Electrical Circuits

I

Introduction – Basic Concepts

CIRCUIT VARIABLES (VOLTAGE, CURRENT, POWER) AND ITS BASIC QUANTITIES. SPECIAL VALUES

Lecture 1 - introduction

- Basic quantities
    - Electric charge
    - Electric current, current flow convention, 1^{st} Kirchhoff's law (KCL)
    - Voltage, passive sign convention and active sign convention, 2^{nd} Kirchhoff's law (KVL)
    - An energy and power
- Classification of time functions of circuit variables
- Basic active elements (voltage source and current source, dependent sources)
- Special values of circuit variables (mean value, RMS).

Basic quantities

- **Electric charge**

symbol: $\boldsymbol{Q}$

unit: **coulomb [C]**, in the SI standard system [A s]

elementary charge: $e \doteq 1.602176487 \cdot 10^{-19}$ C (carried by a single electron (-) or proton(+))

irreplaceable, uncreateable property of elementary particles – **charge conversation law**

Alongside which quantities we may met with electric charge?

- Electric current
- Capacity of an accumulator (e.g. 2700 mAh)
- The charge stored in a capacitor → voltage across capacitor, stored energy

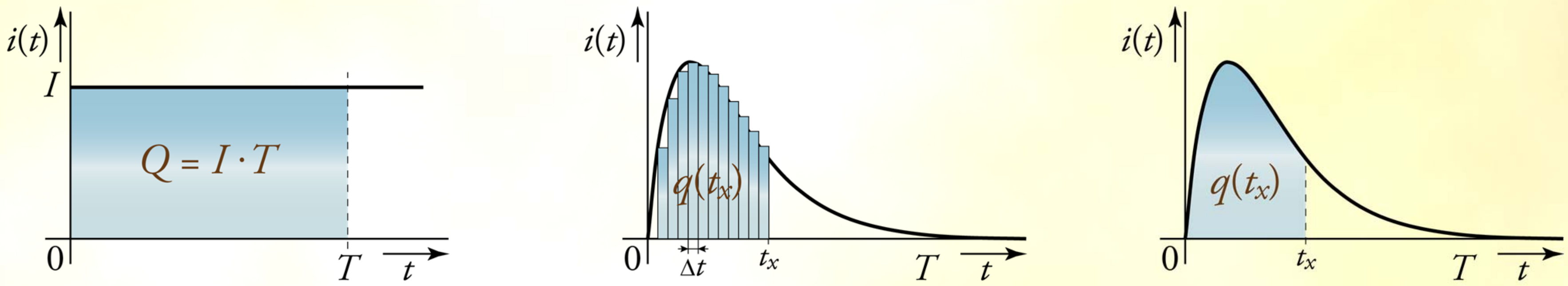
- **Electric current**

symbol: $\boldsymbol{I}$

unit: **ampere [A]**

generally it is **oriented moving of electrically charged particles** – usually electrons (metals, vacuum), but also ions (electrolytes, e.g. in accumulator)

Generally, the current is **an equivalent amount of electric charge, transferred per second**.



Convention: when the quantity has constant value, we will write it in uppercase Q; when it is function of time (and it varies in time), we will use lower case, eventually we will write time in round brackets, e.g. *q(t)*

The charge, delivered by constant electric current *I* within time period *T*: $Q = I \cdot T$

But, when electric current varies in time: $$q(t) = \int_0^t i(\tau)\,\mathrm{d}\tau$$

Or opposite, **the time rate of change of charge constitutes an electric current:** $$i(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t}$$

Physical analogy: *electric current we can imagine as flow of water, electric charge as total volume (weight) of water, which passed through the pipe*

Example

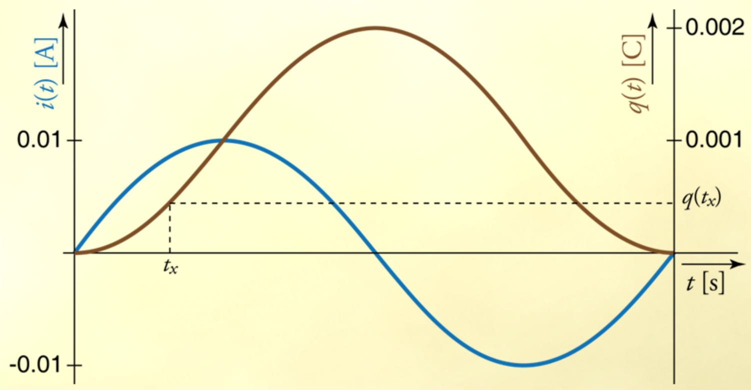
The waveform of current is given as $i(t) = 0.1\sin(100t)$.

