1. **What types of errors we recognize (in general, for a precise number x and its approximation &) and what information they tell us? How do those errors appear in calculations? What impact does it have on numerical computing?**

Roundoff Error: Roundoff errors occur because computers have a limited ability to represent numbers. For example, π has infinite digits, but due to precision limitations, only 16 digits may be stored in MATLAB. While this roundoff error may seem insignificant, if your process involves multiple iterations that are dependent on one another, these small errors may accumulate over time and result in a significant deviation from the expected value. Furthermore, if a manipulation involves adding a large and small number, the effect of the smaller number may be lost if rounding is utilized. Thus, it is advised to sum numbers of similar magnitudes first so that smaller numbers are not “lost” in the calculation.

Truncation Error: Truncation errors are introduced when exact mathematical formulas are represented by approximations. An effective way to understand truncation error is through a Taylor Series approximation. Let’s say that we want to approximate some function, f(x) at the point xi+1, which is some distance, h, away from the basepoint xi, whose true value is shown in black in Figure 1. The Taylor series approximation starts with a single zero order term and as additional terms are added to the series, the approximation begins to approach the true value. However, an infinite number of terms would be needed to reach this true value.

2. **Describe some method (good candidates are bisection or Newton method) for finding roots (that is, solving equations) numerically. Discuss how we recognize when to stop the algorithm.**

The simplest root-finding algorithm is the bisection method. Let f be a continuous function, for which one knows an interval [a, b] such that f(a) and f(b) have opposite signs (a bracket). Let c = (a +b)/2 be the middle of the interval (the midpoint or the point that bisects the interval). Then either f(a) and f(c), or f(c) and f(b) have opposite signs, and one has divided by two the size of the interval.

Newton's method assumes the function f to have a continuous derivative.

Set the accuracy so that the error is less than the accuracy after many iterations.

3. **Describe some method (most likely the Euler method) for solving initial value problems (an ODE with initial condition) numerically. Discuss the notion of the order of method.**

The assumption is that over a time-span Δt=tn+1=tn. The original differential equation by

$$\frac{d\vec{y}}{dt} = f(t,\vec{y}) \implies \frac{\vec{y}_{n+1}-\vec{y}_n}{\Delta t} \approx f(t,\vec{y})$$ and get $$\vec{y}_{n+1} = \vec{y}_n + \Delta t.\, f(t,\vec{y})$$

Thus, the Euler method gives an iterative scheme by which the future values of the solution can be determined. Graphically, the slope (derivative) of a function is responsible for generating each subsequent approximation of the solution $\vec{y}(t)$. It is important to note that the truncation error in the above equation is O(Δt^2)

4. **Discuss the notion of computational complexity and show how it applies to solving large systems of linear equations using elimination.**

Computational complexity, a measure of the amount of computing resources (time and space) that a particular algorithm consumes when it runs. Direct methods such as Gauss elimination methods. Matrix is symmetric matrix.

5. **What is a general solution of an ODE? What is a particular solution of an ODE? How can we determine (choose) a certain particular solution?**

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6. **What can we say about the set of solutions of a homogeneous linear ODE? How does it help us find a general solution? Where do we get the fundamental system? (It is enough to know the basic answer with characteristic numbers, no need to go into details about higher multiplicity.)**

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7. **What can we say about solutions of a general (non-homogeneous) linear ODE? How does it help us in solving such an equation?**

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8. **What can we say about the set solutions of a system of linear ODEs? How can we find a general solution?**

(Again, the simplest case of simple real eigenvalues is enough).

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