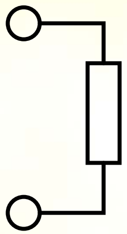
Electrical Circuits

II

Basic Passive Circuit Elements

BASIC IDEAL PASSIVE CIRCUIT ELEMENTS. ELECTRICAL DEVICES AND ITS MODELS, PRACTICAL CIRCUIT ELEMENT.

Fundamental passive circuit elements

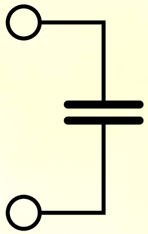


R – resistor (circuit element); Ohm's law $$u = Ri$$

Power – heat, which is (irreversibly) dissipated in time 1 s $$P = UI = RI^2 = \frac{U^2}{R}$$

It is **non-inertial element**, since the relation between voltage and current is not time dependent

R (property) – electrical resistance, measured in ohms [Ω]; the voltage across a resistance is *directly proportional* to the current flowing through it



C – capacitor, it stores charge.

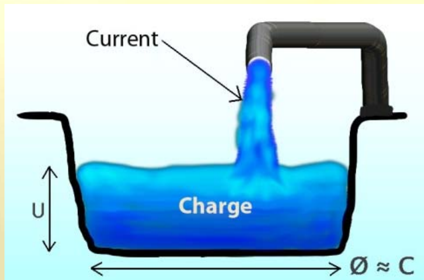
The charge has to be delivered into the capacitor by flowing electrical current – and it lasts distinct time *(remember again our „water" physical analogy)*.

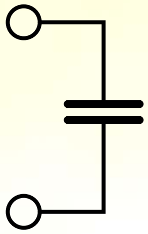
The relation between voltage and current has to be time dependent, it is **"inertial" element**. The charge on the capacitor is proportional to the voltage across it such that

$$Q = Cu, \quad C = \frac{Q}{u}, \quad u = \frac{Q}{C}$$

When the current, delivering the charge into capacitor is constant, (and supposing , the capacitor had no charge before we start to charge it), than

$$u(t) = \frac{I \cdot t}{C}$$





But, when the current, flowing into the capacitor, varies with time, we have to use integral

The charge at the time instant t (supposing, at $t = 0$ the capacitor stored the initial charge $q(0)$)

$$q(t) = \int_0^t i(\tau)\,\mathrm{d}\tau + q(0)$$

So the voltage is

$$u(t) = \frac{q(t)}{C} = \frac{1}{C}\int_0^t i(\tau)\,\mathrm{d}\tau + u_c(0)$$

Reversely, the current is flowing into the capacitor only if voltage across its terminals varies with time

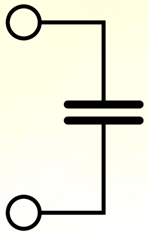
$$i(t) = C\,\frac{\mathrm{d}u(t)}{\mathrm{d}t}$$

What happens, when we connect dc voltage source to the capacitor?

⇒ The voltage across terminals of the capacitor has to be equal to the source voltage. It has to deliver distinct charge into capacitor (with ideal circuit elements source has to deliver infinite current in infinitely short time instant when there is some resistor between voltage source and capacitor, the resistor limits the current, and then it take some time to deliver the charge into capacitor) – **the capacitor is an open circuit to dc** (or block dc), **but not until the capacitor is fully charged**; *later in semester we will calculate this and we will call it transient*

What happens, when we connect dc current source to the capacitor?

⇒ Current source continually delivers constant current, stored electrical charge increases for ever, and hence the voltage across capacitor growths too *(more quickly than when we connect voltage source, practical application e.g. USB 2.0)* – in theory ad infinitum; so the capacitor is not open circuit to dc (or, if you like, is valid just after infinitely long time)



The energy stored in the electric field is

$$W_c = \frac{1}{2} C U^2$$

Note

Of course, even here is hidden the power theorem:

$$\begin{aligned} W_e(t) &= A(t) = \int_0^t u(\tau) i(\tau) \mathrm{d}\tau = \int_0^{q(t)} u(q) \mathrm{d}q \\ &= \int_0^{q(t)} \frac{q}{C} \mathrm{d}q = \frac{1}{2C} q^2(t) = \frac{1}{2} C u^2(t) \end{aligned}$$

Quiz question: how differs the first integral in calculation of energy in the equation above from the calculation of power?

