Greybody Light Approximation

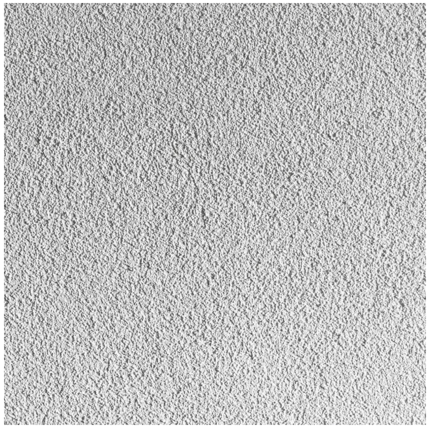
Tianle Yuan

Framework & Methodology

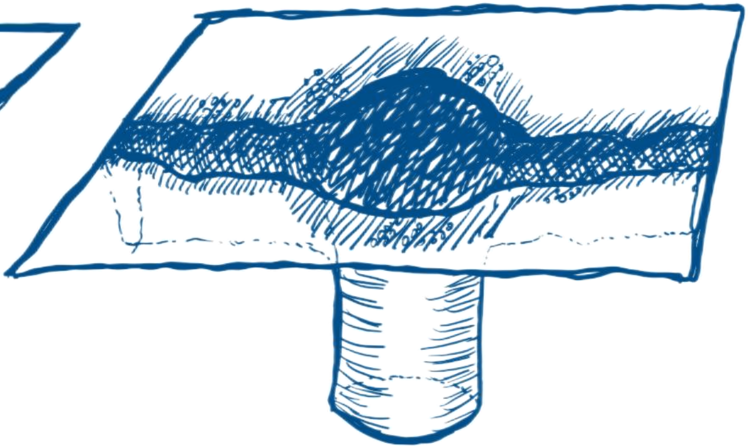
Framework - Rendering

Lighted geometry:

Terrian: generic curved surfaces (also surfaces has roughness and damage, i.e. broken geom)



Crack: Core shape geometry

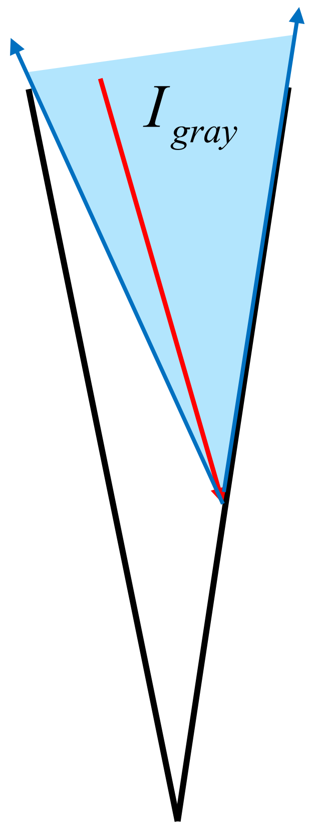


Framework - Rendering

Lighted rays for crack:

Incoming rays: Any rays can shoot on the inside surface of cavity

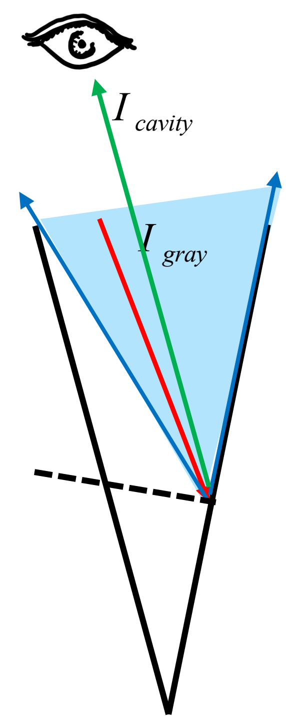
Outcoming rays: Represent the reflected light with average brightness at each point on the cravity surface



Framework - Rendering

Crack obervation:

With average brightness at each point on the cravity surface of out comming light, we implement Lambercian law as known, the cosin law with surface normal and viewer eyes, to attenuate the reflection of gray body:



$$I_{cavity}(x) = \pi \cdot I_{gray}(x) \cdot \cos(< view, normal >)$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

Gray body:
Imperfect black body, a physical object that partially absorbs incident electromagnetic radiation [https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/].

With Gouffé's paper [Corrections d'ouverture des corps-noirs artificiels compte tenu des diffusions multiples internes (Corrections of emissivity for the artificial black-body considering multiple internal diffusions)], the classical gray body can have some radiation theory with it's geometry charactor

Thus realife cavity like deep crack and hole can be treated as gray bodys

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- ## Render -- Crack & Hole

According to Gouffe (A. Gouffé, "Corrections d'ouverture des corps-noirs artificiels compte tenu des diffusions multiples internes (Corrections of emissivity for the artificial black-body considering multiple internal diffusions)" (in French), Revue d'Optique, t. 24, no. 1–3, 1945.), a **cavity** of arbitrary shape with uniformly reflected radiation over the cavity walls can have following formula for the effective emissivity by using infinite geometric progression:

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \tag{14}$$

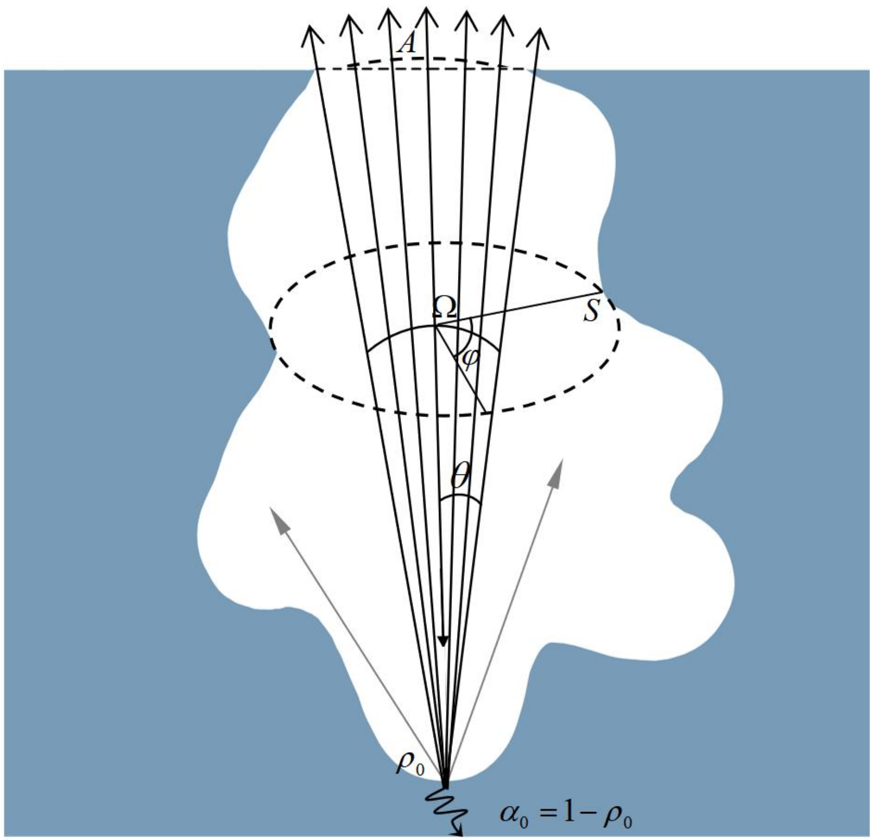
NOW, let's prove it !

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

For the model below, we <u>simplify</u> the mark of **material absorptivity** α, **material emissivity ε** and **material reflectivity** ρ:

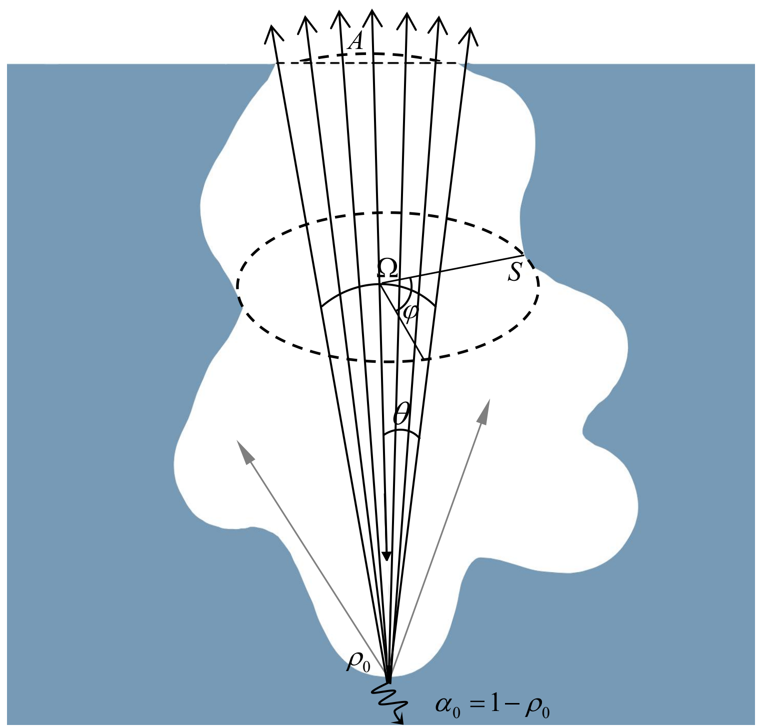


$$\alpha_0 \approx \varepsilon_0 \tag{2}$$

$$1 = \rho_0 + \alpha_0 \tag{3}$$

By combining the equation (2) and (3) and focus on Cavity model as below, we can get:

$$\varepsilon_0 \approx \alpha_0 = 1 - \rho_0 \tag{4}$$



A
Ω
S
φ
θ
ρ_0
$\alpha_0 = 1 - \rho_0$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

For **Kirchhoff's law** we can have the approximation between the gray body's **material absorpitivity** and **material emissivity**.

$$\alpha_{\lambda_0} \approx \varepsilon_{\lambda_0} \qquad (2)$$

Here the λ_0 can be constrained in the range of visible light for the light rendering.

For opaque materials, by following the conservation law of energy, we have relationship between cavity **material reflectivity** ρ and **material absorptivity** α:

$$1 = \rho_{\lambda_0} + \alpha_{\lambda_0} \qquad (3)$$

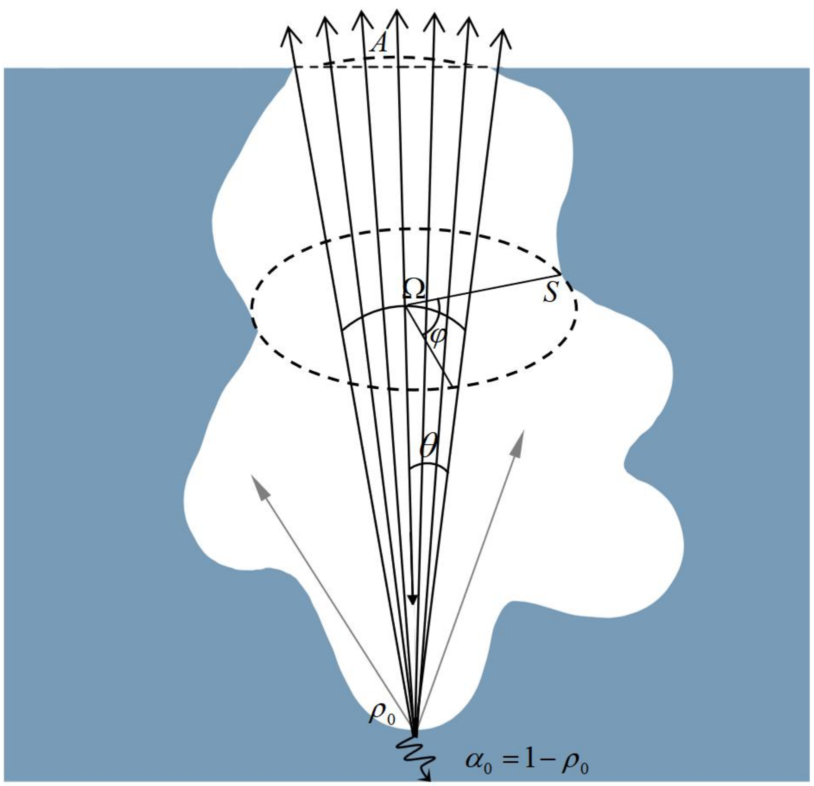
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

The reflection can be catched up by the camera

We want to:
calculate how much **apparent reflection** we get out of an incident light of the energy of unity



