Voronoi Diagram & A Method for Creating Mosaic Images

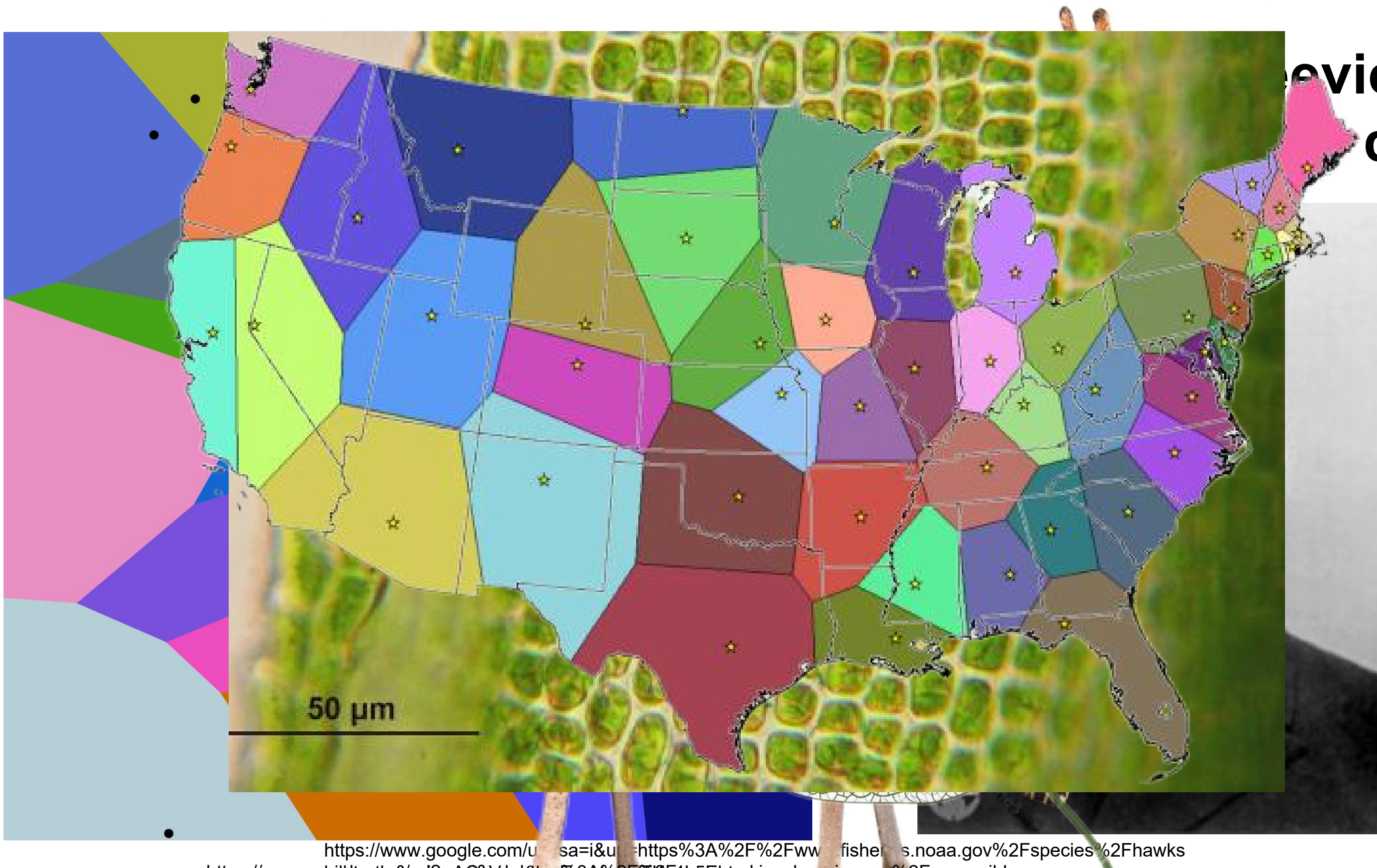
Tianle Yuan

Catagory

1. Background
2. Introduction of the theory
3. Mosaic method

1. Background

What is the Voronoi Diagram?



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https://towardsdatascience.com/how-to-find-the-nearest-hospital-with-voronoi-diagram-63bd6d0b7b75

https://www.google.com/url?sa=i&url=https%3A%2F%2Fwallpapersafari.com%2Fw%2FOUYkc7&psig=AOvVaw36

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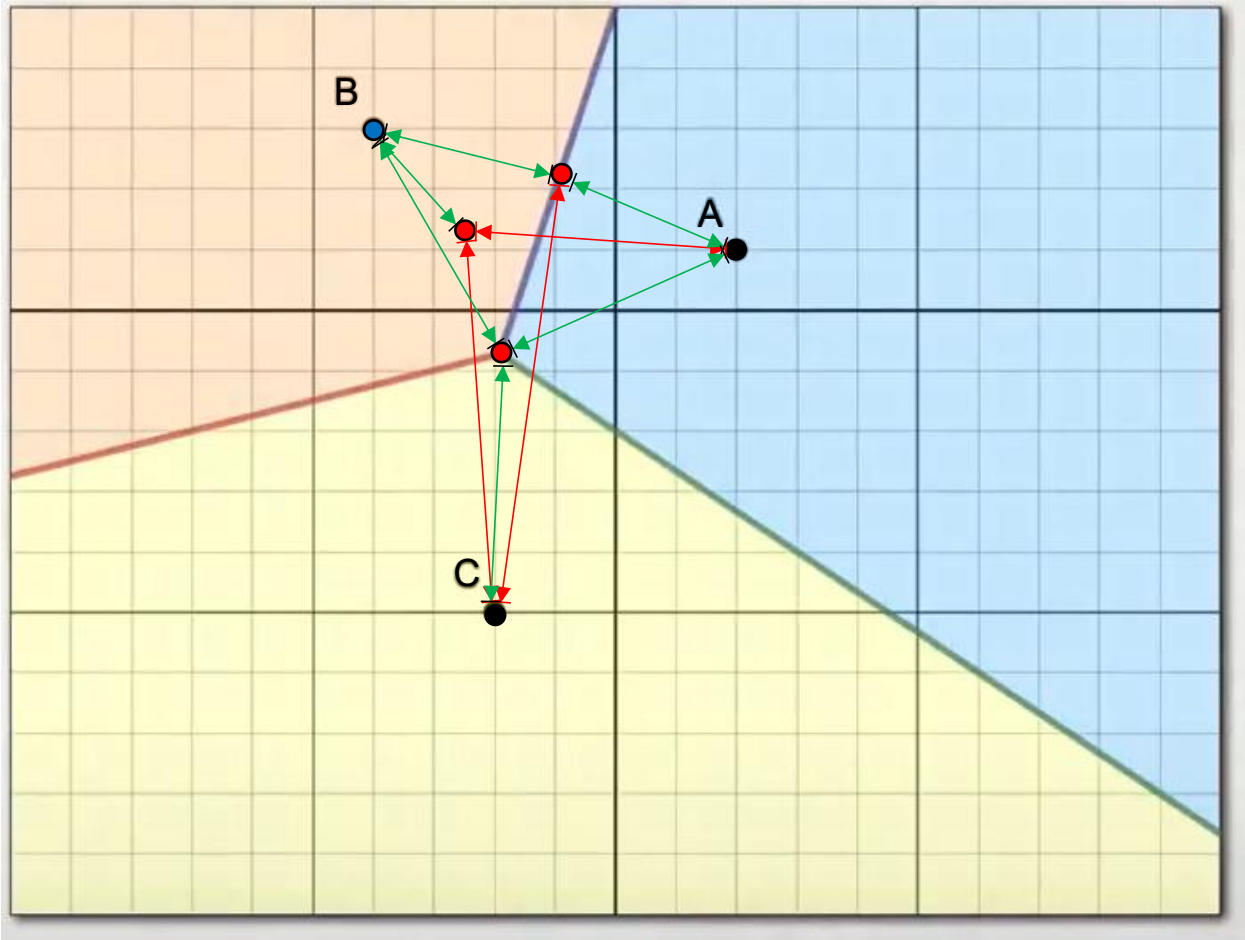
1. Background

