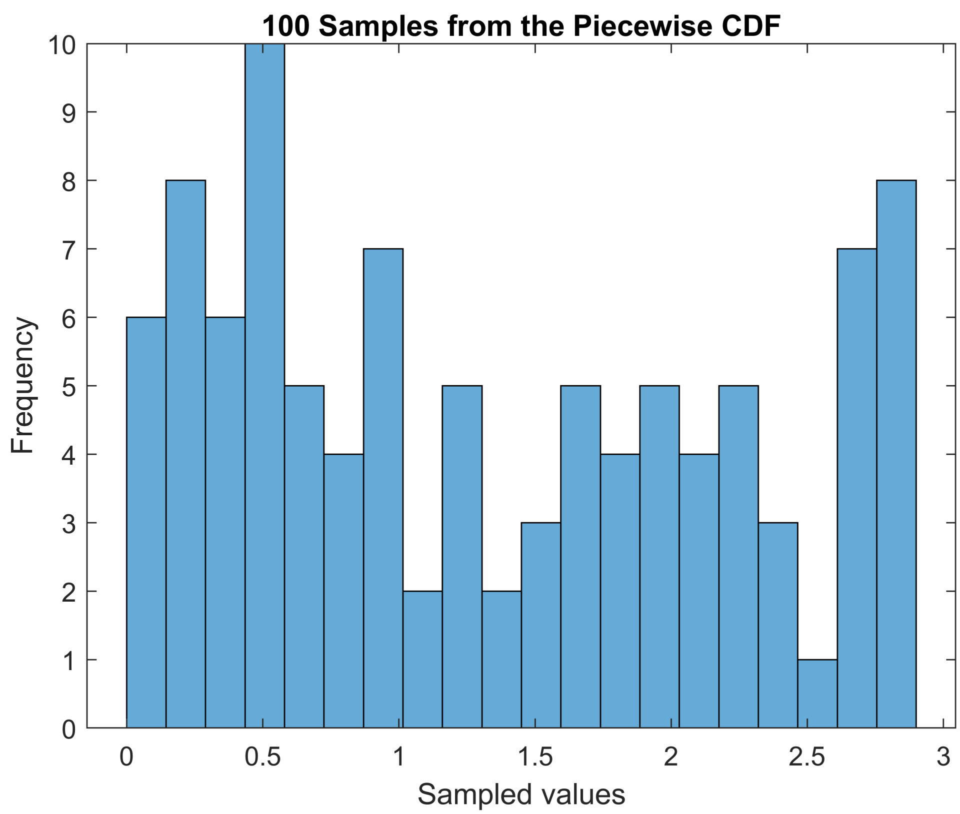
ECE537 Programming Assignment 1

Yuan Sui

1a) Generate N=100 independent sample and plot histogram

```
n = 100;

U = rand(n, 1);

samples = F_inv(U);

average = mean(samples);
variance = var(samples);
histogram(samples, 20); % 10 bins for visualization
xlabel('Sampled values');
ylabel('Frequency');
title('100 Samples from the Piecewise CDF');
```



1b) Expected value is 1.25, emprical average is:

```
fprintf('Average: %.4f\n', average);
```

```
Average: 1.3452
```