Exciting by a sinusoidal current:

$$i(t) = I_m \sin \omega t$$

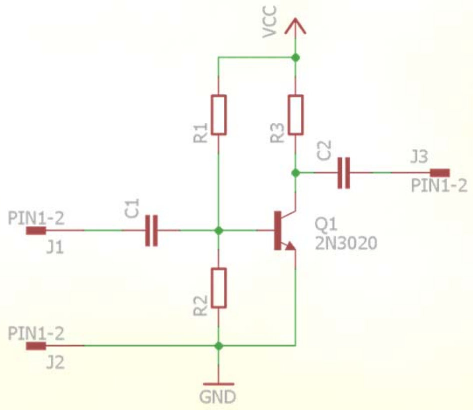
$$u(t) = \frac{1}{C}\int_0^t I_m \sin \omega\tau \, \mathrm{d}\tau + u_c(0) =$$

$$= \frac{I_m}{\omega C}(1 - cos\omega t) + u_c(0) = \frac{I_m}{\omega C}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{I_m}{\omega C} + u_c(0)$$

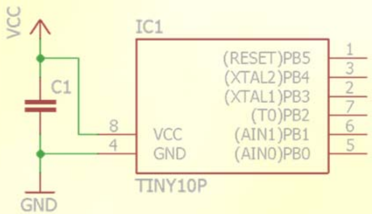
- The magnitude of **sinusoidal** voltage is related to magnitude of current by the term: $\frac{1}{\omega C}$
- We can write a relation: $U_c = X_c I_c$
  - Where U_c is magnitude of voltage, I_c magnitude of current and X_c is reactance – the term above
  - Formally, it is the same relation as the Ohm's law
  - However, the reactance is a frequency dependent. At low frequencies, the capacitor acts like open circuit while at high frequencies as a short circuit.

Applications:

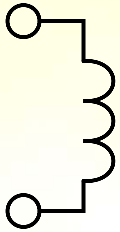
- **Input coupling capacitor**: since it acts like an open circuit at DC and (almost) short circuit excited by AC at high frequencies, it can separate different part of circuits operating at different DC levels and pass AC signals only.



- **Decoupling capacitor**: The purpose is opposite to the coupling capacitor. It is connected between supply pins of the digital ICs. It shorts the high frequency noise away from the chip. The capacitors of typical capacitance 100 nF are combined with lager one which are reservoirs of charge to supply the instantaneous power requirements of the chips.



- **Energy reservoirs** – stores and supplies electrical energy
- **Switched capacitors** – replaces resistors in IC filters
- **Frequency filtering** – will be explained on 9^{th} lecture (frequency response)



L – inductor (idealized model), in practice implemented e.g. as a coil

Electric current flowing through the inductor generate magnetic flux. In the magnetic field is stored the energy. So either inductor is **“inertial” element** (in “real” circuit is not possible deliver energy in infinitely short time instant).

The relationship between magnetic flux and electric current is

$$\Phi = L \cdot i, \quad i = \frac{\Phi}{L}, \quad L = \frac{\Phi}{i}$$

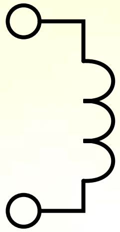
Where **Φ** is magnetic flux, measured in Webers [Wb] and
L is inductance, measured in Henry [H]

Physical analogy: *Going on with our water analogy, we may consider inductance as equivalent of moment of inertia of water turbine (water wheel)*

Consider our analogy of rotating water turbine is clear, that the inductor resists to the changes of current – when connected resistivity change (or, we disconnect the source) the voltage across inductor’s terminals changes so, the inductor enforce its passing current to other circuit elements; such voltage may be (theoretically) even infinite; because stored energy is not infinite, both voltage and current will decrease after mentioned change

Even if the inductor is passed by magnetic flux, when it doesn’t vary with time, there is no voltage across its terminals. Only when magnetic field varies with time (e.g. in alternator in car, or generator in power plant) there will be some voltage given by

$$u = \frac{\Delta\Phi}{\Delta t}, \quad \text{resp. } u(t) = \frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} \quad \longrightarrow \quad u(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$



Frequently we are interested in the current passing the inductor, when we connect voltage source. Now, we have to integrate. Also, the inductor could be passed by some initial magnetic flux (or, corresponding electric current $i_L(0)$), so

$$i(t) = \frac{1}{L}\int_0^t u(\tau)\mathrm{d}\tau + i_L(0)$$

What happens, when we connect dc voltage source to the inductor?