- Calculate total charge transferred in a time interval of $t_x = 10\ \mathrm{ms}$.
- Determine general relation for a charge, transferred in a time interval of t.

$$q(0.01) = \int_0^{0.01} 0.1\sin(100t)\,\mathrm{d}t = \left[\frac{-0.1\cos(100t)}{100}\right]_0^{0.01} =$$

$$= 0.01\,[-\cos(100\cdot 0.01) + \cos(0)] \doteq \underline{\underline{459.7\ \mu\mathrm{C}}}.$$

$$q(t) = \int_0^{t} 0.1\sin(100\tau)\,\mathrm{d}\tau = \left[\frac{-0.1\cos(100\tau)}{100}\right]_0^{t} = 0.001\,[1-\cos(100t)]\ \mathrm{C}.$$



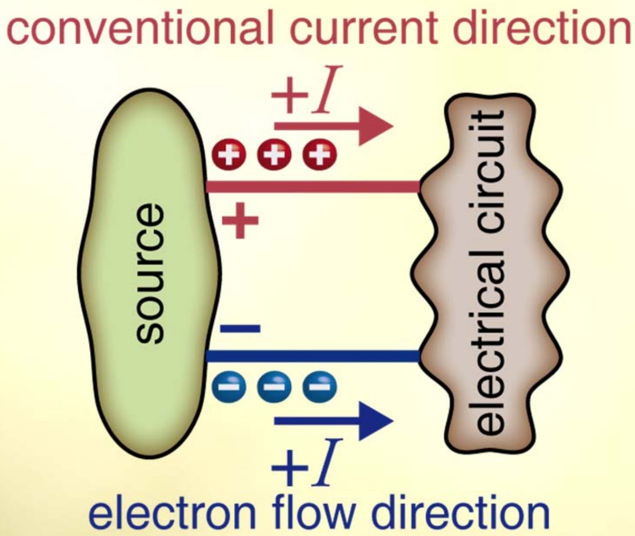
Direction of current – two conventions

- Consider device, which delivers an electrical charge to other parts of an electrical circuit – the current source
- The terminals, among which the charge flows are distinguished by + and – symbols
- With exception of positive charged ions in electrolyte en electric charge is transferred by a moving electrons, that exhibits negative electrical charge

  ☞ **Electron flow direction**: a negatively charged particles – electrons – leaves negative terminal of the source and flows into the positive terminal – preferred by many in fields of electrical technology especially in English speaking countries, but **not generally adopted in Europe**

  ☞ **Conventional current (flow) direction**: although currently we know it has no physical basis, from historical reasons we assume that **positive charged particles leaves positive terminal of the source – it is universally adopted**

- *The results of electric circuit analysis are not affected by the direction of current that is assumed for analytical purposes*



Marked positive direction of current matches positive movement direction of electrical charge