We assume that
1. the **material reflectivity** is uniform over the internal surface of the cavity

2. the reflection takes place according to Lambert's law; i.e., the **intensity of the reflection** is

$$I_r = \rho_0 \cos\theta \qquad (5)$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

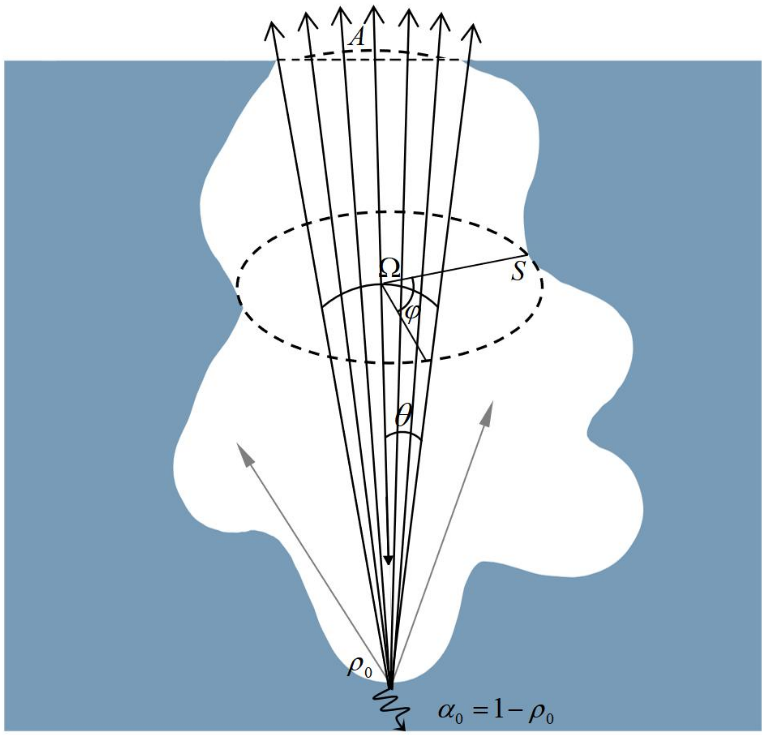
Methodology

- ## Render -- Crack & Hole

The **apparent reflection** from the bottom of the cavity is:

$$\rho = \rho_0 F \qquad (6)$$

F is the view factor, which use to express that the camera only recieve part of the material reflection.



the view factor is the normalized **solid angle** to the opening from the first reflection point. The solid angle is calculated as

$$\begin{aligned} F &= norm(\Omega) \\ &= norm\left(\int_0^{2\pi}\int_0^{\theta} \sin\theta' \cos\theta' d\theta' d\varphi'\right) \qquad (7) \\ &= norm(\pi \sin^2\theta) \\ &= \sin^2\theta \end{aligned}$$

*For the normalization, because the Lambertian factor is cosθ, the total solid angle of the hemisphere is π

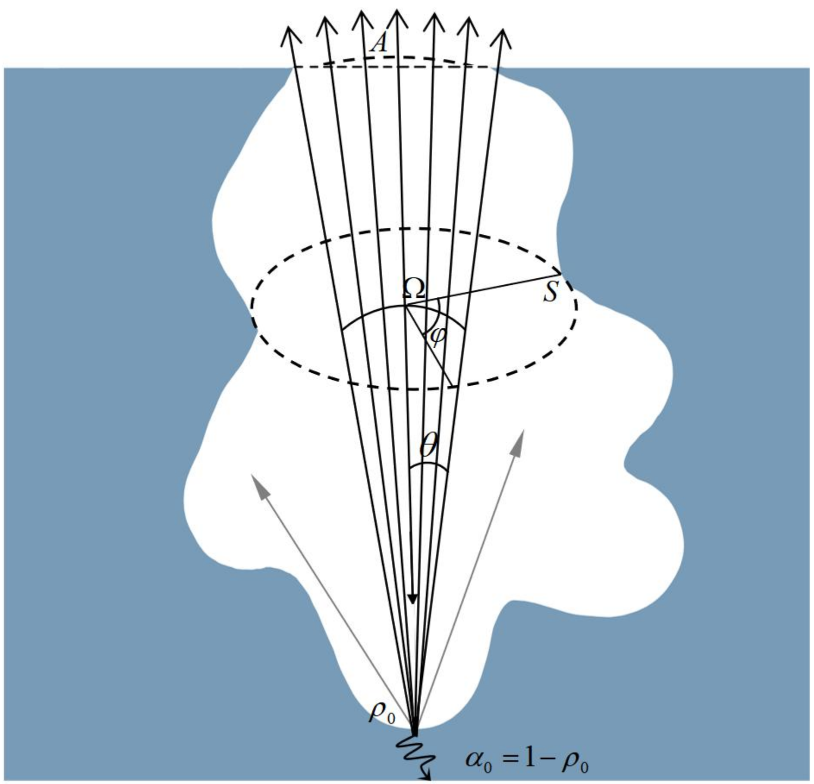
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

Thus, combing (7) the **apparent reflection (the first order apparent reflection)** from the bottom of the cavity is:

$$\rho^{(1)} = \rho_0 F = \rho_0 \sin^2\theta \qquad (8)$$



Based on (4), the for apparent property, (4) can be write down as:

$$\boxed{\varepsilon \approx \alpha = 1 - \rho \qquad (4')}$$

Thus, by combing (8) the **apparent emissivity (first order approximation)** is:

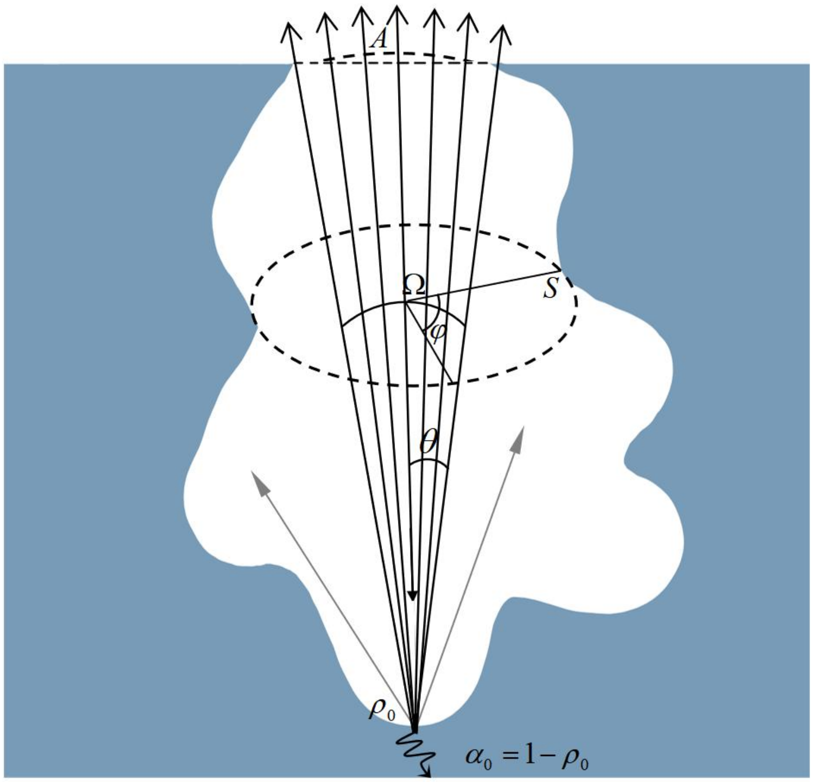
$$\varepsilon^{(1)} \approx 1 - \rho^{(1)} = 1 - \rho_0 \sin^2\theta \qquad (9)$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

● Render -- Crack & Hole

Roughly speaking, the smaller the opening, the more the cavity becomes a black body **due to** the **view factor: F (or θ)**



$$\varepsilon^{(1)} \approx 1 - \rho^{(1)} = 1 - \rho_0 \sin^2\theta \qquad (9)$$

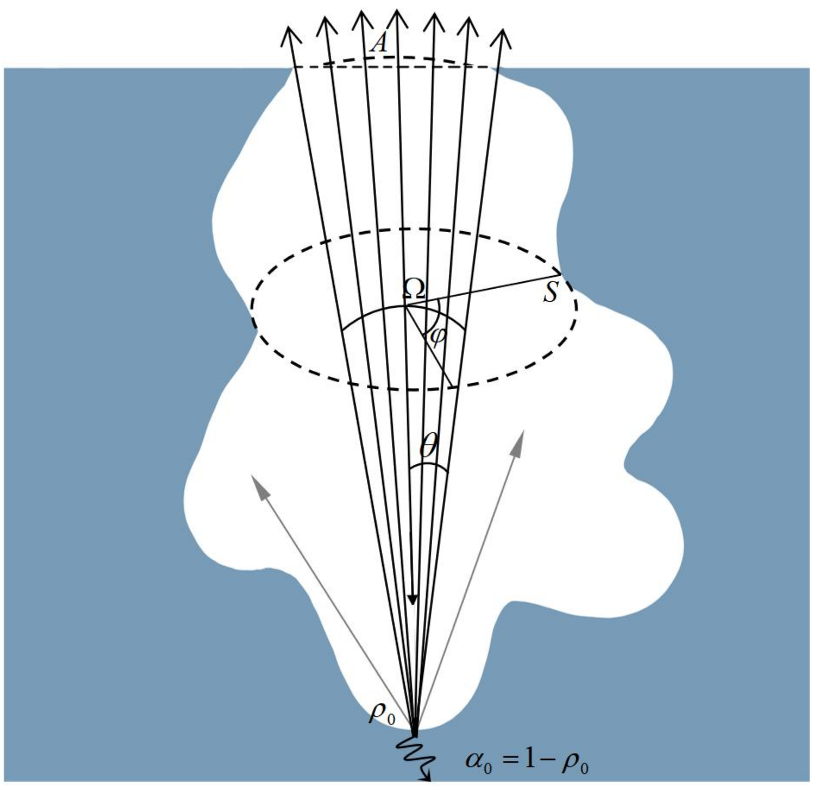
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

$$\varepsilon^{(1)} \approx 1 - \rho^{(1)} = 1 - \rho_0 F = 1 - \rho_0 \sin^2\theta \qquad (9)$$

Right now, the <u>approximation</u> can be improved:



<u>After the first reflection</u>, which we already calculated in (9), **the rest** is absorbed by the cavity material <u>or</u> contributes to further reflections.

The material absorption α0 is expressed by (4):

$$\alpha_0 = 1 - \rho_0 \qquad (4)$$

The energy left for the **subsequent reflections** is (9) - (4):

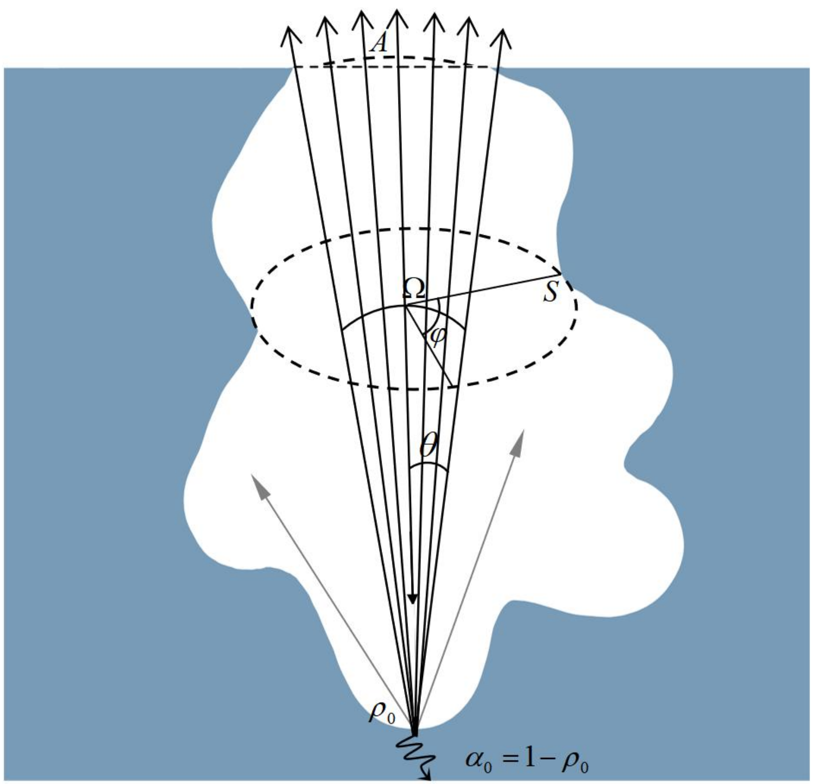
$$\begin{aligned} \rho_{sub} &= (1 - \rho_0 \sin^2\theta) - \alpha_0 \\ &= (1 - \rho_0 \sin^2\theta) - (1 - \rho_0) \qquad (10) \\ &= \rho_0 (1 - \sin^2\theta) \\ &= \rho_0 (1 - F) \end{aligned}$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

