

Fundamentals of electrical Circuits

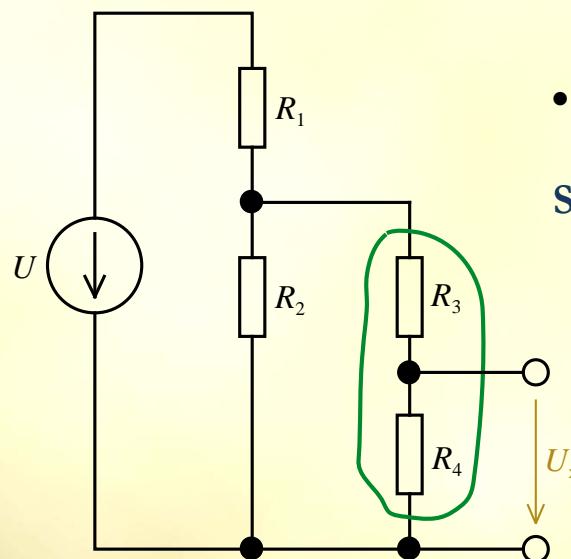
IV

Elementary analysis of linear resistive circuits

STEP BY STEP SIMPLIFICATION METHOD. SUPERPOSITION. LOAD LINE.
MAXIMUM POWER TRANSFER.

Step by step simplification method

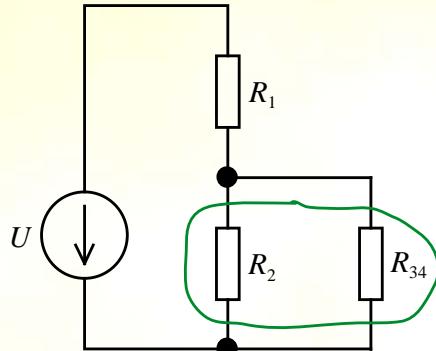
- ◆ Intuitively we used this method before
- ◆ In the circuit we replace series and parallel connected resistors by equivalent resistance (impedance)
- ◆ Start from most distant branch from source and proceed toward the source; step by step the circuit is simpler and simpler, till at last I got simple circuit in which is possible use elementary methods of analysis (Ohm's law, voltage divider, current divider)
- ◆ In simple circuit calculate voltage and / or current.
- ◆ Next, step by step return back to original circuit – voltages and / or currents are divided among distinct circuit elements, till I got original circuit.
- ◆ Example:



- Calculate voltage U_x

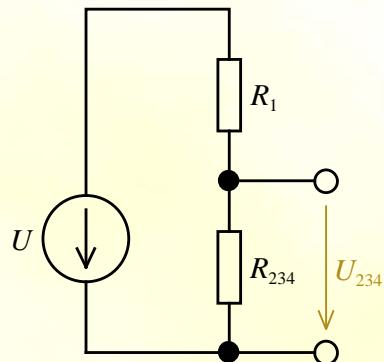
Step 1: Replace series connected resistors R_3 and R_4 with equivalent resistance

$$R_{34} = R_3 + R_4$$



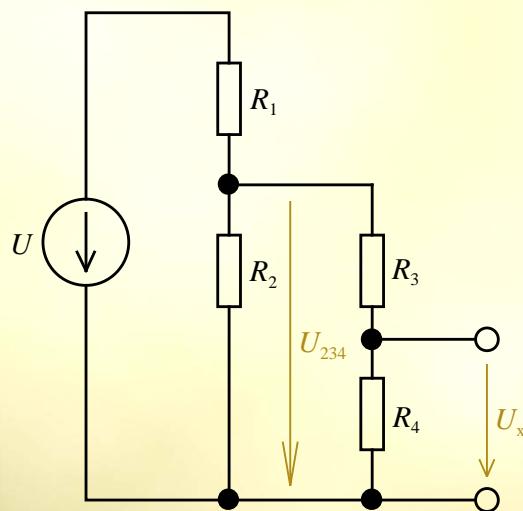
Step 2: Replace parallel connected resistors R_2 and R_{34} with equivalent resistivity

$$R_{234} = \frac{R_2 \cdot R_{34}}{R_2 + R_{34}}$$



Step 3: resistors R_1 and R_{234} are in series, acts like a voltage divider

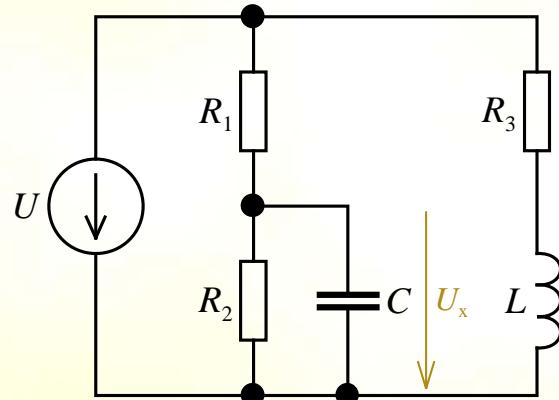
$$U_{234} = U_1 \frac{R_{234}}{R_1 + R_{234}}$$



Now we will return back to the original circuit – in this example is enough to divide voltage U_{234} across series connected resistors R_3 and R_4

$$U_x = U_{234} \frac{R_4}{R_3 + R_4}$$

Example: circuit in the figure below is in a steady state. Calculate voltage across resistor R_2 , power delivered to the circuit by voltage source and energy accumulated in the circuit.



$$R_1 = 2 \text{ k}\Omega, \quad R_2 = 4 \text{ k}\Omega, \quad R_3 = 3 \text{ k}\Omega$$

$$L = 0.2 \text{ H}, \quad C = 10 \mu\text{F}, \quad U = 12 \text{ V}$$

- Voltage across resistor R_2 :

Steady state: both capacitor and inductor are fully charged, since the voltage source is DC, all circuit variables are constant – capacitor acts like open circuit, inductor like short circuit.

$$U_x = U \frac{R_2}{R_1 + R_2} = 12 \cdot \frac{4000}{2000 + 4000} = 8 \text{ V}$$

- Power, delivered by source:

Capacitor and inductor are fully charged – they don't absorb any energy; delivered power is limited just on the heat, dissipated on resistors.

$$I_{12} = \frac{U}{R_1 + R_2} = \frac{12}{2000 + 4000} = 2 \text{ mA}$$

$$I_3 = \frac{U}{R_3} = \frac{12}{3000} = 4 \text{ mA}$$

$$P = R_1 I_{12}^2 + R_2 I_{12}^2 + R_3 I_3^2 = 6000 \cdot 0.002^2 + 3000 \cdot 0.004^2 = 72 \text{ mW}$$

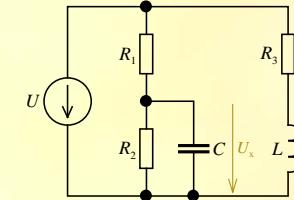
- Energy stored in the circuit:

Capacitor is parallel connected to the resistor R_2 , the voltage is 8 V.

$$W_C = \frac{1}{2} C U_x^2 = \frac{1}{2} \cdot 10^{-5} \cdot 64 = 0.32 \text{ mJ}$$

Inductor is in series with R_3 , so the current is the same, 4 mA

$$W_L = \frac{1}{2} L I_3^2 = \frac{1}{2} \cdot 0.2 \cdot 0.004^2 = 1.6 \mu\text{J}$$



$$L = 0.2 \text{ H}, \quad C = 10 \mu\text{F}, \quad U = 12 \text{ V}$$

$$R_1 = 2 \text{ k}\Omega, \quad R_2 = 4 \text{ k}\Omega, \quad R_3 = 3 \text{ k}\Omega$$

Example: consider 400 kV high-voltage dc transmission facility with parameters: $R = 23.1 \text{ m}\Omega/\text{km}$, $L = 0.858 \text{ mH/km}$, $C = 13.3 \text{ nF/km}$. Equivalent model of load is represented by a resistor of value 183.5Ω . The length of the line is 600 km, input power is 800 MW. Calculate power loss in transmission line and stored energy.

