Navier-Stokes Equations Application:

Incompressible Newtonian fluid simulation

Speaker: Mark (Tianle Yuan)

OUTLINE

1. Project Topic

2. Background Knowledge

3. Preliminary

4. Schedule of the Project

1. Project Topic

We all have know how to express the motion of the particle.

How does those expressions apply to macroscopic fluids?

1. Project Topic

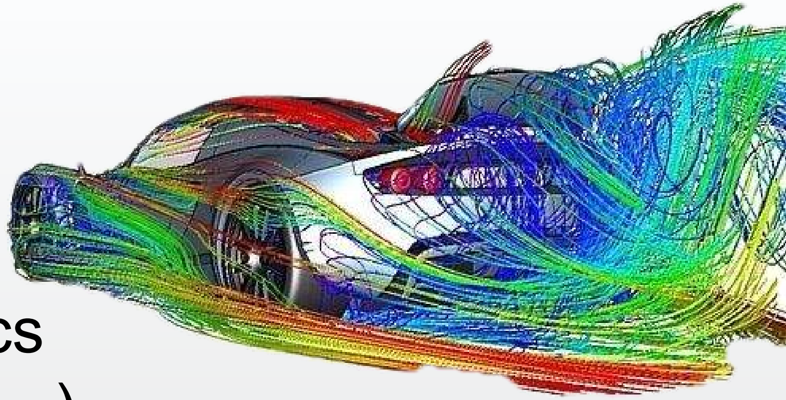
This project will mainly go back to the nature of fluid molecular motion, which would be the basic of computer animation theory.

The motion trajectory of the fluid will be simulated by using the given boundary conditions and initial value conditions.

1. Project Topic

This project will focus on the Incompressible Newtonian fluid simulation. Common laminar and turbulent flow phenomena are the main research targets.

2. Background



Hydrodynamics
(gas,water,etc...)

Essence: Laws of Conservation

The Conservation Law of:

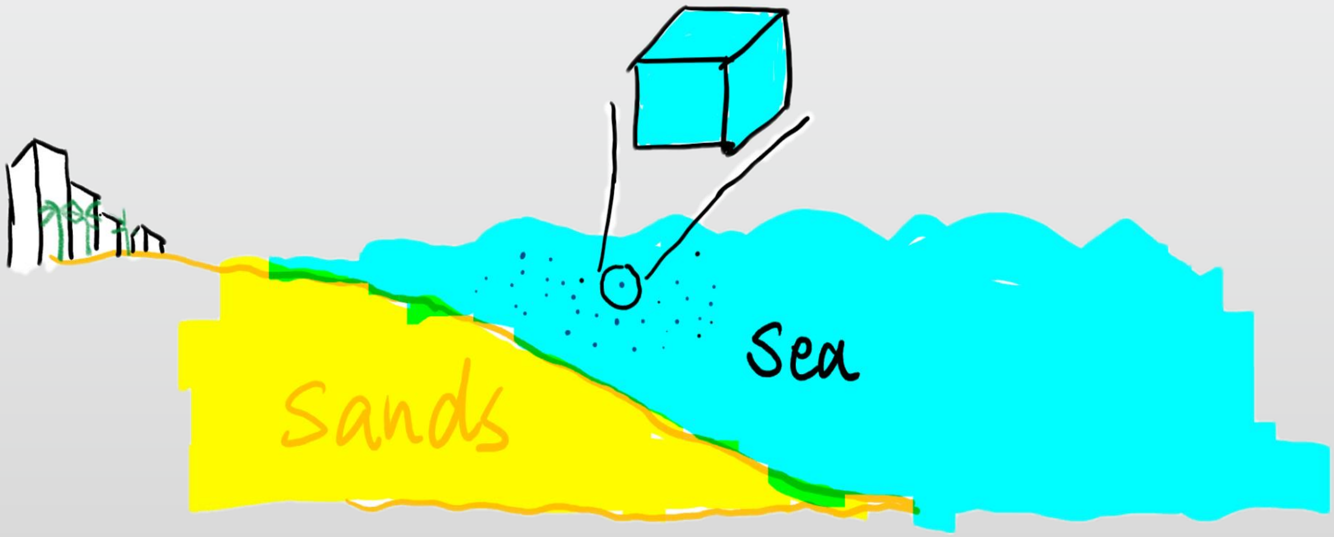
Mass: $F = m*a$

Momentum: $I = F*t = m*v$

Energy: $\Delta E = Q + W$

2. Background

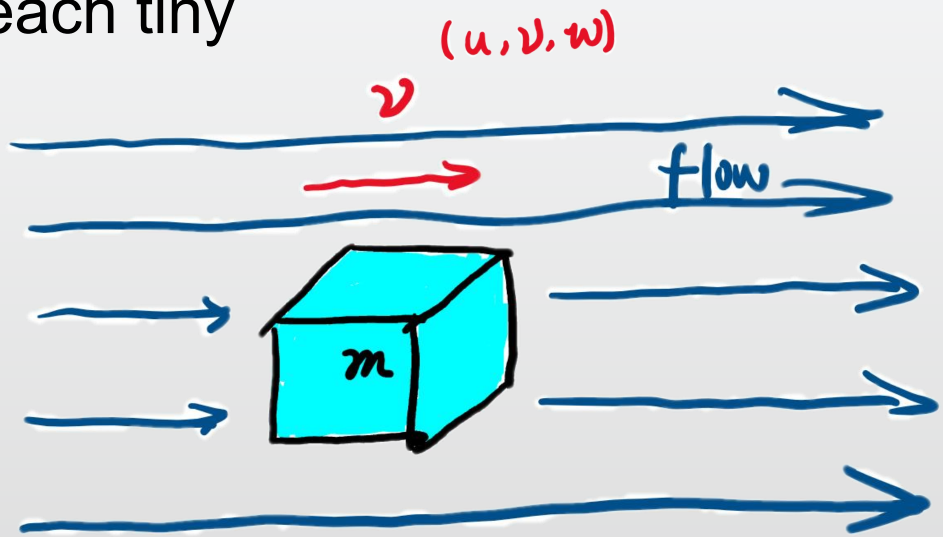
Try to imagine, there is a sea.
The sea is made up of
countless tiny drops of water...



So how to discribe the motion status of each tiny drop of water?

2. Background

The motion status of each tiny drop of water?

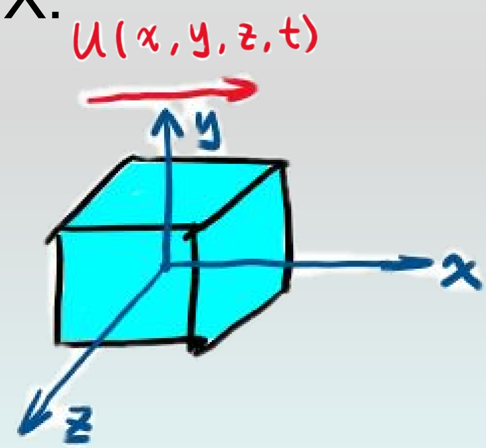


$F = m*a$

$\Rightarrow F = m* (dv/dt)$

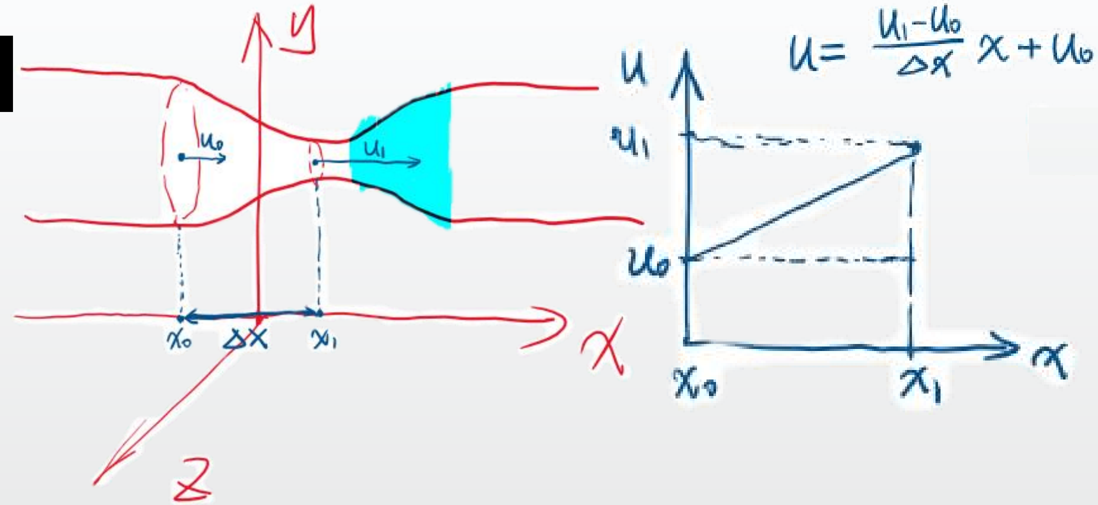
$\Rightarrow F = (\rho*1)*(dv/dt)$

In 3D, If we want to analys the direction of X:



$$\Rightarrow a_X = du/dt = (\partial u/\partial t)*(dt/dt) + (\partial u/\partial x)*\underline{(dx/dt)} \rightarrow u + (\partial u/\partial y)*\underline{(dy/dt)} \rightarrow v + (\partial u/\partial z)*\underline{(dz/dt)} \rightarrow w$$

2. Background



Thus, we get:

$$F_x = \rho * [\underbrace{(\partial u/\partial t)}_{\text{Local Acceleration}} + \underbrace{(\partial u/\partial x)*u + (\partial u/\partial y)*v + (\partial u/\partial z)*w}_{\text{Convective Acceleration}}]$$

$$= \rho * [(\partial u/\partial t) + V \cdot \nabla u]$$

Now let's talk about the **force (F)** on the water cube :

$\mathbf{F_x}$ = $f_{xinside}$ $\longrightarrow$ τ (viscous shear stress) + σ (viscous normal stress)

+ $F_{xoutside}$ $\longrightarrow$ $m*g$ (can be ignored if we don't think gravity)

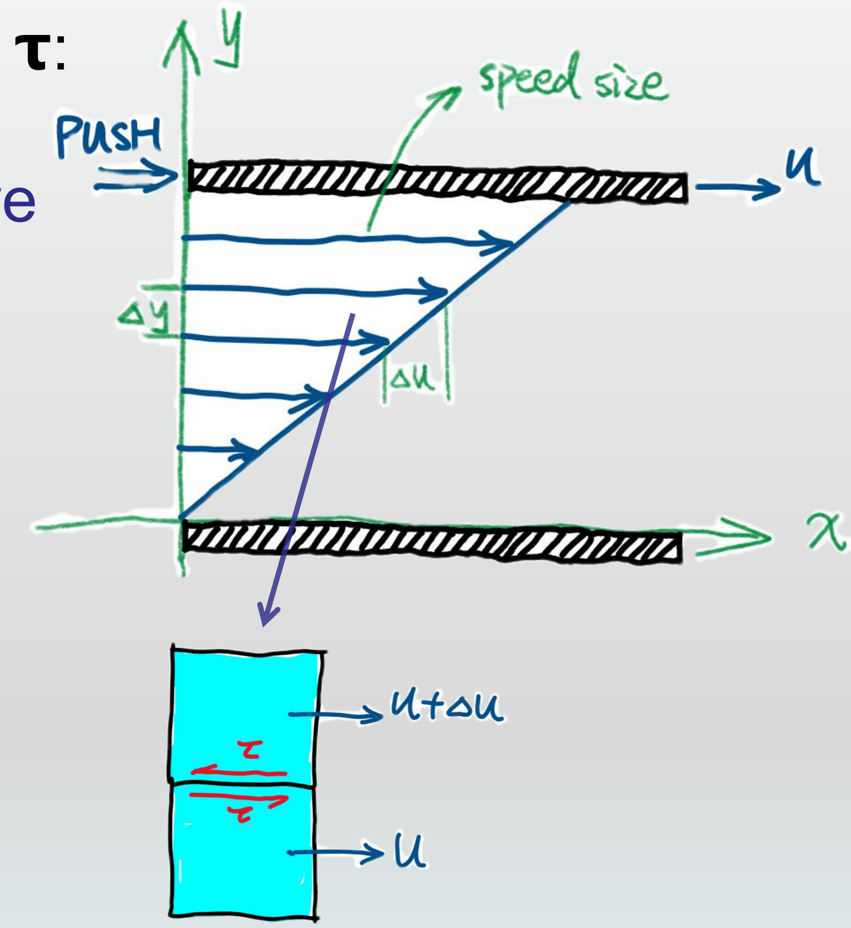
2. Background

Let's talk about the **shear stress τ**:

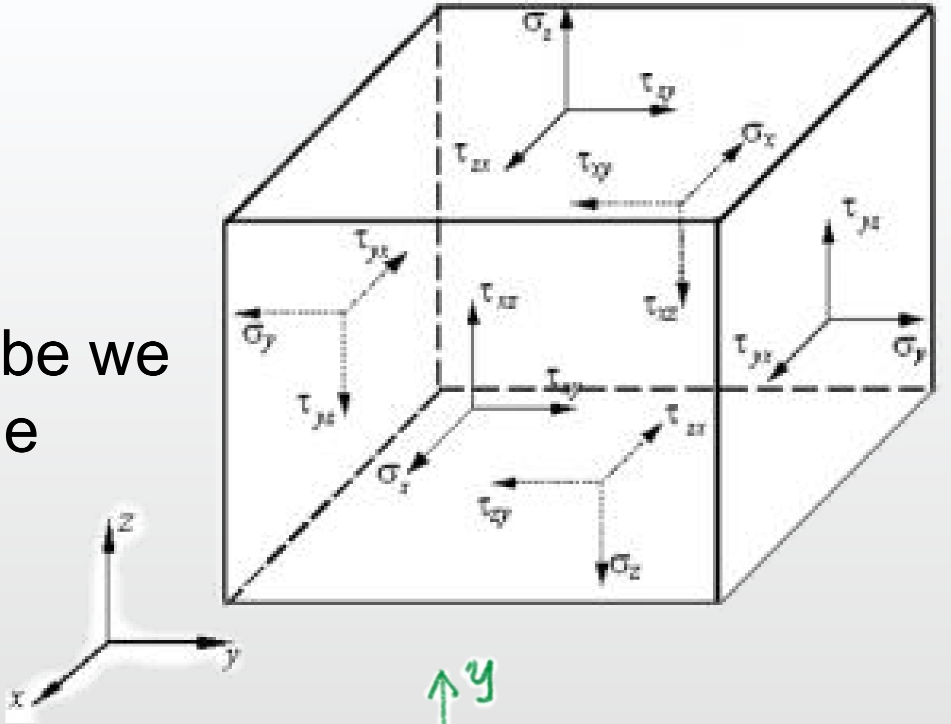
For the viscous Newton fluid, if we think about the 2D model, we will get the **viscous shear stress τ** between fluid molecules:

$$\tau = \mu*(du/dy)$$

μ is the coefficient of viscosity



2. Background



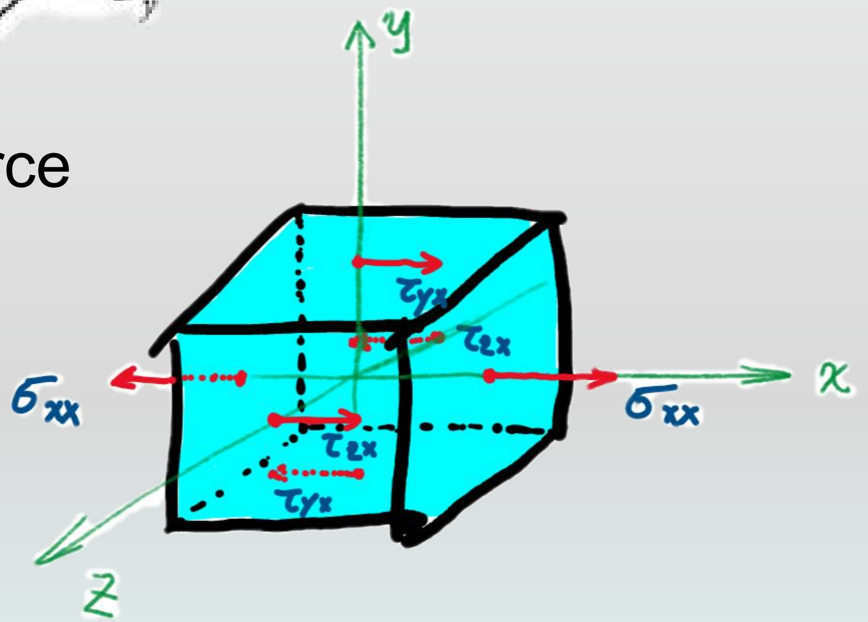
So for the 3D model, for the cube we not only need to considerate the **shear stress τ**, but also the **normal stress σ**.

