Phasors, impedances and admittances

1.
   a) $u(t) = 67.439\sin(100\pi t + 0.7546)\,\mathrm{V}$
   b) $\mathbf{U} = 47.69\,e^{j\cdot 0.7546}\,\mathrm{V}$, $f = 50\,\mathrm{Hz}$
   c) $\mathbf{U}_{1m} = 81.06\,\mathrm{V}$, $\mathbf{U}_{2m} = -9 = 9\,e^{j\pi}\,\mathrm{V}$, $\mathbf{U}_{3m} = 3.24\,\mathrm{V}$
   It is not possible to sum them up, since they have different frequency.
2.
   a) $\mathbf{Z} = 150\,e^{-j\frac{\pi}{6}} = 130 - 75j\,\Omega$
   b) $R = 130\,\Omega, C = 11.99\,\mu\mathrm{F}$
3. $i(t) = 8.98\sin\left(1000t + 0.33\right)\ \mathrm{mA}$
4.
   a) $U_C = 196.2\,\mathrm{V}$
   b) $C = 2.03\,\mu\mathrm{F}$
   c) $U_C = 229.68\,\mathrm{V}, C = 20.79\,\mu\mathrm{F}$
   In the moment, when we connect circuit to the voltage source, the voltage on the incandescent lamp can be higher, than $U_z = 12\,\mathrm{V}$. It depends on the phase of the source voltage. The maximum voltage can be the same, as the source voltage. As the maximum voltage is almost 20 times greater, than the rated voltage, it might seem the incandescent lamp should be destroyed. However, this maximum voltage falls down exponentially – it is a transient. And the time constant is about 166 µs, so even the initial instantaneous power can be few kW, the total delivered energy is just about 1 J, which the incandescent lamp should tolerate.
5.
   a) $\mathbf{Z} = 100 + 300j = 316.23\,e^{j1.25}\,\Omega$
   b) $i(t) = 0.537\ \sin(500t - 1.25)\,\mathrm{A}$
   c) $u_R(t) = 53.7\ \sin(500t - 1.25)\,\mathrm{V}$, $u_L(t) = 160.8\ \sin(500t + 0.32)\,\mathrm{V}$
6.
   a) $i(t) = 7.8\ \sin(1000t + 0.11)\,\mathrm{mA}$
   b) $u(t) = 1.56\ \sin(1000t + 0.11)\,\mathrm{V}$
7.
   a) On the left is capacitor C_1 and resistor R_1 in this order, from top to bottom. On the right is the order from top to bottom R_2, C_2.
   $U_{R_1} = U_{R_2} = U_{C_1} = U_{C_2} = \sqrt{50^2 + 50^2} = 70.71\,\mathrm{V}$
   $R_1 = R_2 = 5000\,\Omega, C_1 = C_2 = 0.2\,\mu\mathrm{F}$
   b) The order of circuit elements from top to bottom is R_1, C_1 and C_2, R_2.
   $U_R = 38.27\,\mathrm{V}, U_C = 92.39\,\mathrm{V}$
   $R_1 = R_2 = 2706.5\,\Omega, C_1 = C_2 = 0.153\,\mu\mathrm{F}$
8. $\mathbf{U}_i = 86.21 - 34.48j + 8.62 - 3.45j = 94.83 - 37.93j = 102.13\,e^{-0.38j}$,
   $\mathbf{Z}_i = 250\,e^{0.81j}\,\Omega$
9. $\mathbf{Z}_v = 1000 - 90.\overline{90}j = 1004.1\,e^{-0.09j}\,\Omega$

Sinusoidal steady state power analysis

1. Current without compensation: $\mathbf{I} = 0.0824 - 0.5214j = 0.5278\, e^{-1.414j}$ A
   Power without compensation: $\mathbf{S} = 18.95 + 119.92j = P + jQ$
   Power factor: $\lambda = 15.6\,\%$
   $C = 6.84\ \mu$F - parallel.
   Current after compensation: $\mathbf{I} = 0.0824 - 0.027j = 0.0867\, e^{-0.318j}$ A
2. The current, drawn by single TV panel is $I = 0.2\,\mathrm{A}$, so the apparent power is $S = 46\,\mathrm{VA}$. The power factor is $\lambda = 6.52\,\%$. For power factor compensation we will use a choke with impedance $L = 62.5\,\mathrm{mH}$. Total current after power factor compensation drops from 12 A to $I_2 = 0.824$A.
3.
   a) P = 0.449 W, Q = -1.43 var, S = 1.449 VA
   b) P = 4.99 W, Q = -0.159 var, S = 4.993 VA
   c) $\dfrac{P}{S} = \dfrac{\omega RC}{\sqrt{1+(\omega RC)^2}}, \dfrac{Q}{S} = \dfrac{1}{\sqrt{1+(\omega RC)^2}}$
4.
   a) P = 4.99 W, Q = 0.157 var, S = 4.992 VA.
   b) P = 0.46 W, Q = 1.445 var, S = 1.516 VA.
   c) $\dfrac{P}{S} = \dfrac{R}{\sqrt{R^2+(\omega L)^2}}, \dfrac{Q}{S} = \dfrac{\omega L}{\sqrt{R^2+(\omega L)^2}}$
5.
   a) P = 4.9 mW, Q = 0.157 var, S = 0.157 VA.
   b) P = 4.54 W, Q = 1.45 var, S = 4.76 VA.