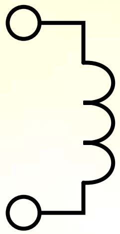
$\Rightarrow$ **The inductor is short circuit to DC**. BUT:

$$U = \frac{\Delta\Phi}{\Delta t} = L\frac{\Delta i}{\Delta t}, \qquad \text{is constant}$$

so the current passing the inductor increases linearly *(practical application in voltage converters and switched sources)*, in the case of ideal inductor and ideal voltage source up to the infinity – so the inductor is short circuit just after infinitely long time,

$$I = \frac{Ut}{L}$$

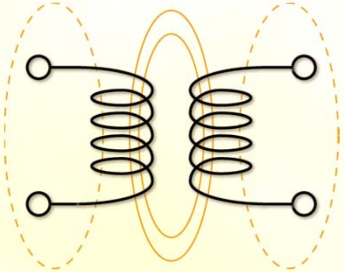
$\Rightarrow$ When we connect some resistivity between the source and inductor, the maximum possible current will be limited by resistivity, and **the inductor will be short circuit just after distinct time instant, necessary to fully charge the inductor**… *(This necessary time instant we will calculate when we will study transients)*



The energy stored in magnetic field is

$$W_L = \frac{1}{2} L I^2$$

Coupled inductors



When two or more coils are geometrically aligned in such way, the magnetic flux generated in the first coil passes the second one, there will be induced voltage across terminals of the second coil (of course when the flux varies with time)

The voltage across first coil affects both the current, passing the first one, and the coupled one

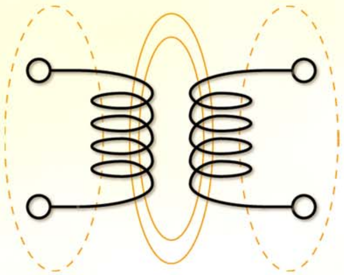
$$u_1(t) = L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t} + M \frac{\mathrm{d}i_2(t)}{\mathrm{d}t} \qquad u_2(t) = L_2 \frac{\mathrm{d}i_2(t)}{\mathrm{d}t} + M \frac{\mathrm{d}i_1(t)}{\mathrm{d}t}$$

An example of coupled inductors is transformer. Instead of general calculations there are proposed methods of design using tables and special design equations.

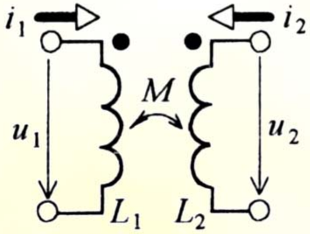
Note – inverse equations:

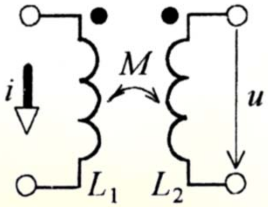
$$i_1(t) = i_1(0_+) + \Gamma_1 \int_0^t u_1(\tau)\mathrm{d}\tau + \Gamma_M \int_0^t u_2(\tau)\mathrm{d}\tau \qquad i_2(t) = i_2(0_+) + \Gamma_2 \int_0^t u_2(\tau)\mathrm{d}\tau + \Gamma_M \int_0^t u_1(\tau)\mathrm{d}\tau$$

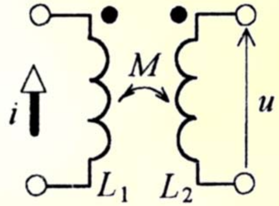
$$\Gamma_1 = \frac{L_2}{L_1 L_2 - M^2}; \qquad \Gamma_2 = \frac{L_1}{L_1 L_2 - M^2}; \qquad \Gamma_M = \frac{-M}{L_1 L_2 - M^2} \qquad M = \kappa\sqrt{L_1 L_2}, \quad \kappa \in \langle 0, 1 \rangle$$



When we reverse one of the inductors, the way in which magnetic flux passes the coils also reverse, so the signs in equations also changes. That is why the beginning and the end of each wiring has to be marked (dot in circuit diagrams).

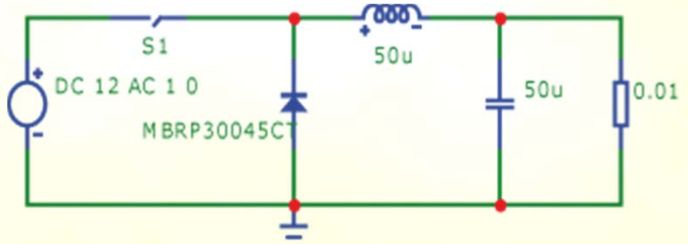






Applications:

- **Transformer** – AC / AC voltage converter, which usually changes one AC voltage level to another. It also converts an impedance which source sees between its terminals. It has two or more coils with common magnetic flux.
- **Switched source** – DC / DC voltage converter, which converts one DC voltage level to another. It maintains constant mean value of current passing through the inductor.