Assume that the second reflection take place in a ***uniform*** way, thus we should have another view factor **G**, which should be the area ratio of the Cavity



The energy left for the **subsequent reflections**

$$
\begin{aligned}
&(1-\rho_0 \sin^2\theta) - \alpha_0 \\
&= (1-\rho_0 \sin^2\theta) - (1-\rho_0) \qquad (10) \\
&= \rho_0 (1-\sin^2\theta) \\
&= \rho_0 (1-F)
\end{aligned}
$$

Multiply with material reflectivity and area ratio view factor, we get the **second order apparent reflection:**

$$\rho^{(2)} = \rho_0 (1-F) \cdot \rho_0 \cdot G \qquad (11)$$

$$G = \frac{A}{S} \qquad (11')$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

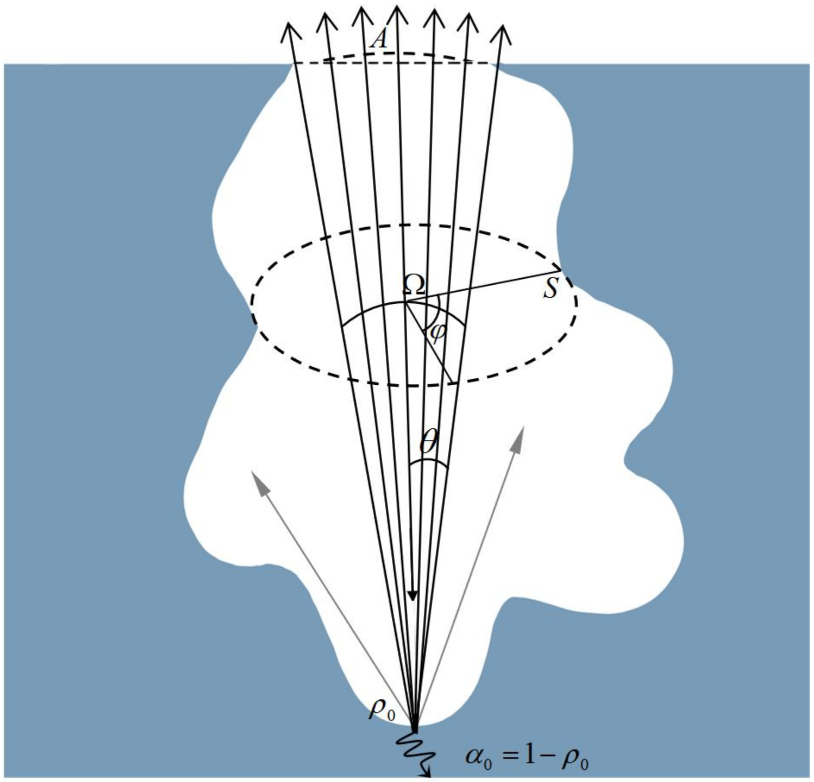
Methodology

- **Render -- Crack & Hole**

$$\boxed{\varepsilon \approx \alpha = 1 - \rho \qquad (4')}$$

$$\rho^{(1)} = \rho_0 F = \rho_0 \sin^2\theta \qquad (8)$$

$$\rho^{(2)} = \rho_0 (1 - F) \cdot \rho_0 \cdot G \qquad (11)$$



Now according to (4'), combining (11) and the first order of apparent reflection, the **second order approximation of apparent emissivity** is:

$$\begin{aligned} \varepsilon^{(2)} &\approx 1 - \rho \\ &= 1 - (\rho^{(1)} + \rho^{(2)}) \qquad (12) \\ &= 1 - \rho_0 F - \rho_0{}^2 (1 - F) G \end{aligned}$$

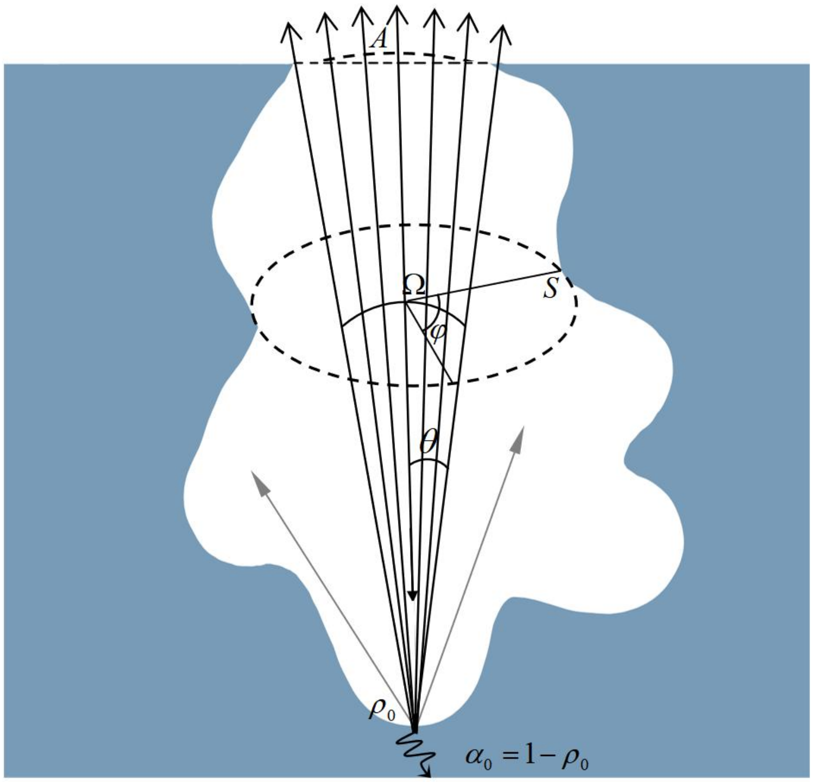
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

● Render -- Crack & Hole

Now let's do the same thing for higher oder of sub-reflections, we still use (10) an

$$\rho_{sub} = \rho_0(1-F) \qquad (10)$$



For the second order reflection, we did:

$$\rho_2 = \rho_{sub} \cdot \rho_0 G \qquad (11)$$

So for the third order reflection, we do:

$$\begin{aligned}\rho_3 &= \rho_{sub} \cdot \rho_0(1-G) \cdot \rho_0 G \\ &= \rho_0{}^3(1-F)G(1-G) \qquad (12)\end{aligned}$$

So for the n order reflection, we have:

$$\rho_n = \rho_0{}^{(n)}(1-F)G(1-G)^{(n-1)} \qquad (13)$$

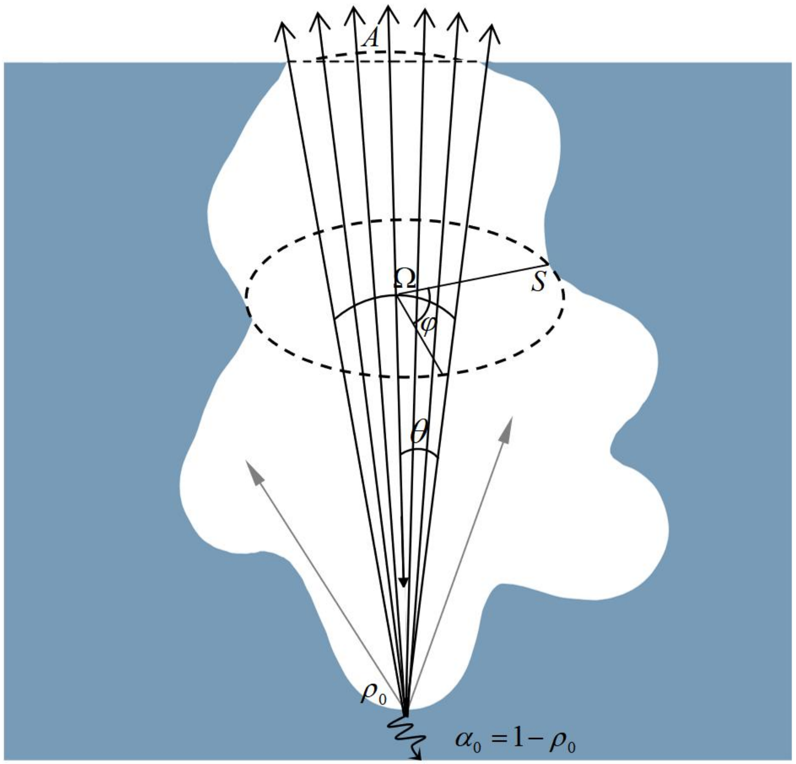
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

$$\rho_n = \rho_0^{(n)}(1-F)G(1-G)^{(n-1)} \qquad (13)$$

So based on (13) we have **n order approximation of apparent emissivity**, as our final expression of **apparent emissivity approximation**:



$$\begin{aligned}\varepsilon^{(\infty)} &\approx 1-\rho \\ &= 1-(\rho^{(1)}+\rho^{(2)}+\ldots+\rho^{(n)}+\ldots) \qquad (14) \\ &= 1-\rho_0 F-\rho_0{}^2(1-F)G-\ldots-\rho_0{}^n(1-F)G(1-G)^{(n-1)}-\ldots\end{aligned}$$

The infinite series is converged:

$$\begin{aligned}\varepsilon^{(\infty)} &\approx 1-\rho_0 F-\rho_0{}^2(1-F)G-\ldots-\rho_0{}^n(1-F)G(1-G)^{(n-1)}-\ldots \\ &= \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)\end{aligned}$$

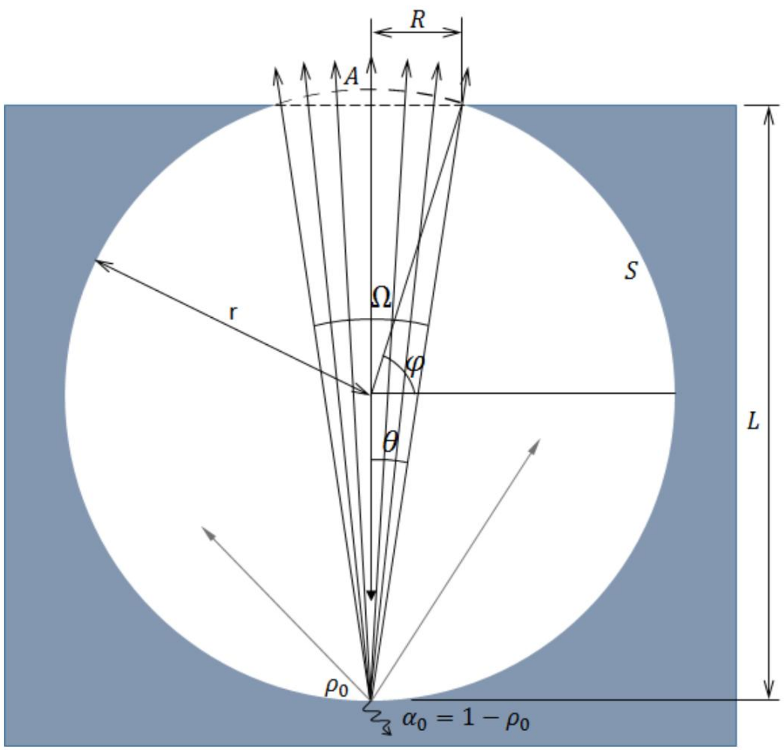
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- ## Render -- Crack & Hole

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

(14) can be simplified if we think about the **geomertry relationship** between **F** and **G** for the **sphere** case:



Surface structure for calculating the apparent reflectivity.

$$F = \sin^2\theta = \left(\frac{R}{\sqrt{R^2+L^2}}\right)^2 = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G = \frac{A}{S} = \frac{2\pi r(2r\text{-}L)}{4\pi r^2} = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (11')$$