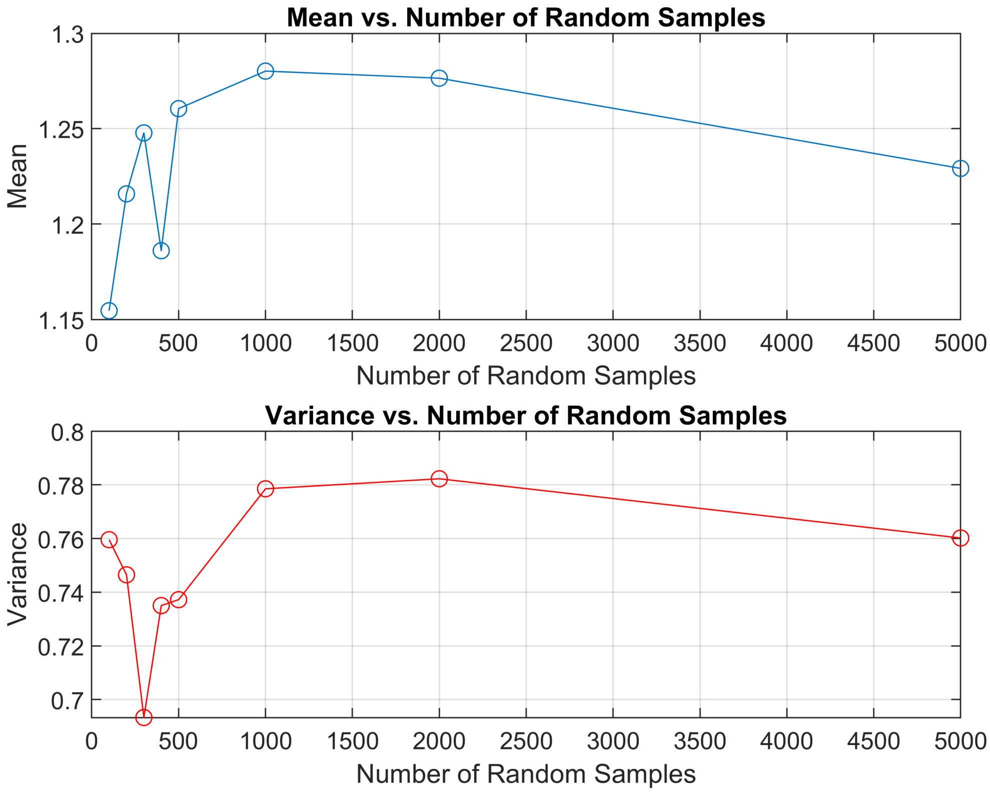
1c) Variance is 0.77, emprical variacance is:

```
fprintf('Variance: %.4f\n', variance);
```

```
Variance: 0.8318
```

1d) Find N=100, 200, ... samples of Z. Plot average and variance as function of N. They eventually converge to calculated expected value and variance as N increases.