Let's think about those internal force in the direction of x:

$$f_x = (\partial\sigma_{xx}/\partial x)dxdydz \rightarrow (\partial\sigma_{xx}/\partial x)*1$$

$$+(\partial\tau_{yx}/\partial y)dxdydz \rightarrow (\partial\tau_{yx}/\partial y)*1$$

$$+(\partial\tau_{zx}/\partial z)dxdydz \rightarrow (\partial\tau_{zx}/\partial z)*1$$



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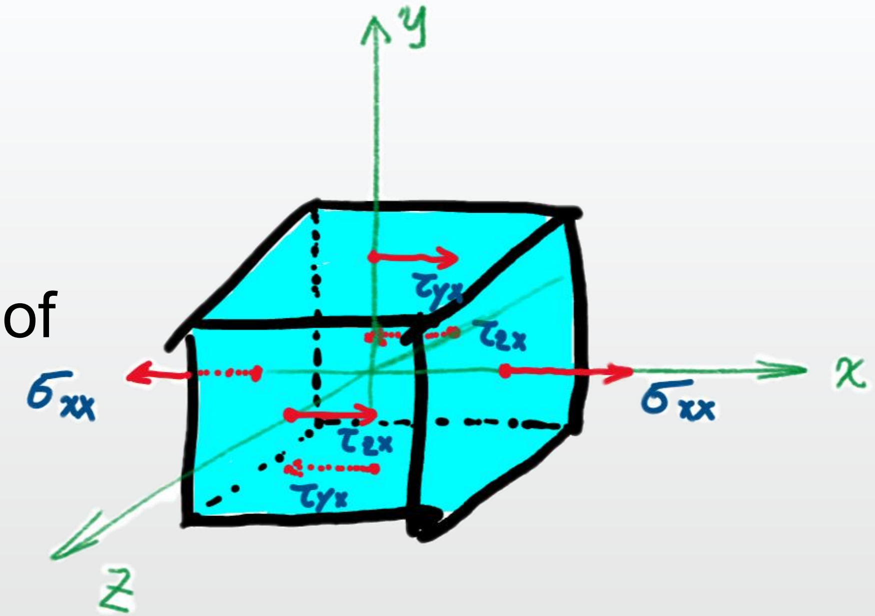


The internal force in the direction of x:

$$f_x = (\partial\sigma_{xx}/\partial x)+(\partial\tau_{yx}/\partial y)+(\partial\tau_{zx}/\partial z)$$

Based on the **Constitutive Equation** (for in/compressible newton fluid) we've talken in 2D situation:

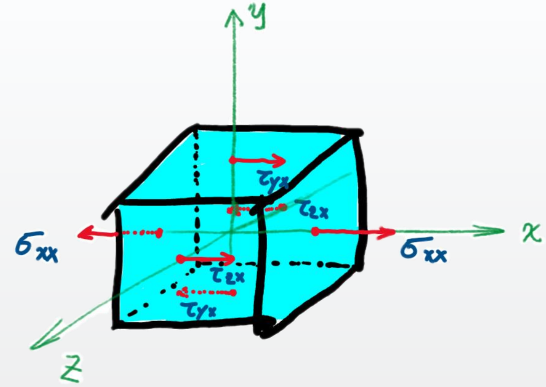
$$\boldsymbol{\sigma_{xx}} = -p + \underbrace{\triangle\sigma_{xx}}_{\text{(Additional viscous stress)}} = -p + \underbrace{2\mu^*(\partial V_x/\partial x)}_{\text{(Incompressible part)}} - \underbrace{(2/3)\mu^*[(\partial V_x/\partial x)+(\partial V_y/\partial y)+(\partial V_z/\partial z)]}_{\text{(Compressible part)}}$$

$$= -p + 2\mu^*(\partial u/\partial x) - \underline{(2/3)\mu^*(\triangledown V)}$$

$$\boldsymbol{\tau_{yx}} = \mu^*[(\partial V_y/\partial x) + (\partial V_x/\partial y)] = \mu^*[(\partial v/\partial x) + (\partial u/\partial y)]$$

$$\boldsymbol{\tau_{zx}} = \mu^*[(\partial V_z/\partial x) + (\partial V_x/\partial z)] = \mu^*[(\partial w/\partial x) + (\partial u/\partial z)]$$

2. Background



Thus, for the incompressible newton fluid we have:

$$F_x = f_x + F_{xoutside}$$

$$= [(\partial\sigma_{xx}/\partial x)+(\partial\tau_{yx}/\partial y)+(\partial\tau_{zx}/\partial z)] + [(\partial/\partial x)*\rho*g]$$

$$= \{-(\partial/\partial x)*p+2\mu*(\partial^2u/\partial x^2) + \mu*[(\partial^2v/\partial x\partial y) + (\partial^2u/\partial y^2)]$$

$$+ \mu*[(\partial^2w/\partial x\partial z) + (\partial^2u/\partial^2z)]\} + \{(\partial/\partial x)*\rho*g\}$$

If we ignore the gravity:

$$F_x = -(\partial/\partial x)*p + \mu*[2*(\partial^2u/\partial x^2)+(\partial^2v/\partial x\partial y)+(\partial^2u/\partial y^2)+(\partial^2w/\partial x\partial z)+(\partial^2u/\partial^2z)]$$

$$= -(\partial/\partial x)*p +\mu*[(\partial^2u/\partial x^2)+(\partial^2u/\partial y^2)+(\partial^2u/\partial^2z)$$

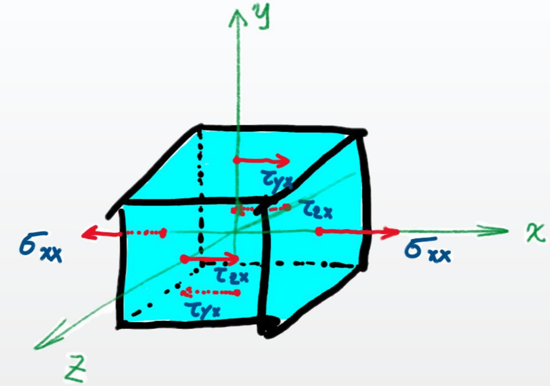
$$+(\partial^2u/\partial x^2)+(\partial^2v/\partial x\partial y)+(\partial^2w/\partial x\partial z)]$$

$$= -(\partial/\partial x)*p +\mu*[\triangledown^2u+(\partial^2u/\partial x\partial x)+(\partial^2v/\partial x\partial y)+(\partial^2w/\partial x\partial z)]$$

$$= -(\partial/\partial x)*p +\mu*[\triangledown^2(\partial V/\partial x)+(\partial^2\triangledown V/\partial x^2)]$$

$$= -(\partial/\partial x)*p+\triangledown\mu*[\triangledown u+(\partial u/\partial x)]$$

2. Background



In conclusion, if we have x-y-z directions, combining motivation equations, we will have:

$$\begin{cases} F_x = \rho^*[(\partial u/\partial t) + V \cdot \nabla u] \\ F_y = \rho^*[(\partial v/\partial t) + V \cdot \nabla v] \\ F_z = \rho^*[(\partial w/\partial t) + V \cdot \nabla w] \\ F_x = -(\partial/\partial x)^*p + \nabla \mu^*[\nabla u + (\partial u/\partial x)] \\ F_y = -(\partial/\partial y)^*p + \nabla \mu^*[\nabla v + (\partial v/\partial x)] \\ F_z = -(\partial/\partial z)^*p + \nabla \mu^*[\nabla w + (\partial w/\partial x)] \end{cases}$$

Let's put the system in vector form:

$$\begin{cases} F = \rho^*[(\partial V/\partial t) + V \cdot \nabla V] \text{ ....①} \\ F = -\nabla p + \nabla \mu^*(\nabla V + \nabla V^T) \text{ ....②} \end{cases}$$

Of course if you wanna talk about the compressible fluid with gravity, ② can be expressed as:

$$F = -\nabla p + \nabla \mu^*(\nabla V + \nabla V^T) - \nabla(2/3)\mu^2(\nabla V)^*I + \rho^*g \text{ ....②'}$$

2. Background

Navier–Stokes Equations
3 – dimensional – unsteady
Glenn Research Center

Coordinates: (x,y,z)
Velocity Components: (u,v,w)
Time : t
Density: ρ
Total Energy: Et
Pressure: p
Stress: τ
Heat Flux: q
Reynolds Number: Re
Prandtl Number: Pr

Continuity:
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:
$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:
$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum
$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:
$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] + \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

Thus, combining ① and ②, the Navier-Strokes Equations can be expressed as below:

$$\rho*[(\partial V/\partial t) + V\cdot\triangledown V] = -\triangledown p + \triangledown\mu*(\triangledown V + \triangledown V^T)$$

$$\underbrace{\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{1} = \underbrace{-\nabla p}_{2} + \underbrace{\nabla\cdot\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)}_{3} + \underbrace{\mathbf{F}}_{4}$$

https://www.comsol.com/multiphysics/navier-stokes-equations?parent=modeling-conservation-mass-energy-momentum-0402-432-302

Don't forget! our aim is researching a SEA not only one tiny water cube. Thus, we need to have continuity equation to confine our NS equation:

$$\partial\rho/\partial t + \triangledown(\rho V) = \partial\rho/\partial t + \partial\rho u/\partial x + \partial\rho v/\partial y + \partial\rho w/\partial z = 0$$

2. Background

Continuity equation shows our fluid is always a whole with a balanced mass distribution, which will eliminate the compressible part of the NS equation.

$$\rho*[(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p + \nabla\mu*(\nabla V + \nabla V^T)$$

$$\underbrace{\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{1} = \underbrace{-\nabla p}_{2} + \underbrace{\nabla\cdot\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)}_{3} + \underbrace{\mathbf{F}}_{4}$$

https://www.comsol.com/multiphysics/navier-stokes-equations?parent=modeling-conservation-mass-energy-momentum-0402-432-302

Compressibility can be measured by Match Number:

$M = |V|/C$, (C is the sound speed in the fluid)

Hight Match Number means compressible flow;
Low Match Number means incompressible flow;

2. Background

Navier-Stokes Equations
3 - dimensional - unsteady

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X - Momentum: $\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$

Y - Momentum: $\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$

Z - Momentum $\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$
$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

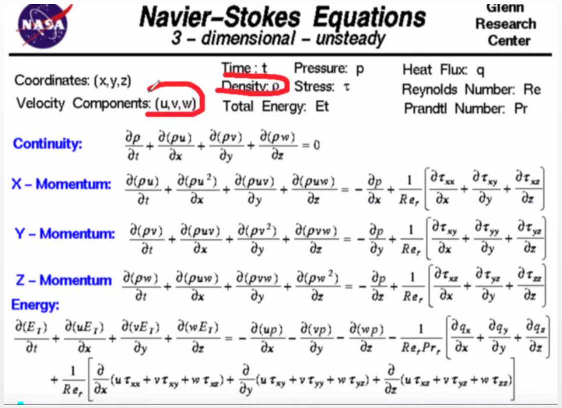
By the way, we can also have the restricted condition by adding the energy equation like:

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}u^2\right)\right] + \nabla \cdot \left[\rho \mathbf{u}\left(e + \frac{1}{2}u^2\right)\right] = \nabla \cdot (k\nabla T) + \nabla \cdot (-p\mathbf{u} + \tau \cdot \mathbf{u}) + \mathbf{u} \cdot \mathbf{F} + Q$$

https://www.comsol.com/multiphysics/navier-stokes-equations?parent=modeling-conservation-mass-energy-momentum-0402-432-302

However, for incompressible flows, the temperature equation is completely decoupled from the NS equation, so we won't talk about the equation here.

2. Background



Navier–Stokes Equations
3 – dimensional – unsteady

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X – Momentum: $$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum: $$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}+\frac{\partial(\rho vw)}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum $$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho uw)}{\partial x}+\frac{\partial(\rho vw)}{\partial y}+\frac{\partial(\rho w^2)}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:
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$$+\frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx}+v\tau_{xy}+w\tau_{xz})+\frac{\partial}{\partial y}(u\tau_{xy}+v\tau_{yy}+w\tau_{yz})+\frac{\partial}{\partial z}(u\tau_{xz}+v\tau_{yz}+w\tau_{zz})\right]$$

Next for the NS equations, let's eliminate the dimensions of the equation:

$$\rho^*[(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p+\nabla\mu^*(\nabla V+\nabla V^T)$$

We can multiply the both side of the equation by a factor $L/(\rho^*\tilde{V}^2)$. L is the characteristic length, ρ is the fluid density, $\tilde{V}$ is the mean velocity. Then we will get:

$$(L/\tilde{V}^2)^*[(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p^*L/(\rho^*\tilde{V}^2) + \nabla\mu L/(\rho^*\tilde{V}^2)^*(\nabla V+\nabla V^T)$$

Set $\underline{V' = V/\tilde{V}}$, $\underline{p' = p^*(1/(\rho^*\tilde{V}^2))}$, $\underline{(\partial/\partial t') = (L/\tilde{V})^*(\partial/\partial t)}$, $\underline{\nabla' = L\nabla}$:

$$\Rightarrow [(\partial V'/\partial t') + V'\cdot\nabla' V'] = -\nabla' p' + \boldsymbol{\mu/(\rho^*L^*\tilde{V})}^*\nabla'(\nabla' V'+\nabla' V'^T)$$
$$\Rightarrow [(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p + \underbrace{\boldsymbol{\mu/(\rho^*L^*\tilde{V})}}_{\uparrow\ \text{Inverse of Reynolds number}}{}^*\nabla(\nabla V+\nabla V^T)$$

$$\Rightarrow [(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p + \boldsymbol{1/Re}^*\nabla(\nabla V+\nabla V^T)$$

2. Background

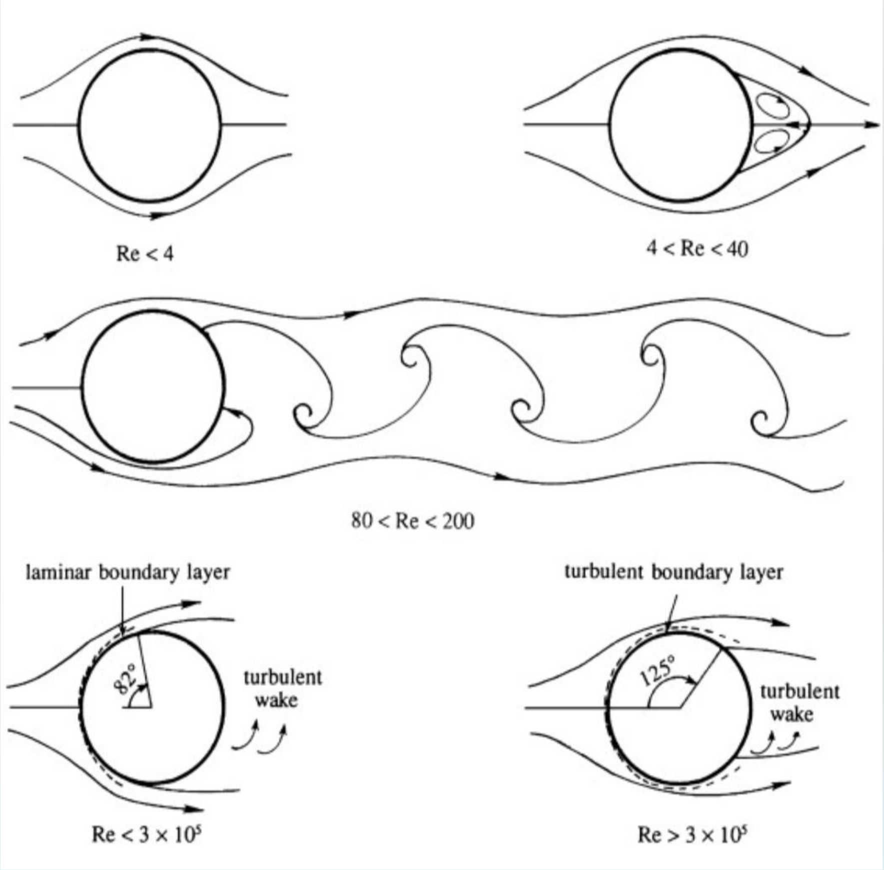
Reynolds number: A number shows the ratio between internal forces and viscous forces.

$$Re = (\rho * L * \tilde{V}) / \mu$$

1. Because there is no dimension, incompressible Newtonian flows with the same Reynolds number are comparable.

2. High Reynolds number will cause turbulent. At this time flow is inviscid.

3. Low Reynolds number will cause laminar. At this time flow is viscous.



https://www.sciencedirect.com/topics/engineering/creeping-flow

2. Background

$$\begin{cases} \partial\rho/\partial t + \nabla(\rho V) = 0 \\ [(\partial V/\partial t) + V\cdot\nabla V] = -\nabla p + \mathbf{1/Re}*\nabla(\nabla V + \nabla V^{T}) \end{cases}$$

Navier–Stokes Equations

3 – dimensional – unsteady

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X – Momentum:

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Z – Momentum

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$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

3. Preliminary

Normally, the **NS equation** is classed as Nonlinear Partial Differential Equation, which means the equation is hard to find the analytical solution.