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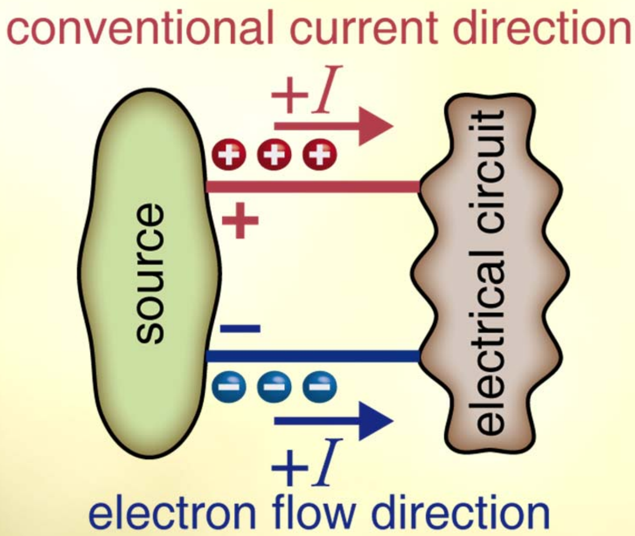
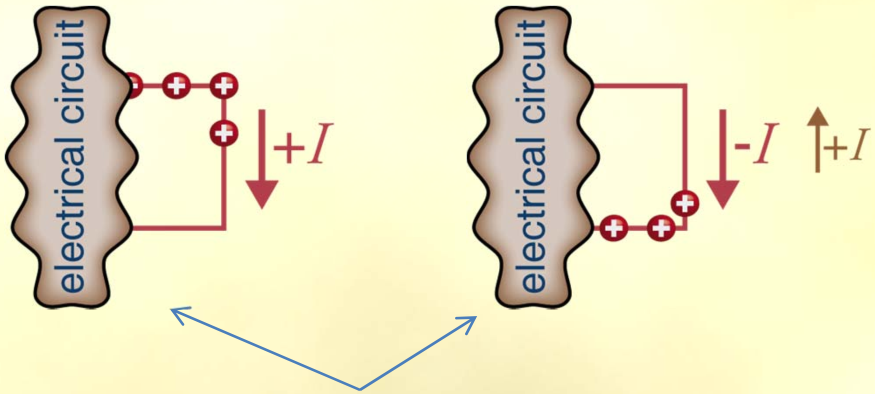
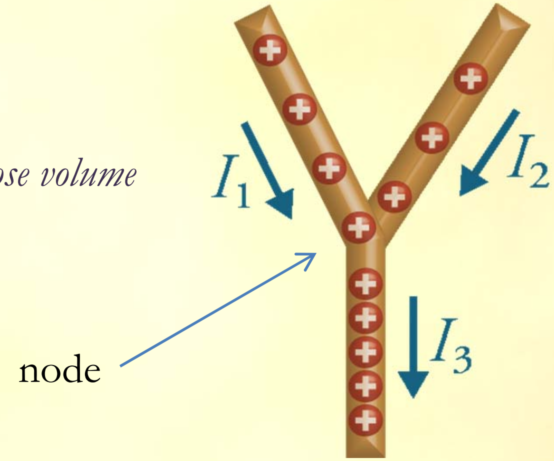




Marked positive direction of current matches positive movement direction of electrical charge

1st Kirchhoff's law (KCL): $$\sum_{k=1}^{n} i_k = 0$$



According to the physical analogy – same volume (weight) of water, which flow in close volume (full of water, of course), shall flow out this closed volume

Electrical meaning: **the charge cannot accumulate in a node**

$$I_1 + I_2 = I_3 \quad \rightarrow \quad -(I_1 + I_2) + I_3 = 0$$

Sign convention:

- Current **entering** the node is **negative**
- Current **leaving** the node is **positive**

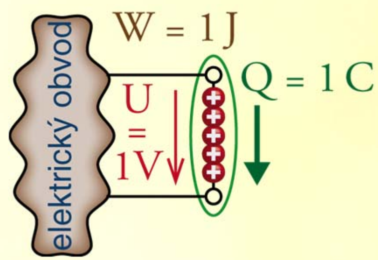
Notes *(informational, not required for examination)*:

- *The velocity of oriented movement of electrons in copper: in 1 m^3 of copper is $8.5 \cdot 10^{28}$ atoms, in 1 mm^3 is than charge 13.617 C in the form of free electrons; if the wire with cross-section 1 mm^2 is passed by the current 1 A, than each 1 s must passed this cross-section approx. 1/13 of free electrons contained in this volume – the velocity of oriented movement is approx. 75 $\mu m/s$!*
- *Why the velocity of propagation of electricity is much higher? – Each moving electron creates magnetic field, among them is electric field – the „carrier" is electromagnetic field…*
- *Conducting of electric current in dielectric materials – electric induction.*