Why is the Voronoi Diagram?



https://www.youtube.com/watch?v=1iYkEv6AxzU

1. Background

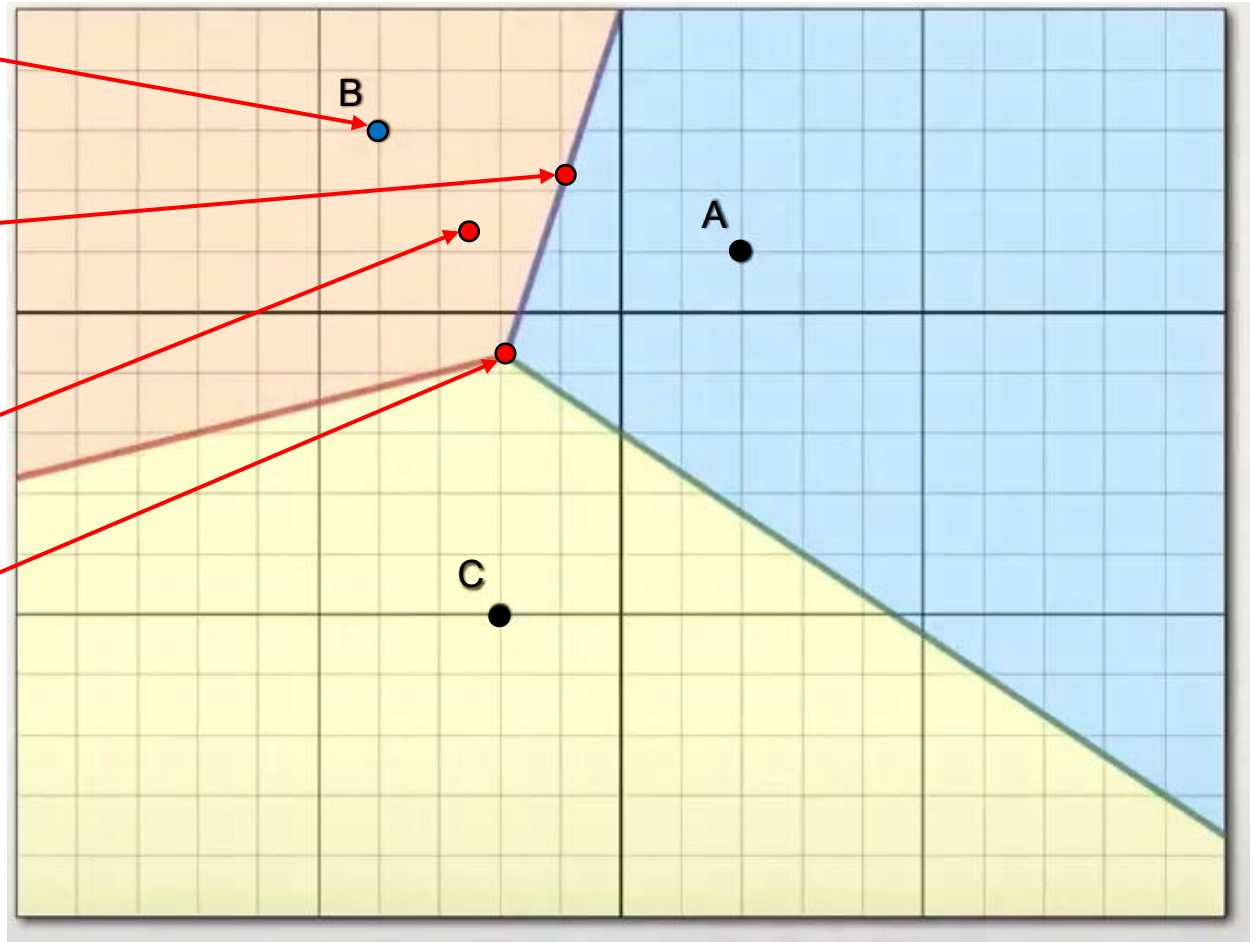
Why is the Voronoi Diagram?

Site of the diagram

Point on the **edge**

Point in the **cell region**

Vertex of the diagram



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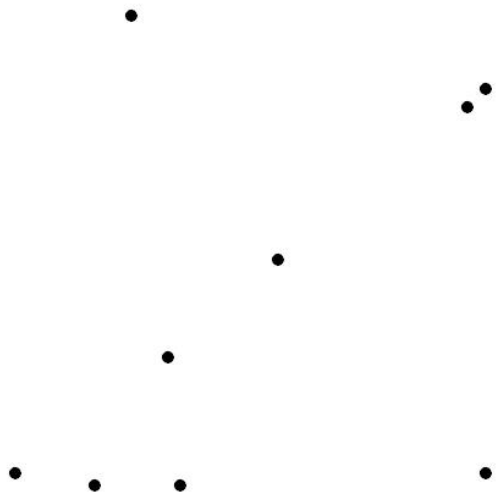
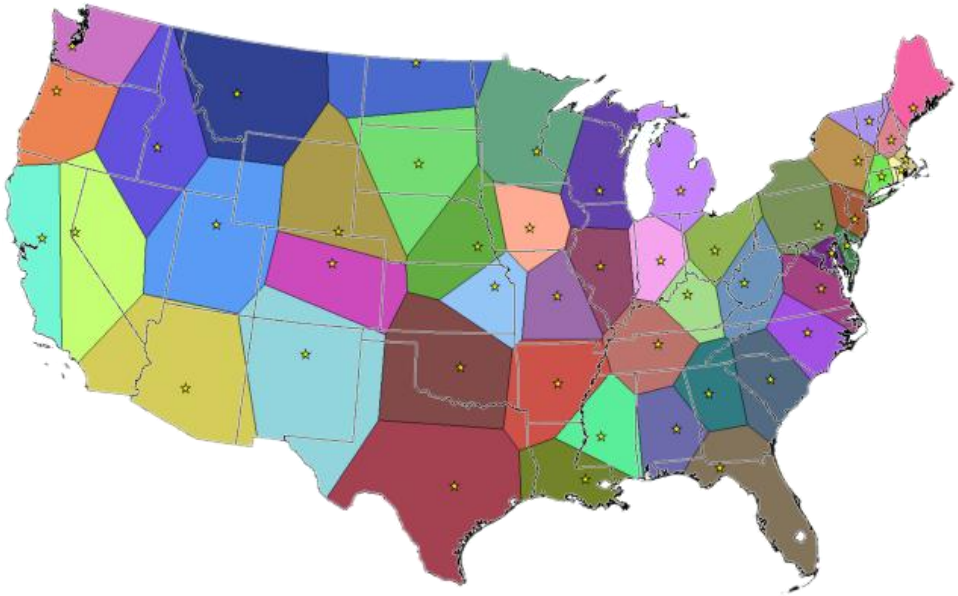
1. Background

Why is the Voronoi Diagram?



After we have set the **Sites** in the region of the diagram canvas, the region of each Voronoi cell provides the **optimized distance** (the global shortest distance).



For point in its site's region, it is more cost-efficient for the point to reach back to this site **compared with** heading for other cell's sites.

https://en.wikipedia.org/wiki/Voronoi_diagram#:~:text=In%20mathematics%2C%20a%20Voronoi%20diagram,%2C%20sites%2C%20or%20generators).&text=Voronoi%20cells%20are%20also%20known%20as%20Thiessen%20polygons.