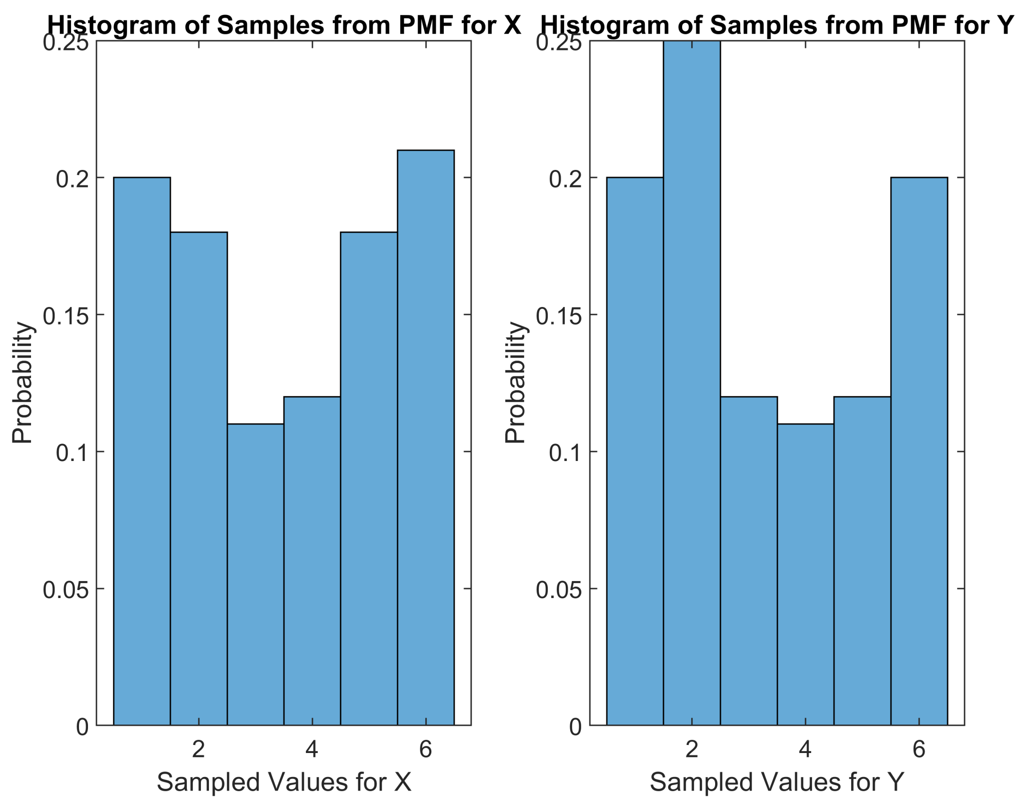
```
random_counts = [100, 200, 300, 400, 500, 1000, 2000, 5000];

means = zeros(length(random_counts), 1);
variances = zeros(length(random_counts), 1);

for i = 1:length(random_counts)

    n = random_counts(i);

    U = rand(n, 1);

    samples = F_inv(U);

    average = mean(samples);
    variance = var(samples);

    means(i) = mean(samples);
    variances(i) = var(samples);

end

% Plot means and variances
figure; % Create a new figure window
subplot(2, 1, 1); % First subplot for means
plot(random_counts, means, '-o'); % Plot means with markers
xlabel('Number of Random Samples');
ylabel('Mean');
title('Mean vs. Number of Random Samples');
grid on;

subplot(2, 1, 2); % Second subplot for variances
plot(random_counts, variances, '-o', 'Color', 'r'); % Plot variances with markers
xlabel('Number of Random Samples');
ylabel('Variance');
title('Variance vs. Number of Random Samples');
grid on;
```



2a) Generate N= 100 independent samples of (X,Y)

```
% Define the number of samples
n = 100; % Number of samples to generate

% Generate uniform random samples for X and Y
U_X = rand(n, 1); % Random values for X
U_Y = rand(n, 1); % Random values for Y

% Apply the inverse CDF to generate samples for X and Y
samples_X = F_inv2(U_X);
samples_Y = F_inv2(U_Y);

% Calculate average and variance for X and Y
average_X = mean(samples_X);
variance_X = var(samples_X);
average_Y = mean(samples_Y);
variance_Y = var(samples_Y);

% Plot histograms of the samples for X and Y
figure;

subplot(1, 2, 1);
histogram(samples_X, 'Normalization', 'probability', 'BinEdges', 0.5:1:6.5);
xlabel('Sampled Values for X');
ylabel('Probability');
```

```
title('Histogram of Samples from PMF for X');

subplot(1, 2, 2);
histogram(samples_Y, 'Normalization', 'probability', 'BinEdges', 0.5:1:6.5);
xlabel('Sampled Values for Y');
ylabel('Probability');
title('Histogram of Samples from PMF for Y');
```




```
% Joint Counts Matrix
joint_counts = zeros(6, 6); % 6x6 matrix for the joint distribution

% Count occurrences of each (X, Y) pair
for i = 1:n
    joint_counts(samples_X(i), samples_Y(i)) = joint_counts(samples_X(i), samples_Y(i)) + 1;
end

% Convert counts to probabilities (joint empirical PMF)
joint_pmf = joint_counts / n;
disp(array2table(joint_pmf, ...
    'VariableNames', {'Y=1', 'Y=2', 'Y=3', 'Y=4', 'Y=5', 'Y=6'}, ...
    'RowNames', {'X=1', 'X=2', 'X=3', 'X=4', 'X=5', 'X=6'}));
```

	Y=1	Y=2	Y=3	Y=4	Y=5	Y=6
X=1	0.05	0.03	0	0.02	0.04	0.06
X=2	0.01	0.05	0.04	0.03	0.02	0.03
X=3	0.02	0.04	0.01	0	0.02	0.02

X=4	0.03	0.05	0	0	0.01	0.03
X=5	0.03	0.05	0.03	0.05	0	0.02
X=6	0.06	0.03	0.04	0.01	0.03	0.04

2b) X and Y are independent variable. By definition, X and Y are independent if P[X=x & Y=y]=P[X=x]*P[Y=y].