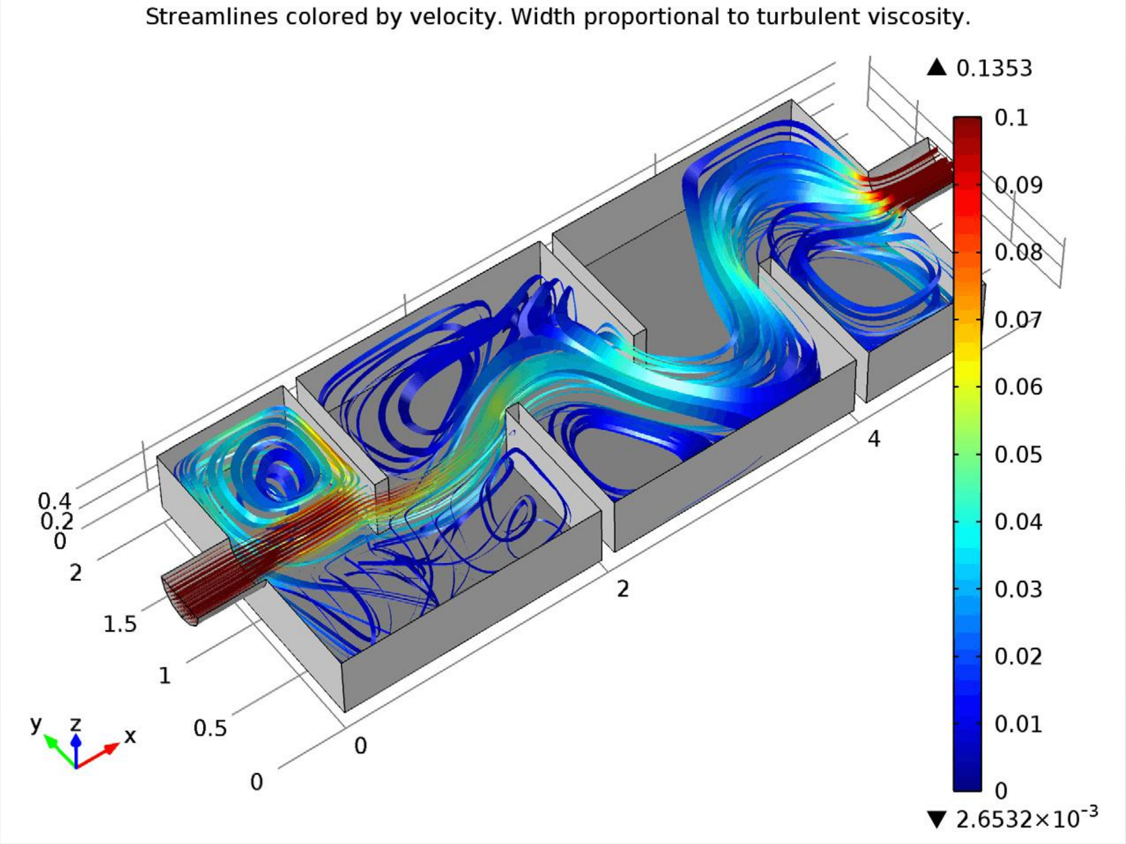
Furthermore, because of the complexity of the NS equation, fluid movement without any constraints takes up a huge amount of computing resources.

Thus, the simulation should be based on the approximated equation and the spectial boundary conditions.

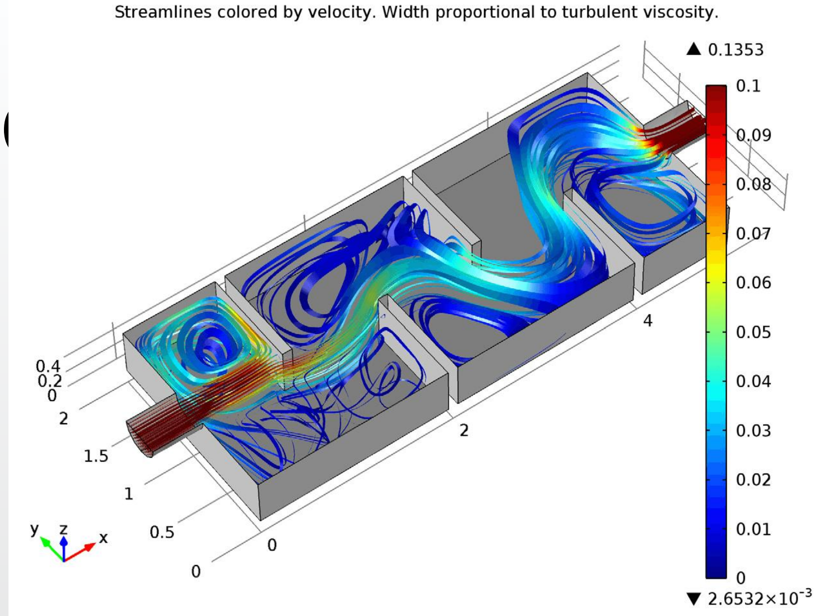
3. Cosmol Experiement

Cosmol would be a useful modeling tool.

The NS equation based on the approximated equation and the spectial boundary conditions (inlet, outlet, walls, etc..) can be simulated in Cosmol.

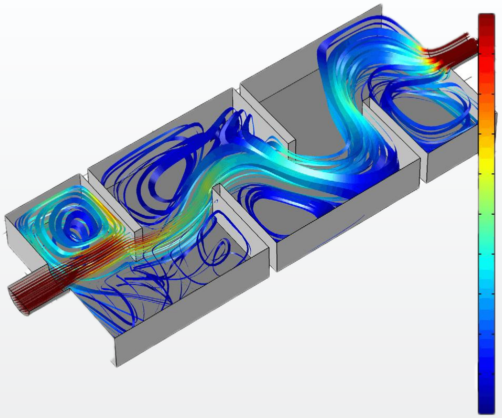


3. Cosmol Experieme



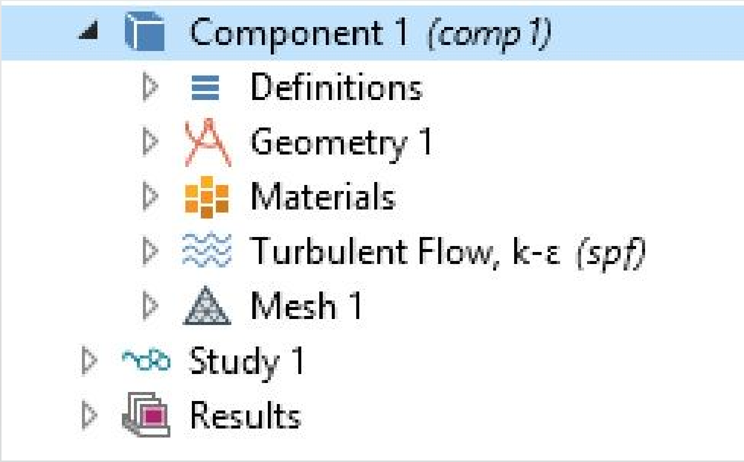
The initial phase of the project will be used to simulate and compare different NS family equation systems.

3. Cosmol Experiement



Comsol has its simulating pipeline for single-phase flow. The pipeline can be simplified as below:

Geometry
↓
Material
↓
Equation
↓
Mesh
↓
Study

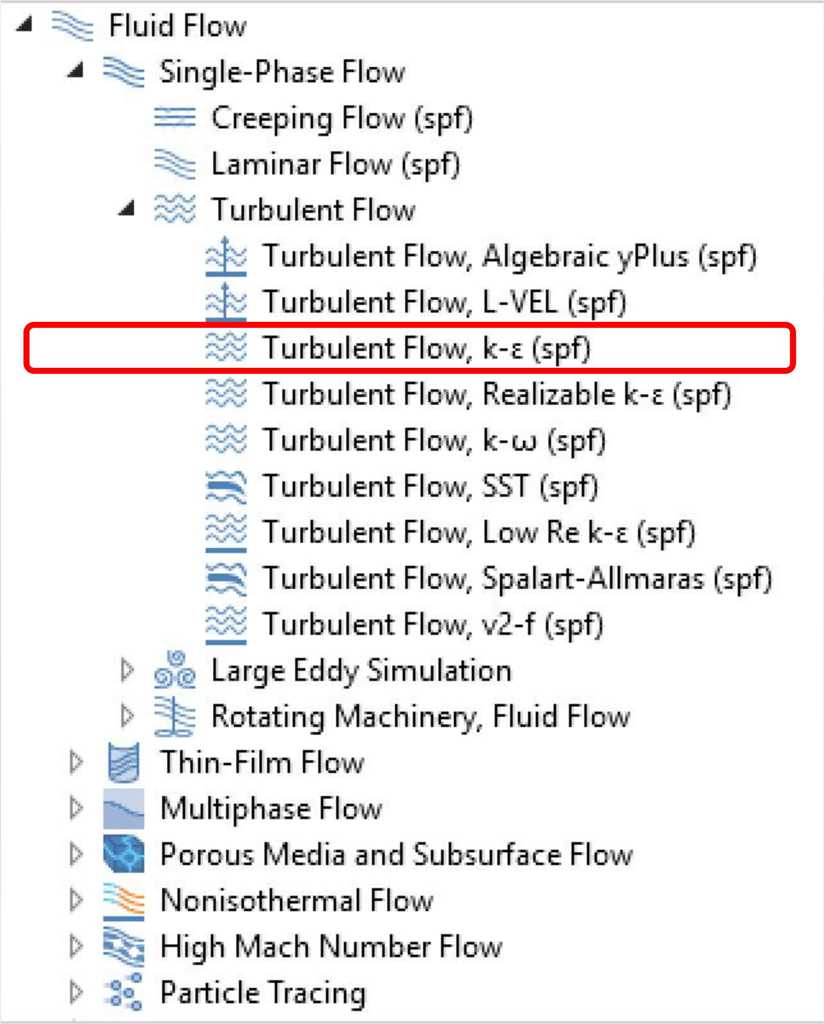


3. Cosmol Experiement

Equation

Comsol provides different kinds of equations for different conditions of flow:

- Laminar Flow
  - Navier-Stokes Equations
- Creeping Flow
  - Stokes Equations
- Non-Newtonian
  - Carreau, Power-Law, and user-defined models
- Turbulent Flow
  - k-ε, k-ω, Low Re k-ε, Shear Stress Transport, and Spalart-Allmaras turbulence models



3. Parameter Experiments

$$Re = (\boxed{\rho}*\boxed{L}*\boxed{\tilde{V}})/\boxed{\mu}$$

Fluid Density

Characteristic Length (V/S)

Mean Velocity

Dynamic Viscocity

INCREASE Reynolds number

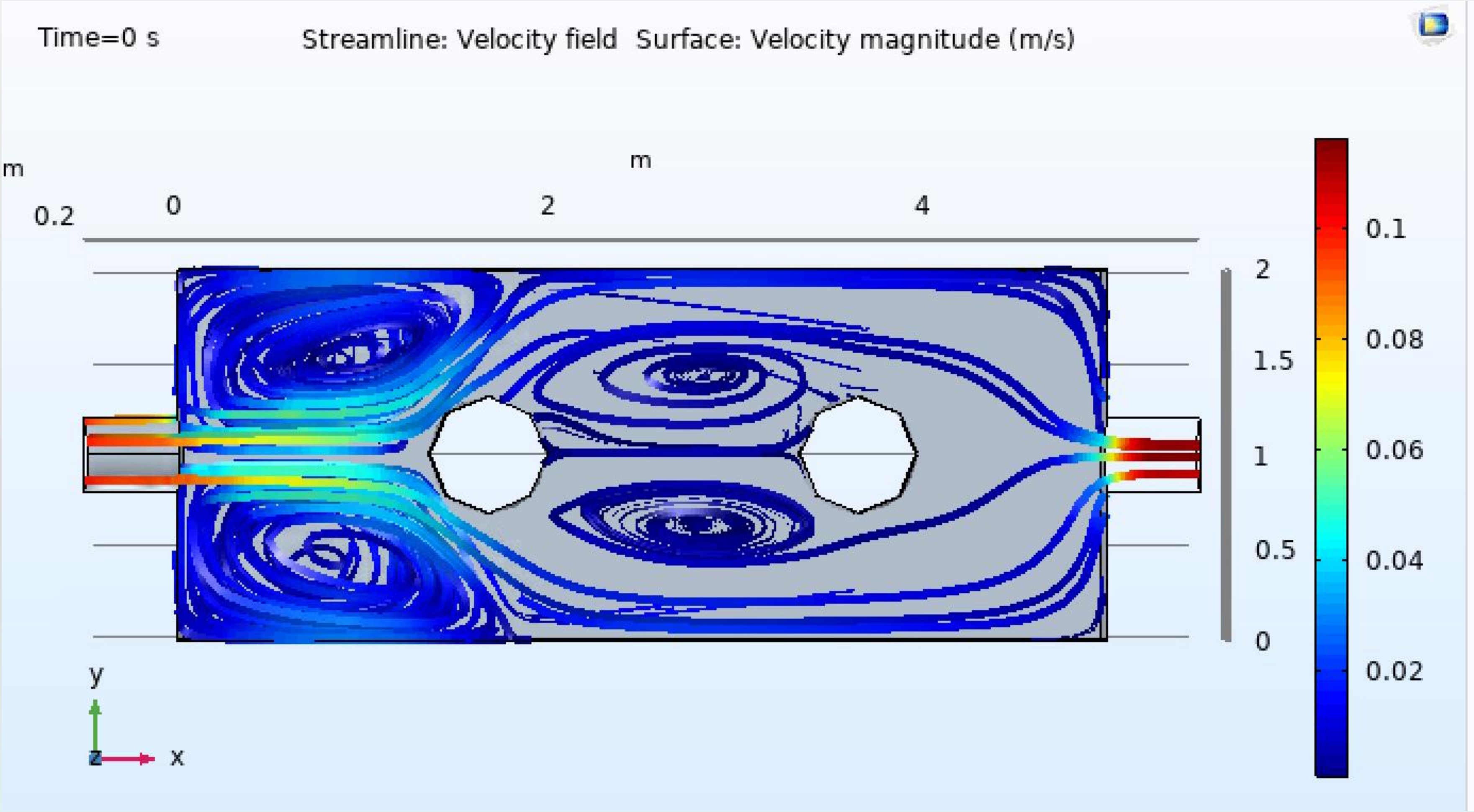
DECREASE Reynolds number

3. Parameter Experiments

EXP1: Temperature

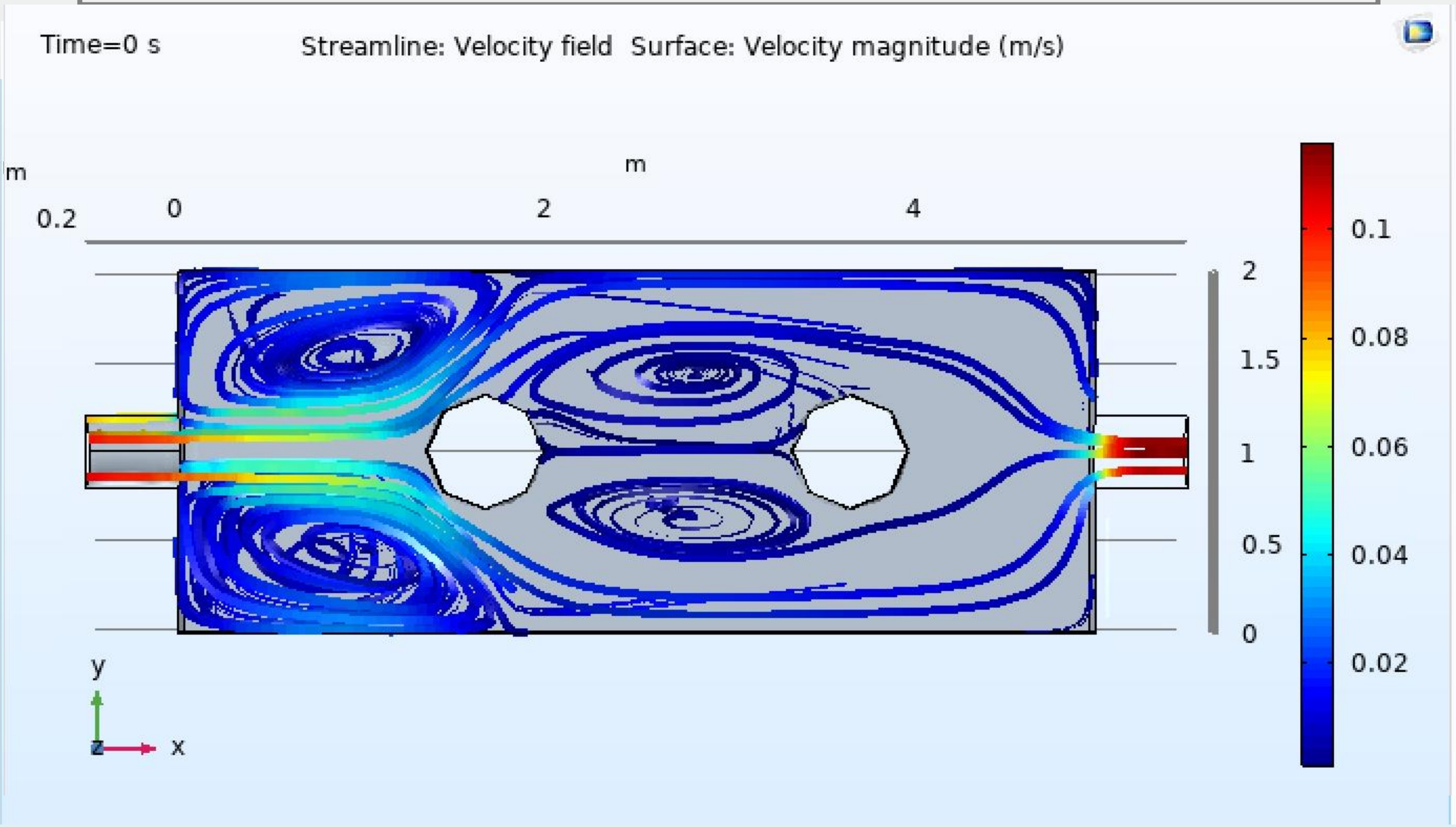
Experiment 1: Experimental Group

Material: Water	Density: 997.05 kg/m³
Temperature: 25℃	Characteristic Length: Normal
	Mean Velocity: 0.1 m/s
	Dynamic Viscosity $\mu = \nu\rho$: 8.90×10^{-4} pa·s



Experiment 1: Controlled Group 1 - Low Temp.