Total resistivity, inductance and capacitance:

$$R_l = R \cdot l = 0.0231 \cdot 600 = 13.86 \Omega$$

$$L_l = L \cdot l = 0.000858 \cdot 600 = 0.5148 \text{ H}$$

$$C_l = C \cdot l = 13.3 \cdot 10^{-9} \cdot 600 = 7.98 \mu\text{F}$$

Power loss in the line: $P = R_l \cdot I^2 = R_l \cdot \left(\frac{P_z}{U} \right)^2 = 13.86 \cdot \left(\frac{8 \cdot 10^8}{4 \cdot 10^5} \right)^2 = 55.44 \text{ MW}$

The load voltage is:

$$U_2 = U \cdot \frac{R_z}{R_l + R_z} = 40000 \cdot \frac{183.5}{13.86 + 183.5} = 371909 \text{ V}$$

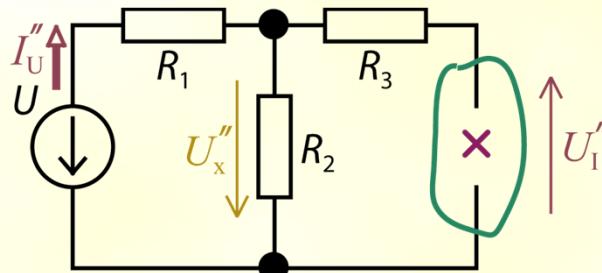
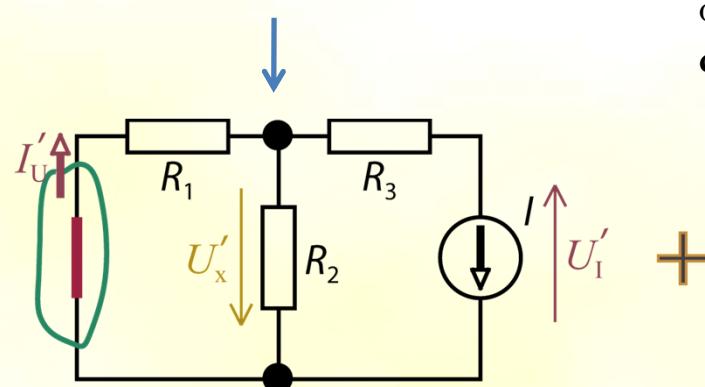
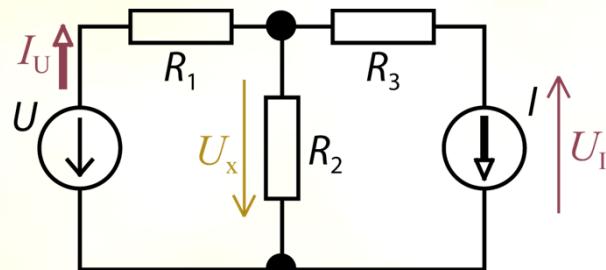
The energy stored in the transmission line:

$$P_s = \frac{1}{2} L_l I^2 + \frac{1}{2} C_l U^2 = \frac{1}{2} 0.5148 \cdot 2000^2 + \frac{1}{2} 7.98 \cdot 10^{-6} \cdot 400000^2 = 1.668 \text{ MJ}$$

Note: the calculation is just approximation of this issue (it is transmission line with losses...)

Consider an 200 kV transmission facility – the current would be 2x higher, the power loss 4x higher- about 220 MW...

Superposition



- If a **linear** circuit has two or more independent sources, one way how to determine the value of a specific variable (voltage or current) is to determine the contribution of each independent source to that variable separately and then add them up.
- If the circuit has N independent sources, the analysis is separated on N independent steps – in each step we **remove from the circuit $N - 1$ independent sources**

- In the example above we have 2 independent sources – analysis is divided on 2 steps – in the first one we remove the voltage source, in the second one the current source

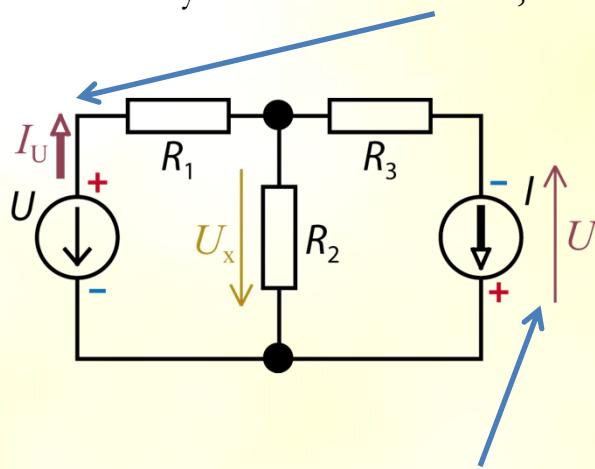
- The voltage across resistor R_2 : $U'_x = -I \cdot \frac{R_1 R_2}{R_1 + R_2}$, $U''_x = U \cdot \frac{R_2}{R_1 + R_2}$, $U_x = U'_x + U''_x$

- The power, dissipated on R_2 : the superposition principle is not generally valid for powers, it is not linear hence

$$P \neq P' + P''$$

But still, it is valid expression $P_{R_2} = R_2 \cdot I_{R_2}^2 = \frac{U_x^2}{R_2}$

- To calculate powers, delivered to the circuit by distinct independent sources, is again possible to use superposition – using superposition principle, in each step, distinct source **delivers power** to the circuit, but, it also may **absorb power** from other sources
- Power, delivered by distinct independent source is determined from general relation $P = UI$
- In the case of voltage source it is necessary to evaluate current, leaving its positive terminal