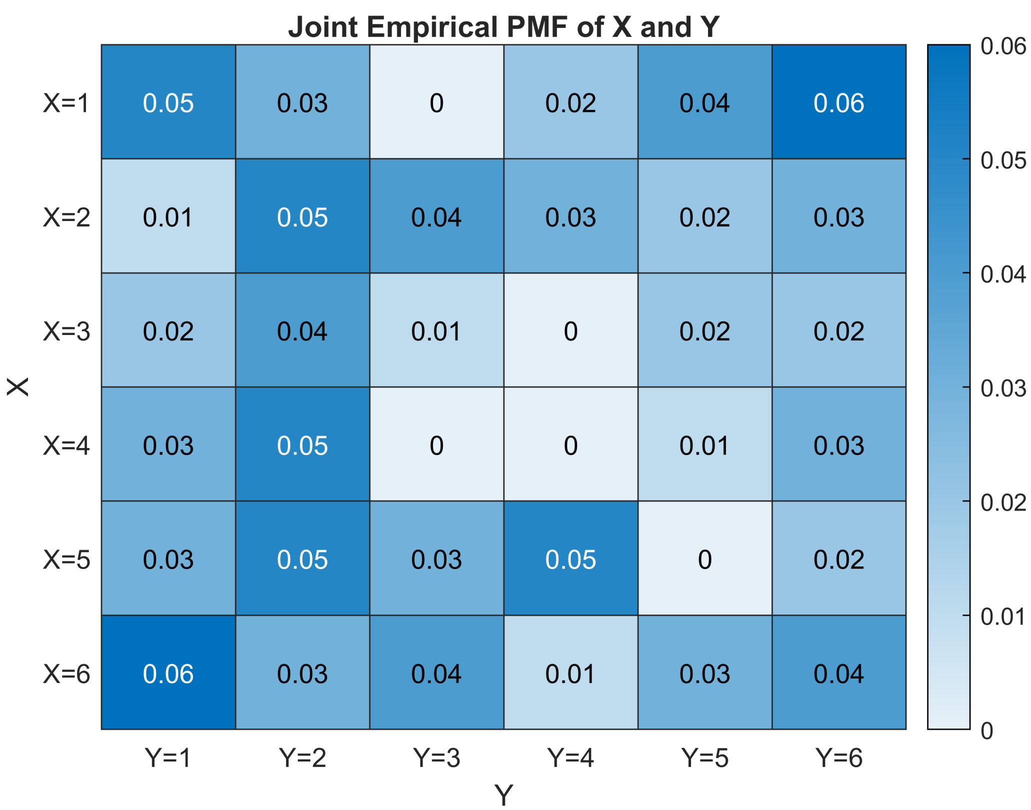
From the following heat map, we can tell that in the top left area (1<=X<=2 & 1<=Y<=2) the joing PMF is around 0.25*0.25=0.0625

In the bottom right area (3<=X<=6 & 3<=Y<=6) the joing PMF is around 0.125*0.125=0.0156

Same for top right and bottom left, (3<=X<=6 & 1<=Y<=2) and (1<=X<=2 & 3<=Y<=6) the joing PMF is around 0.125*0.25=0.05125

Therefore. X and Y are indpendent variable.