Material: Water	Density: 1000 kg/m³
Temperature: 20℃	Characteristic Length: Normal
	Mean Velocity: 0.1 m/s
	Dynamic Viscosity $\mu = \nu\rho$: 1.01×10^{-3} pa·s

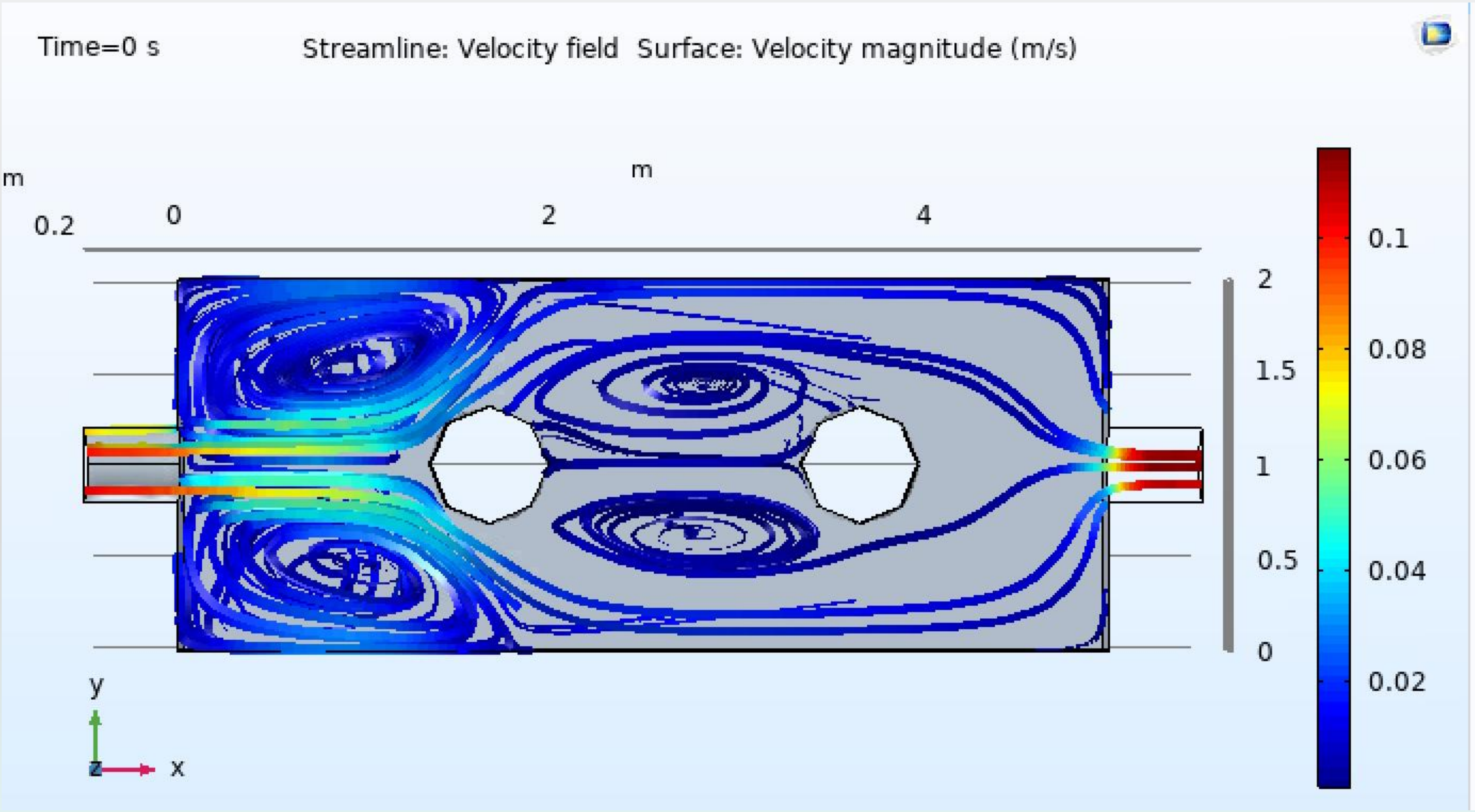


Experiment 1: Controlled Group 2 - High Temp.

Material: Water
Temperature: 100℃

Density: 958.4 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.1 m/s

Dynamic Viscosity $\mu = \nu\rho$: 2.8×10^{-4} pa·s



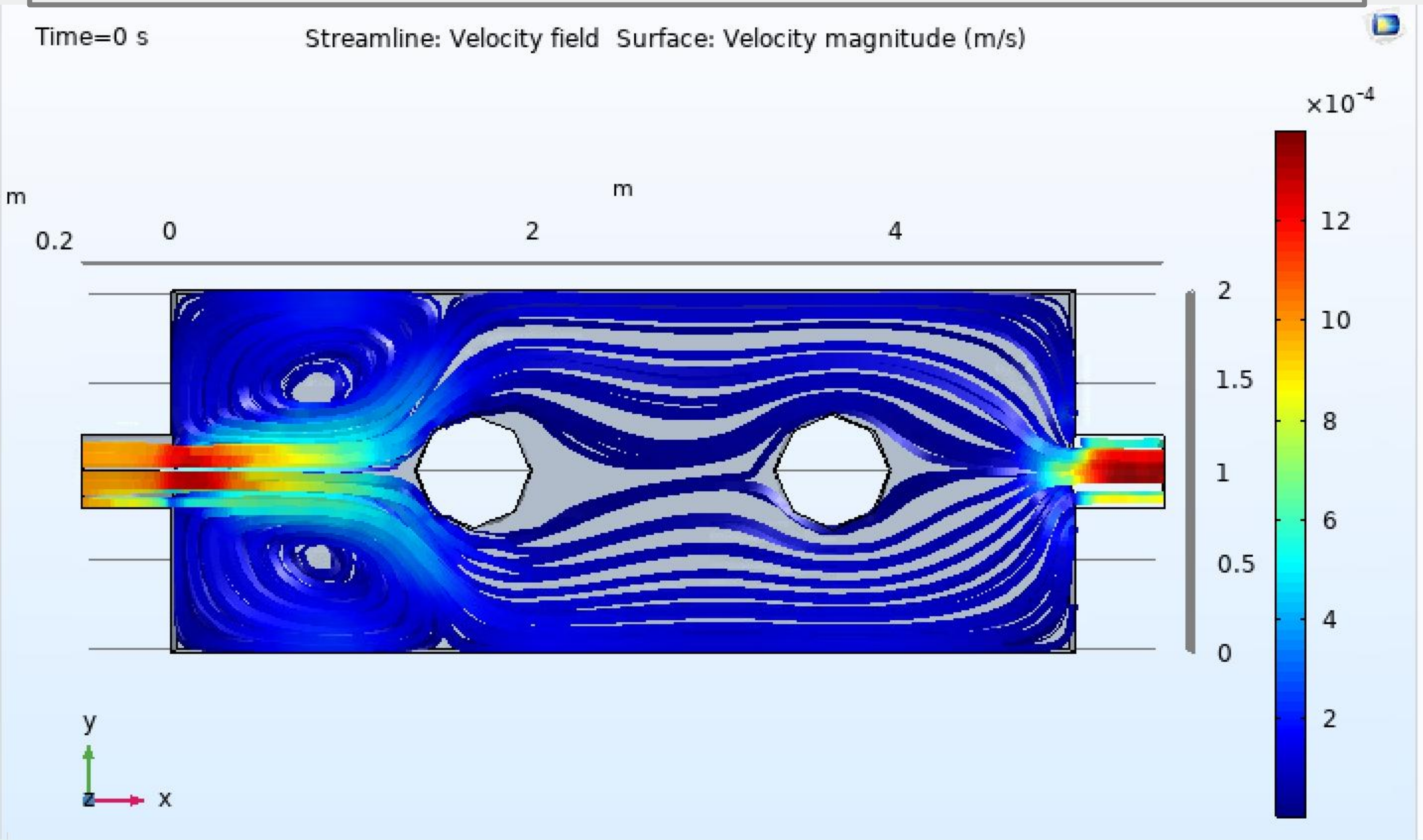
3. Parameter Experiments

EXP2: Density

Experiment 2: Experimental Group

Material: Ethyl Alcohol
Temperature: 20℃

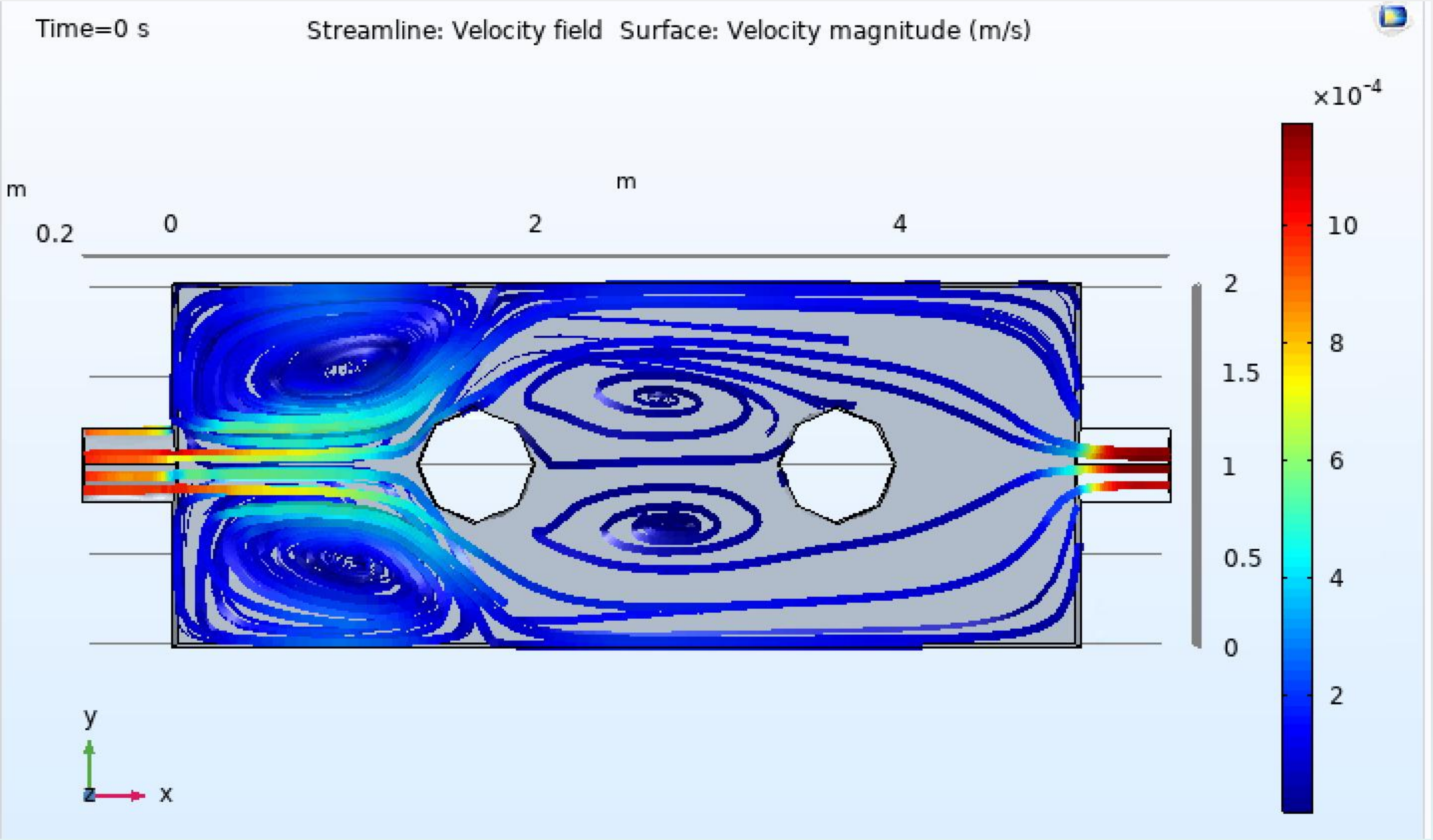
Density: 789 kg/m^3
Characteristic Length: Normal
Mean Velocity: 0.001 m/s
Dynamic Viscosity $\mu = \nu\rho$: 1.19×10^{-3} pa·s



Experiment 2: Controlled Group 1 - High Density

Material: Mercury
Temperature: 20℃

Density: 13600 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.001 m/s
Dynamic Viscosity $\mu = \nu\rho$: 1.57×10^{-3} pa·s