- **Electric voltage**

symbol: ***U***

unit: **volt [V]**



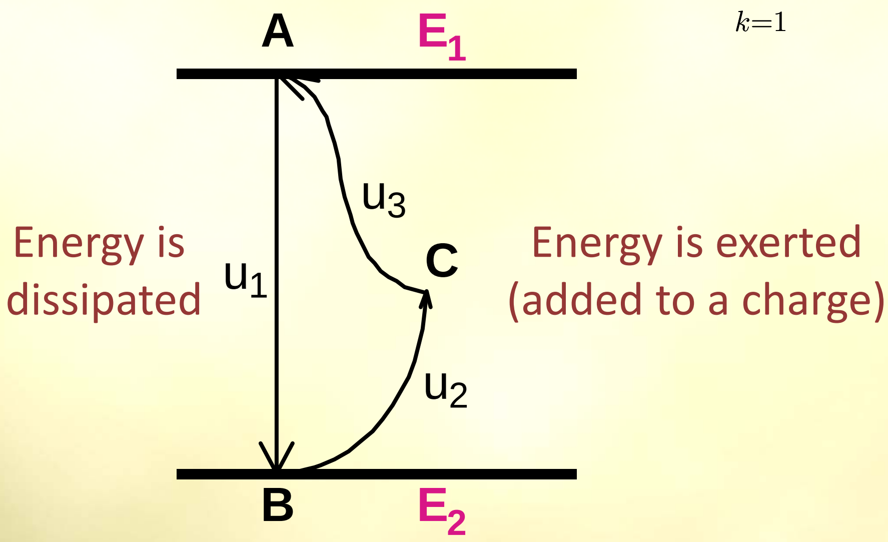
Scalar quantity, is defined as energy or work W, per unit charge – work, which is necessary to move unit charge Q along a (arbitrary) path from the point A to the point B

$$U_{AB} = \frac{W}{Q}$$

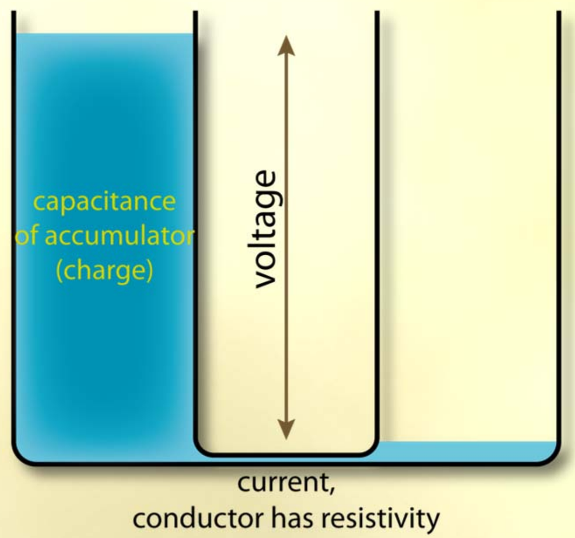
It is the difference in potential energy per charge, voltage forces an electrical charge to move from one point to another, some energy is exerted or dissipated, it is a **sake of electrical current**

Physical analogy: *from the analogy of force action we can imagine intensity of electric field like acceleration of free fall, electric potential as altitude above sea level and voltage as difference of altitudes*

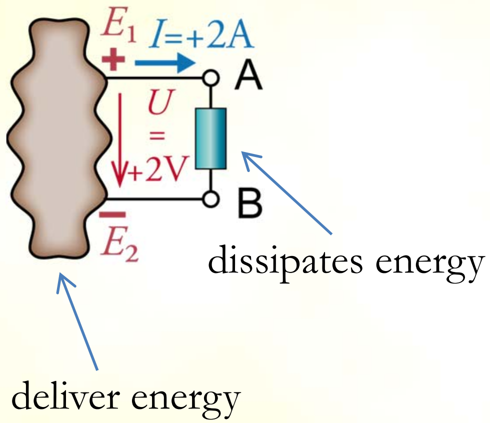
2nd Kirchhoff's law (KVL) $\sum_{k=1}^{n} u_k = 0$