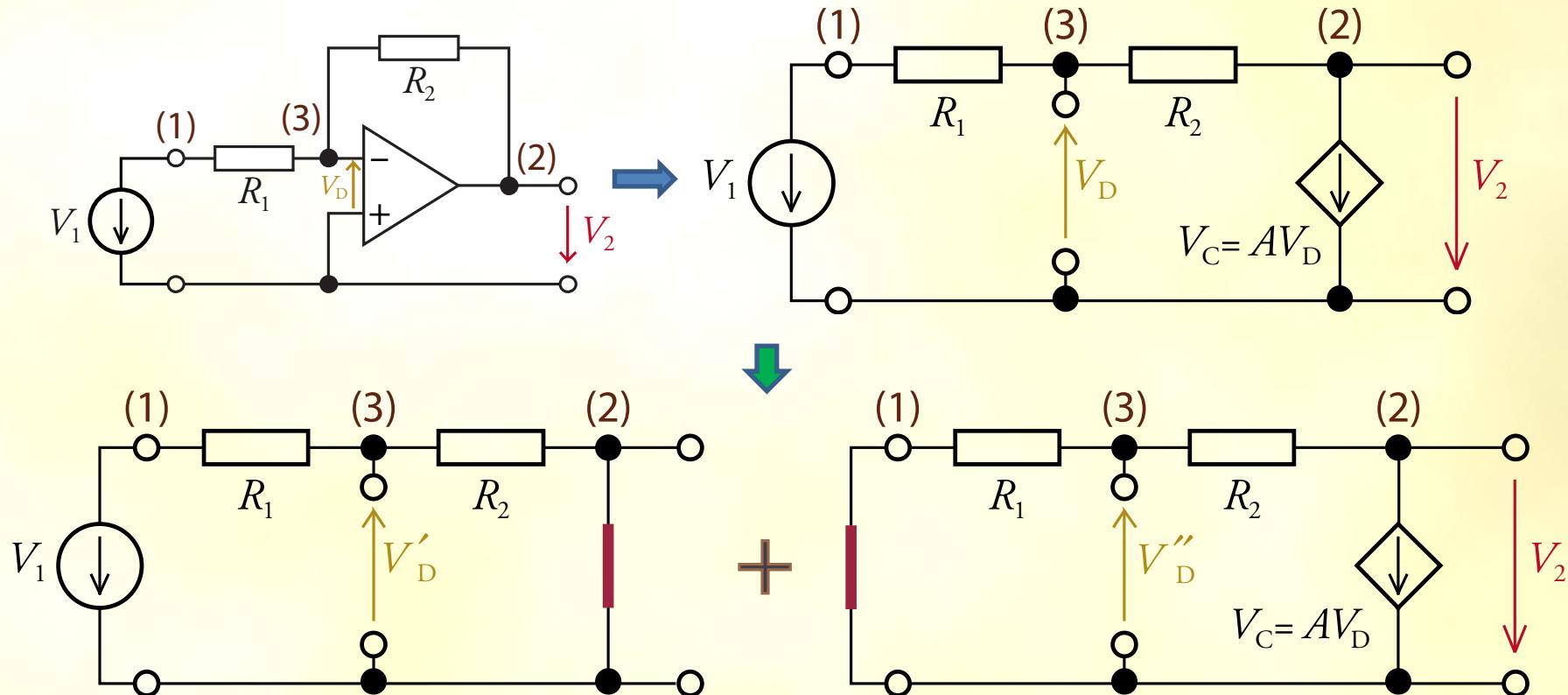


- In the case of current source it is necessary to evaluate voltage, direction of its polarity is given as follows: voltage across current source has positive value on terminal, from which the current is leaving the source, negative polarity has terminal, into which the current flows

$$I'_U = I \cdot \frac{R_2}{R_1 + R_2}, \quad I''_U = \frac{U}{R_1 + R_2}, \quad I_U = I'_U + I''_U, \quad P_U = U \cdot I_U$$

$$U'_I = -U \cdot \frac{R_2}{R_1 + R_2}, \quad U''_I = I \cdot \left(\frac{R_1 R_2}{R_1 + R_2} + R_3 \right), \quad U_I = U'_I + U''_I, \quad P_I = I \cdot U_I$$

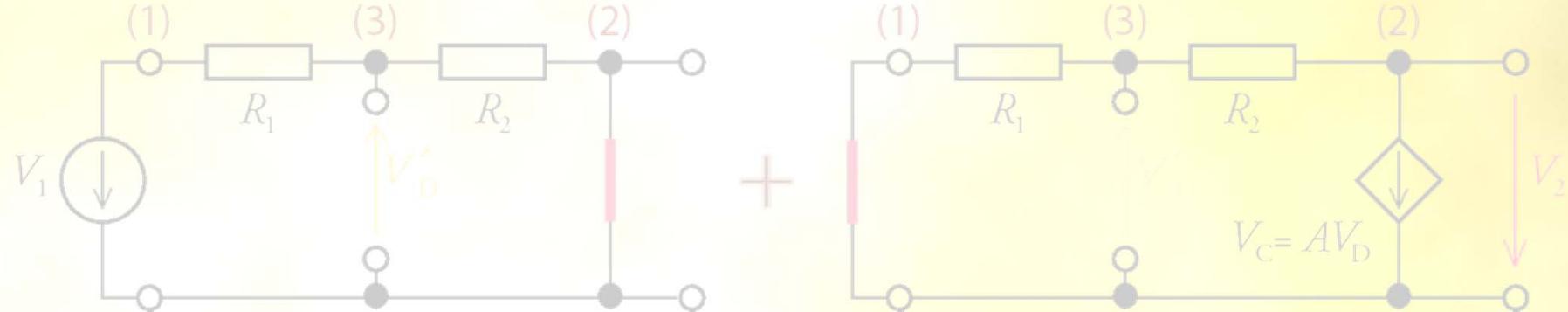
- Superposition in circuits with dependent sources is also possible (even sometimes more complicated and other methods like circuit equations are then recommended)
 - Example: in the circuit according to the figure below we should determine output voltage V_2



- We remove controlled source V_C :

The voltage in node (3) can be determined from voltage divider rule:

$$V'_D = -V_1 \frac{R_2}{R_1 + R_2}$$



2. We remove independent source V_1 :

The voltage in node (3) can be determined from voltage divider rule (V_2 is unknown):

$$V_D'' = -V_2 \frac{R_1}{R_1 + R_2} = -AV_D \frac{R_1}{R_1 + R_2}$$

3. Combine both results together

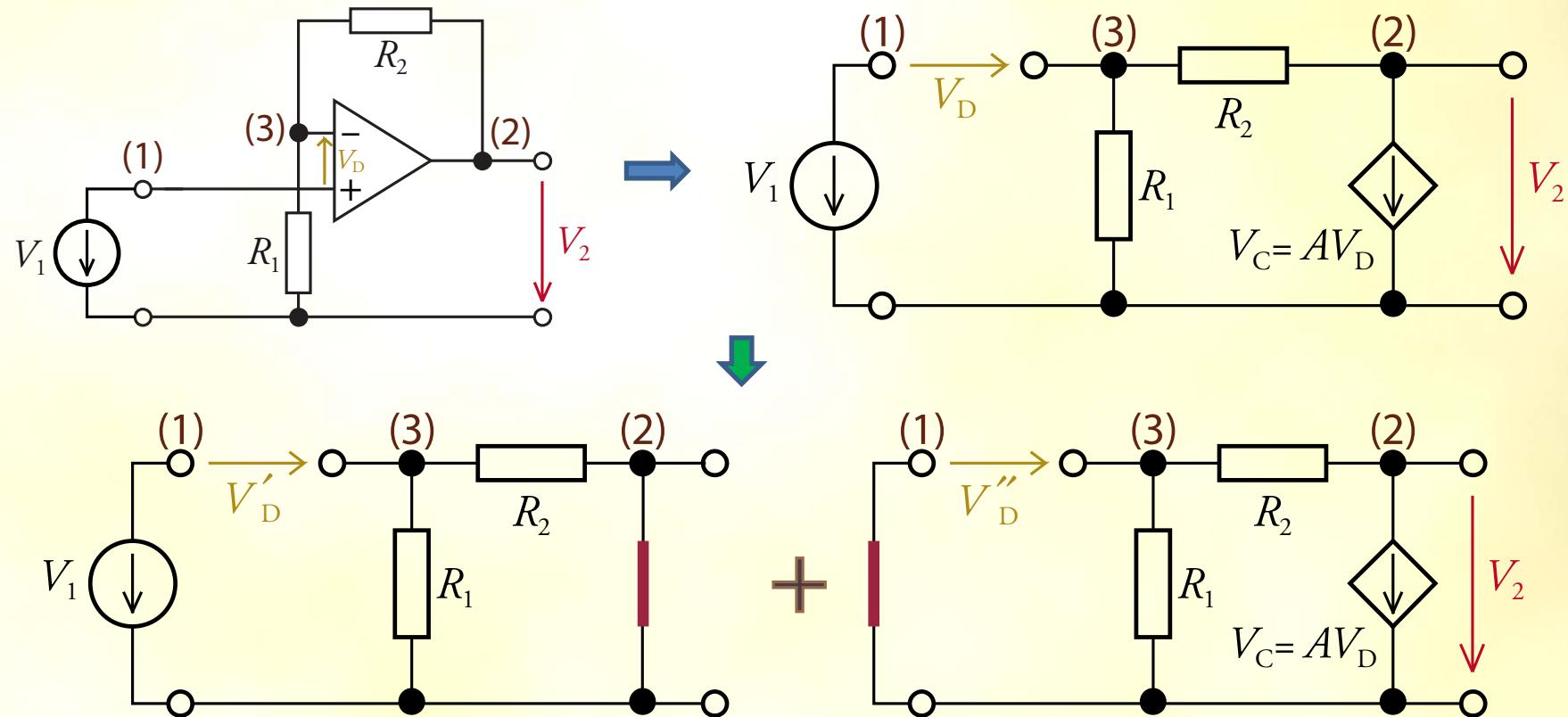
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$$V_D = V_D' + V_D'' = -V_1 \frac{R_2}{R_1 + R_2} - V_2 \frac{R_1}{R_1 + R_2} = -V_1 \frac{R_2}{R_1 + R_2} - AV_D \frac{R_1}{R_1 + R_2}$$

$$V_D \left(1 + \frac{AR_1}{R_1 + R_2} \right) = -V_1 \frac{R_2}{R_1 + R_2}$$

$$V_D = -V_1 \frac{R_2}{R_2 + R_1(1 + A)} \quad \longrightarrow \quad V_2 = -V_1 \frac{AR_2}{R_2 + R_1(1 + A)}$$