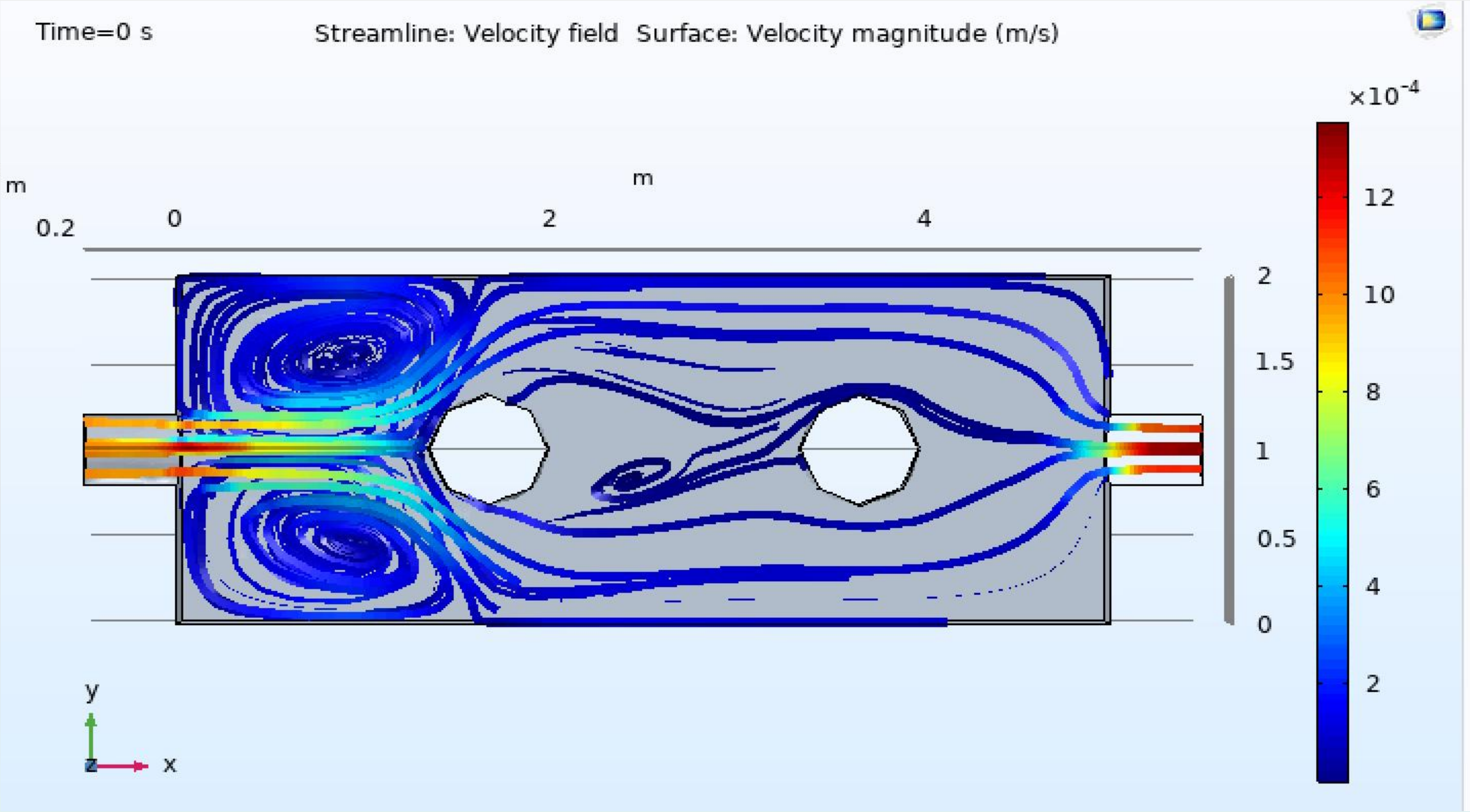


3. Parameter Experiments

EXP3: Characteristic Length

Experiment 3: Experimental Group

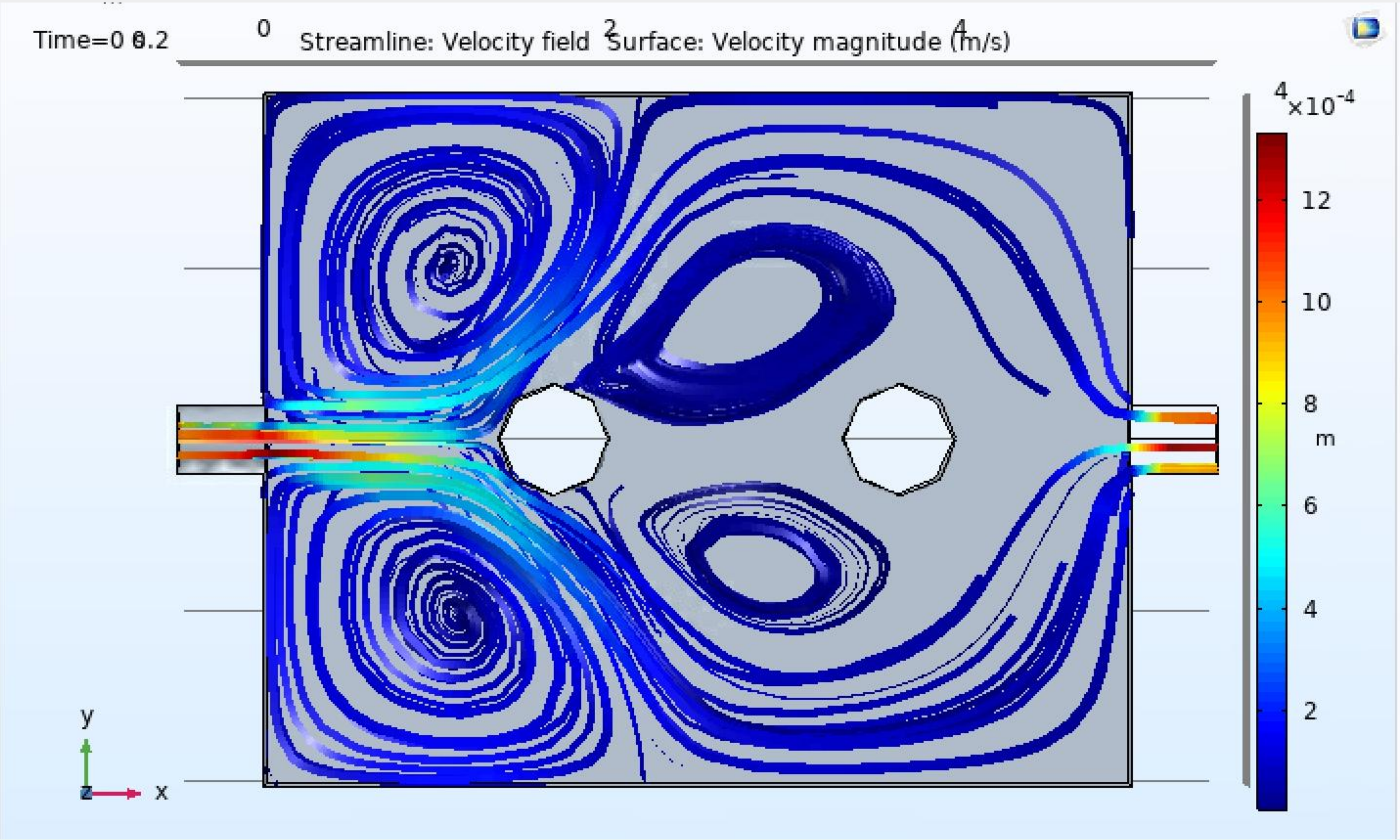
Material: Water	Density: 997.05 kg/m³
Temperature: 25℃	Characteristic Length: Normal
	Mean Velocity: 0.001 m/s
	Dynamic Viscosity $\mu = v\rho$: 8.90×10^{-4} pa·s



Experiment 3: Controlled Group 1 - Square Size

Material: Water
Temperature: 25℃

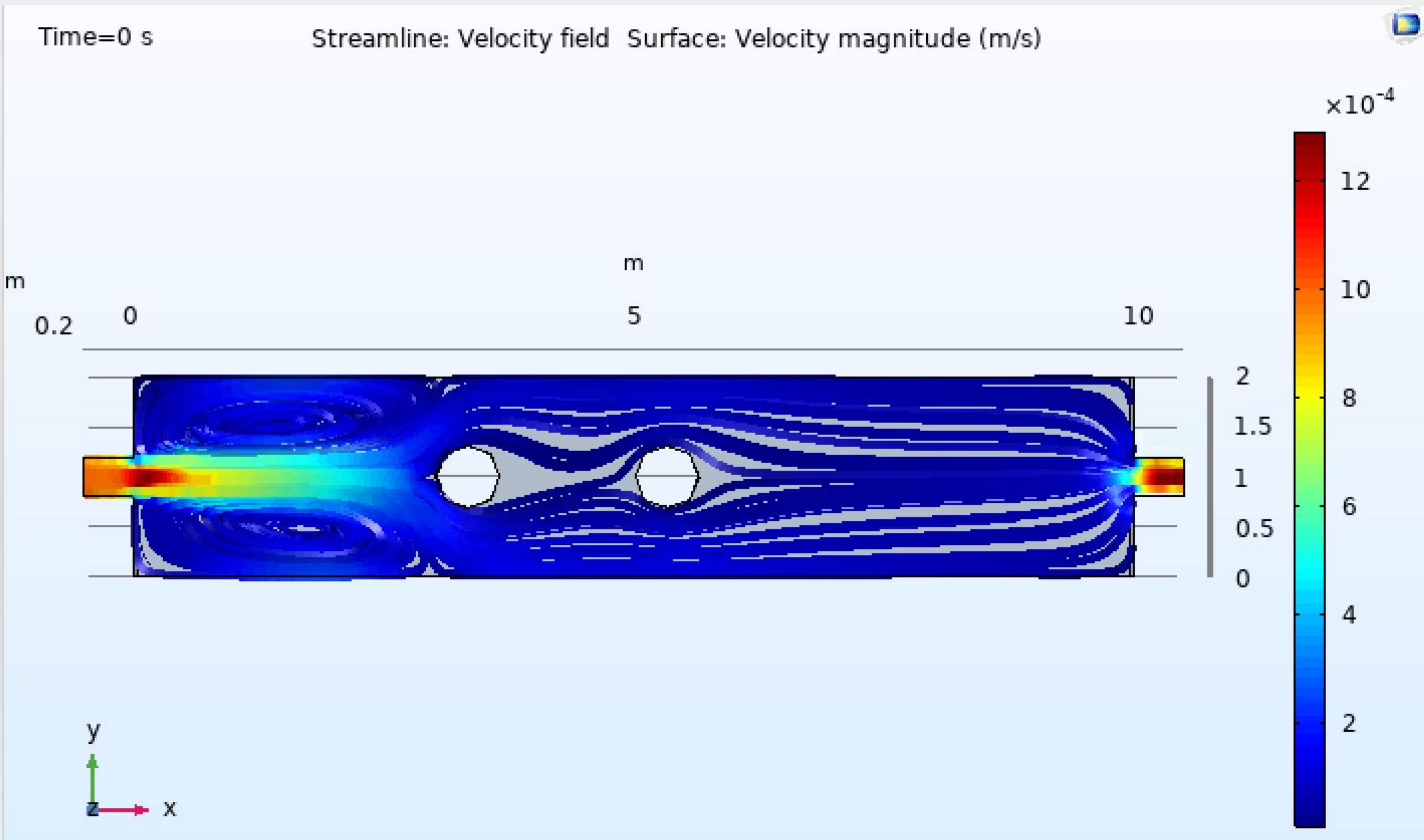
Density: 997.05 kg/m^3
Characteristic Length: Square
Mean Velocity: 0.001 m/s
Dynamic Viscosity $\mu = v\rho$: 8.90×10^{-4} pa·s



Experiment 3: Controlled Group 2 - Long Size

Material: Water
Temperature: 25℃

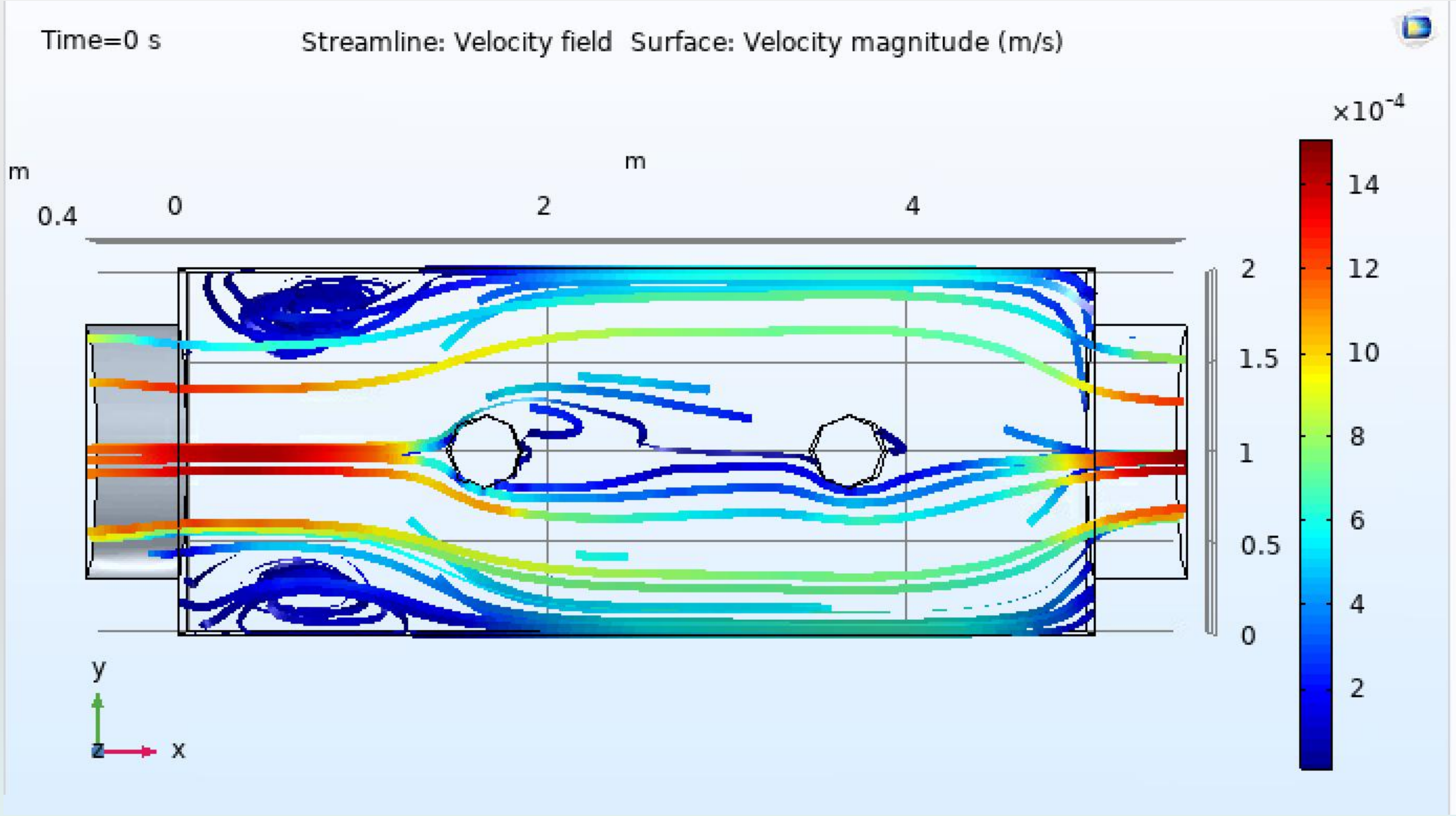
Density: 997.05 kg/m^3
Characteristic Length: Long
Mean Velocity: 0.1 m/s
Dynamic Viscosity $\mu = \nu\rho$: 8.90×10^{-4} pa·s



Experiment 3: Controlled Group 3 - Large Input

Material: Water
Temperature: 25℃

Density: 997.05 kg/m³
Characteristic Length: Large input
Mean Velocity: 0.1 m/s
Dynamic Viscosity $\mu = \nu\rho$: 8.90×10^{-4} pa·s



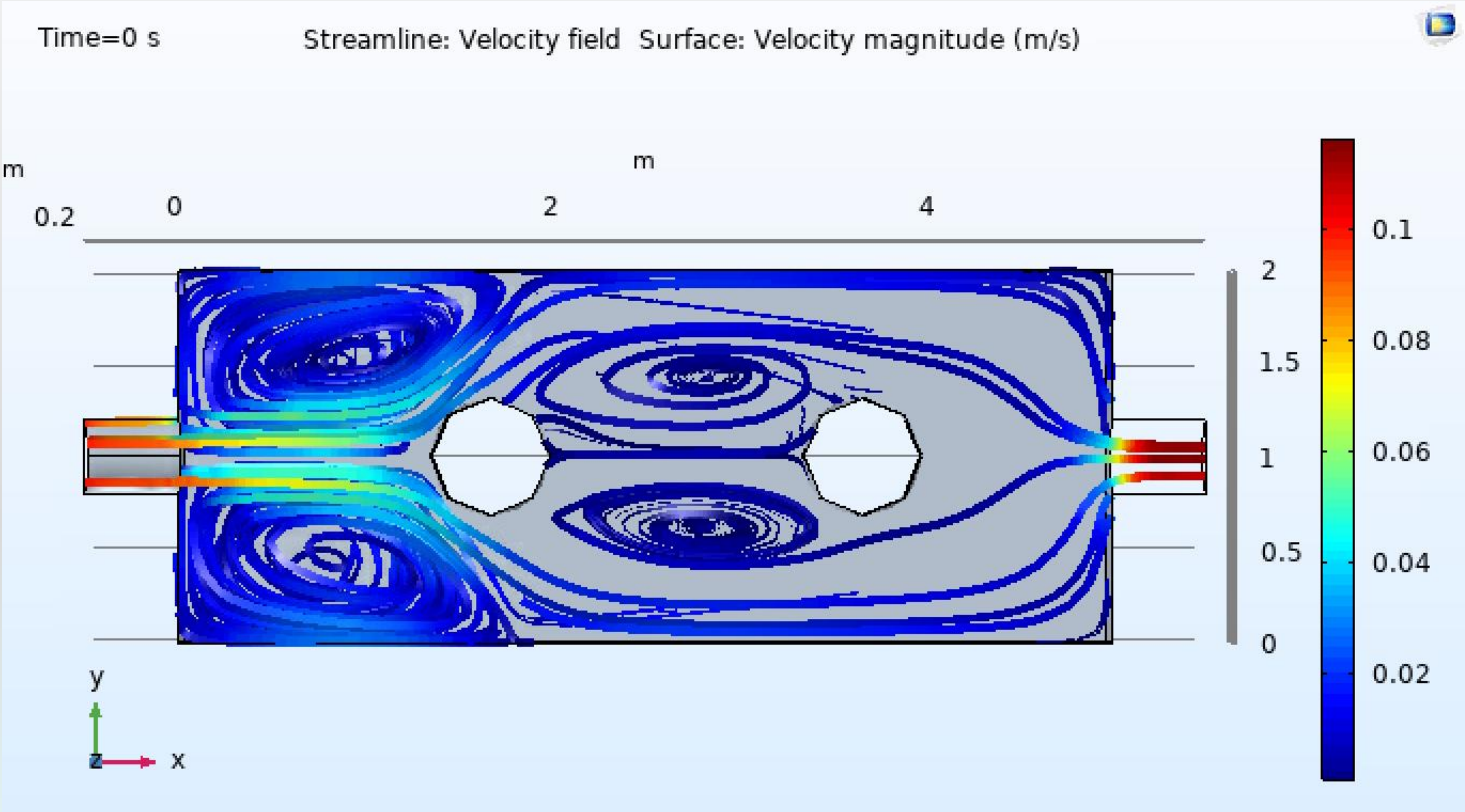
3. Parameter Experiments

EXP4: Average Speed

Experiment4: Experimental Group

Material: Water
Temperature: 25℃

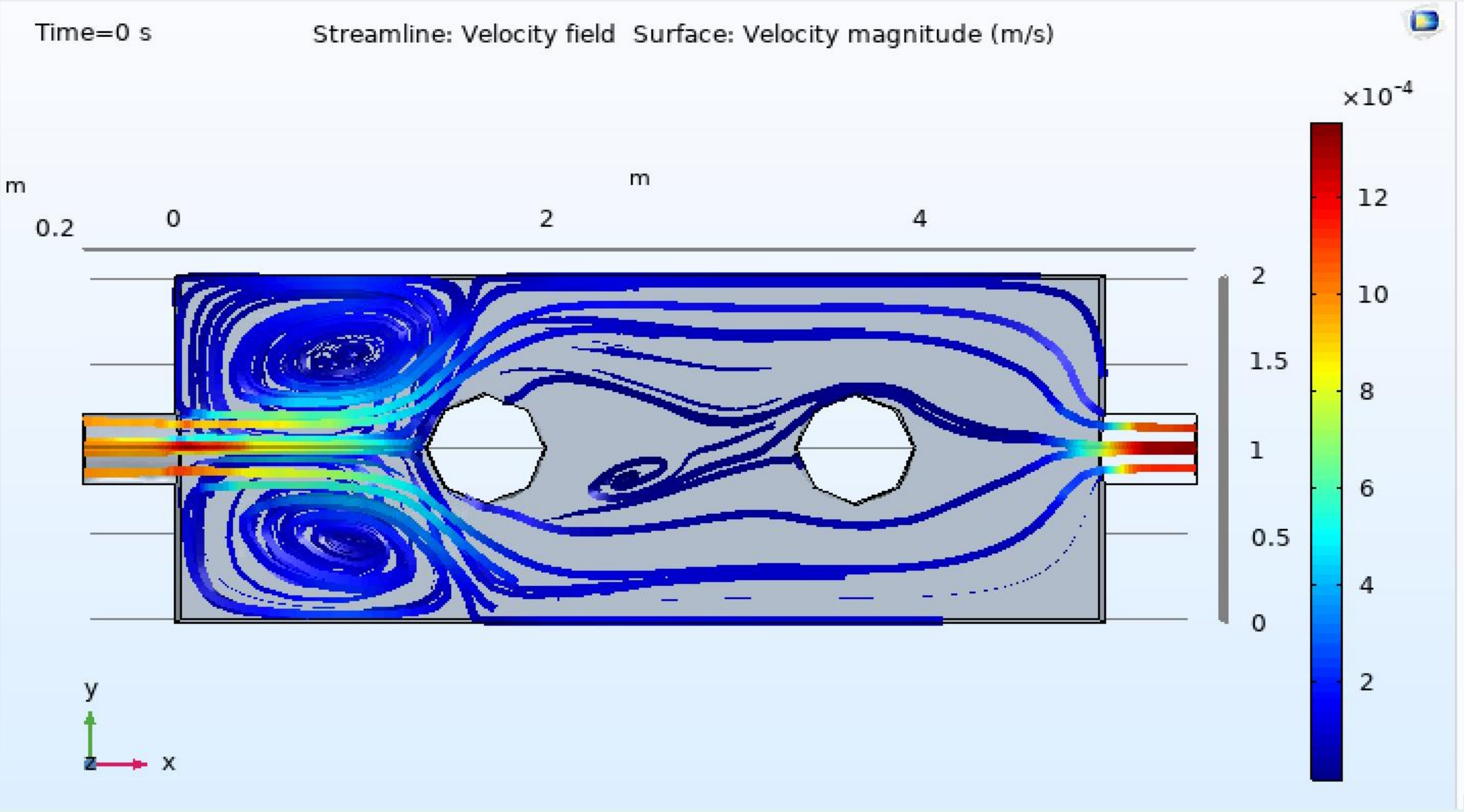
Density: 997.05 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.1 m/s
Dynamic Viscosity $\mu = \nu\rho$: 8.90×10^{-4} pa·s