Physical analogy of accumulator

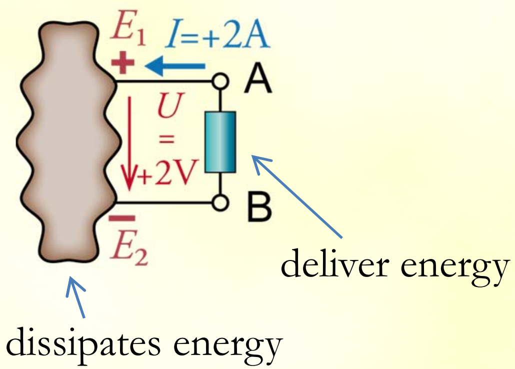


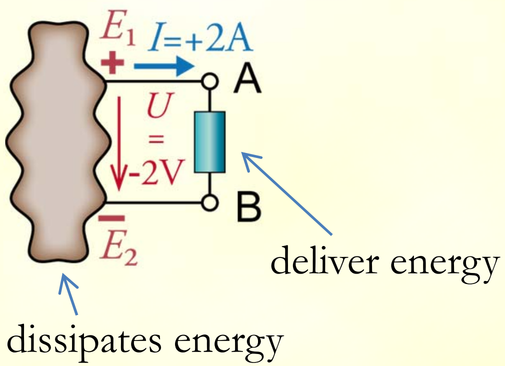
Sign convention



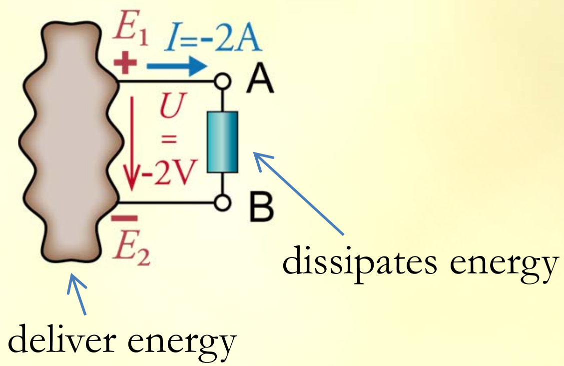
$E_1 > E_2 \quad \rightarrow \quad$ positive voltage from + to -

- („positive“) charge flows from terminal, denoted +, into the terminal –
- each second 2 C charge passes through, with respect to the voltage 2 V each Coulomb each second dissipates work 2 J
- left handed circuit **deliver** each second **work** 4 J $\Rightarrow$ it acts like a **source**
- right-handed circuit **dissipates** each second 4 J $\Rightarrow$ it acts like a power **consumer**





$E_1 < E_2$



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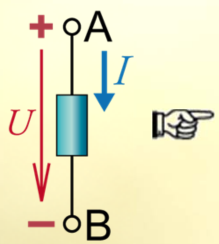
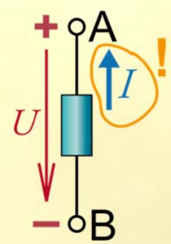


☞ **passive sign convention**



☞ **active sign convention (sources)**; widely, but not generally adopted

- **Power**

symbol: $\boldsymbol{P}$

unit: **Watt [W]**

Work 1 J done within the period of 1 s

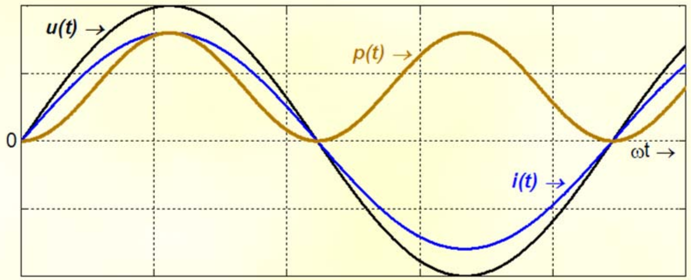
Instantaneous power

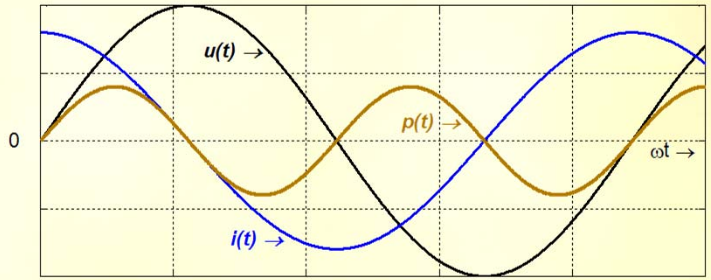
$$p(t) = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}W}{\mathrm{d}q}\frac{\mathrm{d}q}{\mathrm{d}t} = u(t)i(t)$$