We have:

$$F = G \qquad (15)$$

https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

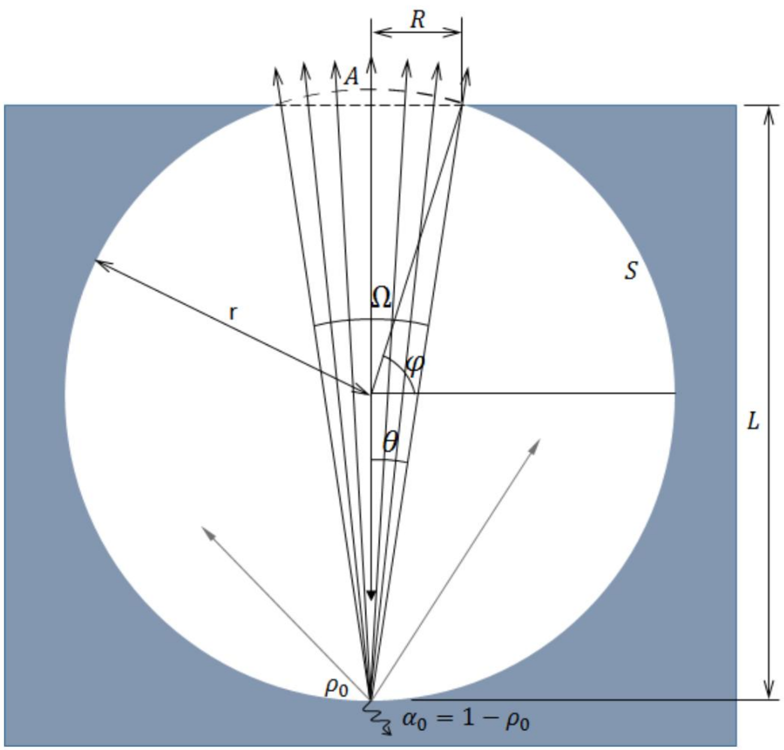
$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

based on (15), (14) can be simplified AS:

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)}{1-\rho_0(1-G)} \qquad (14)$$

$$\boxed{\rho^{(\infty)} \approx 1-\frac{(1-\rho_0)}{1-\rho_0(1-G)} \qquad (15)}$$

$$G = \frac{A}{S} = \frac{2\pi r(2r\text{-}L)}{4\pi r^2} = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (11')$$



Surface structure for calculating the apparent reflectivity.

L↑ => G↓ => ε↑=>ρ↓
R↑ => G↑=> ε↓=>ρ↑

Framework - Rendering

Light transmission [gray body theory]

$$\rho^{(\infty)} = 1 - \varepsilon^{(\infty)} \approx 1 - \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)}$$

Average reflection at a point
on the cavity surface

$$\boxed{\begin{aligned} F &= norm(\Omega') = norm(\int_{ap} d\Omega') \\ &= norm(\int_{ap} \cos(<\overrightarrow{n_p}, \overrightarrow{beam}>) d\Omega) \\ &= norm(\int_0^{2\pi}\int_{\theta_1}^{\theta_2} \cos\theta \sin\theta d\theta d\varphi) \\ &= \sin^2\theta_2 - \sin^2\theta_1 \end{aligned}}$$

$$\boxed{G = \frac{A}{S}}$$

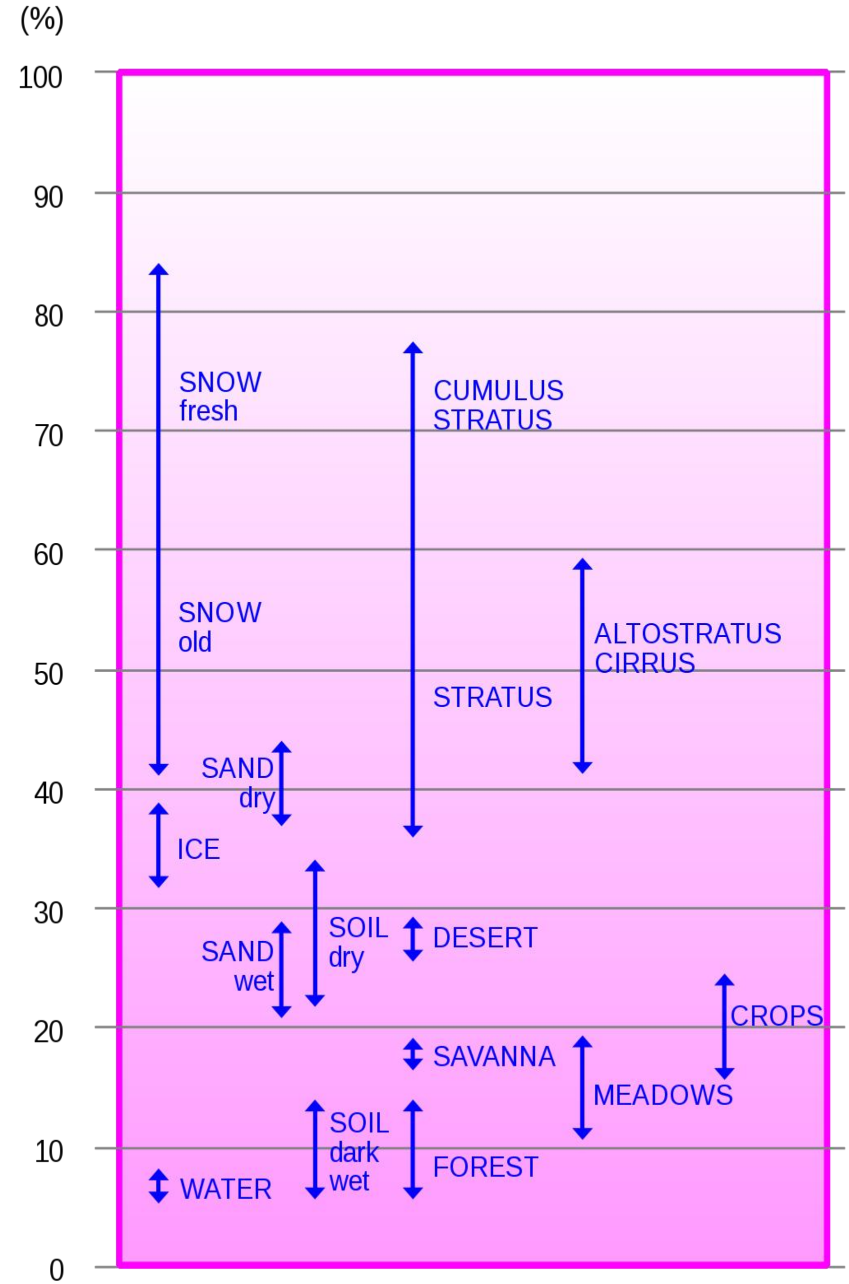
https://zh.wikipedia.org/wiki/%E5%8F%8D%E7%85%A7%E7%8E%87

- ## Albedo Choose

$$\rho^{(\infty)} = 1 - \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)}$$

For the material of **cement and clay**, we choose the **Albedo** with value of 35%:

$$\rho_0 = 0.35$$



https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

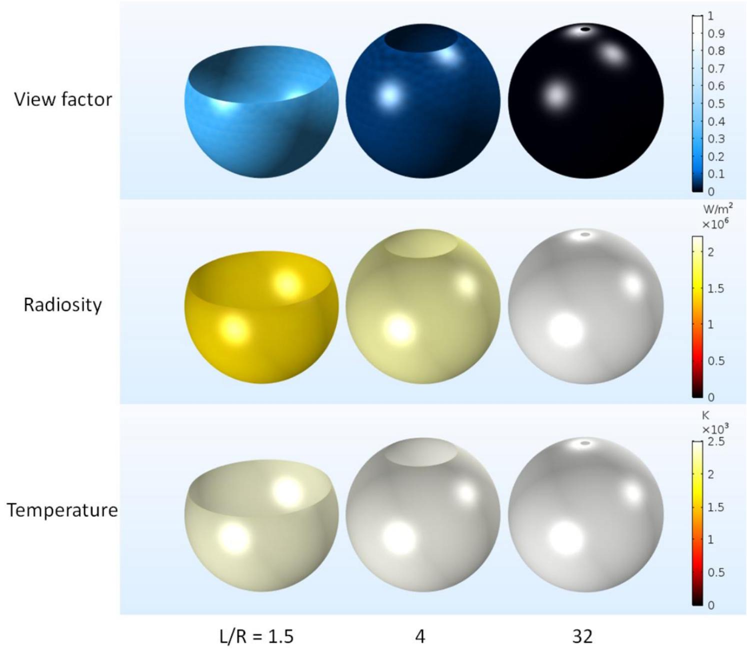
Methodology

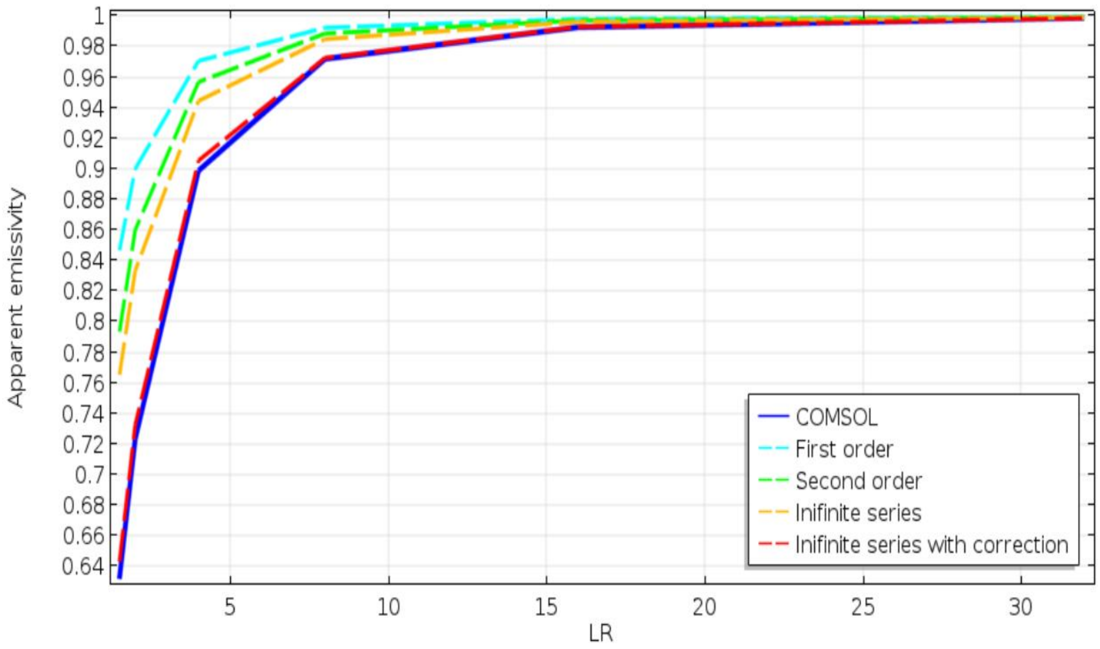
- **Render -- Crack & Hole**

L↑ => G↓ => ε↑=>ρ↓
R↑ => G↑=> ε↓=>ρ↑

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)}{1-\rho_0(1-G)} \quad (14)$$

$$\boxed{\rho^{(\infty)} \approx 1-\frac{(1-\rho_0)}{1-\rho_0(1-G)} \quad (15)}$$





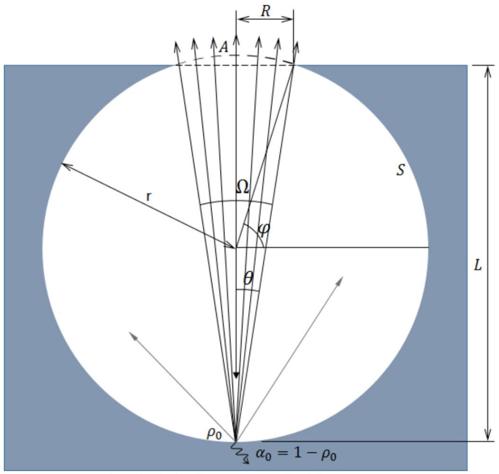
https://www.comsol.com/blogs/understanding-classical-gray-body-radiation-theory/