Experiment 4: Controlled Group 1 - Low Speed

Material: Water
Temperature: 25℃

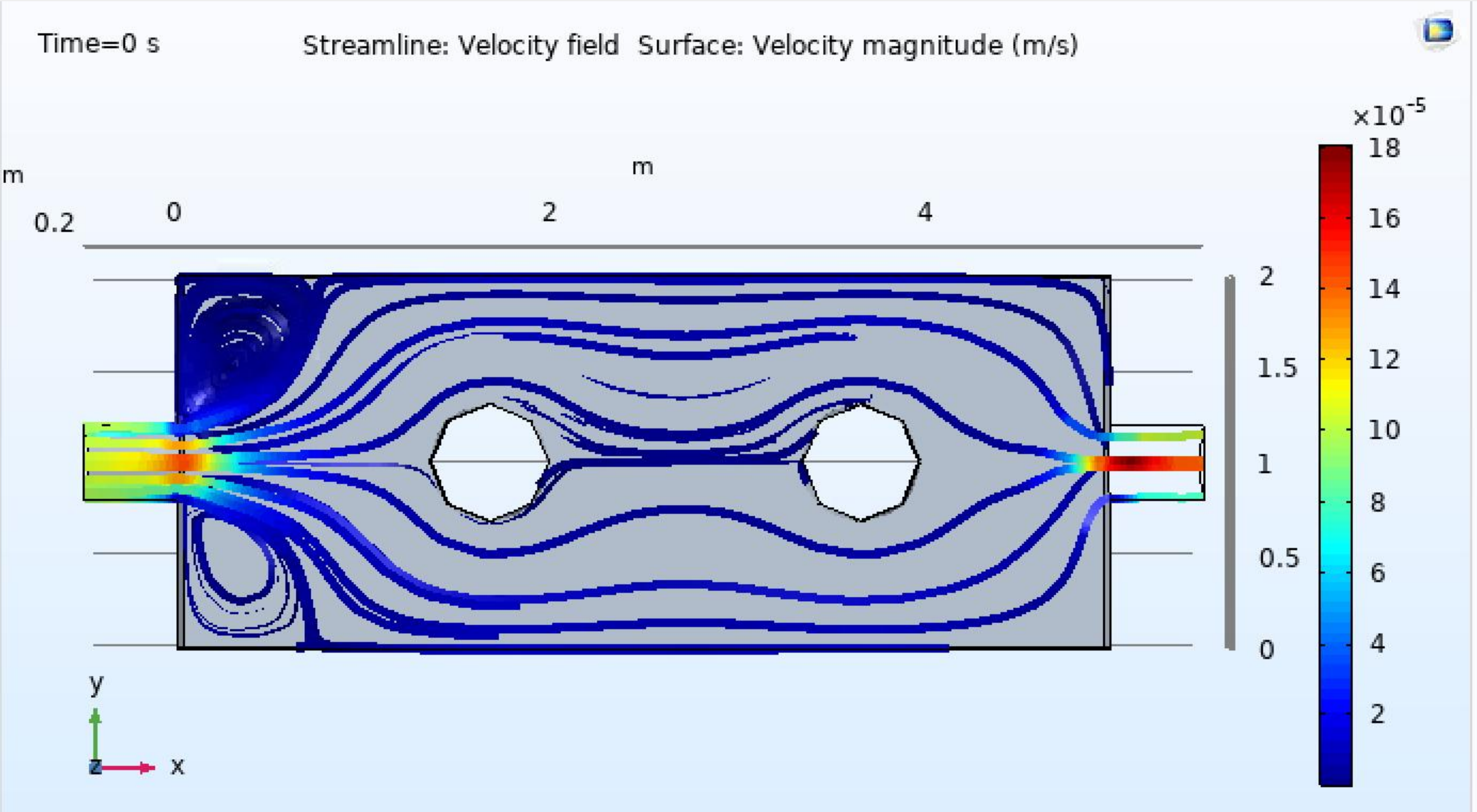
Density: 997.05 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.001 m/s
Dynamic Viscosity $\mu = v\rho$: 8.90×10^{-4} pa·s



Experiment 4: Controlled Group 2 - Lower Speed

Material: Water
Temperature: 25℃

Density: 997.05 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.0001 m/s
Dynamic Viscosity $\mu = v\rho$: 8.90×10^{-4} pa·s



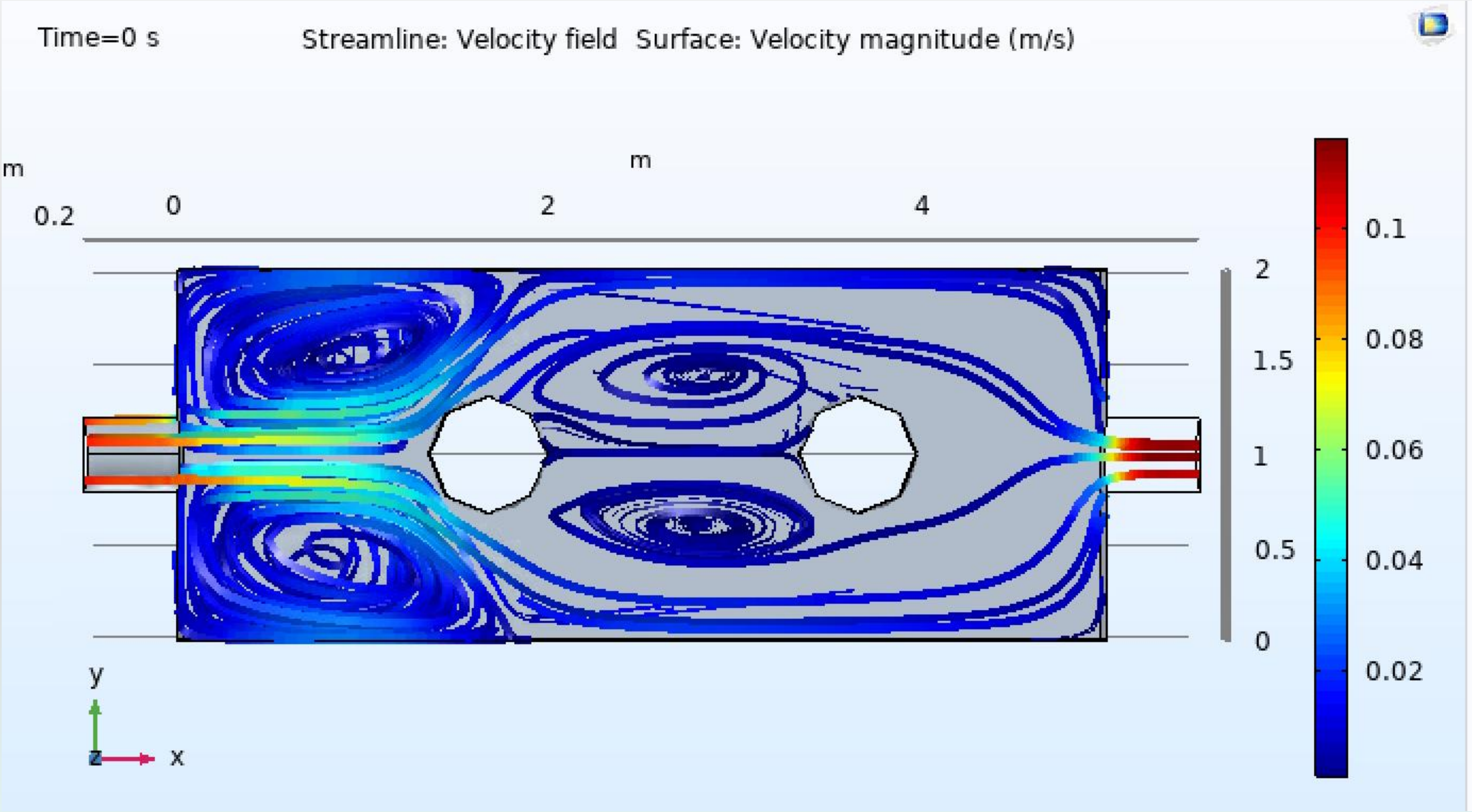
3. Parameter Experiments

EXP5: Dynamic Viscosity

Experiment 5: Experimental Group

Material: Water
Temperature: 25℃

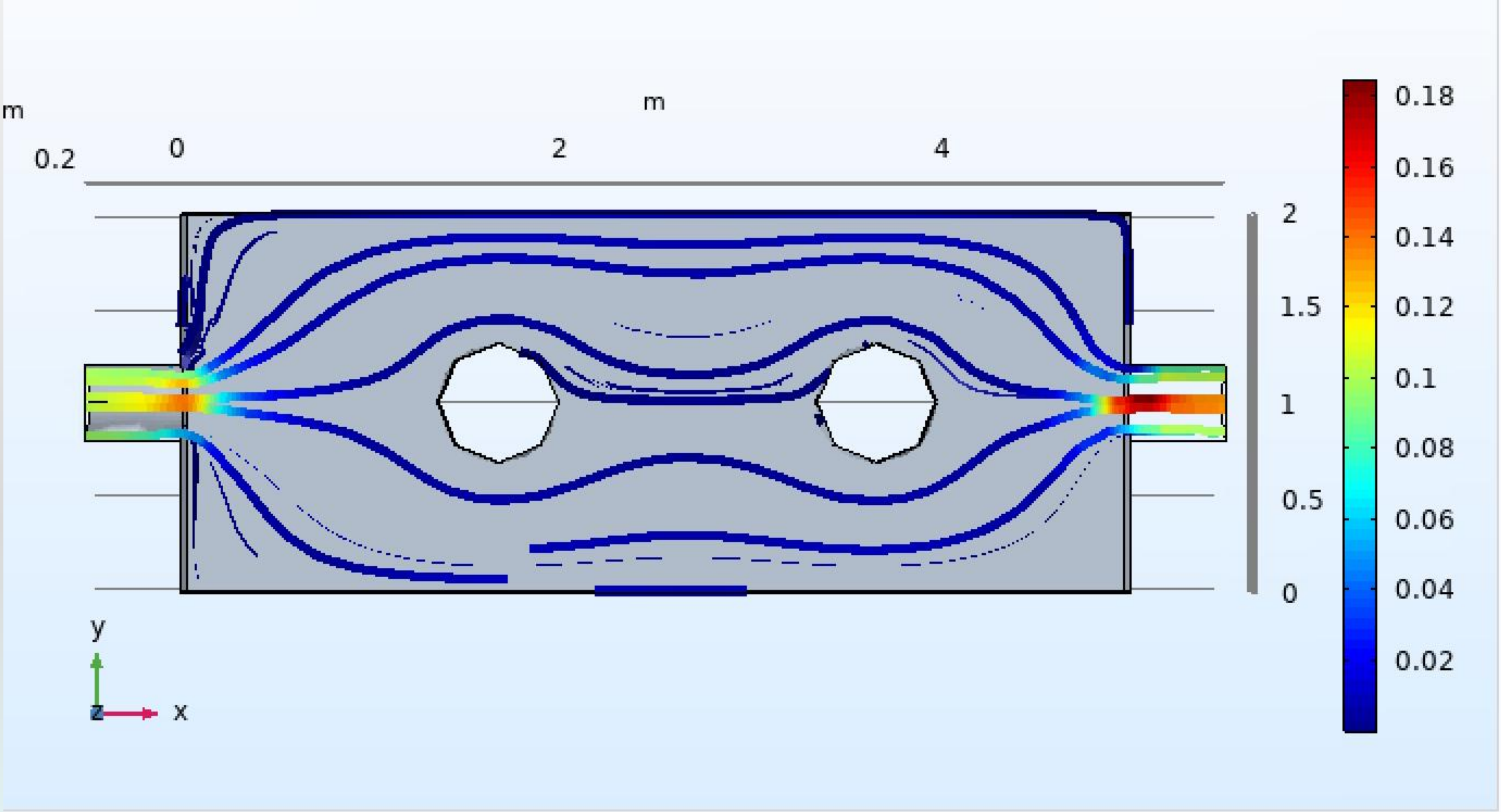
Density: 997.05 kg/m³
Characteristic Length: Normal
Mean Velocity: 0.1 m/s
Dynamic Viscosity $\mu = \nu\rho$: 8.90×10^{-4} pa·s



Experiment 5: Controlled Group 2 - Lower Speed

Material: Honey
Temperature: 25℃

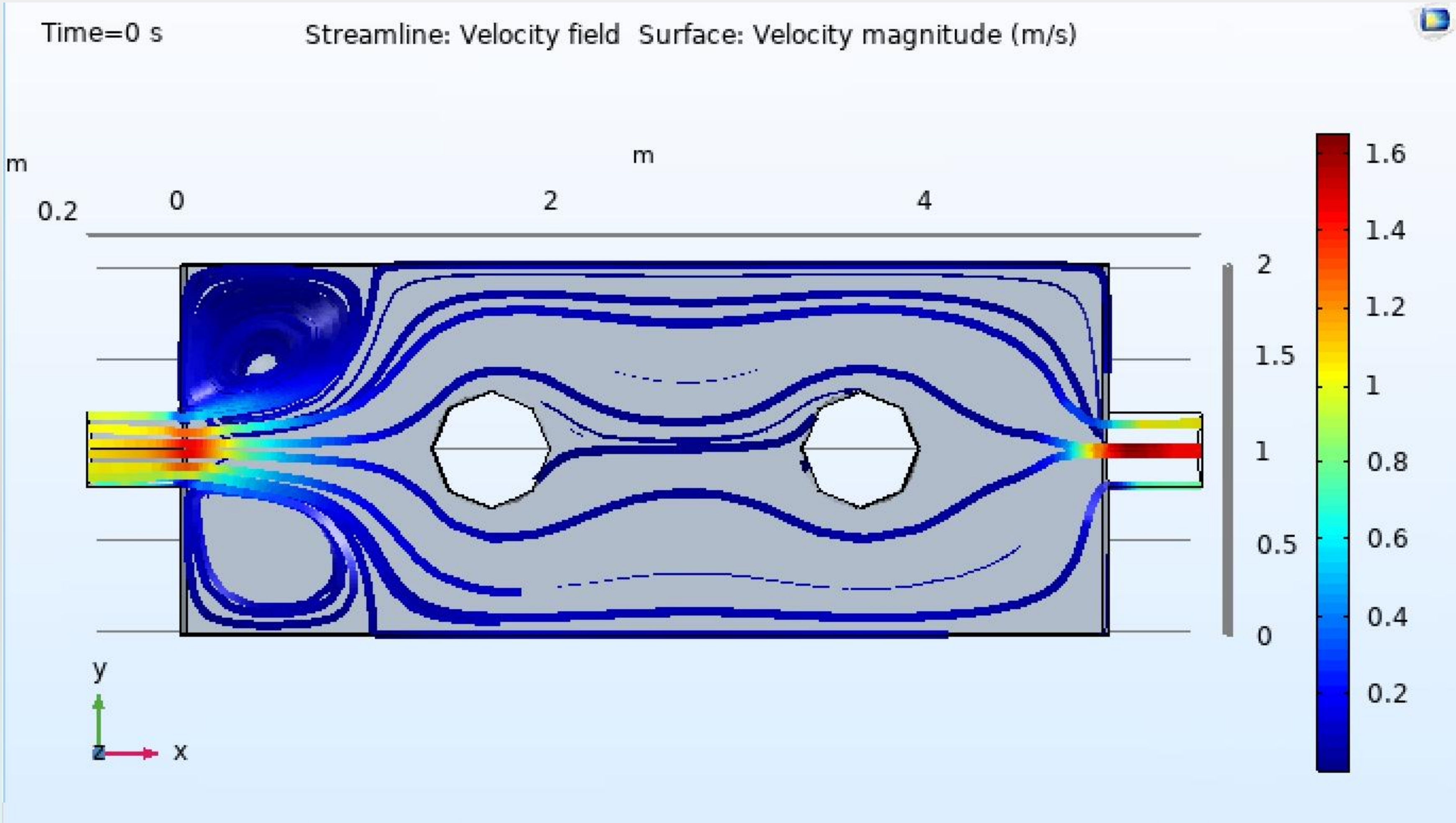
Density: 1450 (1400-1500) kg/m³
Characteristic Length: Normal
Mean Velocity: 0.1 m/s
Dynamic Viscosity $\mu = v\rho$: 7.5 (2-10) pa·s