If both voltage and current are constant $P = UI$

If voltage and / or current varies (periodically)

$$P = \frac{1}{T}\int_0^t p(t)\,\mathrm{d}t$$





- Instantaneous power can be both positive and negative, depending on instantaneous value of voltage and current
- If the voltage and current is AC, the sign depends on phase shift between voltage and current
- Mean value of power P represents an energy, delivered each second to the load and converted irreversibly to another form of energy

Power

- Assuming DC current and using just passive sign convention:
  - Dissipated power is positive
  - Supplied power is negative
  - The sum of dissipated and supplied powers is 0 (Tellegen's theorem)
- When in case of sources active sign convention is used:
  - In Europe both conventions are used – passive in the case of passive elements, and active in the case of sources ☹
  - Then the sum of powers, supplied by the sources, will always equal the sum of the powers, dissipated by power consumers (/*modified*/ Tellegen's theorem)
  - Negative power when we use active sign convention? The source acts like power consumer.

Lecture 1 - introduction

- Electrical device and its model. Circuit diagram, circuit elements
- Basic quantities
  - Electric charge
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Classification of time functions of circuit variables

- Circuit variables – voltage, current, power, …
- Circuit variables may vary with time; some time functions are so important, we talk of specific operating modes, its description uses different mathematical procedures

Circuit variables

Determined
Can be expressed by mathematical functions

Non-determined (stochastic)
Random processes (→ probability density function, …)

Stationary (DC)
Do not change its magnitude and direction during the time interval of observation

Periodic
from time interval of length T (period) repeat their instantaneous values, (special case: AC)

$$u(t + kT) = u(t)$$

Non-periodic

Resistive circuits
Real domain

Sin: Sinusoidal steady state
Complex numbers

Rectangular, triangular, …: Pperiodic steady state
Fourier series

Transients
Integral differential equations
Laplace transform (general)

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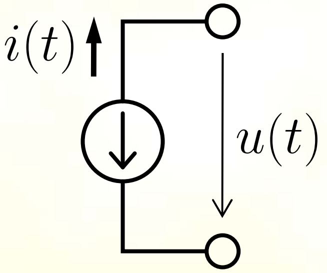
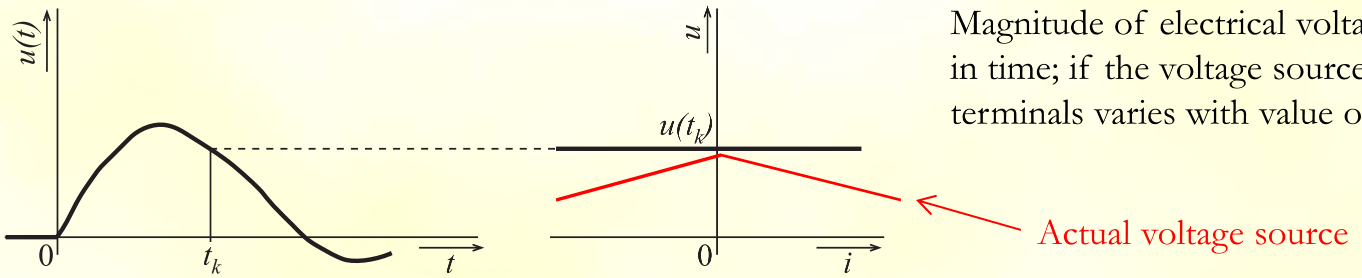
Fundamental active circuit elements

Independent voltage source

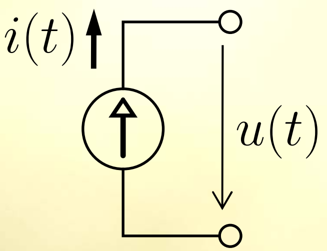


- Across its terminals maintains voltage with a prescribed waveform
- It is able to deliver (or even accept) arbitrary current, even infinite, so that it maintains constant voltage across its terminals (with any attached load)
- It is able to deliver infinite power