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2. Introduction of the theory

What is the Voronoi Diagram in Math?

$$R_k = \{x \in X \mid d(x, P_k) \leq d(x, P_j) \text{ for all } j \neq k\}$$

Minkowski distance of order p (p-norm distance)

$$d\,(\boldsymbol{X}, \boldsymbol{Y}) = \left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{1/p}$$

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https://en.wikipedia.org/wiki/Distance

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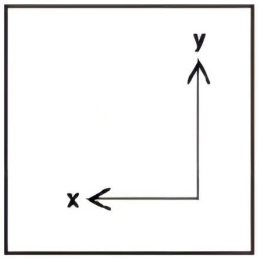
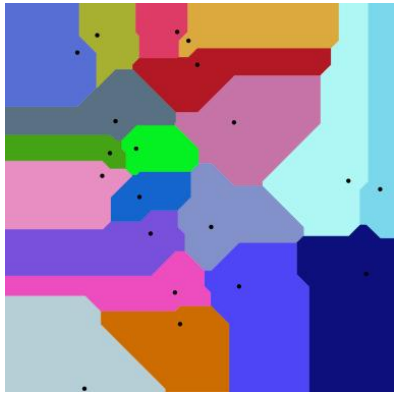
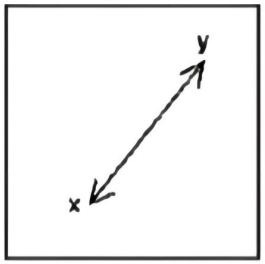
p = 1:



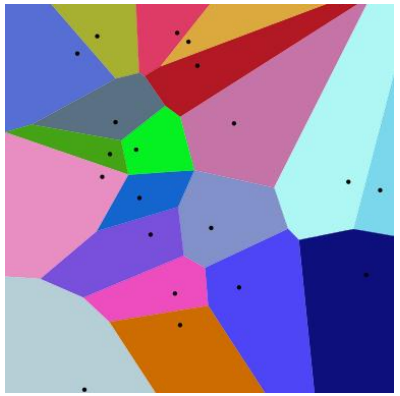
Manhattan



p = 2:



Euclidean



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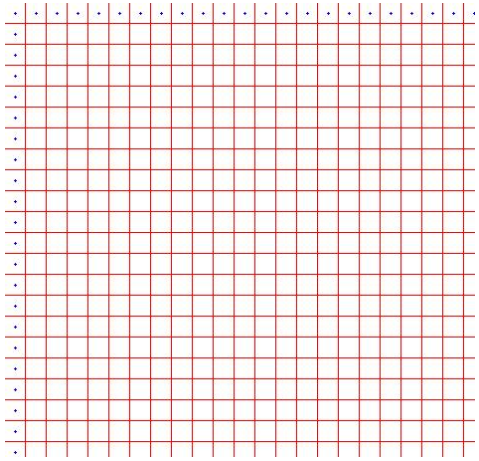

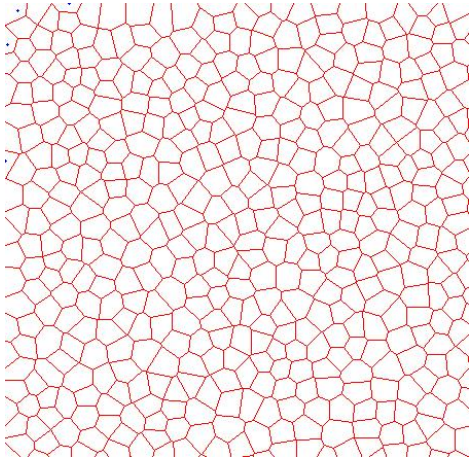

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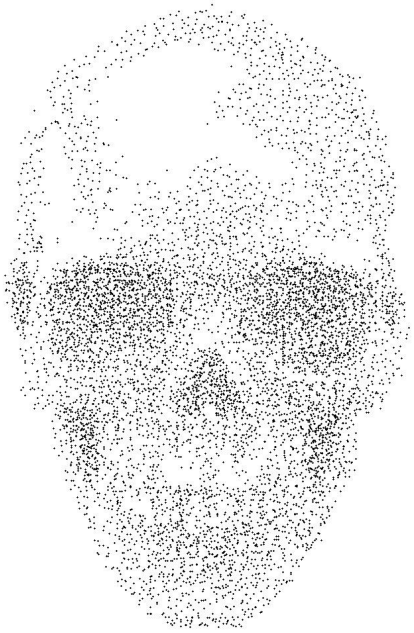

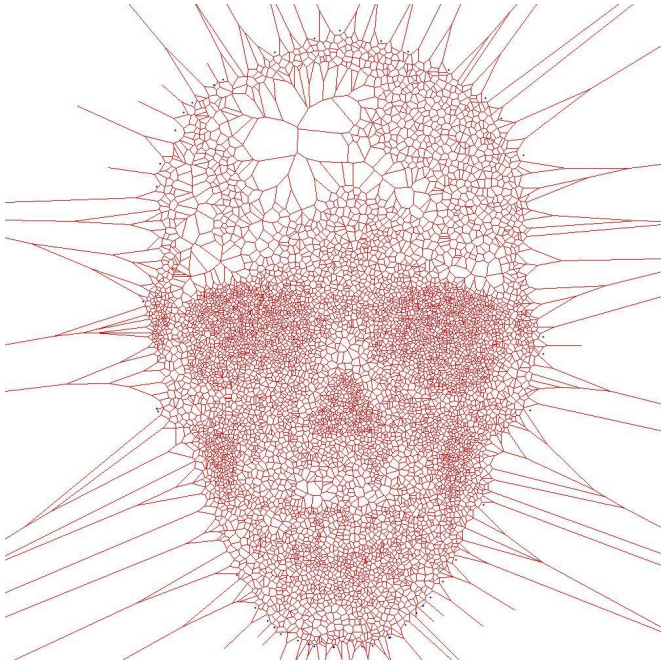

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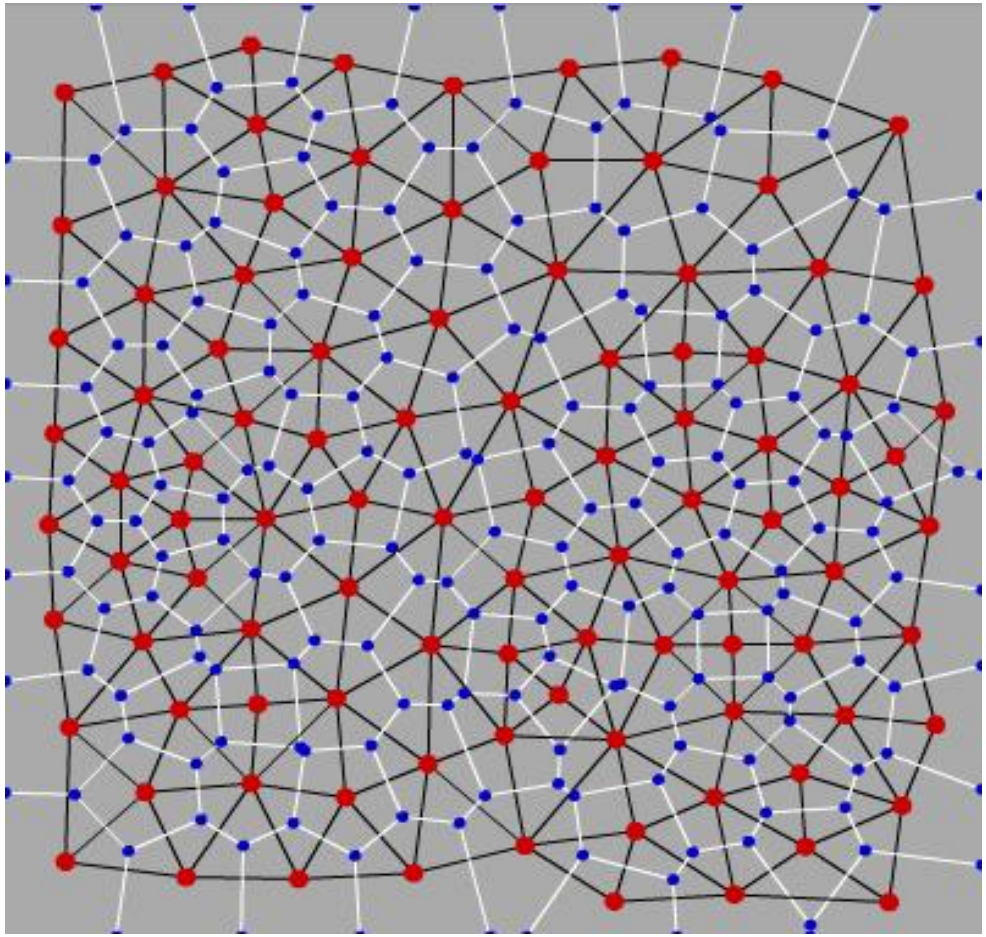
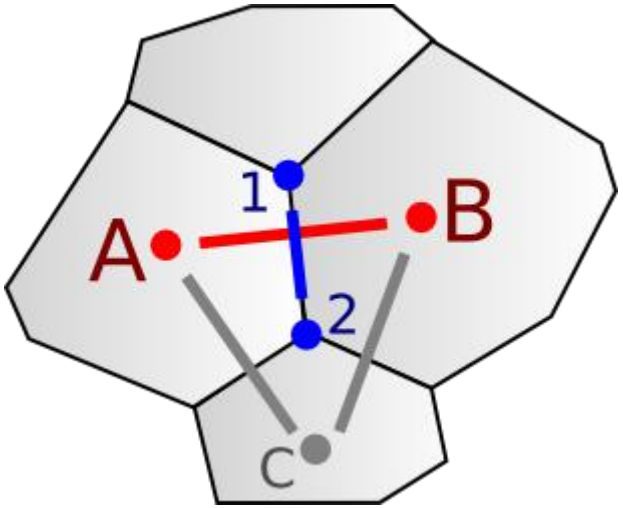
2. Introduction of the theory