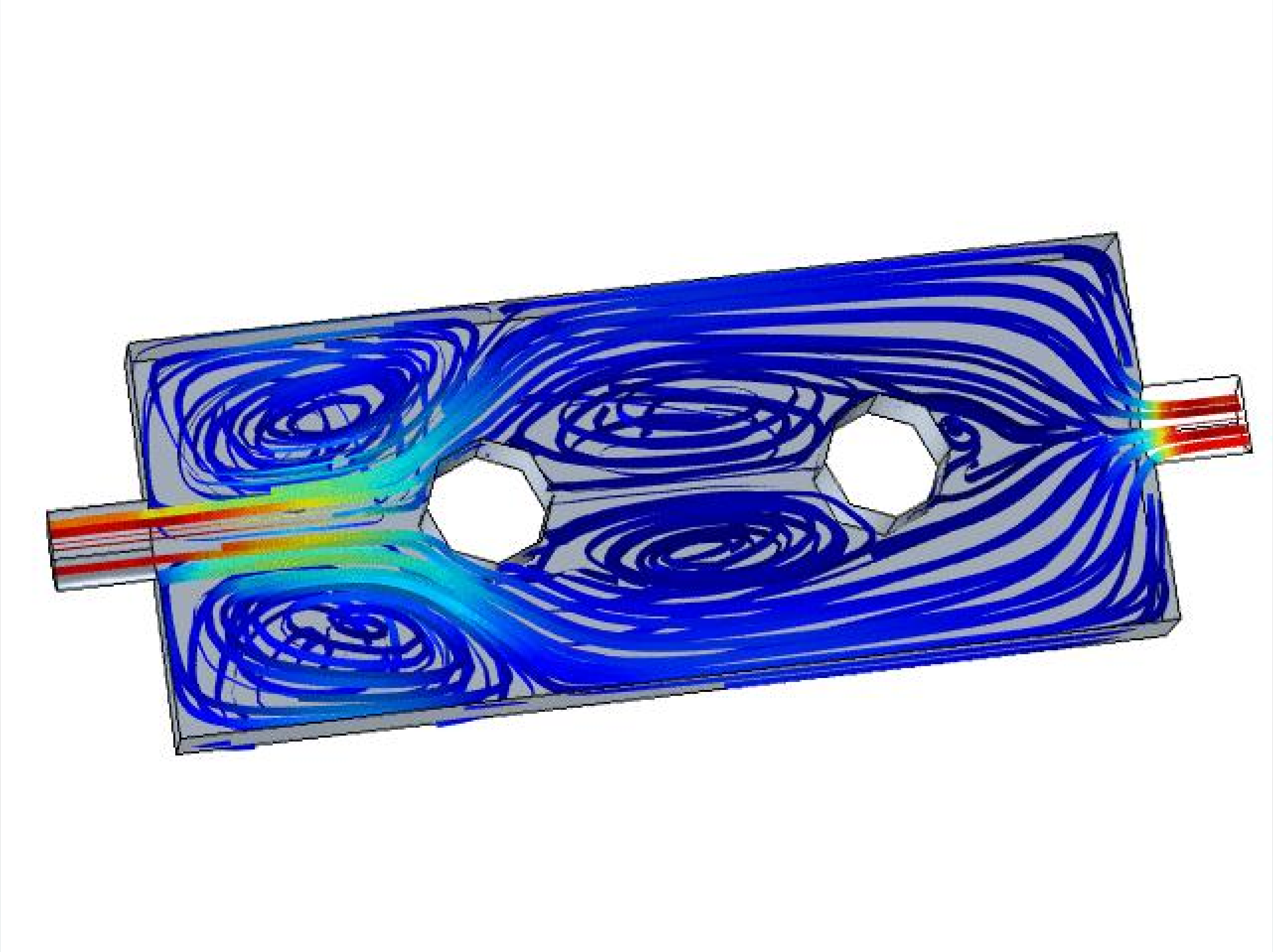
Experiment 5: Controlled Group 2 - Lower Speed

Material: Honey
Temperature: 25℃

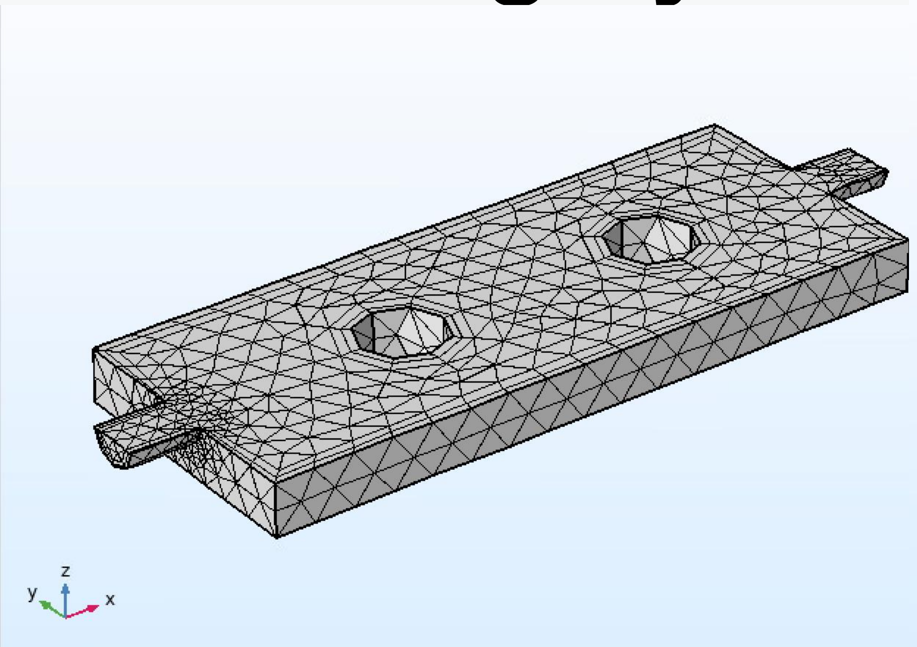
Density: 1450 (1400-1500) kg/m³
Characteristic Length: Normal
Mean Velocity: 1 m/s
Dynamic Viscosity μ = νρ: 7.5 (2-10) pa·s

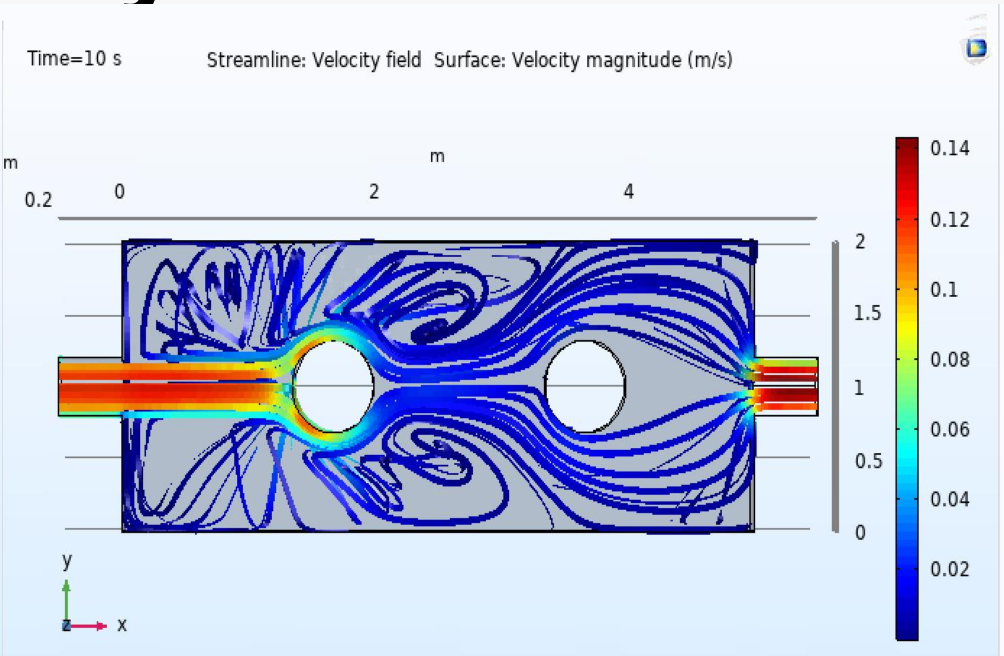


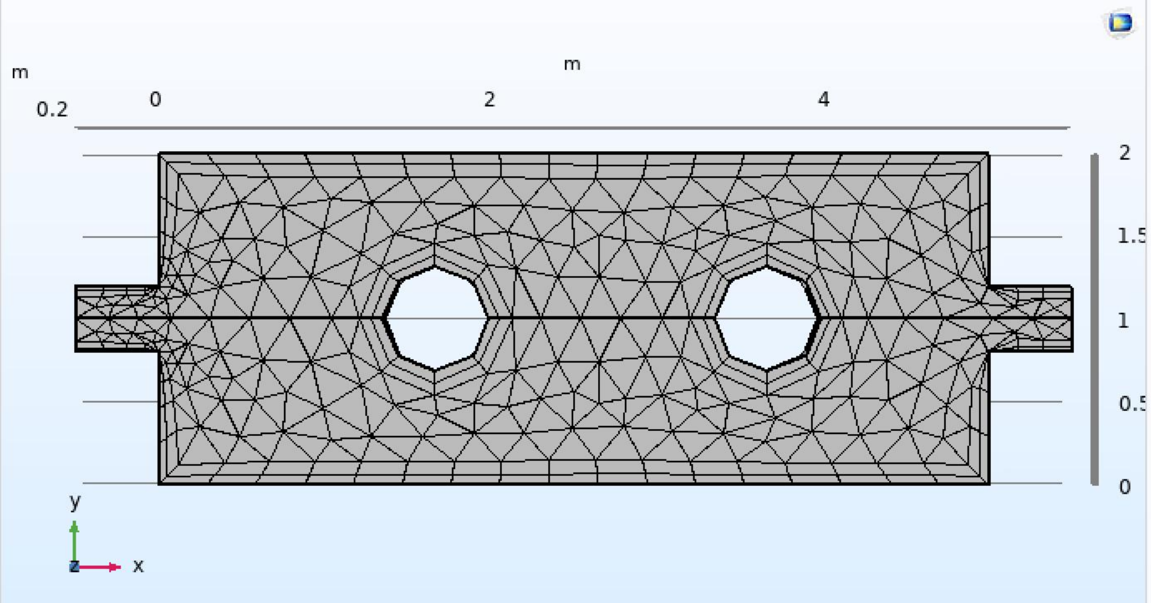
3. Results



4. Ploting Symmetry







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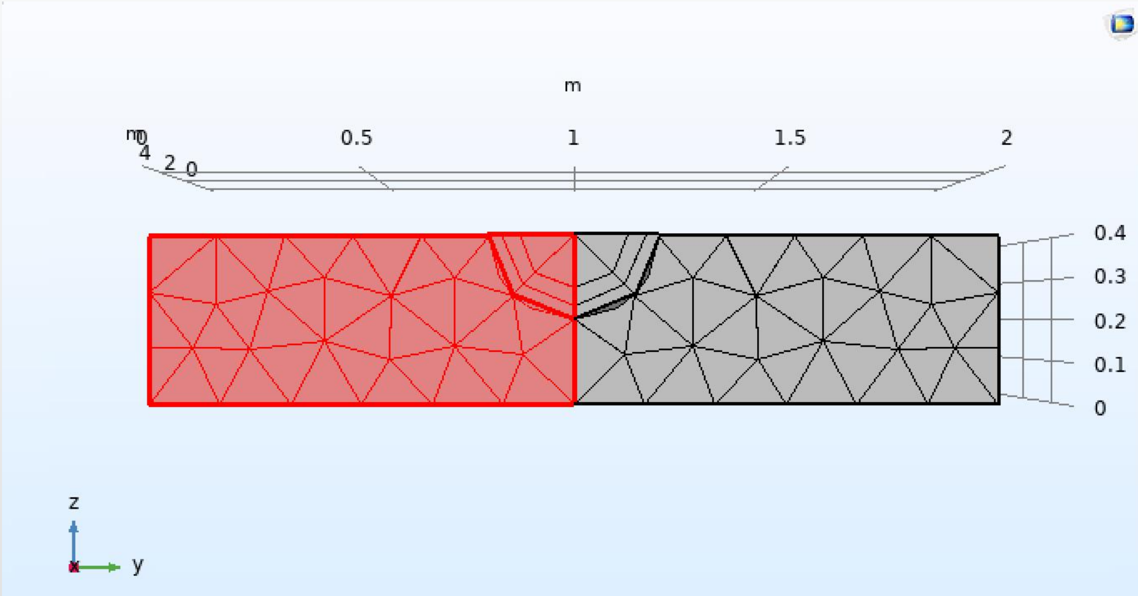
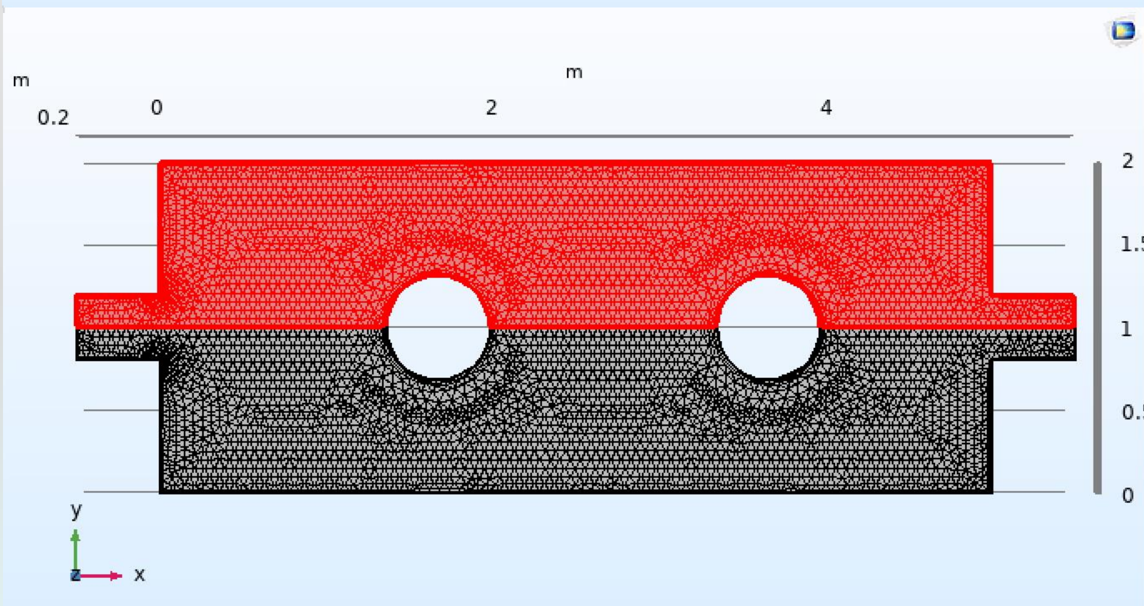
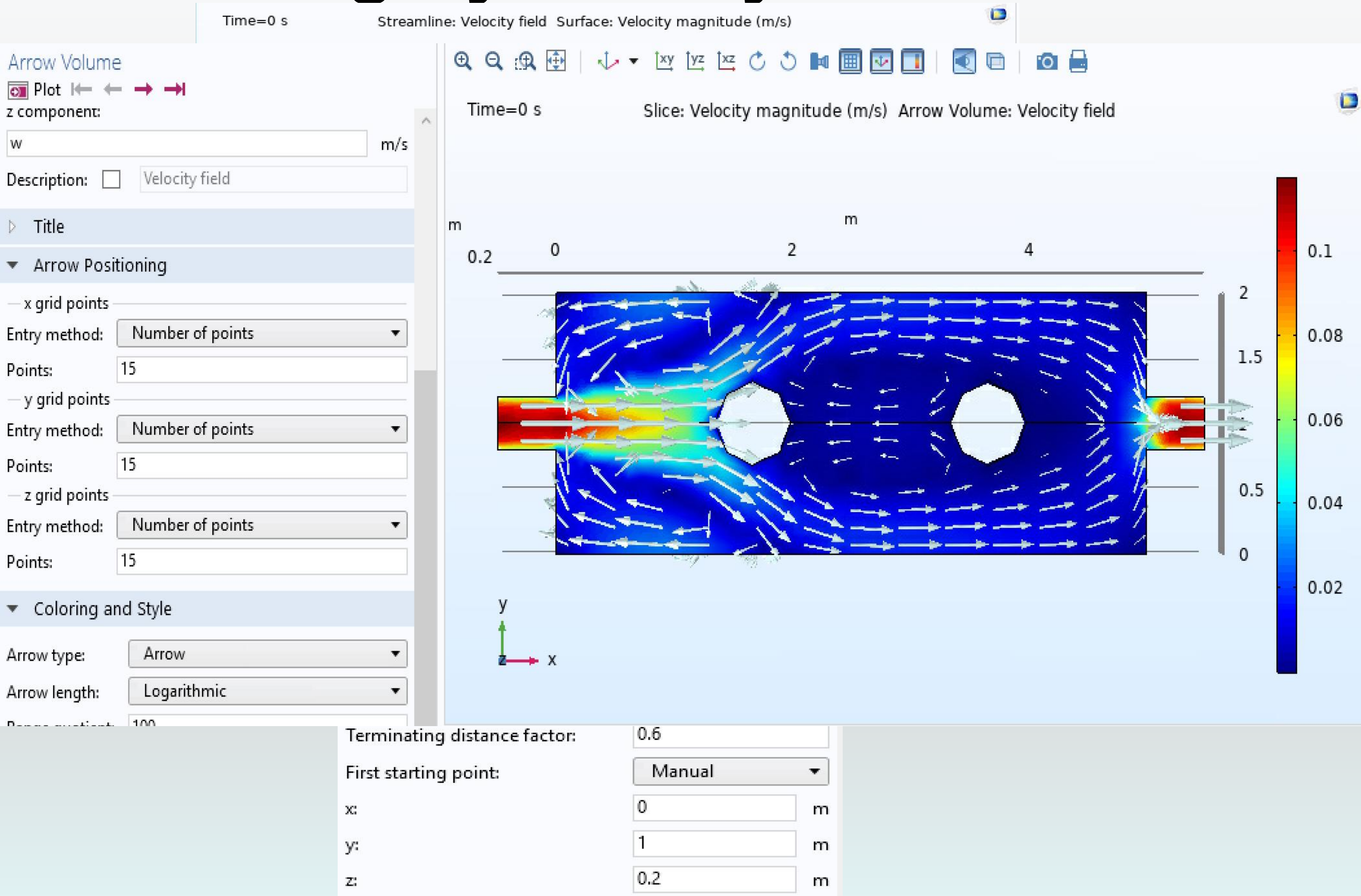
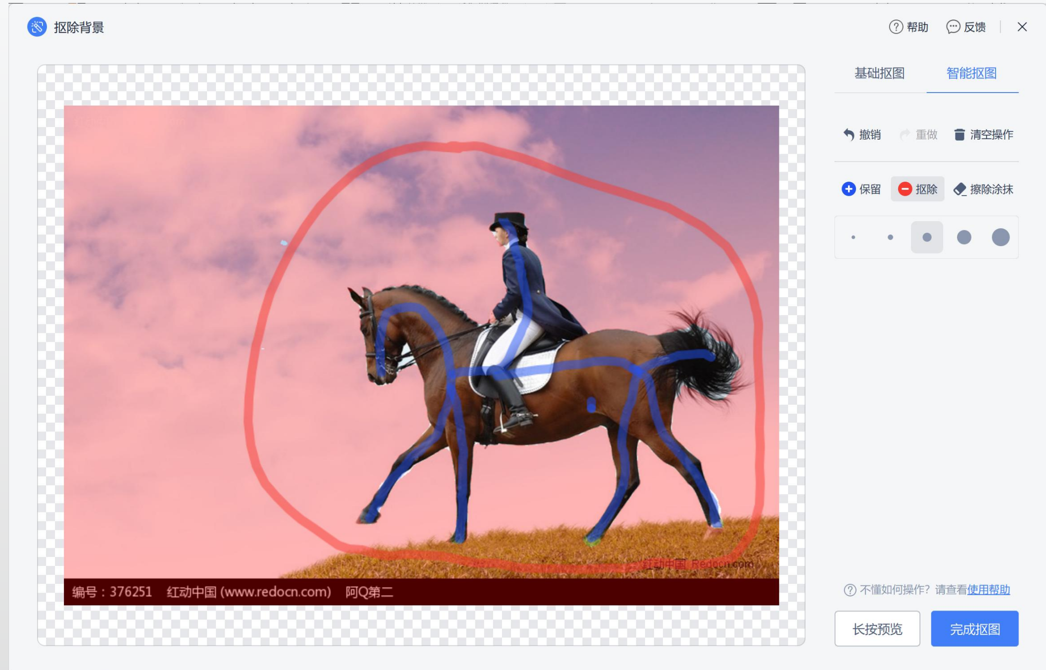