Magnitude of electrical voltage across terminals may vary in time; if the voltage source is not ideal, the voltage across terminals varies with value of loading resistivity

Independent current source



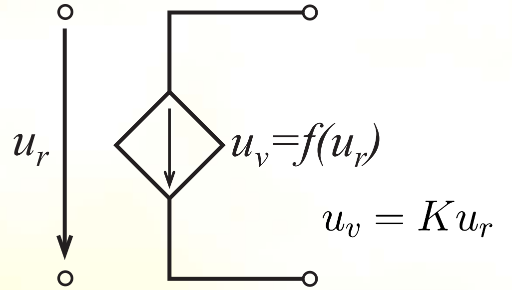
- It maintains a specified current regardless of the voltage across its terminals
- According to the connected circuit, the voltage may be even infinite
- It is able to deliver even infinite power, but, it may also be connected into a circuit such a way that it absorb power

Linear active controlled circuit elements (dependent sources)

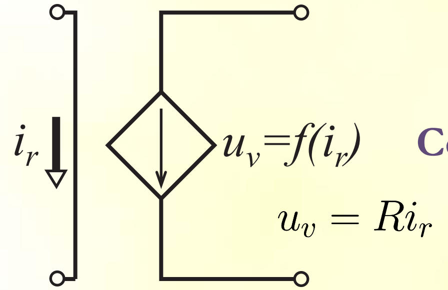
Controlling variable - voltage

Controlling variable - current

Controlled variable - voltage

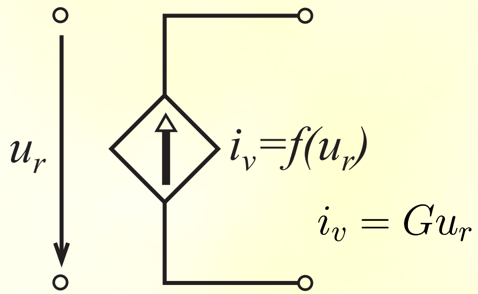


K (μ, A, …) voltage gain
dimensionless constant*

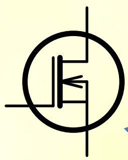


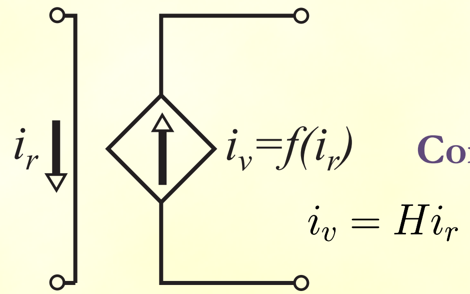
R (Z, …) transresistance
Units – ohms (resistivity)

Controlled variable - current

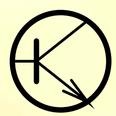


K (G, …) transkonductance
Units – Siemens (conductance)





H (α, β, …) current gain
dimensionless constant



Both metal-oxide-semiconductor field-effect transistors (MOSFETs) and bipolar transistors are modeled with dependent sources

*actually, later you will see, that it is *constant* only in particular *operating point* so generally it is variable

Example

- A car is fitted with starting accumulator battery with capacity $C = 50$ Ah, the battery voltage is $U = 12$ V
- An electric car is fitted with a set of accumulators with capacity $C = 2$ Ah, the total voltage is $U = 400$ V