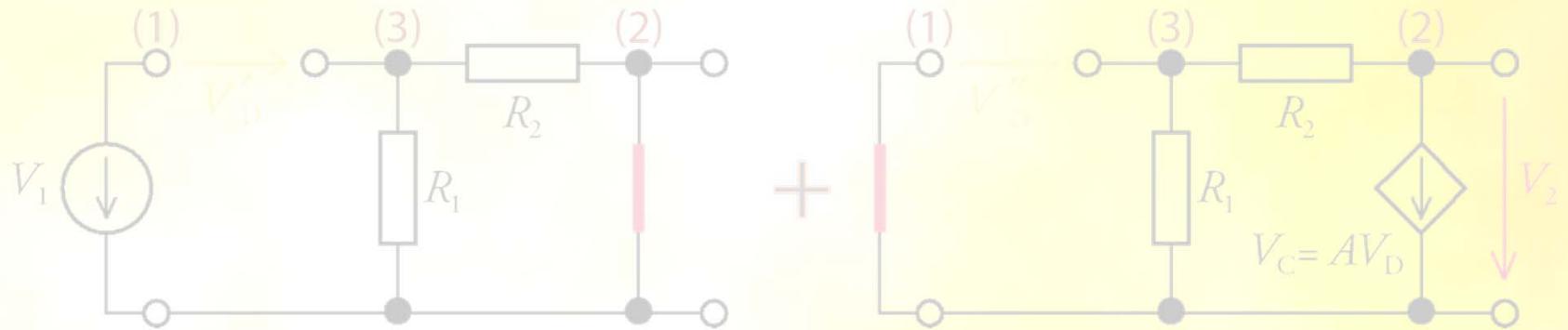
- **Example:** in the circuit according to the figure below we should determine output voltage V_2



1. We remove controlled source V_C :

There is no current in the right-handed part of the circuits, so the voltage in node (3) is the same as the source voltage:

$$V'_D = V_1$$



2. We remove independent source V_1 :

The voltage in node (3) can be determined from voltage divider rule (V_2 is unknown and node (1) is connected to ground):

$$V_D'' = -V_2 \frac{R_1}{R_1 + R_2} = -AV_D \frac{R_1}{R_1 + R_2}$$

!

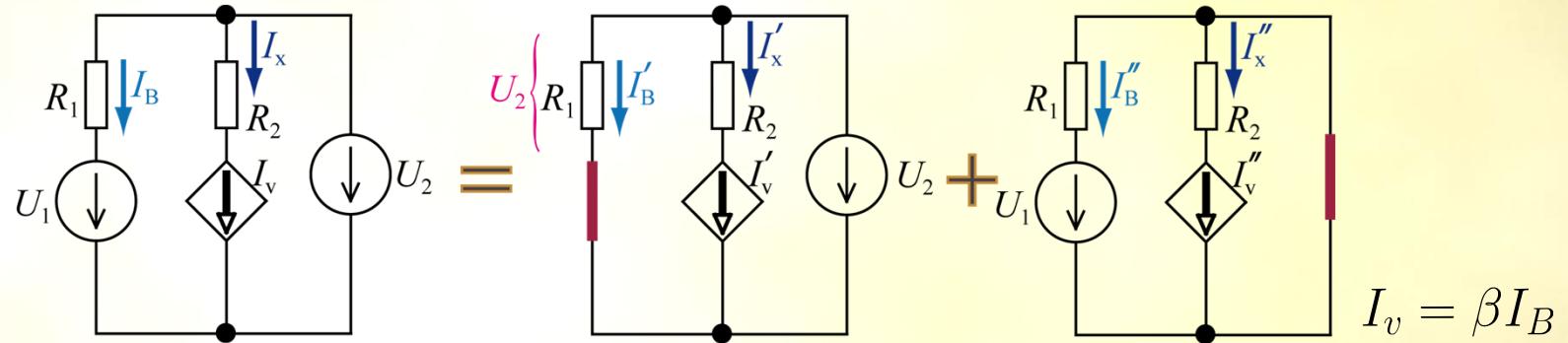
3. Combine both results together

$$V_D = V_D' + V_D'' = V_1 - V_2 \frac{R_1}{R_1 + R_2} = V_1 - AV_D \frac{R_1}{R_1 + R_2}$$

$$V_D \left(1 + \frac{AR_1}{R_1 + R_2} \right) = V_1$$

$$V_D = V_1 \frac{R_1 + R_2}{R_2 + R_1(1 + A)} \quad \longrightarrow \quad V_2 = V_1 \frac{A(R_1 + R_2)}{R_2 + R_1(1 + A)}$$

- **Example:** in the circuit according to the figure below we should determine current I_x



1. We remove voltage source U_1 :

Across resistor R_1 is the voltage U_2 , so the value of current I_B is given by Ohm's law:

$$I'_B = \frac{U_2}{R_1} \quad \rightarrow \quad I'_x = I'_v = \beta I'_B = \frac{\beta U_2}{R_1}$$

2. We will remove voltage source U_2 :

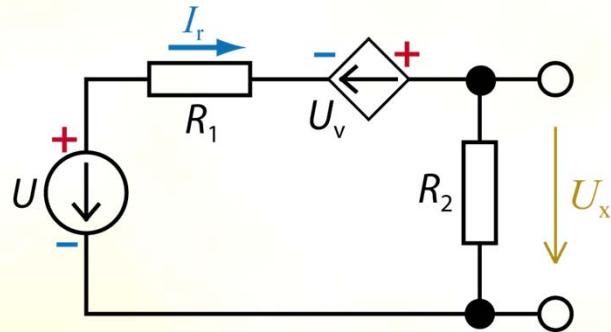
Across resistor R_1 is voltage U_1 , so the value of current I_B can be evaluated using Ohm's law again (keep in mind voltage source polarity!):

$$I''_B = \frac{-U_1}{R_1} \quad \rightarrow \quad I''_x = I''_v = \beta I''_B = \frac{-\beta U_1}{R_1}$$



$$I_B = I'_B + I''_B = \frac{\beta U_2}{R_1} - \frac{\beta U_1}{R_1}$$

- **Example:** in the circuit according to the figure below we should determine voltage U_x
- We can use also Kirchhoff's laws (KVL in this case)



$$U_v = RI_r$$

$$R_1 = 1 \text{ k}\Omega, \quad R_2 = 2 \text{ k}\Omega, \quad R = 1500, \quad U = 12 \text{ V}$$

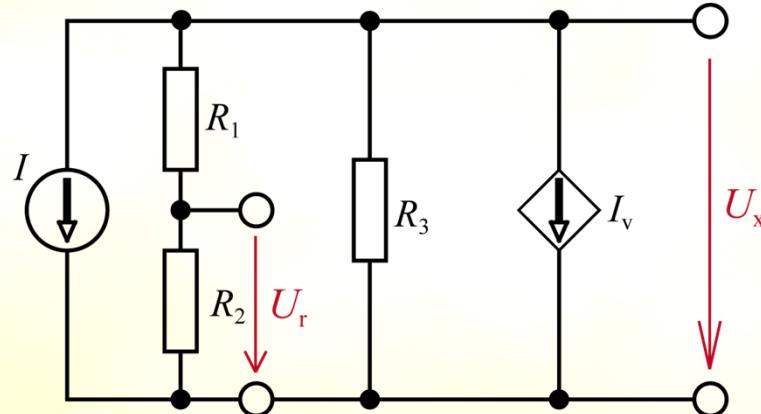
1. Using KVL:

$$-U + R_1 I_r - U_v + R_2 I_r = 0 \quad \rightarrow \quad -U + R_1 I_r - RI_r + R_2 I_r = 0 \quad \rightarrow \quad I_r(R_1 + R_2 - R) = U$$

$$I_r = \frac{U}{R_1 + R_2 - R} = \frac{12}{1000 + 2000 - 1500} = 8 \text{ mA}$$

$$U_x = R_2 I_x = 2000 \cdot 0.008 = 16 \text{ V}$$

- **Example:** in the circuit according to the figure below we should determine voltage U_x
- We can use also Kirchhoff's laws (KCL in this case)



$$I_v = KU_r$$

$$R_1 = 2 \text{ k}\Omega, \quad R_2 = 4 \text{ k}\Omega, \quad R_3 = 3 \text{ k}\Omega$$

$$K = 0.0045, \quad I = 20 \text{ mA}$$

1. Using KCL:

$$I + \frac{U_x}{R_1 + R_2} + \frac{U_x}{R_3} + KU_r = 0 \quad \rightarrow \quad I + \frac{U_x}{R_1 + R_2} + \frac{U_x}{R_3} + KR_2 \frac{U_x}{R_1 + R_2} = 0$$