Methodology

- **Render -- Crack & Hole**

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

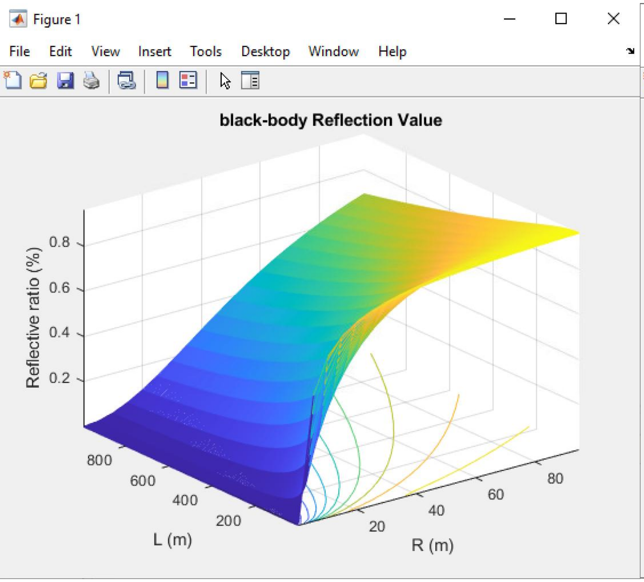
L↑ => G↓ => ε↑=>ρ↓
R↑ => G↑=> ε↓=>ρ↑

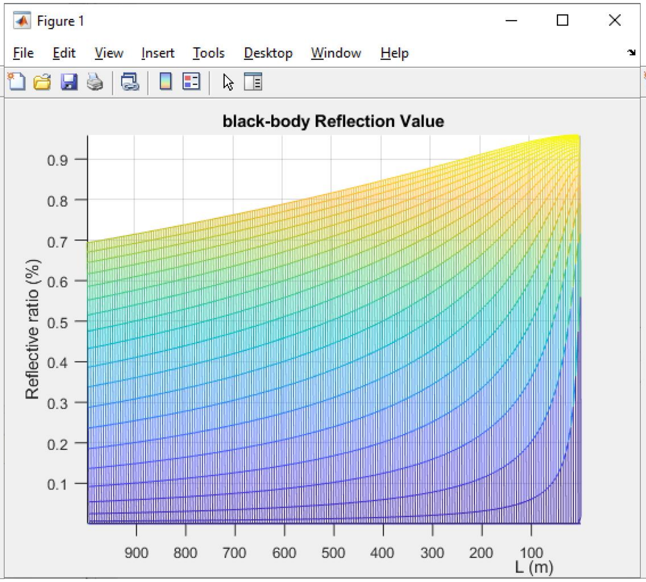


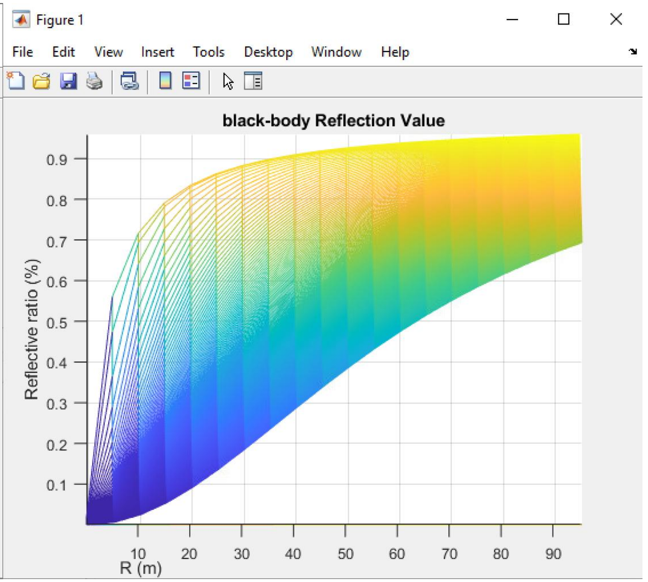
Surface structure for calculating the apparent reflectivity.

$$F = \sin^2\theta = \left(\frac{R}{\sqrt{R^2+L^2}}\right)^2 = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G = \frac{A}{S} = \frac{2\pi r(2r\text{-}L)}{4\pi r^2} = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (11')$$





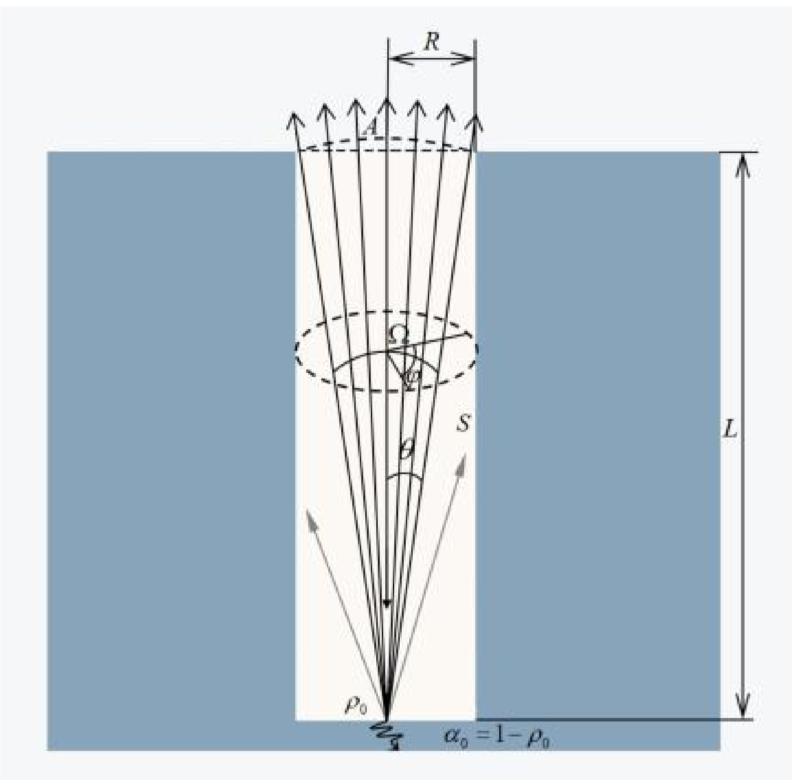


Methodology

- **Render -- Crack & Hole**

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

(14) can be simplified if we think about the **geomertry relationship** between **F** and **G** for the **Cylinder** case:



$$F = \sin^2\theta = \left(\frac{R}{\sqrt{R^2+L^2}}\right)^2 = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G = \frac{A}{S} = \frac{2\pi\left(\frac{L^2+R^2}{2L}\right)\left(\left(\frac{L^2+R^2}{L}\right)-L\right)}{2\pi LR} = \frac{R(L^2+R^2)}{2L^3} \qquad (11')$$

We have:

$$F = \left(\frac{2L^3R}{(L^2+R^2)^2}\right)G \qquad (15)$$

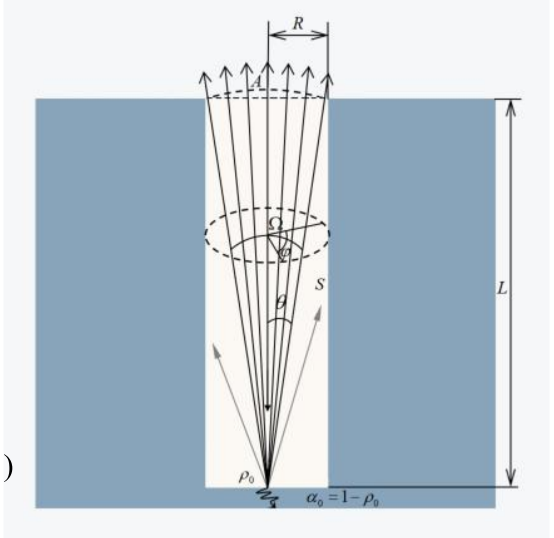
Methodology

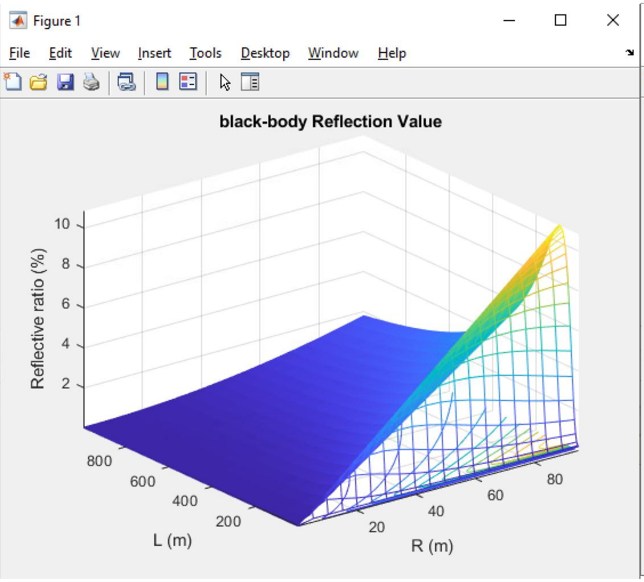
● Render -- Crack & Hole

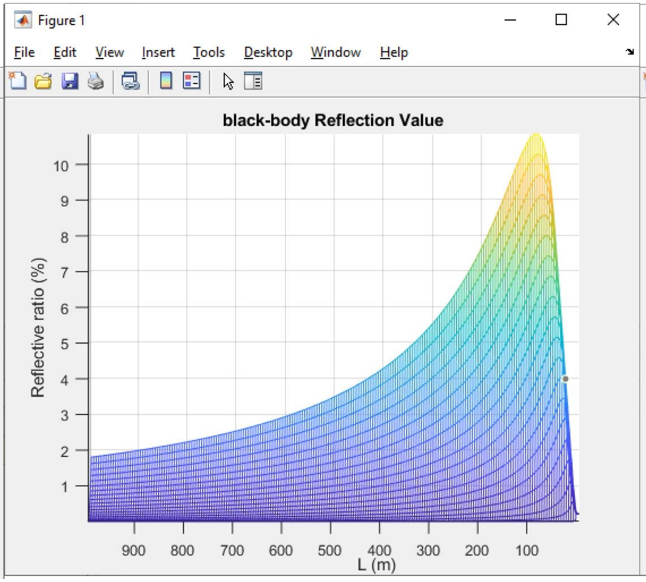
$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

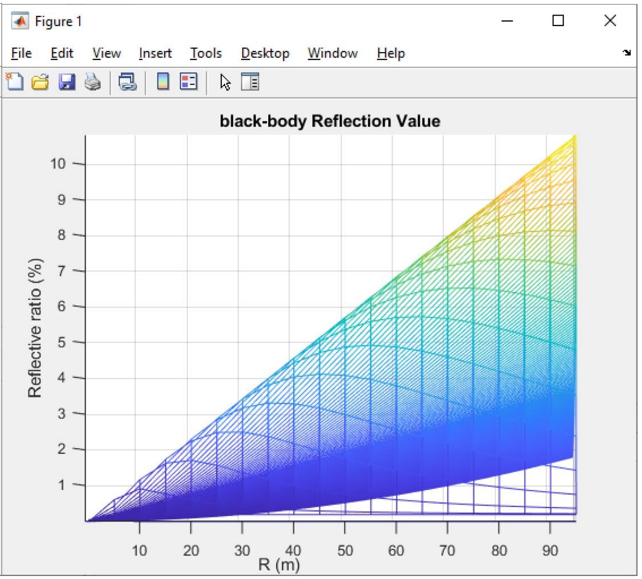
$$F=\sin^2\theta=\left(\frac{R}{\sqrt{R^2+L^2}}\right)^2=\frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G=\frac{A}{S}=\frac{2\pi\left(\frac{L^2+R^2}{2L}\right)\left(\left(\frac{L^2+R^2}{L}\right)-L\right)}{2\pi LR}=\frac{R(L^2+R^2)}{2L^3} \qquad (11')$$