What is the Voronoi Diagram in Math?

Dual graph: Delaunay triangulation





Vertical split to each other !!

http://www-cs-students.stanford.edu/~amitp/game-programming/polygon-map-generation/

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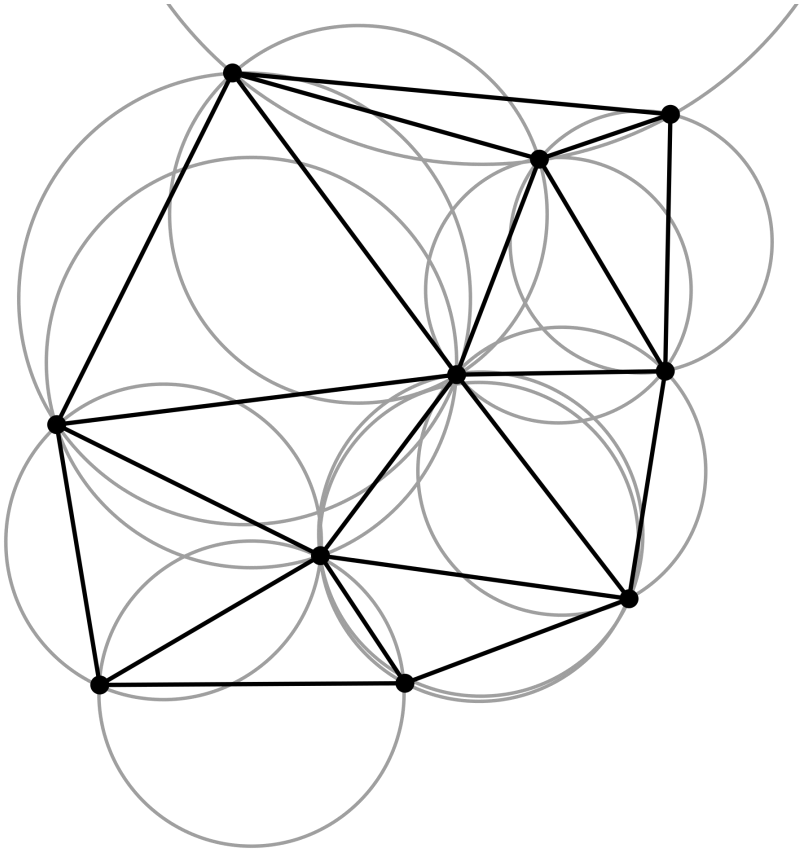
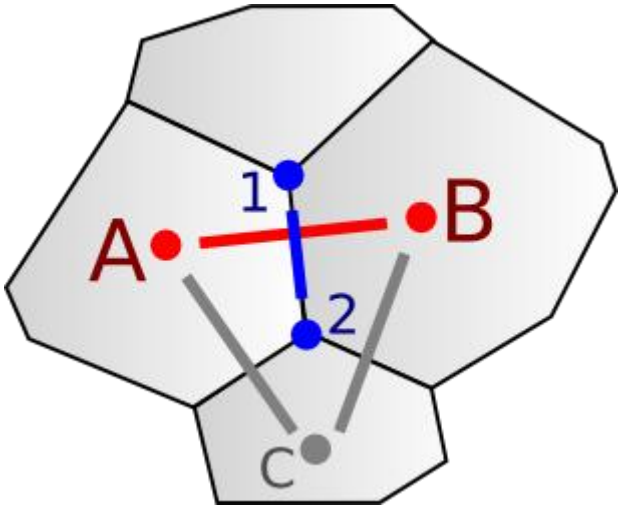
What is the Voronoi Diagram in Math?

Dual graph: Delaunay triangulation





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3. Mosaic method

The are lots of arts are made by Mosaic Tech:

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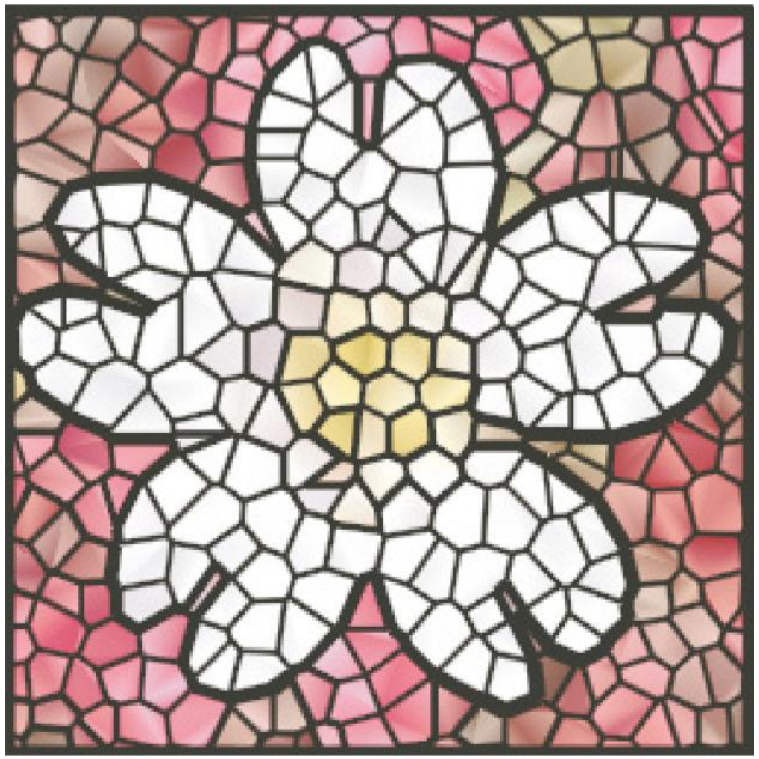
https://www.youtube.com/watch?v=DmmQIiMsVjg

https://mail.google.com/mail/u/0/#inbox/FMfcgzGlkjZJXCMcQgRPcnFHDrDbNKlg?projector=1

3. Mosaic method

Let's talk about an automatic method for mosaicing images by using Voronoi diagrams.

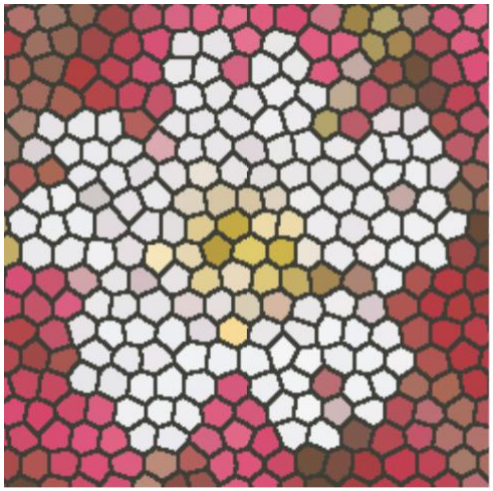


http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

Let's talk about an automatic method for mosaicing images by using Voronoi diagrams.

