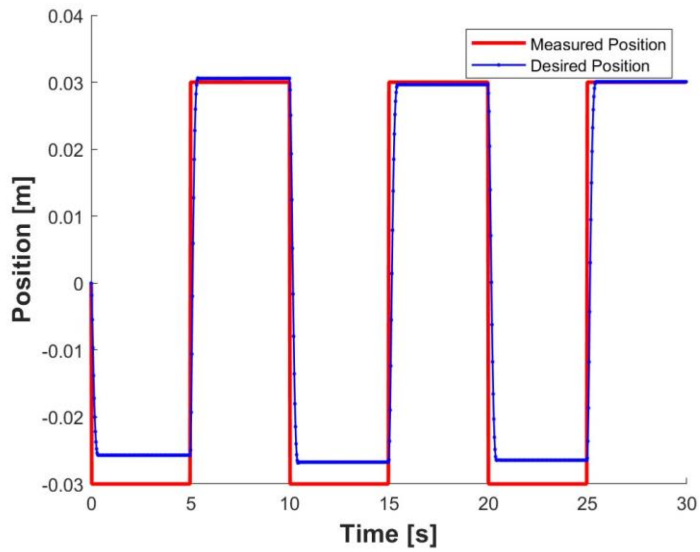
4 Physical Experimentation

In the physical experiment, we started with p_feedback = [-2, -2] p_observer = [-10 -10] and noticed that the cart doesn't have enough force to reach the desired reference signal. We then decided to decrease p_feedback value to find a better performing controller because simulation of a smaller p_feedback value has shown smaller tracking error.

During our trial-and-error approach, we made following observations:

1) p_feedback with smaller poles from –10 to –15 help the model to have smaller tracking error. However, p_feedback smaller than –15 causes abnormal sound of the motor. Thus, we stopped at –15 to avoid damaging the motor.
2) p_observer didn't bring significant improvement in reducing tracking error. Therefore, we kept the value same as our simulation to avoid too much unstableness in the system.

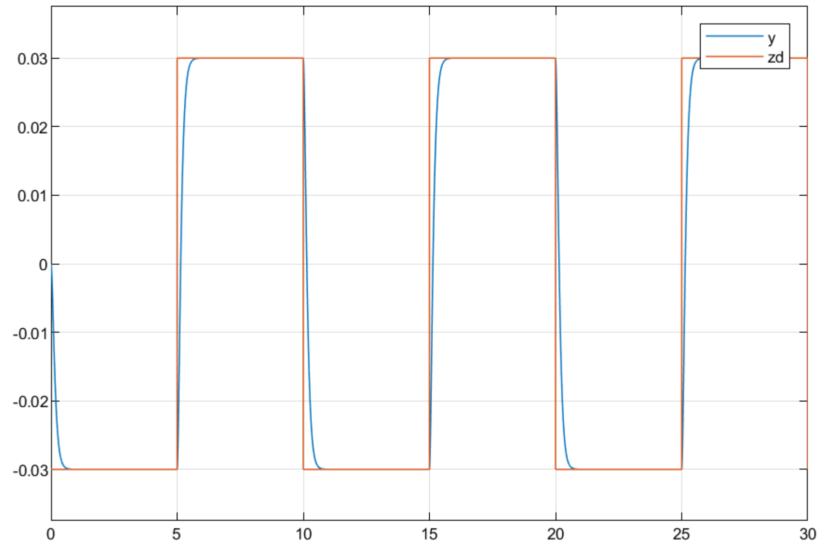
Finally, the optimized solution we find is for p_feedback is [-12, -12] and p_observer = [-20, -20]



After observing through the figure we export from matlab, we find out that $T_s = 0.35s$

From the graph, we can see that the Desired position and Measured position have relatively small error and also the car run smoothly without any noise in the process. For the part lower than 0,we believe part of the reason is that the track is slightly tilted, causing the cart to unable to reach desired location because our plant modeling assumes ideal track condition. This error is consistent in every test we have. Overall, this controller is satisfactory because it can track reference signals without damaging the motor.

The simulation plot for p_feedback = [-12, -12] and p_observer = [-20, -20] is: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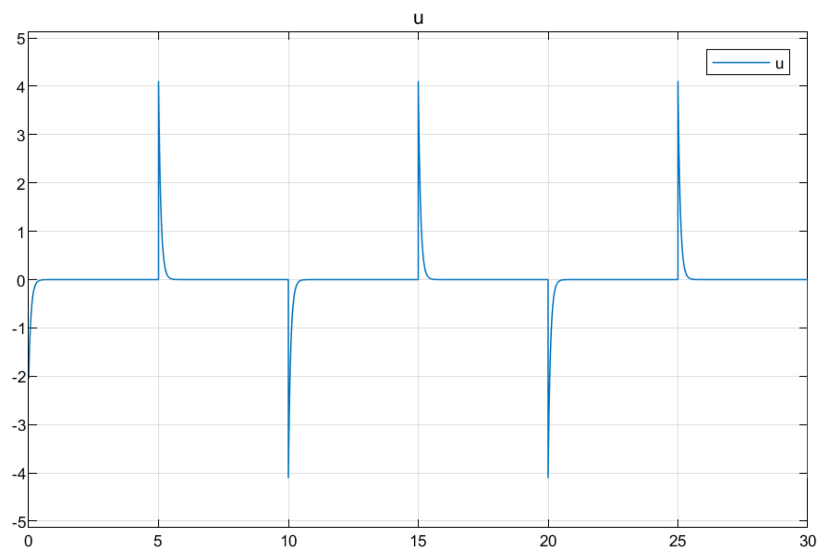
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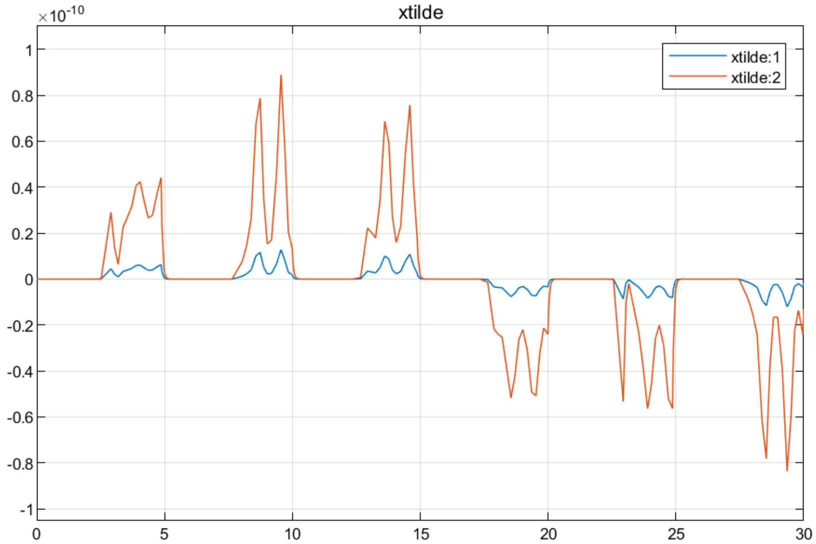

Here in the simulation, we see that the cart is able to almost perfectly track the reference signal. Because the simulation environement is ideal, it doesn't have issues such as uneven track cart that may impact the cart position. The simulation result is expected, because in previous analysis, we have shown that cart systems with smaller p_feedback perform better in tracking reference signal. According to the simulation, the settling time is around 0.5 second, which is not far away from the experimental settling time of 0.35 second.