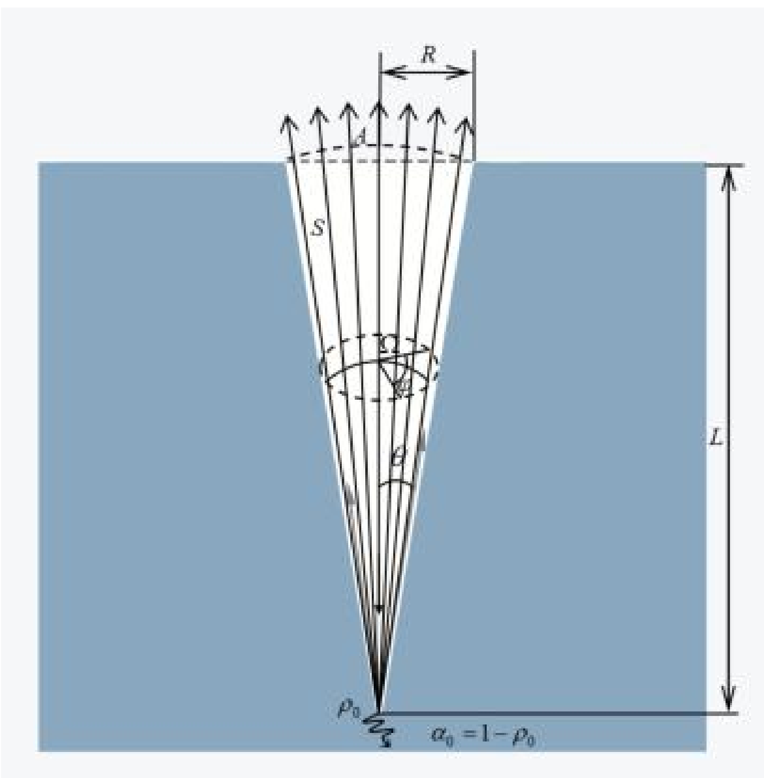


Methodology

- **Render -- Crack & Hole**

$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

(14) can be simplified if we think about the **geomertry relationship** between **F** and **G** for the **Cone** case:



$$F = \sin^2\theta = \left(\frac{R}{\sqrt{R^2+L^2}}\right)^2 = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G = \frac{A}{S} = \frac{2\pi\left(\frac{L^2+R^2}{2L}\right)\left(\left(\frac{L^2+R^2}{L}\right)-L\right)}{\pi R\sqrt{L^2+R^2}} = \frac{R(L^2+R^2)}{L^2\sqrt{L^2+R^2}} \qquad (11')$$

We have:

$$F = \left(\frac{L^2 R}{(L^2+R^2)^{\frac{3}{2}}}\right)G \qquad (15)$$

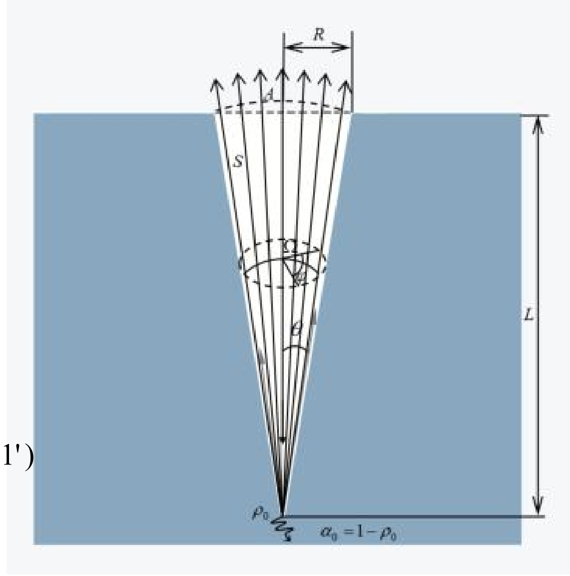
Methodology

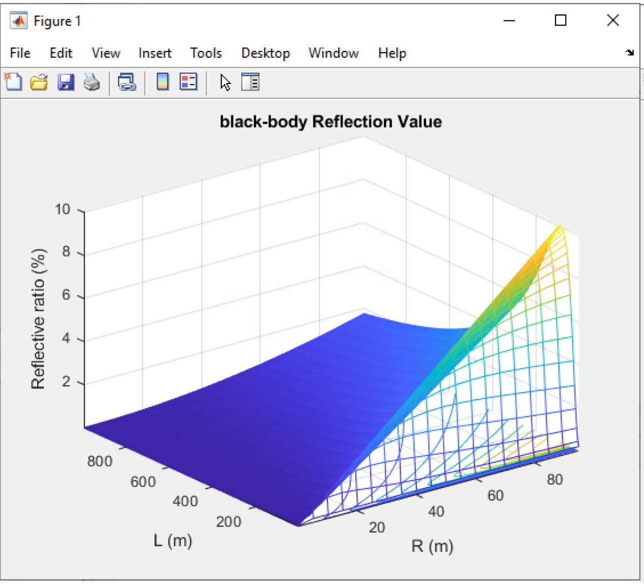
- **Render -- Crack & Hole**

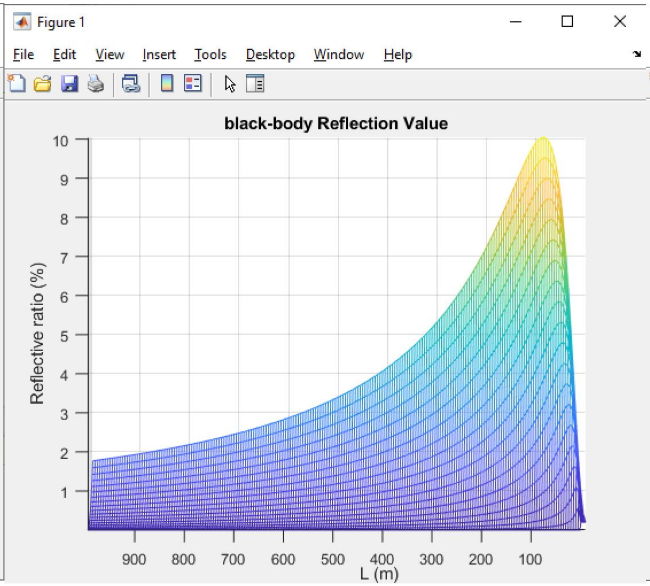
$$\varepsilon^{(\infty)} \approx \frac{(1-\rho_0)(1+\rho_0(G-F))}{1-\rho_0(1-G)} \qquad (14)$$

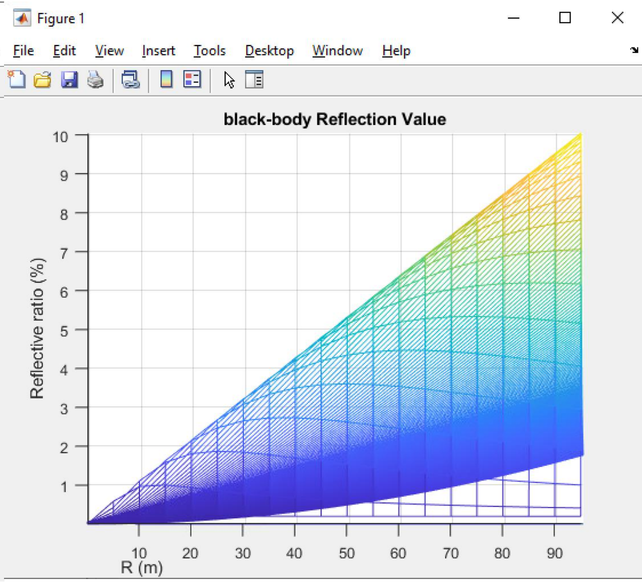
$$F = \sin^2\theta = \left(\frac{R}{\sqrt{R^2+L^2}}\right)^2 = \frac{1}{1+\left(\frac{L}{R}\right)^2} \qquad (7)$$

$$G = \frac{A}{S} = \frac{2\pi\left(\frac{L^2+R^2}{2L}\right)\left(\left(\frac{L^2+R^2}{L}\right)-L\right)}{\pi R\sqrt{L^2+R^2}} = \frac{R(L^2+R^2)}{L^2\sqrt{L^2+R^2}} \qquad (11')$$



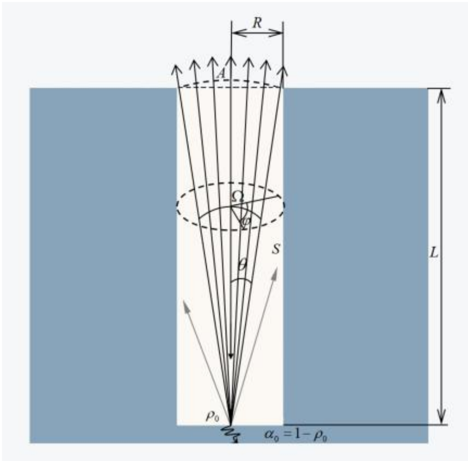


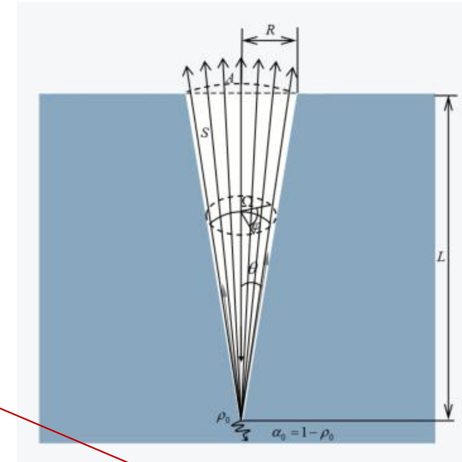




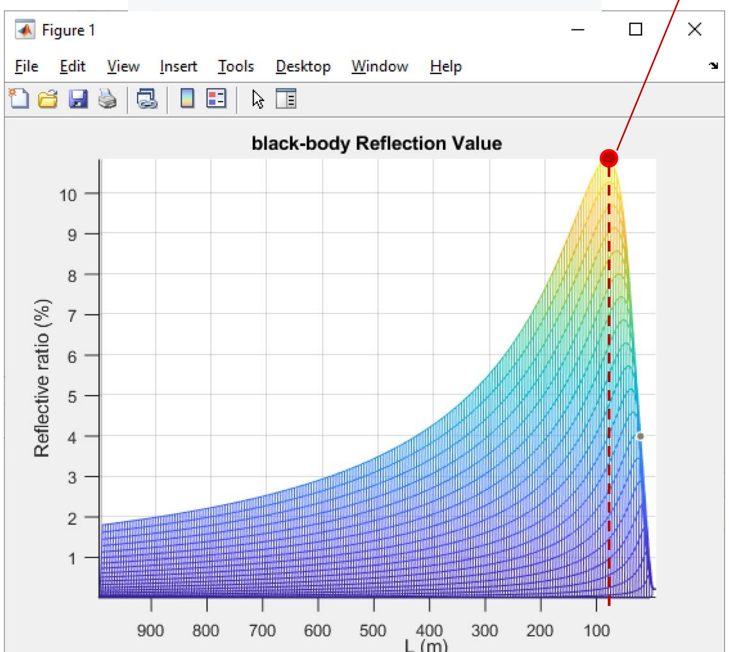
Methodology

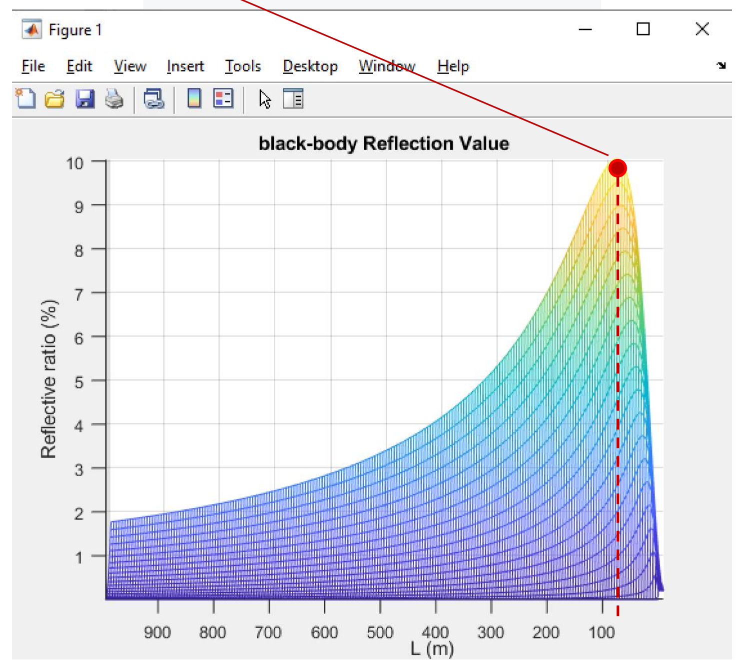
- **Render -- Crack & Hole**





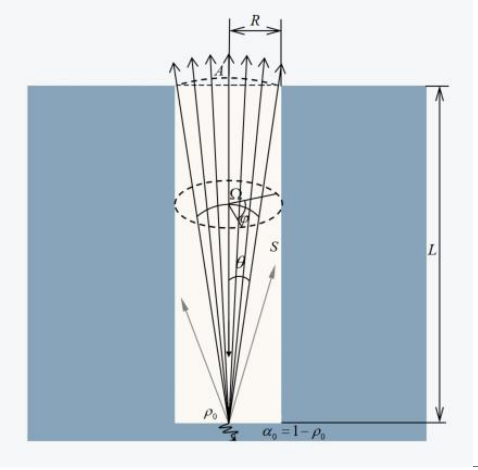
The **critical line** at which the gray body effect occurs