? Which accumulator is able to supply more energy? *(for sake of simplification we will consider accumulator as an ideal source of constant voltage, which doesn't exhibit temperature or any other dependencies)*

$$Q_1 = C_1 \cdot t = 50 \cdot 3600 = 180\,000 \text{ C}$$

$$Q_2 = C_2 \cdot t = 2 \cdot 3600 = 7\,200 \text{ C}$$

… second accumulator stores much less charge …

$$W_1 = Q_1 \cdot U_1 = 180000 \cdot 12 = 2.16 \text{ MJ}$$

$$W_2 = Q_2 \cdot U_2 = 7200 \cdot 400 = 2.88 \text{ MJ}$$

… but due to greater voltage it delivers much more energy…

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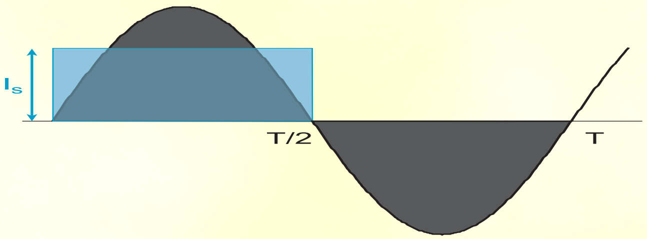
Special values

- When we describe periodical waveforms often is enough to introduce just few constants, which describes heat effects, DC component etc.
- E.g. in the case of electric plug is often enough to know it has 230 V (110 V, …) sinusoidal, 50 Hz (60 Hz)

Mean value

- It is defined by several ways
  - From electrochemical point of view it is equivalent value of DC current which transfer the same electric charge as alternating current. This value is equivalent to the height of rectangle of the same area like the area of waveform of $i(t)$ within one period T.
  - From the electrical point of view it is DC component ☺



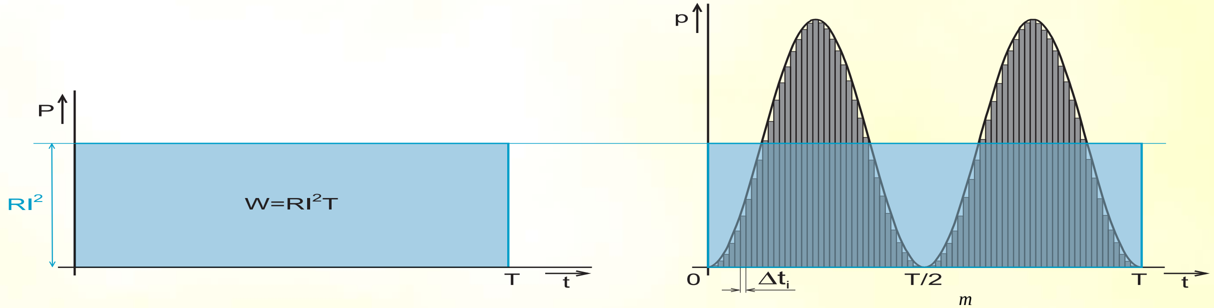
$$I_S = \frac{1}{T}\int_{t_0}^{t_0+T} i(t)\mathrm{d}t \qquad U_S = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)\mathrm{d}t$$

  - Some measuring instruments measures from its physical principle mean value of (rectified) waveform; then is necessary calculate mean value just on first half period (if the area of both is the same), or absolute value (mean rectified value)

$$I_{sar} = \frac{1}{T}\int_{t_0}^{t_0+T} |i(t)|\mathrm{d}t \qquad I_s = \frac{2}{T}\int_{0}^{\frac{T}{2}} i(t)\mathrm{d}t \qquad I_s = \frac{4}{T}\int_{0}^{\frac{T}{4}} i(t)\mathrm{d}t$$

Root mean square

- The voltage 230 V in the electric plug, it is RMS value
- **Meaning:** The value of DC current I, that converts an equivalent amount of heat in the same linear conducting medium of resistance R within a period T



$$W = \lim_{m\to\infty} \sum_{i=1}^{m} p_i \Delta t_i$$

1. The heat generated by DC current within a period T: $\quad W = RI^2T$
2. Instantaneous power of periodic current: $\quad p(t) = Ri^2(t)$
3. Total amount of heat, generated by periodic current within a period T can be expressed in the form of the "sum" of all instantaneous values in distinct time instants – that is integration (see figure) $W = \int_0^T Ri^2(t)\mathrm{d}t$
4. Comparing (1) and (3) we get

$$RI^2T = \int_0^T Ri^2(t)\mathrm{d}t \quad \Longrightarrow \quad I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\mathrm{d}t}$$

Sinusoidal waveform has RMS value $U = \dfrac{U_m}{\sqrt{2}}$ so the maximum value in electric plug is 325 V