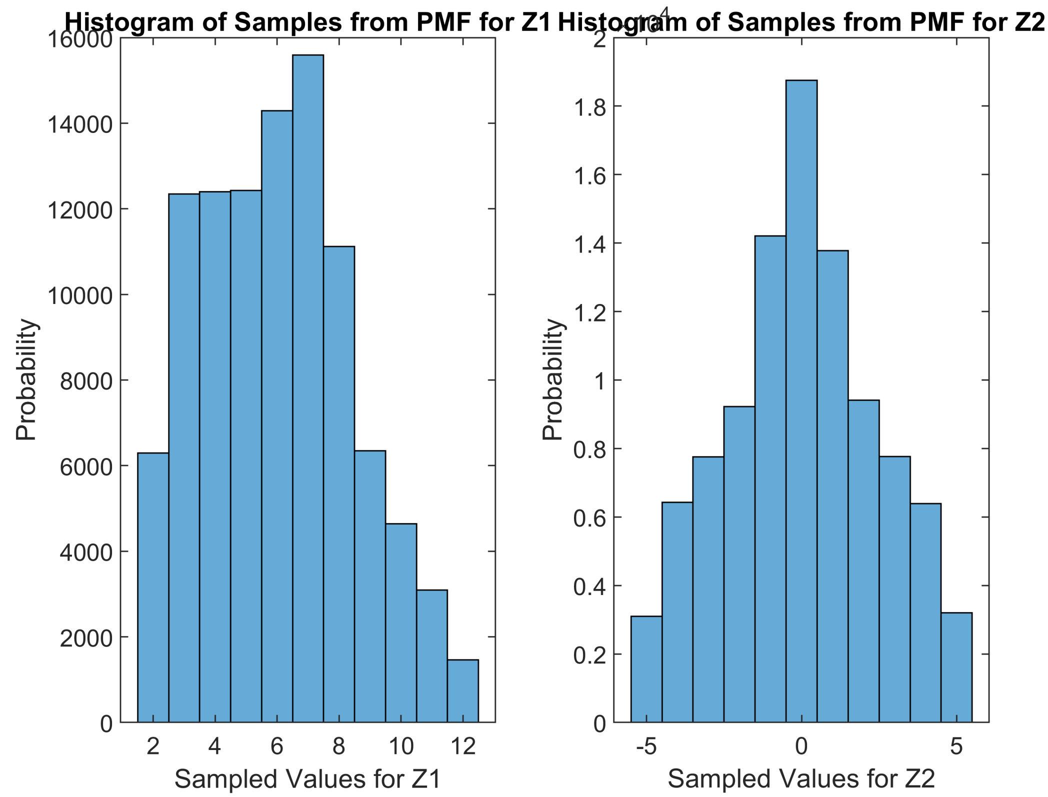
```
figure;
heatmap({'Y=1', 'Y=2', 'Y=3', 'Y=4', 'Y=5', 'Y=6'}, ...
        {'X=1', 'X=2', 'X=3', 'X=4', 'X=5', 'X=6'}, ...
        joint_pmf);
xlabel('Y');
ylabel('X');
title('Joint Empirical PMF of X and Y');
```



2c) Define Z1 = X + Y and Z2 = X-Y

```
% Define the number of samples
```

```
n = 100000; % Number of samples to generate

% Generate uniform random samples for X and Y
U_X = rand(n, 1); % Random values for X
U_Y = rand(n, 1); % Random values for Y

% Apply the inverse CDF to generate samples for X and Y
samples_X = F_inv2(U_X);
samples_Y = F_inv2(U_Y);

Z1 = samples_X + samples_Y;
Z2 = samples_X - samples_Y;

% Plot histograms of the samples for X and Y
figure;

subplot(1, 2, 1);
histogram(Z1);
xlabel('Sampled Values for Z1');
ylabel('Probability');
title('Histogram of Samples from PMF for Z1');

subplot(1, 2, 2);
histogram(Z2);
xlabel('Sampled Values for Z2');
ylabel('Probability');
title('Histogram of Samples from PMF for Z2');
```



Z1 and Z2 are not independent variables. As shown by the joint PMF, the probability of Z1 changes drastically as Z1 changes.