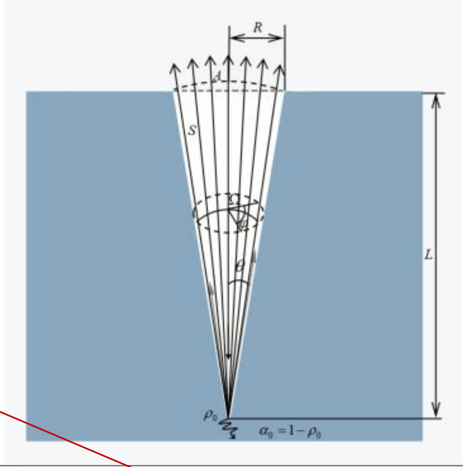




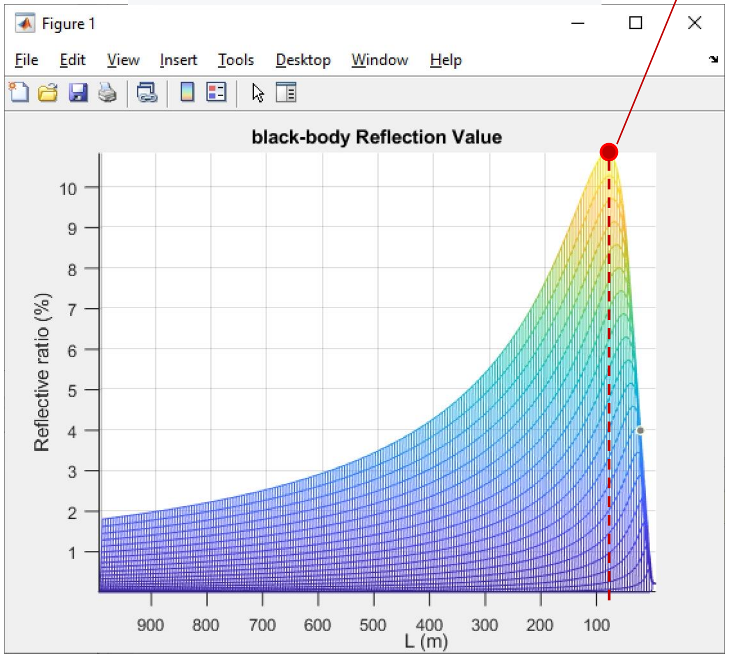
Methodology

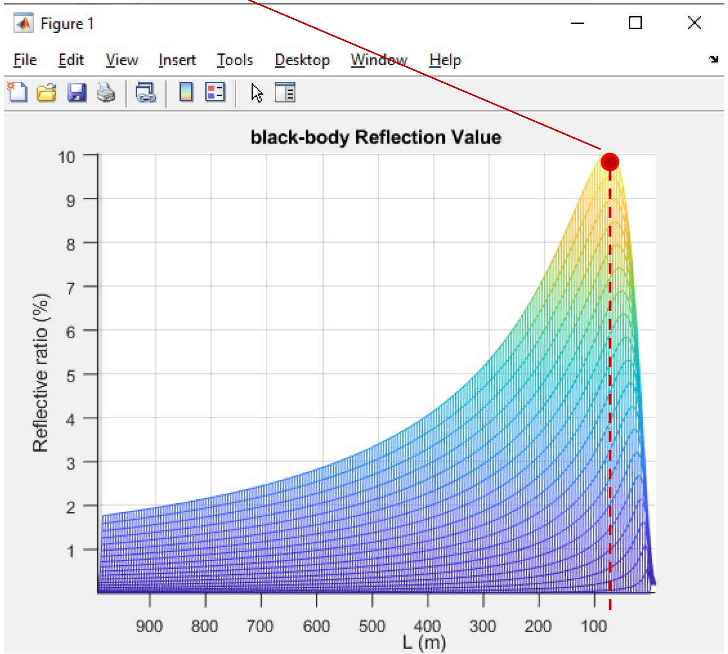
- **Render -- Crack & Hole**





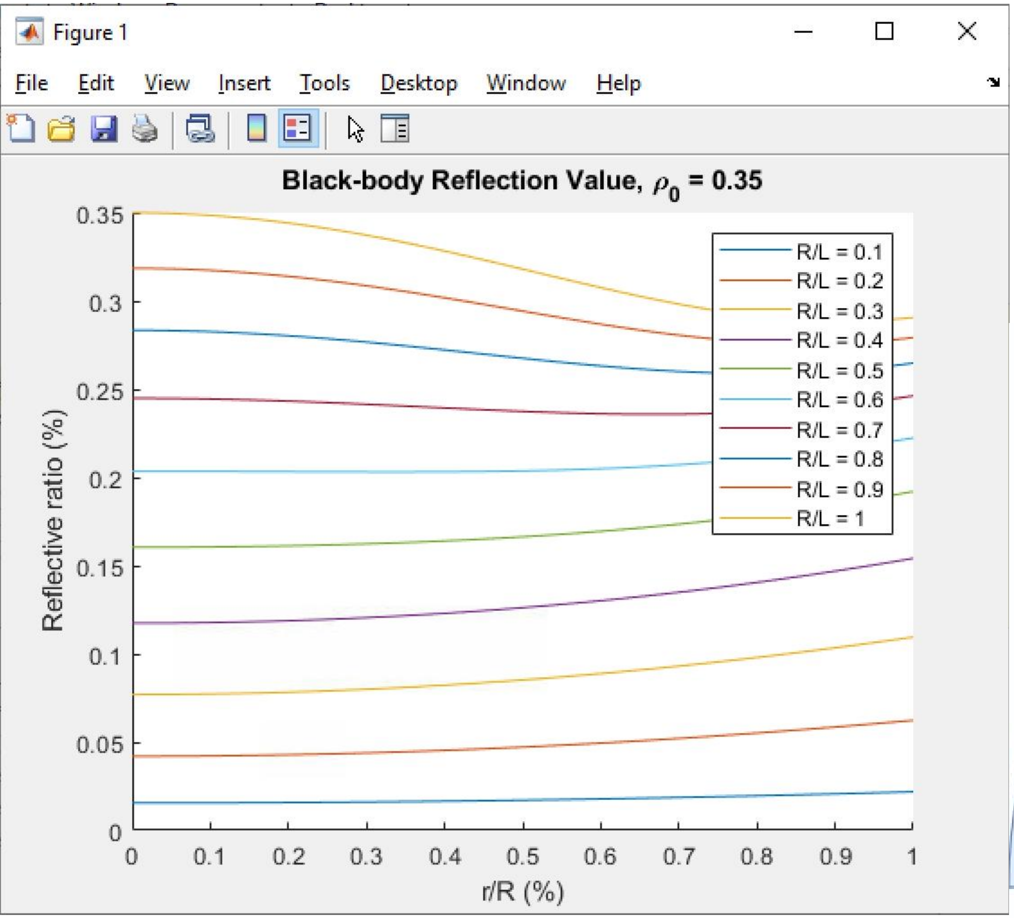
The **critical line** at which the gray body effect occurs