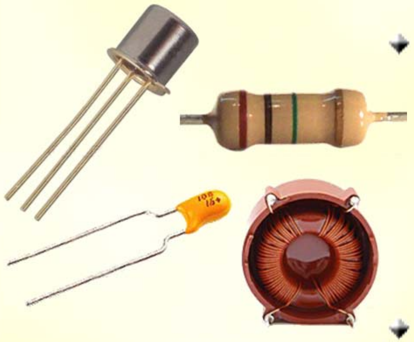
- **Constant-current LED regulators** – operates on the same principle like voltage switched sources, however, the LEDs are connected in series to the inductor.
- **Frequency filtering** – will be explained on 9th lecture (frequency response)

Basic concepts and definitions

Electrical device

(Definition: any equipment used to electric power generation, transformation, transport, distribution or utilization, as are electric machines, transformers, appliances including meters, protective devices, accessories of distribution networks, and other electric appliances)

- It is always physical implementation of electric appliance
- Devices with same functions may be implemented in a variety of ways
- Distinct electric device can be described by different mathematical formulas and its function may be analyzed in many different ways
- *In a very general case we may describe the device using Maxwell's formulas and boundary conditions, but, in practice – it is too complicated…*



Electrical circuit

- Generally it is conductive connection of electrical components (resistors, capacitors, inductors, transistors, …)
- From the point of view of analysis of an electrical devices we will consider electrical circuit idealized model, electrical network of idealized electrical components

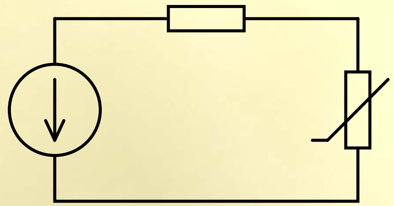
Equivalent circuit diagram

Graphical representation of electrical circuit by agreed graphical symbols (including block diagrams)

Electrical device

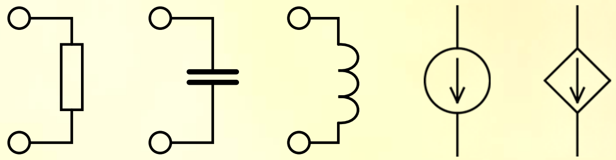
Circuit diagram - functional

Circuit diagram



Elements of electrical circuits

Idealized components, which together forms an **equivalent circuit**, and exhibits capability of limitation of electrical current, accumulate energy in the form of electric or magnetic field, control magnitude of current or voltage, …



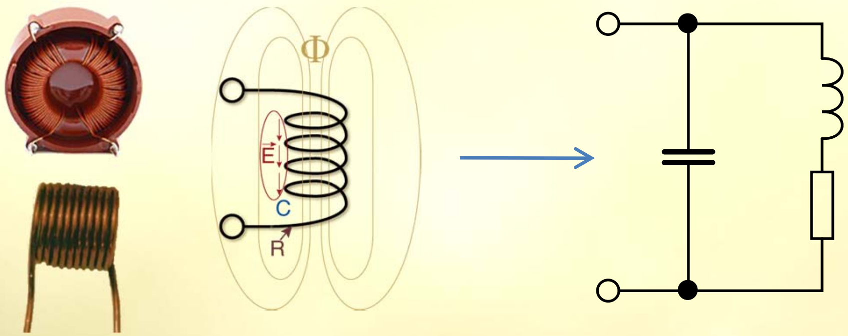
- Passive (resistor, capacitor, inductor)
- Active (An independent (ideal) voltage source, current source)

Circuit model of electrical device

- Using **connection of basic circuit elements** we try to describe properties of actual electrical device (it never could describe it in perfectly)
- **Distinct electrical device could be described by many different models**
  - Each model describes distinct device with acceptable accuracy within specified conditions of application