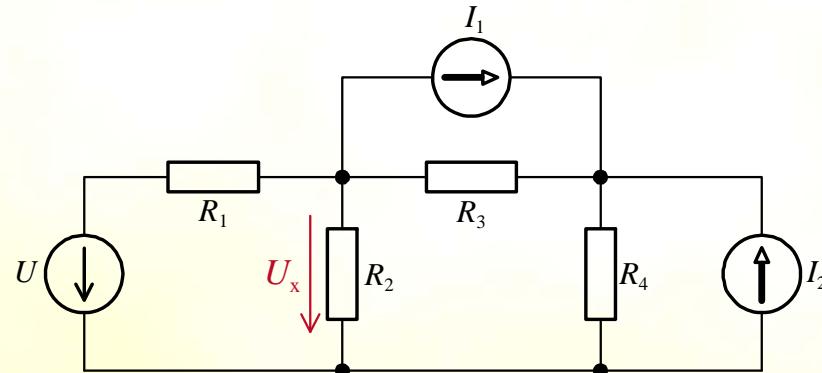
$$U_x \left(\frac{1 + KR_2}{R_1 + R_2} + \frac{1}{R_3} \right) = -I \quad \rightarrow \quad U_x = -0.02 \cdot \left(\frac{1 + 0.0045 \cdot 2000}{2000 + 4000} + \frac{1}{3000} \right)^{-1}$$

$$U_x = -0.02 \cdot \frac{6000}{12} = -10 \text{ V}$$

Application of equivalence of sources

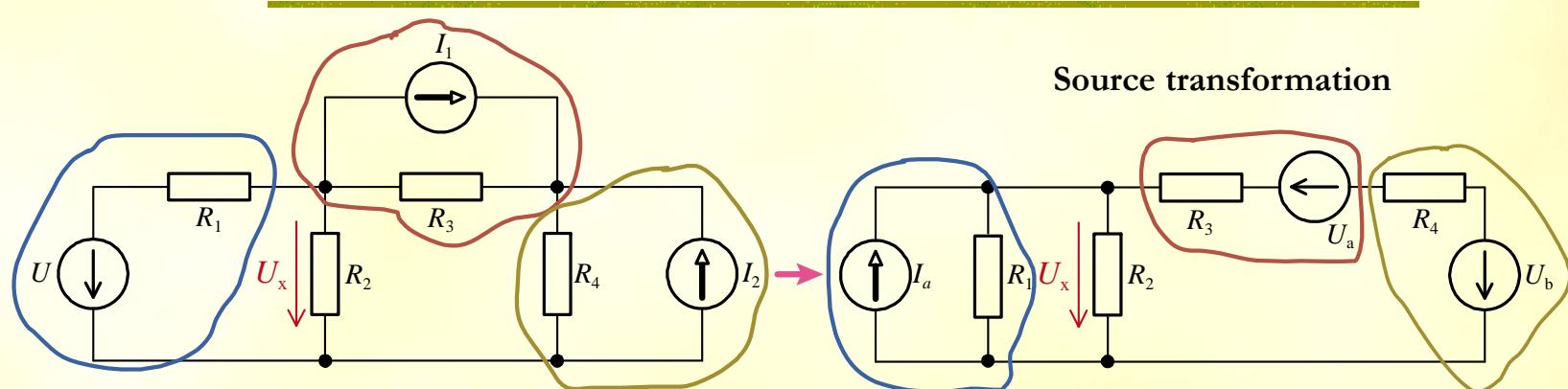
Example: In the circuit in the figure below determine voltage U_x

From the methods of analysis we studied till now we may choose superposition, or source transformation – still, step-by-step simplification will be hidden in all cases...



$$R_1 = 8 \Omega, R_2 = 8 \Omega, R_3 = 10 \Omega, R_4 = 10 \Omega$$

$$U = 40 \text{ V}, I_1 = 3 \text{ A}, I_2 = 2 \text{ A}$$



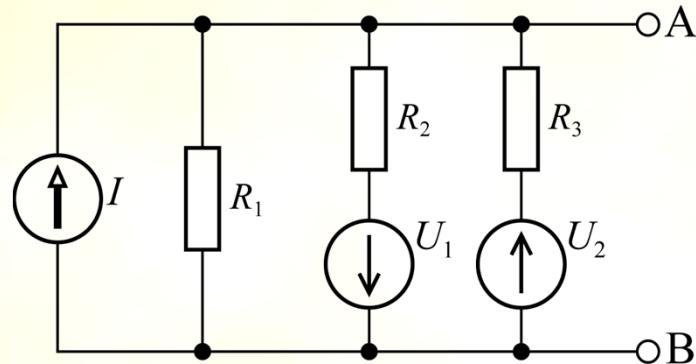
Source transformation

$$I_a = \frac{U}{R_1} = \frac{40}{8} = 5 \text{ A} \quad U_a = I_1 R_3 = 3 \cdot 10 = 30 \text{ V} \quad U_b = I_2 R_4 = 2 \cdot 10 = 20 \text{ V}$$

$$I_b = \frac{U_b - U_a}{R_3 + R_4} = \frac{20 - 30}{20} = -0.5 \text{ A} \quad I = I_a + I_b = 4.5 \text{ A} \quad R = R_1 \parallel R_2 \parallel (R_3 + R_4) = 3.33 \Omega$$

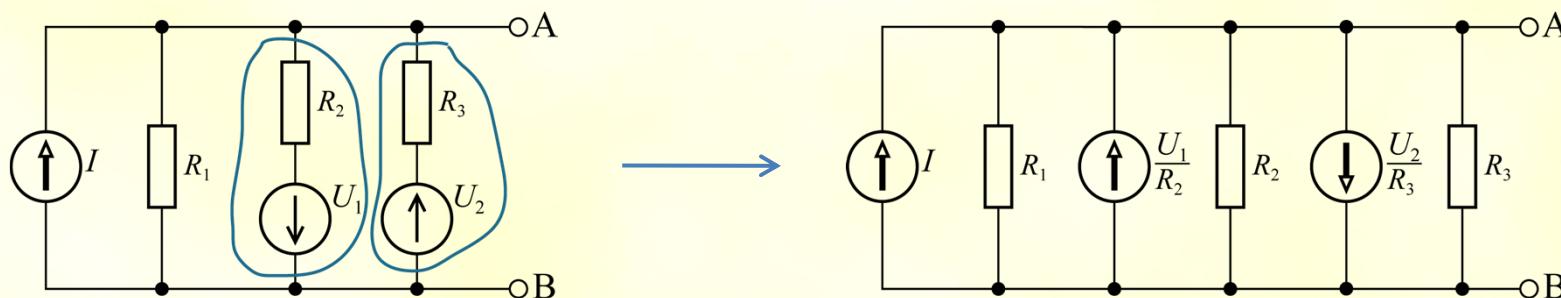
$$U_x = R \cdot I = 3.33 \cdot 4.5 = \underline{\underline{15 \text{ V}}}$$

Example: find the Thévenin equivalent circuit of the circuit in the figure at terminals A, B