Reference image (R)



Voronoi image (V)

$$E_{color} = \sum_{x}\sum_{y}\sum_{c}\left(P^{V}_{(x,y,c)} - P^{R}_{(x,y,c)}\right)^{2}, \qquad (1)$$

Pixel position (x, y)

Pixel color (R, G, B)

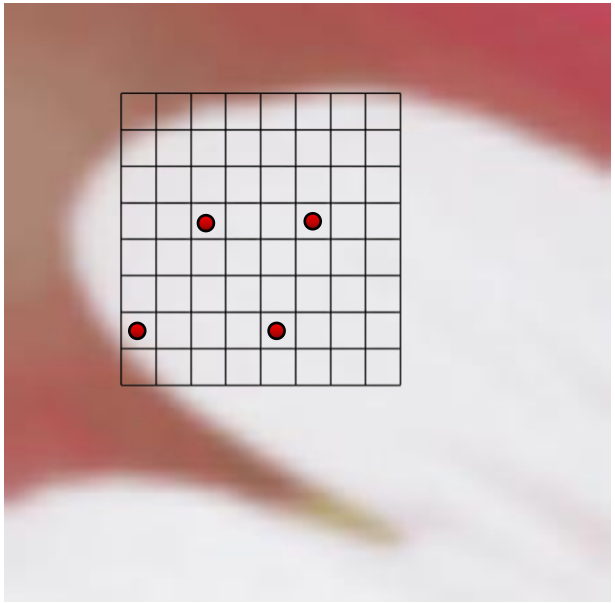
http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

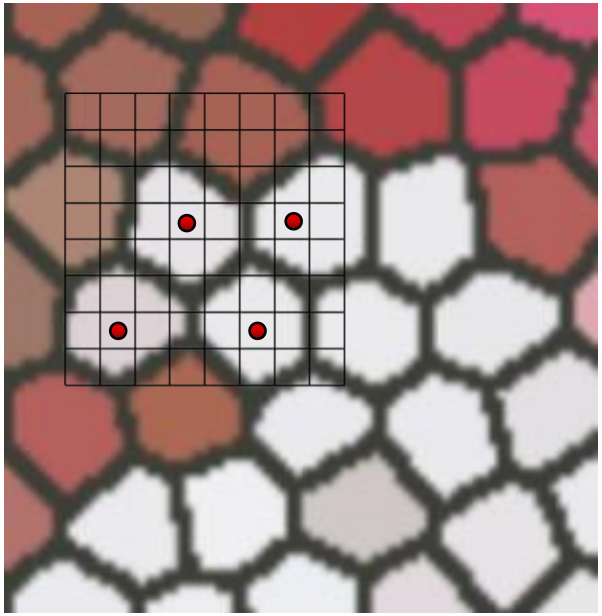
Let's talk about an automatic method for mosaicing images by using Voronoi diagrams.

1. Assume that the **site** of a Voronoi diagram is located at the center of a **pixel**.

2. The color of that Voronoi region refers the color of that pixel in the image R.



Reference image (R)



Voronoi image (V)

$$E_{color} = \sum_x \sum_y \sum_c \left(P^V_{(x,y,c)} - P^R_{(x,y,c)} \right)^2, \qquad (1)$$

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3. Mosaic method

Let's talk about an automatic method for mosaicing images by using Voronoi diagrams.

The whole method can be divided into 2 steps:

I. Pattern Generation

1.1 Initial Positions of the Sites

1.2 Capturing the Global Features of the Reference Image

1.3 Local Adjustment of the Voronoi Image

1.4 Expression of Detailed Features

II. Effects Addition

2.1 Edge Highlight

2.2 Light Effect

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.1 Initial Positions of the Sites

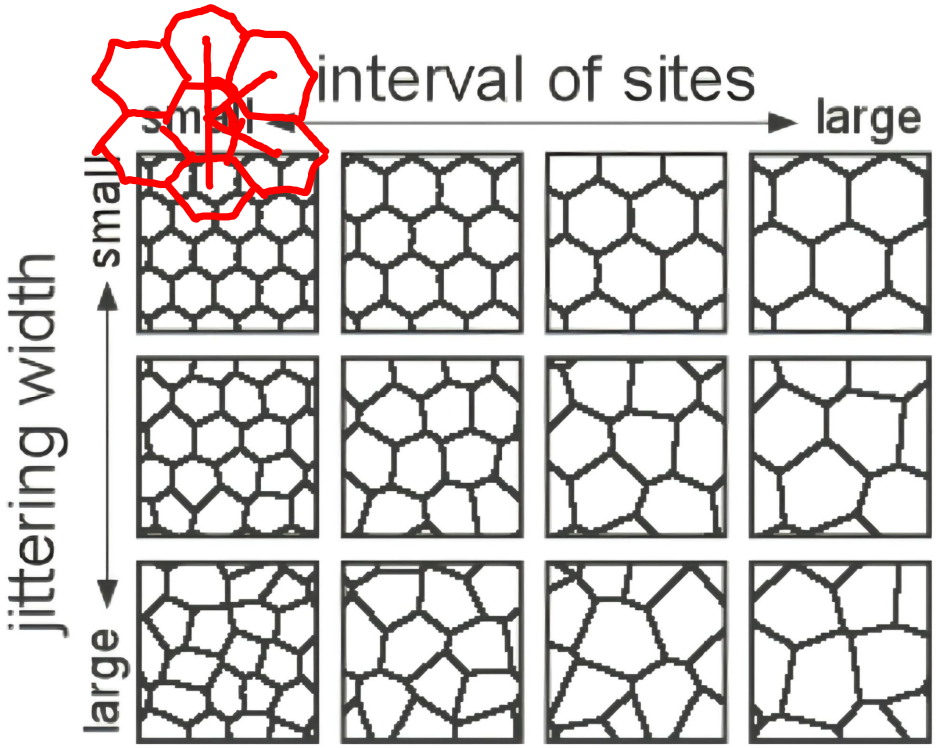
https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.pngwing.com%2Fen%2Ffree-png-zddrw&psig=AOvVaw1jyZA8lrU3pOjwFToilMz1&ust=1635900 44526 9000 &s ource=images &cd=vfe&ved=0CAwQjhxqFwoTCPD49Z26-PMCFQ AAAA AdAAAAAB A J

The initial Voronoi image is created by placing its sites on the **center** of hexagonal lattices.

Then user can adjust:

1. the expected **interval** between sites;

2. the expected **jittering width** from the hexagonal lattices center.



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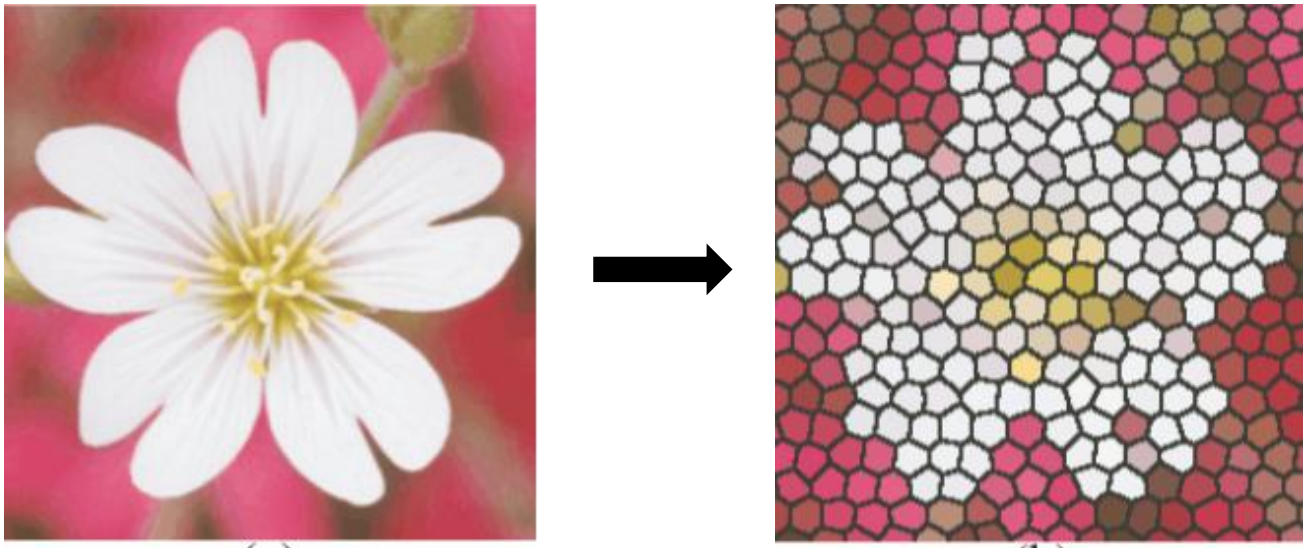
3. Mosaic method