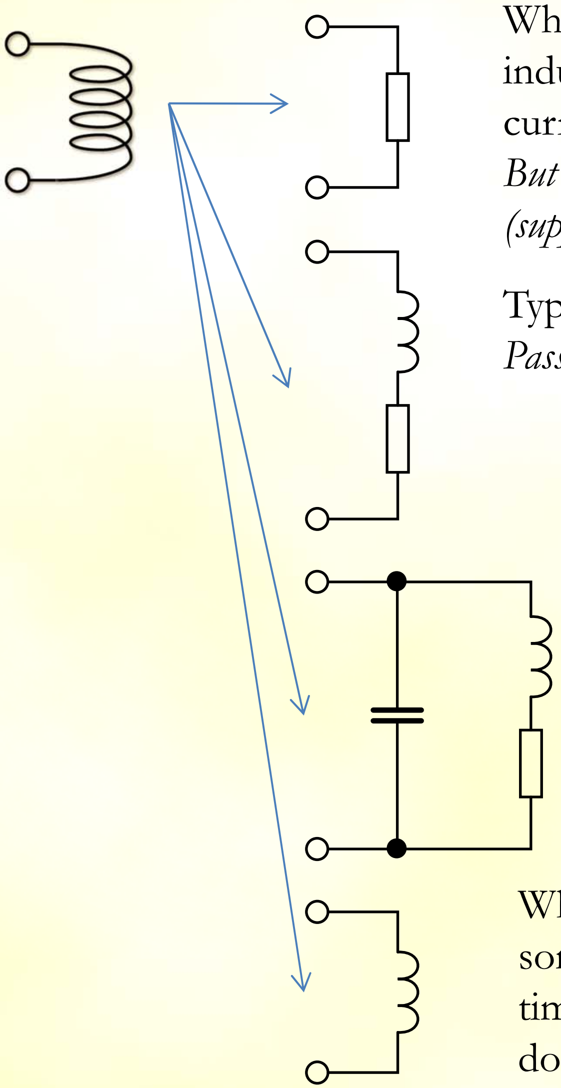
Resistor, capacitor, coil – fundamental electrical devices (any of these devices may create electric, and magnetic field and they have some electric resistivity)

Resistor, capacitor, inductor – idealized elements of electrical circuit (each exhibits only one property)



Electrical device – coil: wire is wound around a core; the wire exhibits some electrical resistivity; between distinct turns is some electrical field, so, coil exhibits some capacitance; the coil is passed by a magnetic flux, so, it has some inductance – it is simulated by inductor, …

Different circuit models of a coil



When the coil is connected to the DC voltage source, after distinct time inductance could not affect passing current any more so that the passing current is affected just by resistivity of a wire;
But electromagnetic field stores energy and it will take effect when the passing current (supplying voltage) changes

Typical model of a real coil when the current (and voltage) varies in time
Passing current and voltage across coil terminals is affected both by resistivity and inductance

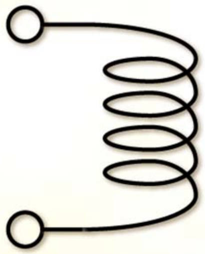
When the frequency of voltage source is high, so far negligible capacity take effect and eventually it may outweigh the inductance

When the resistivity of the wire is negligible, the model has only the inductor; sometimes it is used in the cases, when the circuit is described in a very short time interval (e.g. analysis of voltage convertors), when the inductance dominates

Sometimes it is necessary to use more complicated circuit models, which describes losses in a coil core and another physical phenomena

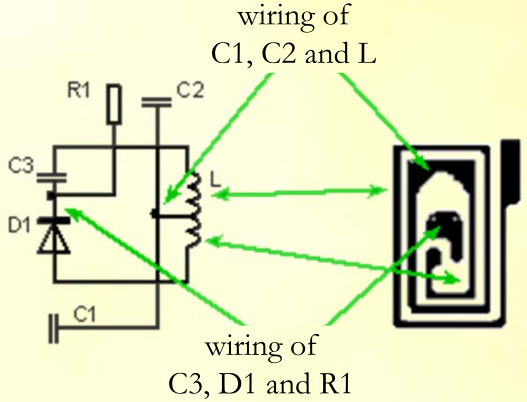
We will always consider ideal circuit elements

As mentioned above, electrical device that exhibits same function may be realized by many ways; described coil may be implemented like wire turns wound around an air or ferrite core, or even a conductive path on printed circuit:



Coil as a wire turns

Coil as a conductive path



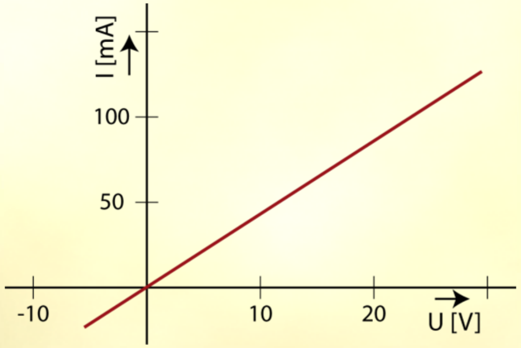
Common wiring and coil may be very similar (even wiring has *parasitic* inductance)

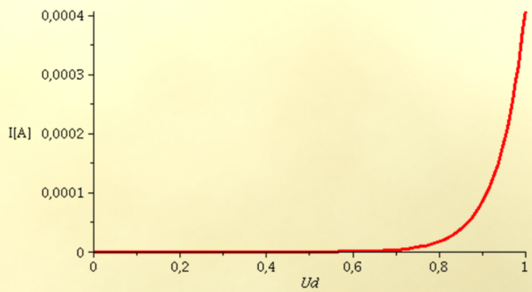
- **Linear circuit**

  Relation between circuit variables (voltage / current, charge, magnetic flux, …) is linear, following general relation is valid $y = Kx$ where K is constant, so, e.g. $U = RI$

- **Non-linear circuit**

  Non linear relation between circuit variables, e.g. $I_D = I_S\left(e^{\frac{U_D}{nU_T}} - 1\right)$

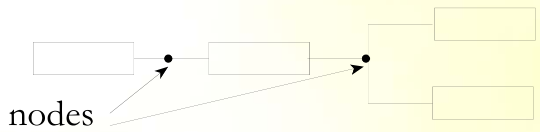




Circuits with lumped elements

Circuit has finite number of circuit elements, realized as distinct parts (resistors, capacitors, inductors, …)

- Distinct circuit elements in circuit diagram are connected by ideal wires, they have no resistivity (inductance, capacitance, …) nor time delays
- Connection of 2 or more wires is called node, graphically it is depicted by a dot

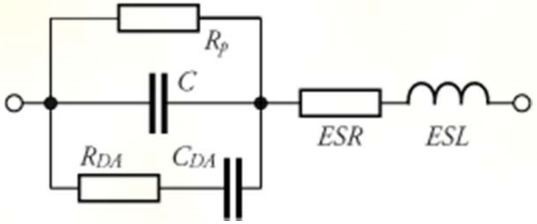


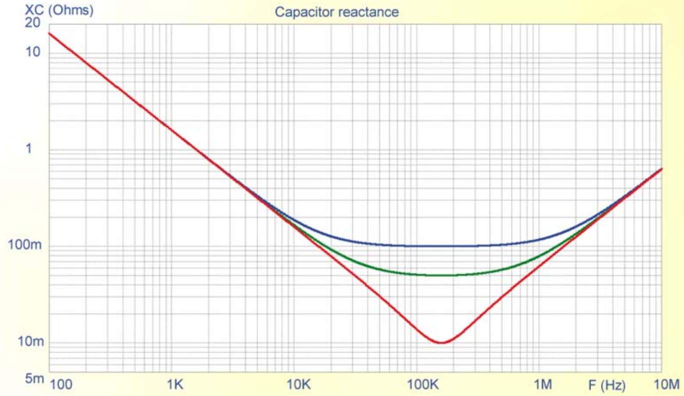
Distributed circuits

It is not implemented from distinct circuit elements (and non-specialist will not consider it is a circuit) – one example is a coaxial cable used with TV, twisted pair in USB or UTP cable, or even conductive path in computer bus (not only) on printed circuit

- It depends on geometrical arrangement of wires
- One of reasons is finite velocity of propagation of a light (and criterion, if a wire is, or is not a distributed circuit is time delay of signals)
- When we derive equations describing distributed circuit, we may use common methods of circuit analysis (using infinitely small circuit elements in infinitely number of lumped circuits), in general it is described using methods of electromagnetic field analysis
- *More information about this topic will be in successive subjects*

Real capacitor





Important parameters:

- **ESR –** Equivalent series resistance – important in switched sources applications, where large currents flows through the capacitor. It produce Ohm's losses – heat. Ceramic capacitors may have ESR less than 10 mΩ, low ESR electrolytic and tantalum less than 100 mΩ, general aluminum few times higher.
- **ESL –** equivalent series inductance - inductance of the leads and plates. At high frequencies the capacitor acts like an inductor.
- Rp – insulation resistance (leakage)
- R_{DA}, C_{DA} - dielectric absorption