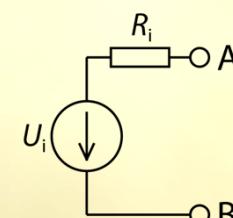
$$\begin{array}{ll} I = 2 \text{ A} & R_1 = 20 \Omega \\ U_1 = 24 \text{ V} & R_2 = 4 \Omega \\ U_2 = 20 \text{ V} & R_3 = 5 \Omega \end{array}$$

Solution: Preferable to other methods we can use source transformation method

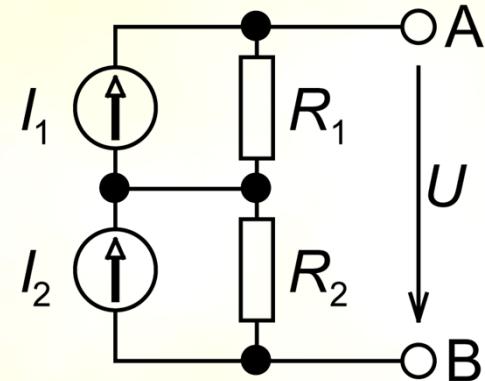


$$I_N = I + \frac{U_1}{R_2} - \frac{U_2}{R_3} = 2 + \frac{24}{4} - \frac{20}{5} = 2 + 6 - 4 = 4 \text{ A}$$

$$R_i = \left(R_1^{-1} + R_2^{-1} + R_3^{-1} \right)^{-1} = \left(\frac{1}{20} + \frac{1}{4} + \frac{1}{5} \right)^{-1} = \underline{\underline{2 \Omega}} \quad U_i = I_N \cdot R_i = 4 \cdot 2 = \underline{\underline{8 \text{ V}}}$$



Example: find the Thévenin equivalent circuit of the circuit in the figure at terminals A, B



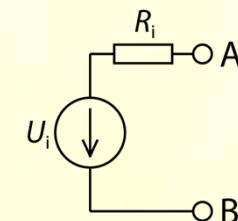
$$\begin{array}{ll} I_1 = 2 \text{ A} & R_1 = 30 \Omega \\ I_2 = 3 \text{ A} & R_2 = 20 \Omega \end{array}$$

Solution: again, transformation of sources is optimal method of solution

☞ current sources transform to voltage sources

$$U_i = I_1 R_1 + I_2 R_2 = 2 \cdot 30 + 3 \cdot 20 = \underline{\underline{120 \text{ V}}}$$

$$R_i = R_1 + R_2 = \underline{\underline{50 \Omega}}$$



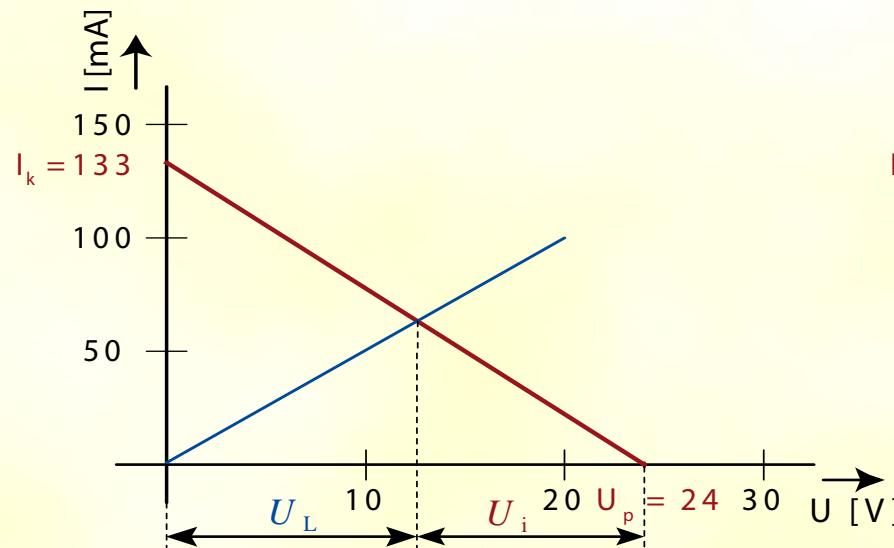
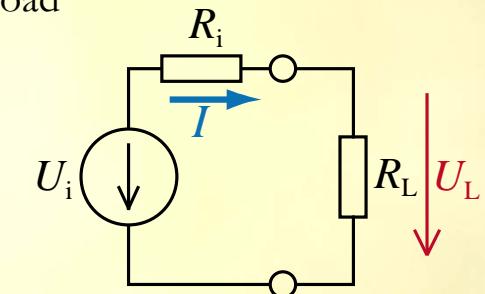
Load line

Once again we return back to the Thévenin's equivalent circuit, with connected load resistor R_L :

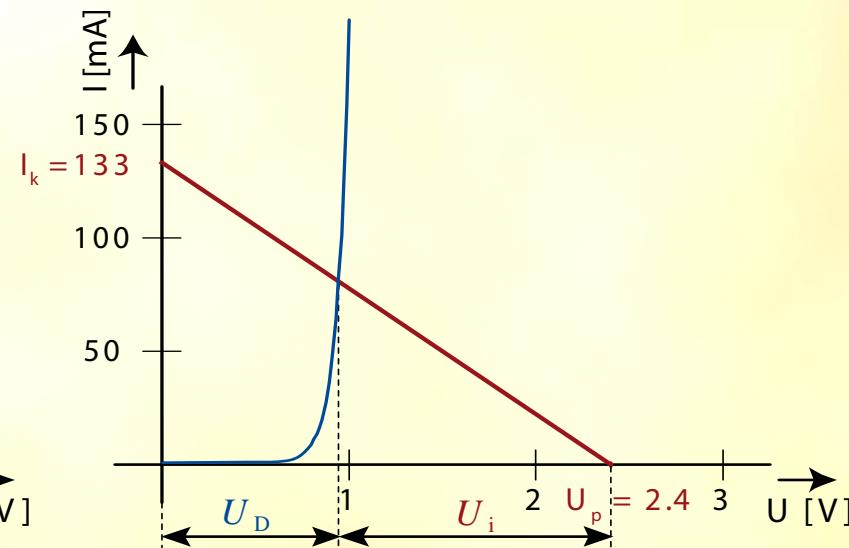
The voltage across the load is less than voltage of the source by voltage drop due to passing current:

$$U_L = U_i - R_i I$$

This equation can be sketched:

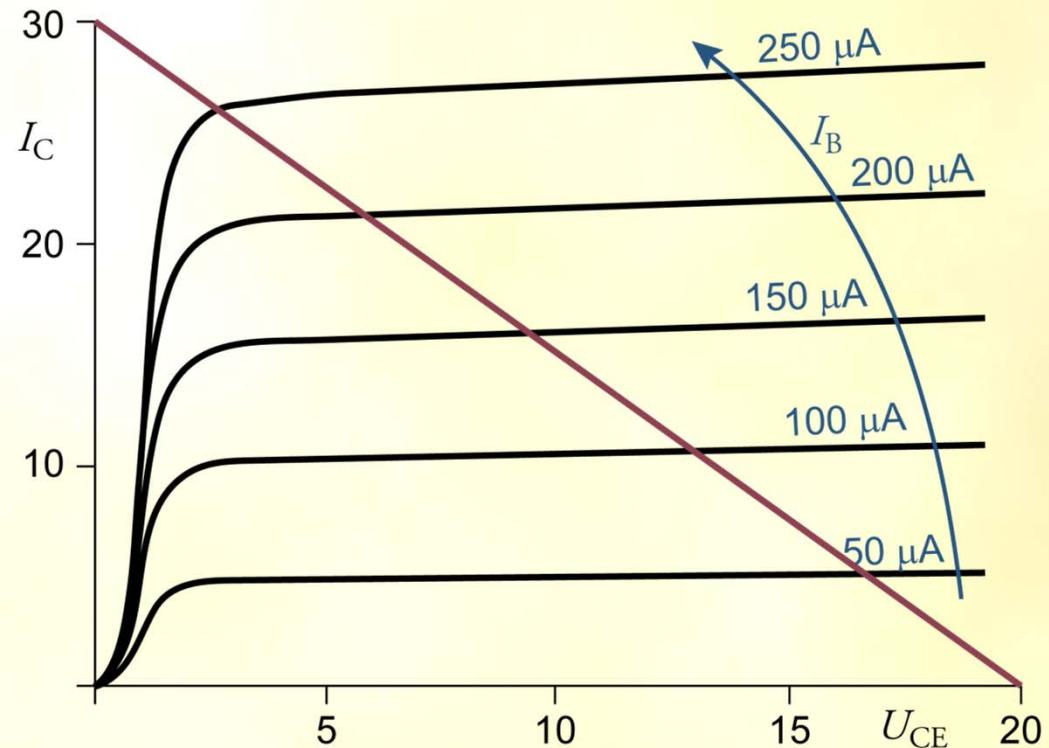
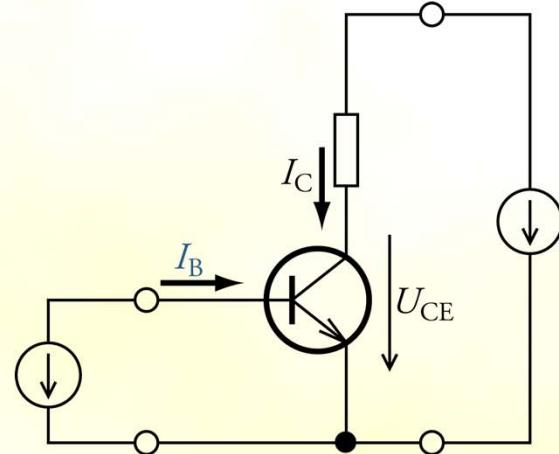


I-V characteristic used to evaluation of voltage across load resistor



I-V characteristic used to evaluation of voltage across one non-linear load

Another example of usage of a load line – transistor parametric set of V-A characteristics

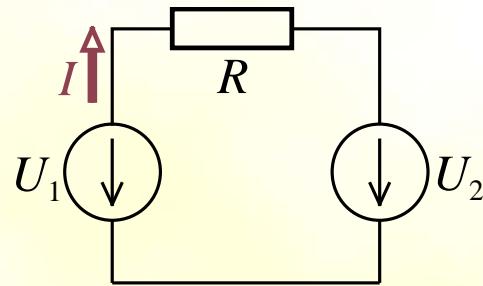


Tellegen's theorem

Law of conservation of energy must be obeyed in any electric circuit.

According to the Tellegen's theorem (one of its reading) the sum of power delivered by (absorbed by) sources must be identical to power absorbed by passive circuit elements.

Example 1: given two voltage sources, $U_1 = 15 \text{ V}$, $U_2 = 5 \text{ V}$ and $R = 10 \Omega$



$$P_{U_1} = U_1 \cdot I = U_1 \cdot \frac{U_1 - U_2}{R} = 15 \cdot \frac{15 - 5}{10} = 15 \text{ W}$$

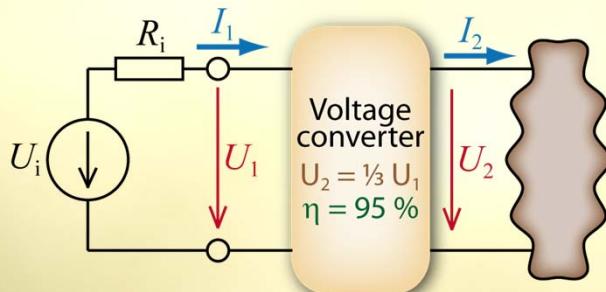
$$P_{U_2} = -U_2 \cdot I = -U_2 \cdot \frac{U_1 - U_2}{R} = 5 \cdot \frac{5 - 15}{10} = -5 \text{ W}$$

$$P_R = R \cdot I^2 = R \cdot \left(\frac{U_1 - U_2}{R} \right)^2 = 10 \cdot \left(\frac{15 - 5}{10} \right)^2 = 10 \text{ W}$$

Total power, delivered by both sources is 10 W, the same, like power absorbed by resistor.

Note: In many English books in the case of sources opposite sign convention is used – supplied power has negative sign, absorbed power has positive sign (conforming to a power absorbed by resistors) – then the Tellegen's theorem is very simple: total sum of all powers in the circuit is 0...

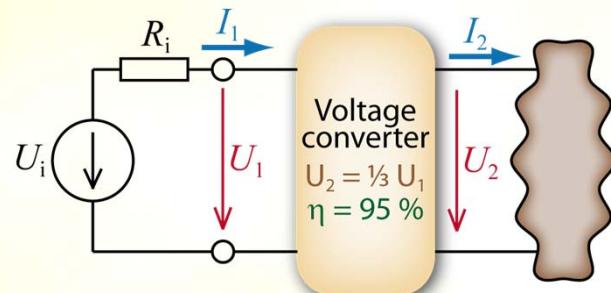
Example 2: step-down DC to DC voltage converter has efficiency 95% and decreases input voltage 3×. Voltage source has 16 V, its internal resistivity is 6Ω . The load draw current 2 A. Determine load voltage and currents in the circuit.



$$P_L = (P_Z - P_i) \cdot \eta \quad \rightarrow \quad U_2 I_2 = U_1 I_1 \eta, \quad \eta = \frac{P_2}{P_1}$$

$$U_2 I_2 = 3U_2 I_1 \cdot \eta \quad \rightarrow \quad I_1 = \frac{I_2}{3\eta} = \frac{2}{3 \cdot 0.95} = 0.701 \text{ A}$$

Example 2: step-down DC to DC voltage converter has efficiency 95% and decreases input voltage 3×. Voltage source has 16 V, its internal resistivity is 6 Ω. The load draw current 2 A. Determine load voltage and currents in the circuit.



$$P_L = (P_Z - P_i) \cdot \eta \quad \rightarrow \quad U_2 I_2 = U_1 I_1 \eta, \quad \eta = \frac{P_2}{P_1}$$

$$U_2 I_2 = \underbrace{3U_2}_{U_1} I_1 \cdot \eta \quad \rightarrow \quad I_1 = \frac{I_2}{3\eta} = \frac{2}{3 \cdot 0.95} = 0.701 \text{ A}$$

$$U_{R_i} = R_i I_1 = 6 \cdot 0.701 = 4.21 \text{ V} \quad \rightarrow \quad U_1 = U_i - U_{R_i} = U_i - R_i I_1 = 16 - 4.2 = 11.79 \text{ V}$$

$$U_2 = \frac{1}{3} U_1 = 3.93 \text{ V}$$

Power supplied by voltage source: $P_Z = U_i I_1 = 16 \cdot 0.701 = 11.23 \text{ W}$

Power loss at internal resistor: $P_i = R_i I_1^2 = 6 \cdot 0.701^2 = 2.955 \text{ W}$

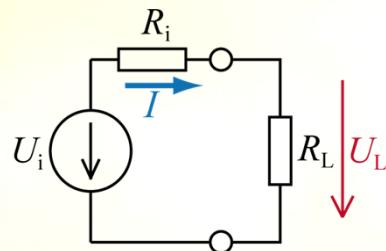
Power delivered to voltage converter: $P_1 = P_Z - P_i = U_1 I_1 = 8.273 \text{ W}$

Power delivered to the load: $P_2 = P_1 \cdot \eta = U_2 I_2 = 3.93 \cdot 2 = 7.86 \text{ W}$

Power loss in voltage converter: $P_m = P_2 - P_1 = 0.42 \text{ W}$

Of course, law of conservation of energy must be obeyed, so: $P_Z = P_i + P_1$
 $P_1 = P_m + P_2$