1.1 Initial Positions of the Sites



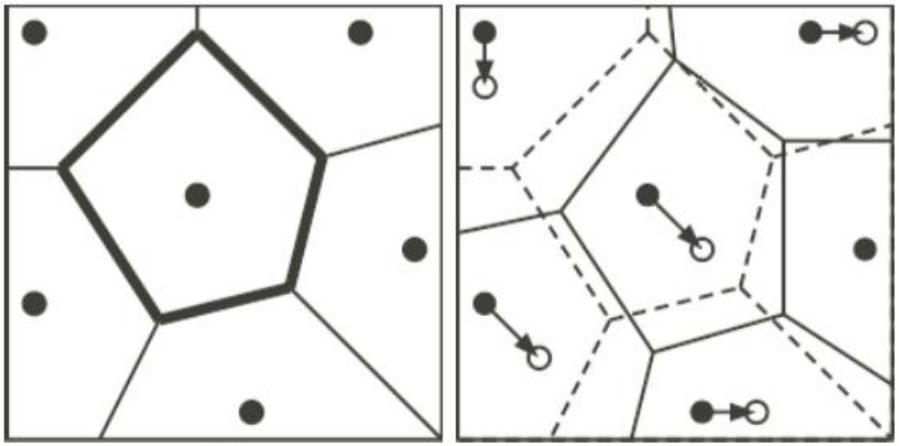
(a) (b)

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.2 Capturing the Global Features of the Reference Image

The **shape** of a Voronoi region (bold line region) is determined by the relative position between its site and the sites of the surrounding Voronoi regions



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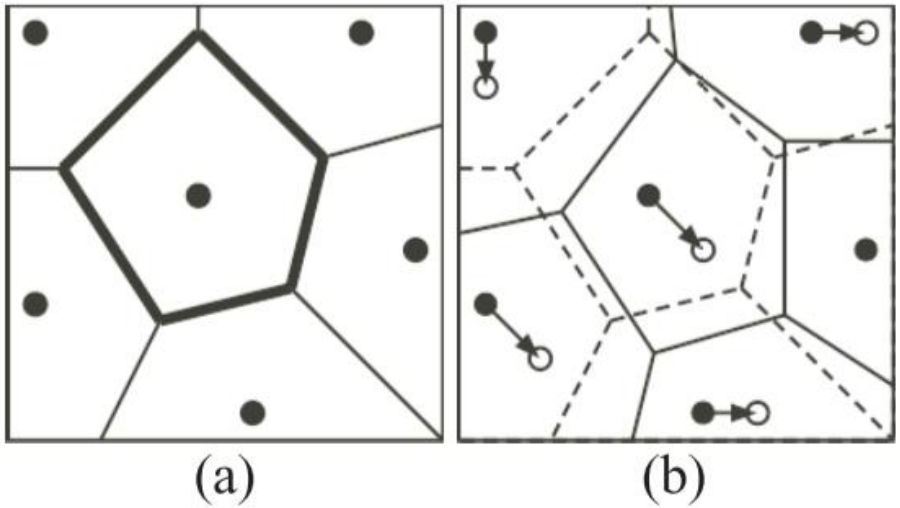
3. Mosaic method

1.2 Capturing the Global Features of the Reference Image

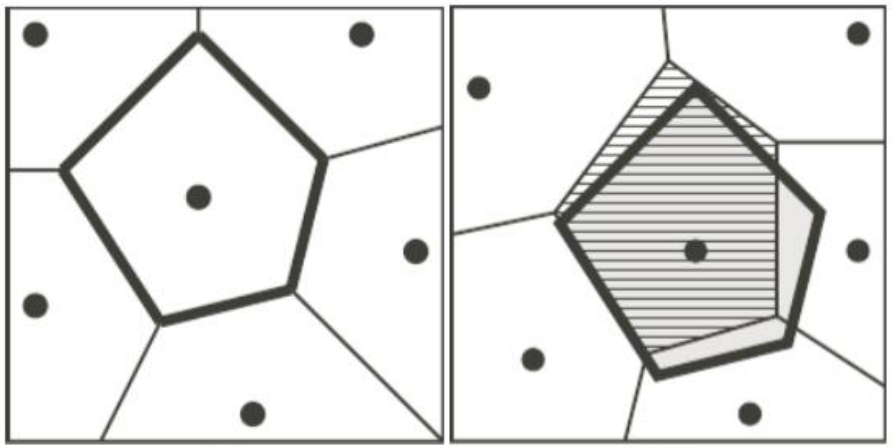


A Voronoi image that expresses the global features of the reference image can be obtained **by** move **all sites** together iteratively until an end condition is triggered.

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.2 Capturing the Global Features of the Reference Image



To simplify the computation, here we assume that the **shape** of each voronoi region will not change after the global movement.

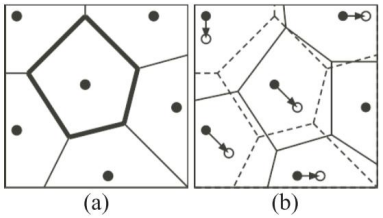
Also when a site is moved, the **color in it's region** will not change

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.2 Capturing the Global Features of the Reference Image

So what is the end condition?



Thus we need to find a global movement of all sites in a direction where $\mathbf{E_{color}}$ decreases the most

<=> Finding the lowest value of the Error function **E**

$$E_{color} = \sum_x \sum_y \sum_c \left(P^V_{(x,y,c)} - P^R_{(x,y,c)} \right)^2, \qquad (1)$$

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.2 Capturing the Global Features of the Reference Image

So what is the activity of each iteration?

In each iteration, **each site's movement traversing** will be limited to its adjoining **eight pixels**.

Hence the site **will be not moved** if the eight-pixel-movement does not decrease the value of E_{color}.

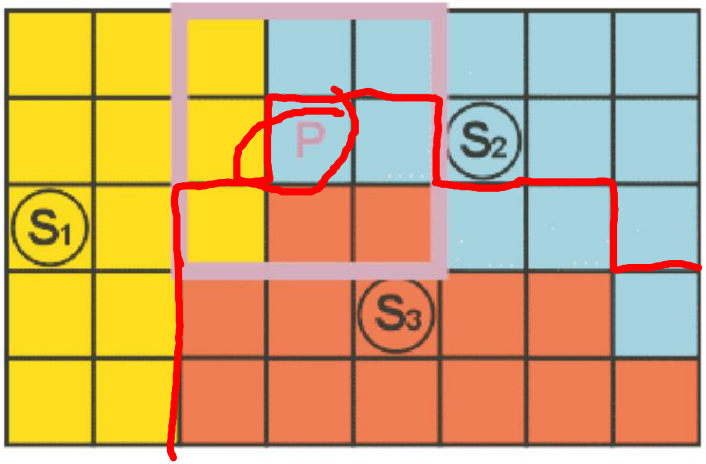


Figure 5: *The calculation for the change of* E_{color}.