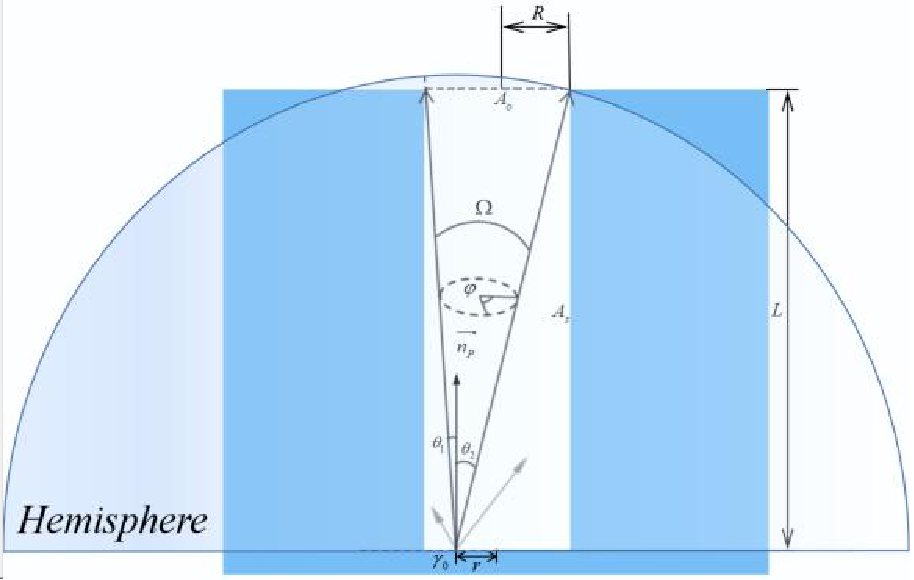
Conclusion

Cylinder Bottom

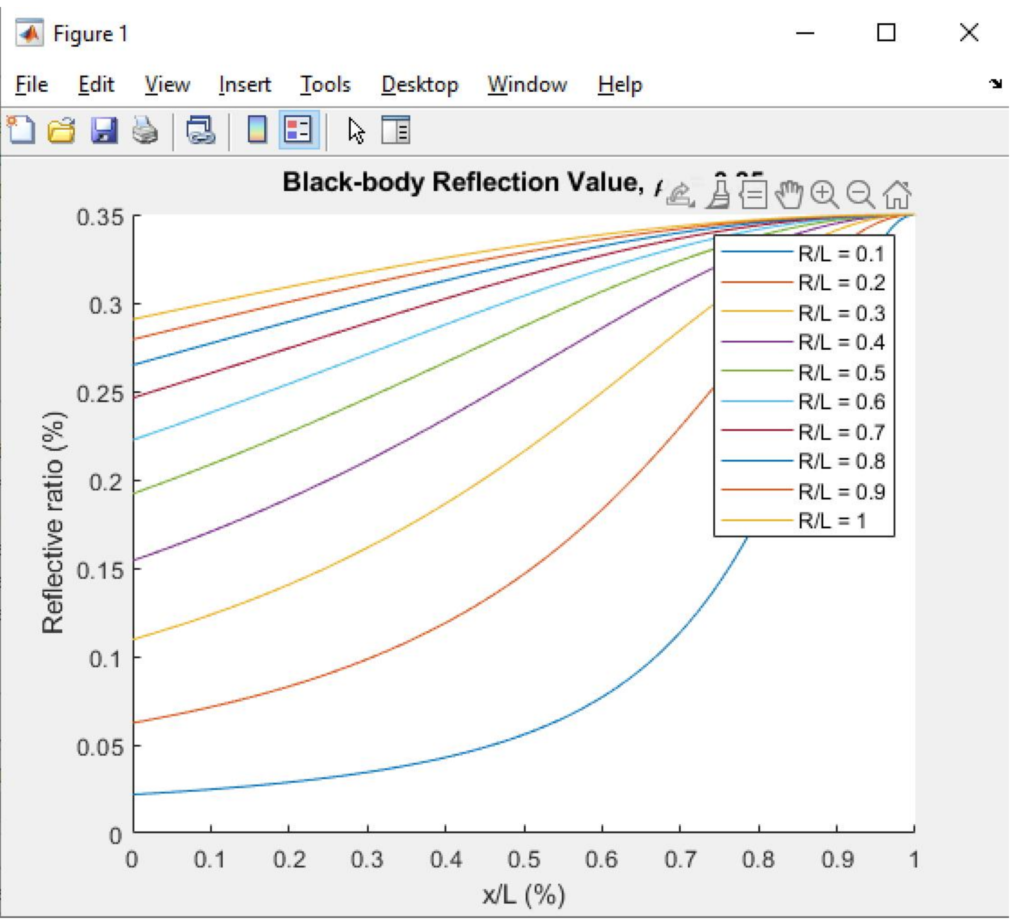


$$\theta_1(r) = \arctan\left(\frac{R-r}{L}\right)$$

$$\theta_2(r) = \arctan\left(\frac{R+r}{L}\right)$$

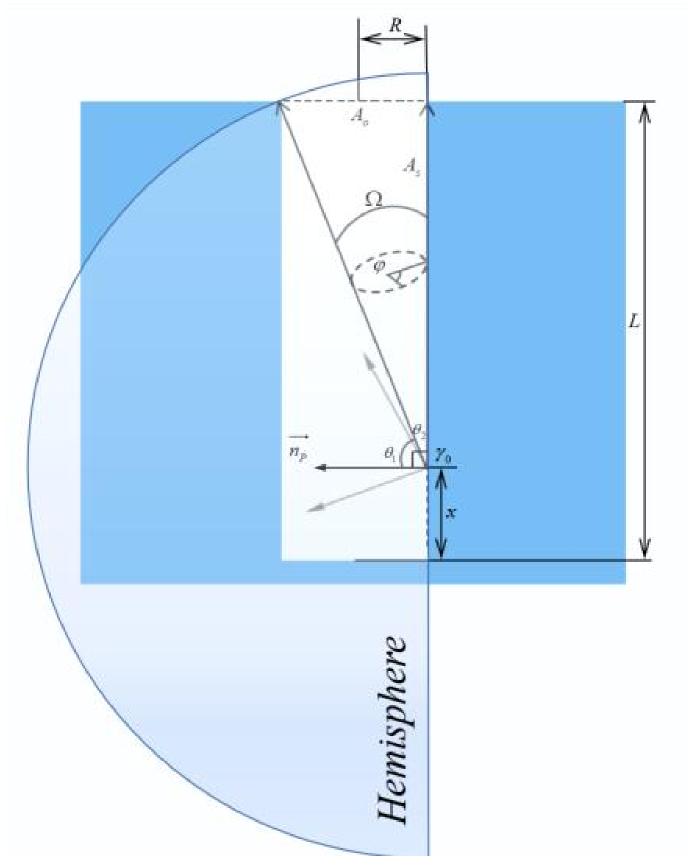


● Cylinder Wall

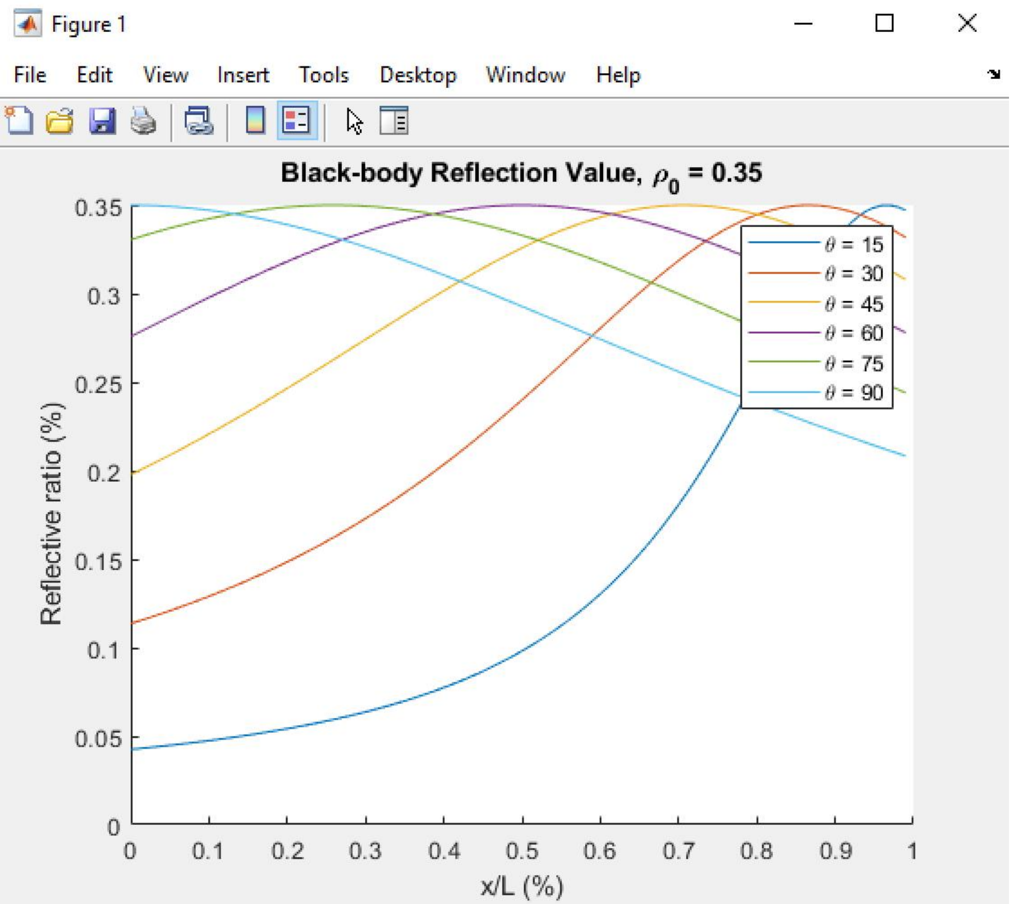


$$\theta_1(x) = \arctan\left(\frac{L-x}{2R}\right)$$

$$\theta_2 = \frac{\pi}{2}$$

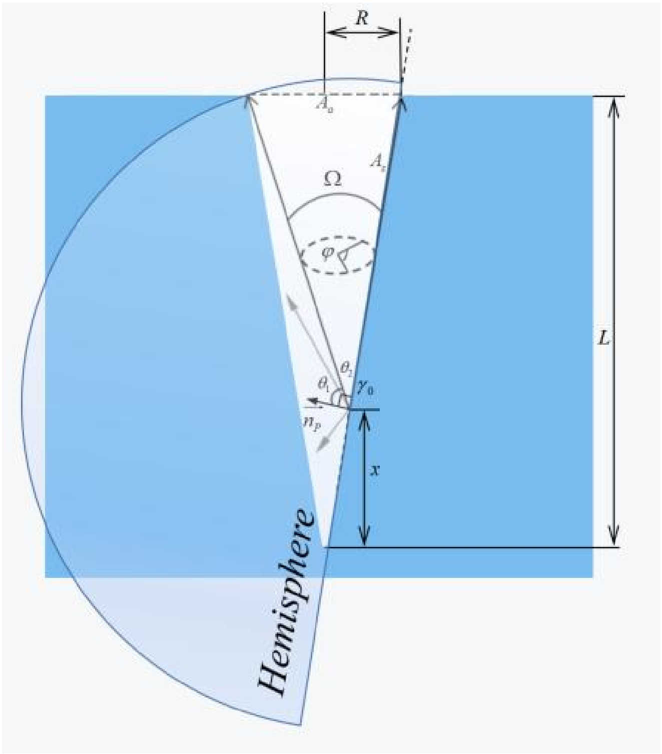


● Cone Wall



$$\theta_1(x) = \frac{\pi}{2} - \arctan\left(\frac{RL + Rx}{L^2 - Lx}\right) - \arctan\left(\frac{RL - Rx}{L^2 - Lx}\right)$$

$$\theta_2 = \frac{\pi}{2}$$



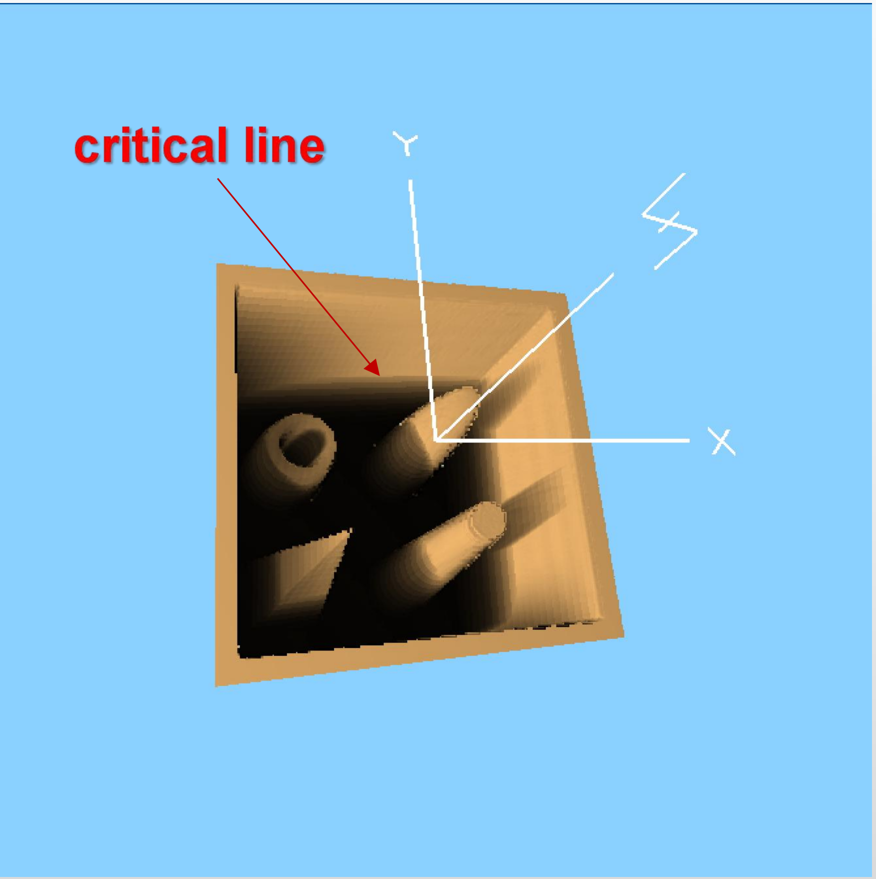
Results

Result

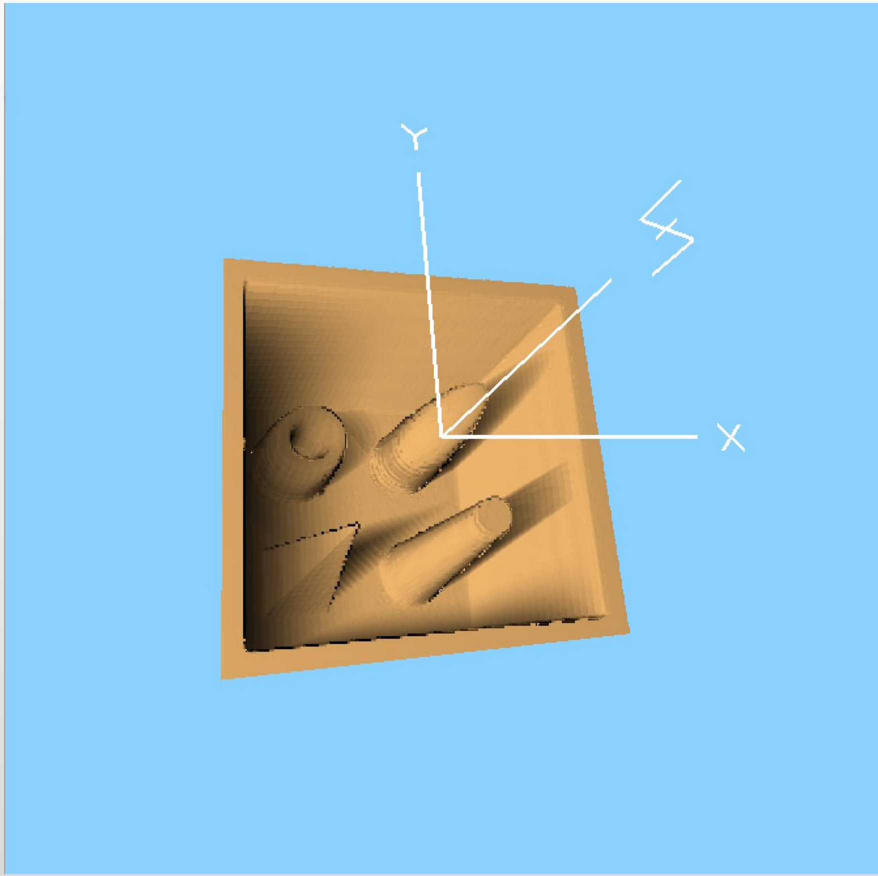
- **Render -- Crack & Hole**

Parallax Occlusion Mapping (POM) with **soft shadow effect**

With gray body effect

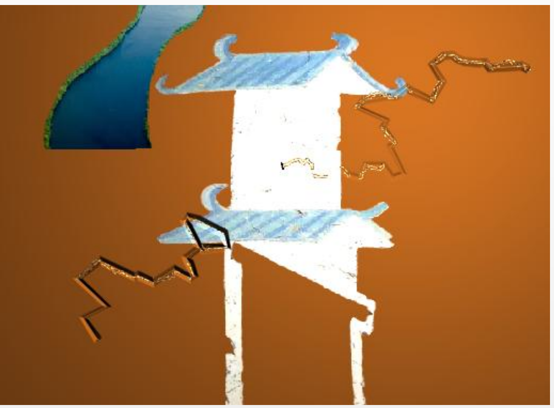


Without gray body effect



Result

+ Terrian Noise



Original wall with Lambercain law

+ Parallel Mapping

+ Black Body Theory