Which mean that the conditional probability P[Z1=z1 & Z2=z2] = P[Z1=z1]*P[Z2=z2] doesn't stand.

```
% Adjust Z2 to avoid negative indices
Z2_shifted = Z2 + 7;  % Shift Z2 so its range starts at 1
Z1_shifted = Z1 - 1; % Shift Z1 so its range starts at 1

% Initialize joint counts matrix
joint_counts = zeros(11, 13); % 11x13 matrix for the joint distribution (Z1: 2-12, Z2: -6 to 6

% Count occurrences of each (Z1, Z2_shifted) pair
for i = 1:n
    joint_counts(Z1_shifted(i), Z2_shifted(i)) = joint_counts(Z1_shifted(i), Z2_shifted(i)) + 
end

% Convert counts to probabilities (joint empirical PMF)
joint_pmf = joint_counts / n;

% Display PMF as a table
disp(array2table(joint_pmf, ...
    'VariableNames', {'Z2=-6', 'Z2=-5', 'Z2=-4', 'Z2=-3', 'Z2=-2', 'Z2=-1', 'Z2=0', 'Z2=1', 'Z
    'RowNames', {'Z1=2', 'Z1=3', 'Z1=4', 'Z1=5', 'Z1=6', 'Z1=7', 'Z1=8', 'Z1=9', 'Z1=10', 'Z1=
```

	Z2=-6	Z2=-5	Z2=-4	Z2=-3	Z2=-2	Z2=-1	Z2=0	Z2=1	Z2=2	Z2=3
Z1=2	0	0	0	0	0	0	0.06293	0	0	
Z1=3	0	0	0	0	0	0.0623	0	0.06114	0	
Z1=4	0	0	0	0	0.03133	0	0.06203	0	0.03065	
Z1=5	0	0	0	0.0308	0	0.03186	0	0.03059	0	0.03
Z1=6	0	0	0.03205	0	0.03088	0	0.01633	0	0.03183	
Z1=7	0	0.03095	0	0.03105	0	0.01617	0	0.01521	0	0.0304
Z1=8	0	0	0.03222	0	0.01532	0	0.01585	0	0.01569	
Z1=9	0	0	0	0.0157	0	0.01631	0	0.01531	0	0.0161
Z1=10	0	0	0	0	0.01473	0	0.01577	0	0.01591	
Z1=11	0	0	0	0	0	0.01542	0	0.01551	0	
Z1=12	0	0	0	0	0	0	0.01459	0	0	

```
function z = F_inv(U)
    % Initialize output array
    z = zeros(size(U));

    % For 0 <= U < 0.5 (first piece)
    idx1 = (U >= 0) & (U < 0.5);
    z(idx1) = 2 * U(idx1);

    % For 0.5 <= U < 1 (second piece)
    idx2 = (U >= 0.5) & (U < 1);
    z(idx2) = 4 * U(idx2) - 1;
end

function z = F_inv2(U)
    % Initialize output array
    z = zeros(size(U));

    idx1 = (U >= 0) & (U < 0.25);
    z(idx1) = 1; % Corresponds to the first piece

    idx2 = (U >= 0.25) & (U < 0.5);
    z(idx2) = 2; % Corresponds to the second piece

    idx3 = (U >= 0.5) & (U < 0.625);
    z(idx3) = 3; % Corresponds to the third piece

    idx4 = (U >= 0.625) & (U < 0.75);
    z(idx4) = 4; % Corresponds to the fourth piece

    idx5 = (U >= 0.75) & (U < 0.875);
    z(idx5) = 5; % Corresponds to the fifth piece

    idx6 = (U >= 0.875) & (U < 1);
    z(idx6) = 6; % Corresponds to the sixth piece
end
```