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3. Mosaic method

1.2 Capturing the Global Features of the Reference Image

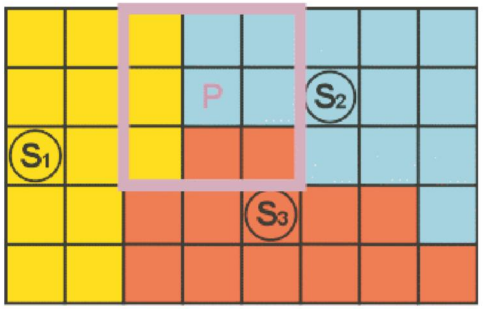


Figure 5: *The calculation for the change of E_{color}.*

While E_{color} > **ε**:

For S in Sites:

For D in 8 neighbors' direction:

Move Site to the neighbors' position

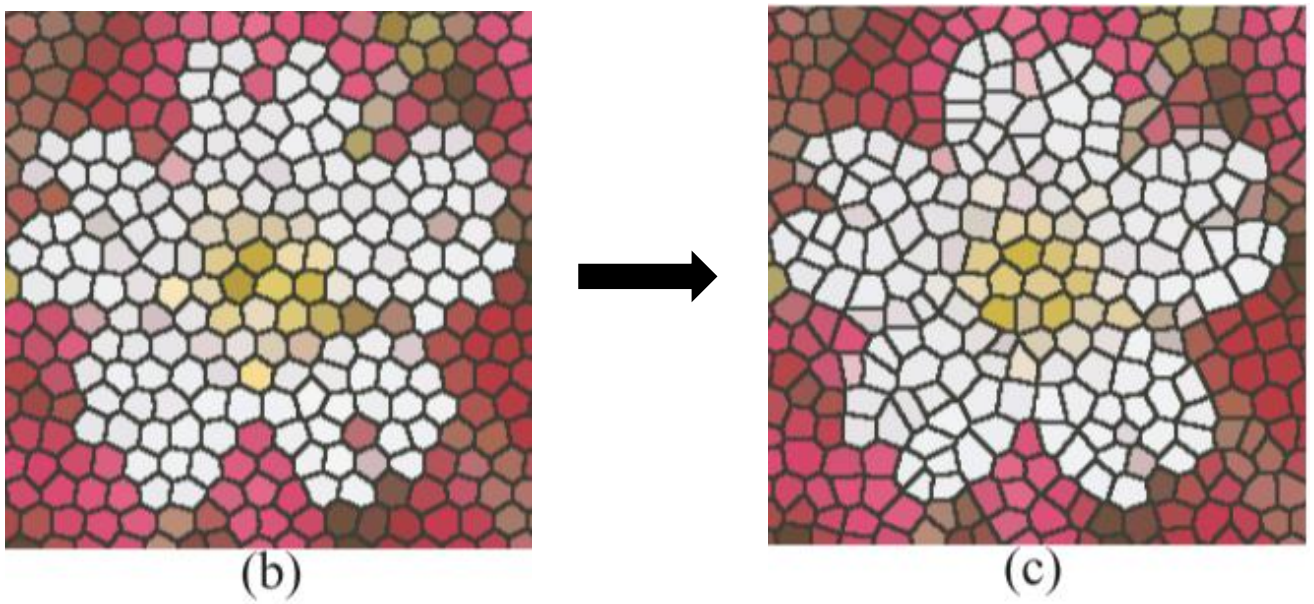
$E_{color}^{now} = \text{Min}\ (E_{color}^{old}, E_{color}^{D})$

* Set threshold ε is because the location of the sites might not converge

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.2 Capturing the Global Features of the Reference Image



http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.3 Local Adjustment of the Voronoi Image

Unlike the global feature adjustment we talked before, in this Local Adjustment Voronoi sites are moved one at a time to adjust the local appearance of the Voronoi image.

For S in Sites:

For D in 8 neighbors' direction:

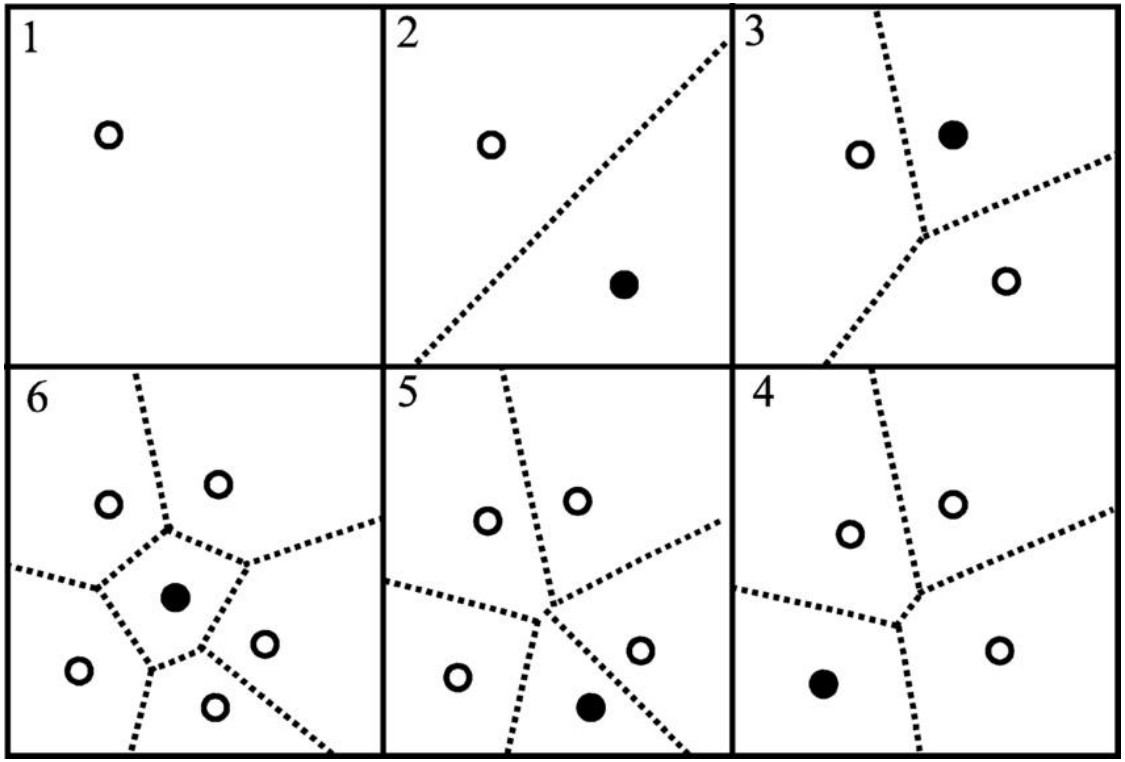
$$E_{color}^{now} = \text{Min}\,(E_{color}^{old}, E_{color}^{D})$$

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

1.4 Expression of Detailed Features

By adding or deleting the sites according to the need of the details.