Note – importance of an adequate power rating



$$U_i = 18 \text{ V}, R_i = 90 \Omega, R_L = 10 \Omega$$



- when an electric charge passes through the resistance, it loses part of its energy, converted to **heat**
- in a long term operation heat balance between delivered heat and dissipated heat must be achieved
⇒ every resistor intensively warms up so it must have adequate power rating (or even use cooling system)

$$I = \frac{U_i}{R_1 + R_2} = \frac{18}{90 + 10} = 0.18 \text{ A}$$

$$P_i = R_i I^2 = 90 \cdot 0.18^2 = 2.916 \text{ W}$$



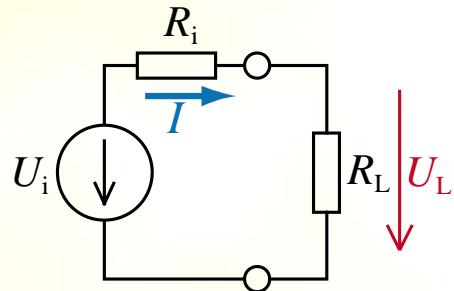
$$P_L = R_L I^2 = 10 \cdot 0.18^2 = 0.324 \text{ W}$$



- ☞ To dissipate delivered heat without damage of the resistor power rating must be at least 3 W
- ☞ When a resistor is used in a circuit, its **power rating must be greater** than the maximum delivered power, so we use at least **the next higher standard value**, in this case 5 W ceramic resistor

To dissipate the energy properly we can use carbon resistor with power rating 0.33 W, but rather we use metallized resistor of higher standard power rating 0.6 W

Maximum power transfer



In some cases it is necessary to evaluate conditions when maximum possible power is delivered to a load. In following section we will investigate two cases – The first, when the value of R_i can be regulated (at least in some degree), the second, when the value of R_L will be regulated.

1. The value of R_i , or both resistors will be changed. Consider the case, when $R_i + R_L = \text{constant}$

$U_i = 100 \text{ V}$	$R_i [\Omega]$	$R_L [\Omega]$	$P_{Ri} [\text{W}]$	$P_L [\text{W}]$
	0	100	0	100
	25	75	25	75
	50	50	50	50
	75	25	75	25
	100	0	100	0

It is evident, the maximum power to the load is delivered in the case of zero internal resistivity of voltage source

It is expected result, since power loss on internal resistor drops to 0

It is the case of power distribution network – every producer of electric energy aims to decrease power losses, and make effort to utmost decrease resistivity of wires in distribution network

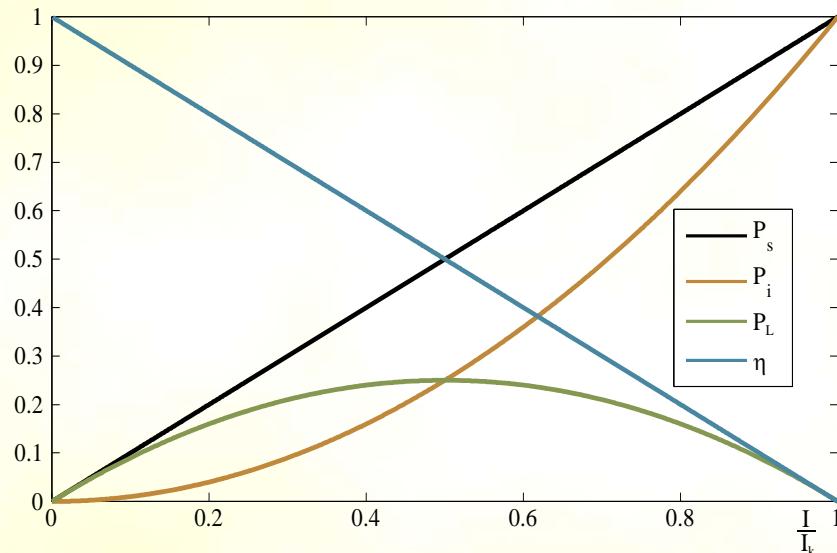
2. But in many practical situations the value of R_i is determined by physical properties of the circuit, but it is possible to regulate value of R_L .

First, consider the value of $R_i = 10 \Omega$, and the value of R_L will be changed.

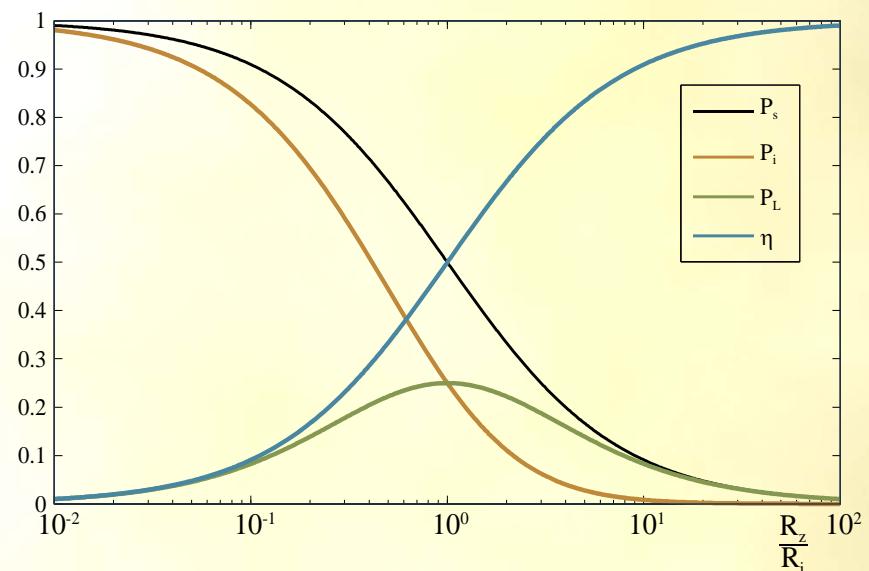
$U_i = 100 \text{ V}$ $R_i = 10 \Omega$	$R_L [\Omega]$	$P_{Ri} [\text{W}]$	$P_L [\text{W}]$
	0	1000	0
	5	444	222
	10	250	250
	15	240	160
	20	222	81

According to this simple evaluation it seems the condition of maximum power transmission to the load is

$$R_i = R_L$$



Dependence of powers in the circuit on relative value of passing current



Dependence of powers in the circuit on relative value of loading resistor

Curve equations:

Depending up the value of connected load, the value of passing current can be $I \in \langle 0, I_k \rangle$ (open circuit / short circuit)

Power supplied by voltage source: $P_z = U_i I$

Power dissipated at internal resistivity on heat: $P_i = R_i I^2$

Power delivered to the load: according to the Tellegen's theorem $P_L = P_z - P_i$

$$P_L = P_z - P_i = U_i I_k \left[\frac{I}{I_k} - \left(\frac{I}{I_k} \right)^2 \right] = P_k \left[\frac{I}{I_k} - \left(\frac{I}{I_k} \right)^2 \right]$$

Derivation:

We may prove this intuitive result mathematically. Power, delivered to the load is:

$$P_L = U_L I = R_L I^2 = R_L \left(\frac{U_i}{R_i + R_L} \right)^2 = U_i^2 \frac{R_L}{(R_i + R_L)^2}$$

The extremum of the function will be determined by its derivative with respect to load resistivity and setting derivative to 0.

$$\frac{dP_L}{dR_L} = \frac{1}{(R_i + R_L)^2} - \frac{2R_L}{(R_i + R_L)^3} = \frac{R_i - R_L}{(R_i + R_L)^3} = 0$$

From this expression we get $R_i = R_L$

Keep in mind the efficiency of power transfer to the load is only 50% - a half of total power, delivered by source, is converted to heat (at its internal resistivity)

Practical application of maximum power transfer theorem:

☞ Audio-amplifiers, ...: The source is an amplifier, the load is resistivity (impedance in fact) of the loudspeaker. The goal is to deliver maximum possible power to the loudspeaker. In past maximum power transfer theorem had significance (but 50% of total power was converted at heat in the amplifier! A-class...) – but most of today's constructions minimize internal resistivity of an amplifier, to achieve maximum voltage transfer to loudspeaker (and minimum losses in amplifier – D-class)

☞ Mechanical example – the horn connected to the loudspeaker.

☞ ??? ☺

☞ Impedance matching