Frequency response

1. $\mathbf{P} = \dfrac{\mathbf{U}_r}{\mathbf{U}_1} = \dfrac{1}{1 + j\omega RC(1-K)}, \omega_p = \dfrac{1000}{12}\,\mathrm{s}^{-1}.$
2. $\mathbf{P} = \dfrac{\mathbf{U}_2}{\mathbf{U}_1} = \dfrac{R_2}{R_1+R_2}\,\dfrac{1}{1+j\omega\frac{CR_1R_2}{R_1+R_2}} = \dfrac{1}{2}\cdot\dfrac{1}{1+j\frac{\omega}{2000}}, \omega_p = 2000\,\mathrm{s}^{-1}$
3. $\mathbf{P} = \dfrac{\mathbf{U}_2}{\mathbf{U}_1} = \dfrac{1+j\frac{\omega}{1000}}{1+j\frac{\omega}{500}}, \omega_0 = 1000\,\mathrm{s}^{-1}, \omega_p = 500\,\mathrm{s}^{-1},$

   $$\lim_{\omega\to\infty}\mathbf{P} = \frac{j\frac{1}{1000}}{j\frac{1}{500}} = 0.5, \qquad 20\log 0.5 = -6.02\,\mathrm{dB}$$
4. $\mathbf{P} = \dfrac{R_2}{R_1+R_2}\,\dfrac{1+j\omega\frac{L}{R_2}}{1+j\omega\frac{L}{R_1+R_2}} = 0.5\cdot\dfrac{1+j\frac{\omega}{10^5}}{1+j\frac{\omega}{2\cdot 10^5}}$

   $\omega_0 = 10^5\,\mathrm{s}^{-1}, \omega_p = 2\cdot 10^5\,\mathrm{s}^{-1}$
   $\lim_{\omega\to 0}\mathbf{P} = 0.5 \Rightarrow -6.02\,\mathrm{dB},$
   $\lim_{\omega\to\infty}\mathbf{P} = 1 \Rightarrow 0\,\mathrm{dB}$
5. $\mathbf{P} = \dfrac{1}{LC}\,\dfrac{1}{(j\omega)^2 + j\omega\frac{R}{L} + \frac{1}{LC}}$

a) $\mathbf{P} = \dfrac{1}{(1 + j\frac{\omega}{199499})(1 + j\frac{\omega}{501})}$

Bode – 0 dB, than decrease by 20 dB per decade from 501 s^{-1} and 40 dB per decade from 199499 s^{-1}.

b) $\mathbf{P} = \dfrac{1}{(1 + j\frac{\omega}{10000})^2}$

Bode – 0 dB, than decrease with slope -40 dB/decade, real response has -6 dB error at break frequency.

c) $\mathbf{P} = \dfrac{1}{1 + 0.01 \cdot j\frac{\omega}{10000} + (j\frac{\omega}{10000})^2}$

Bode – 0 dB, than decrease with slope -40 dB/decade, real response is resonant curve with +40 dB at resonant frequency 10000 s^{-1}.

Resonant circuits

1. $f_r = 503.29\,\mathrm{Hz}$, $R = 3.16\,\Omega$
2.
   a) $\omega_r = 3160.7\,\mathrm{s}^{-1}$ $\qquad f_r = \dfrac{\omega_r}{2\pi} = 503.04\,\mathrm{Hz}$

   b) $\mathbf{Z} = 10000\,\Omega$

   $\mathbf{I} = 100\,\mu\mathrm{A}$, $\mathbf{I}_C = 3.16\,\mathrm{mA}$

   c) $U = 10\,\mathrm{V}$, $\mathbf{I}_C = 31.6\,\mathrm{mA}$