4. Ploting Symmetry

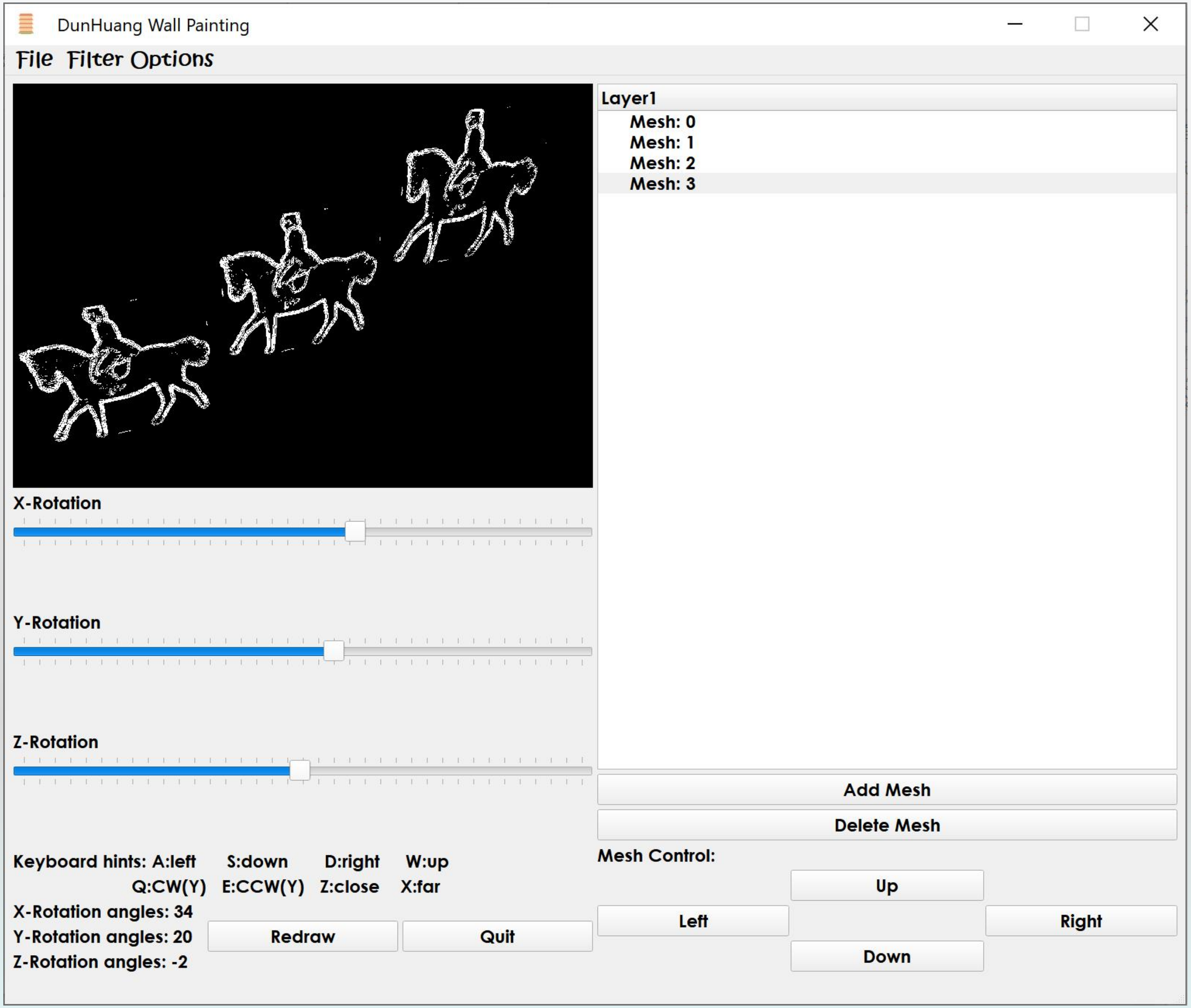


5. Future Talking



Lazy Snapping

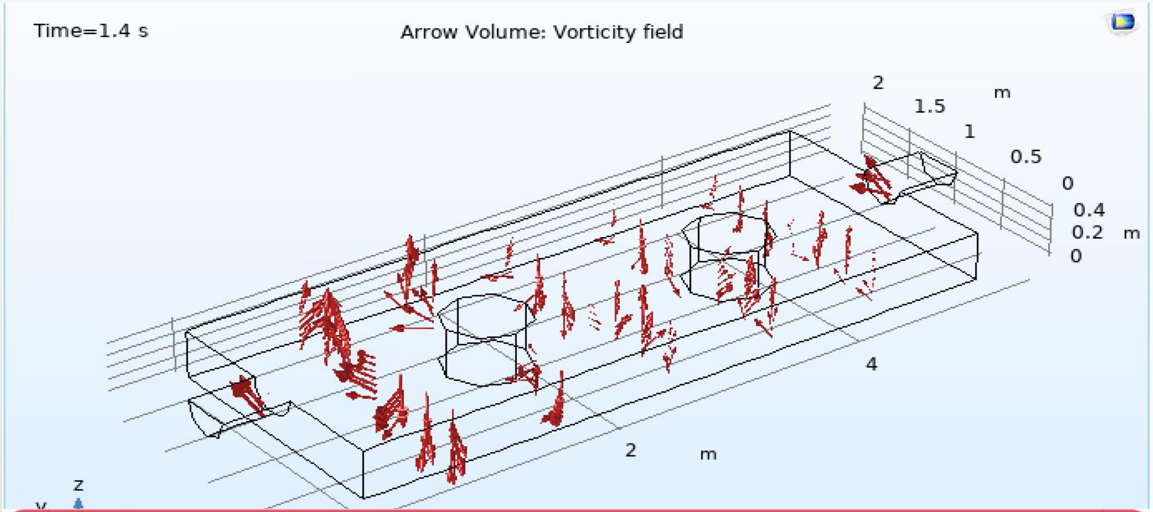
5. Future Talking



5. Future Talking

一四六　九色鹿本生之四　第二五七窟　西壁

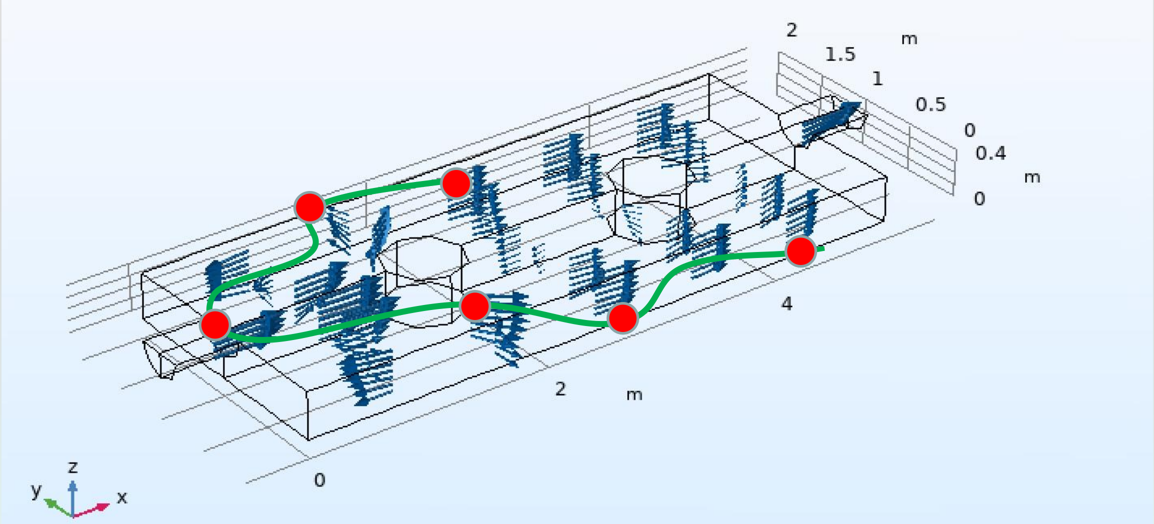
5. Future Talking



Reminder: the Catmull-Rom Sub-curve Equation is:

$$P(t) = 0.5 * \left[2 * P_1 + t * (-P_0 + P_2) + \ t^2(2 * P_0 - 5.* P_1 + 4P_2 - P_3) + t^3(-P_0 + 3P_1 - 3P_2 + P_3)\right]$$

$$0. \le t \le 1.$$



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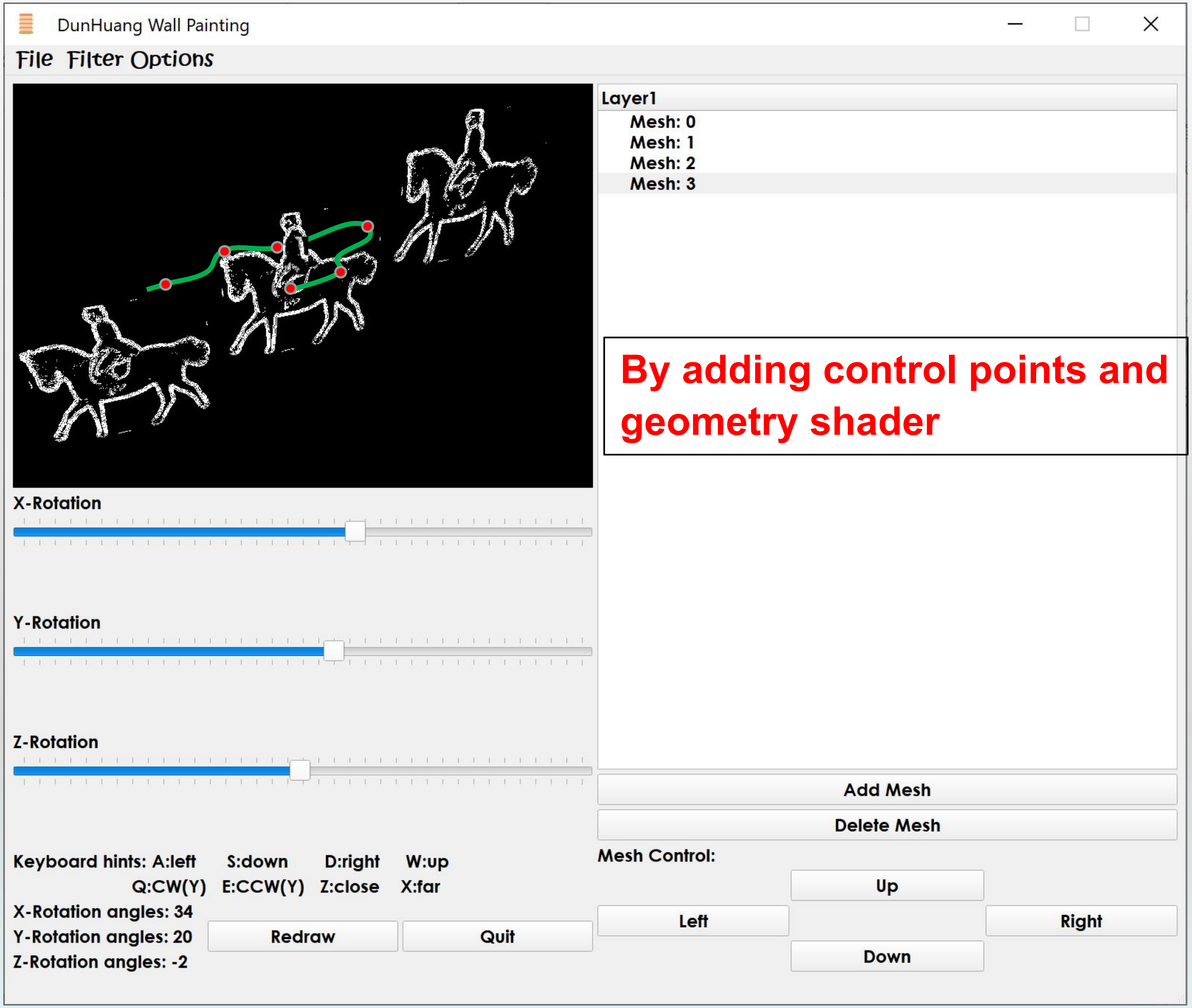

By adding control points and geometry shader

Thank you!