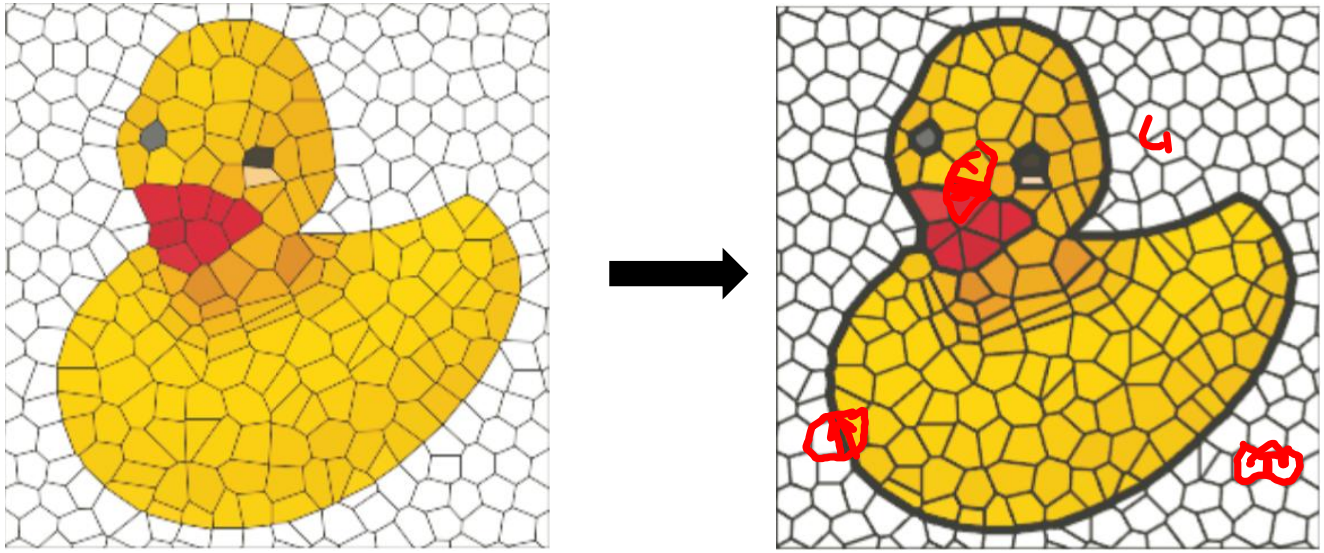


https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.researchgate.net%2Ffigure%2FConstruction-of-a-Voronoi-diagram-using-successive-addition-of-sites-and-local_fig3_13908260&psig=AOvVaw0azSjO9bSpWP1WNIQBS5V6&ust=1635978173795000&source=images&cd=vfe&ved=0CAwQjhxqFwoTCPjL5Yfc-vMCFQAAAAAdAAAAABAD

http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

2.1 Edge Highlighting



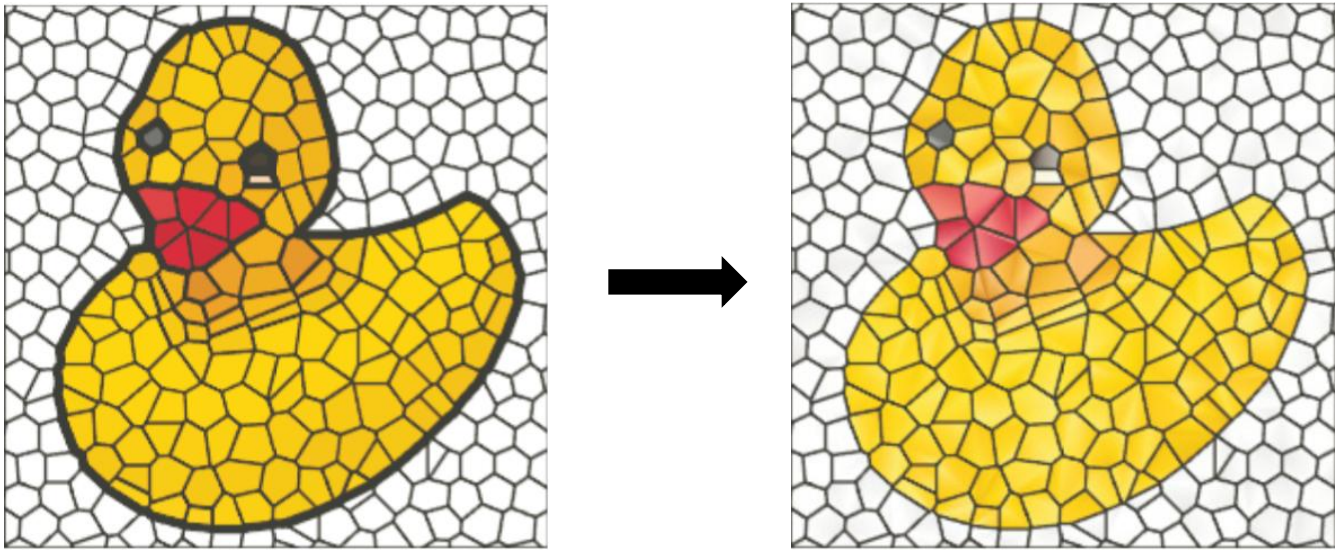
$$width = W_{base} + W_{color} \cdot D_{color}, \qquad (2)$$

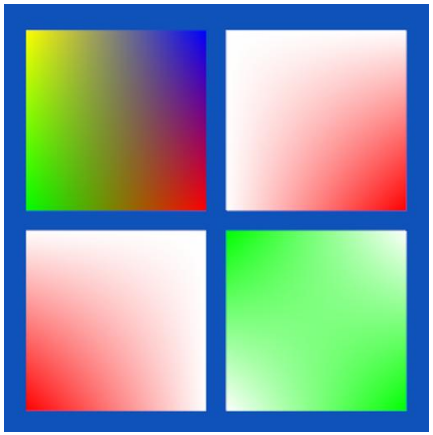
Original Black Added Black Color difference on edge

http://dx.doi.org/10.2312/egs.20021036

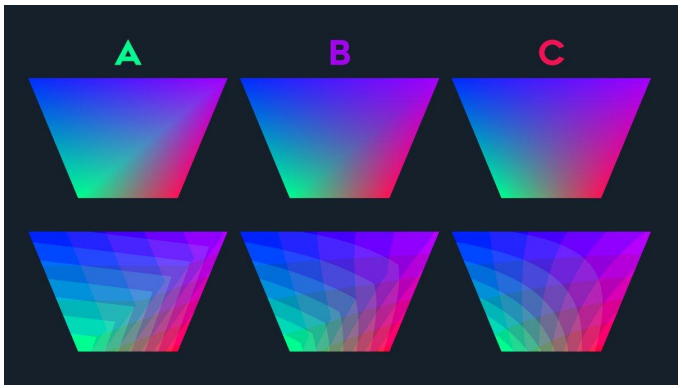
3. Mosaic method

2.2 Light Effect





https://www.google.com/url?sa=i&url=https%3A%2F%2Fasawicki.info%2Fnews_1721_how_to_correctly_interpolate_vertex_attributes_on_a_parallelogram_using_modern_gpus&psig=AOvVaw0x_bVKJkS6JBNgSed9WeHa&ust=1635979063975000&source=images&cd=vfe&ved=0CAwQjhxqFwoTCPjH0L3f-vMCFQAAAAAdAAAAABAD



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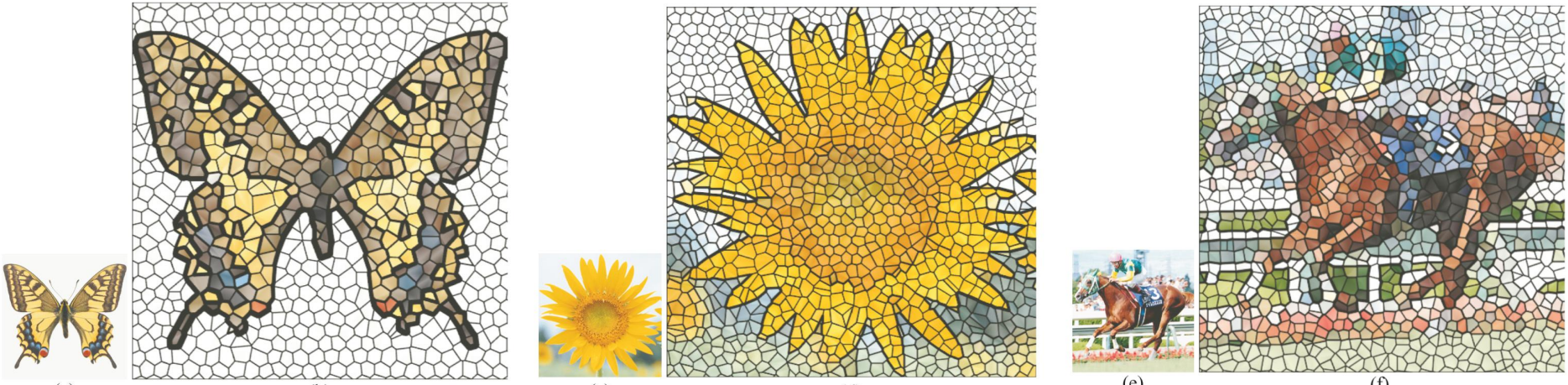
http://dx.doi.org/10.2312/egs.20021036

3. Mosaic method

Some of the result



(a) (b) (c) (d) (e) (f)

http://dx.doi.org/10.2312/egs.20021036