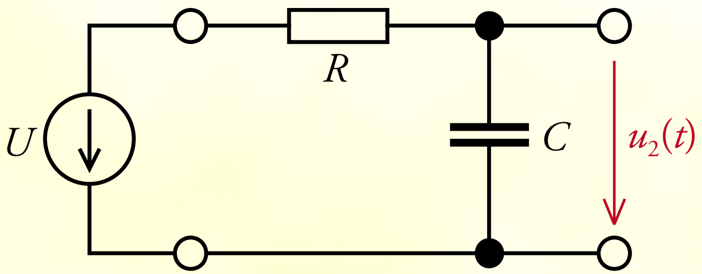
Fundamentals of Electrical Circuits

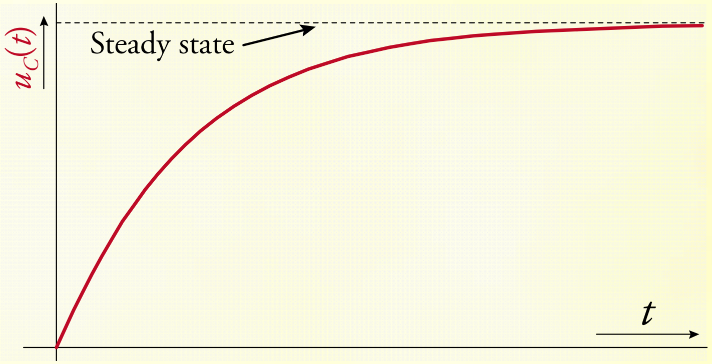
XI

1st order transients

FIRST-ORDER TRANSIENTS

- In previous lectures we already learned, the capacitor and inductor are "inertial" elements, which accumulates an energy
- Thus far we considered steady state – we assumed that in past every capacitor and inductor had been charged and they were in steady state, when *no energy flowed into that circuit elements*
  - What we can say about energy flow in sinusoidal steady state?
    - In sinusoidal steady state *the energy is exchanged among reactive circuit elements and sources* (reactive power) and such energy exchange last all the time the circuit is switched on, but – when we connect source, the circuit variables (voltage / current) needn't have correct phase shift – in sinusoidal steady state the circuit can, but needn't be in steady state immediately after we connect the circuit to the source (in dependence on initial phase of the source)
- After we connect / disconnect source or after another change in the circuit (change of resistivity, …) is necessary to level energy distribution in the circuit (to charge / discharge accumulative circuit elements) – this is called **transient**
- Consider integrating RC circuit, to which we connect at time $t = 0$ DC voltage source; suppose, the capacitor was without any charge





  - Intuitively we can describe the circuit:
    - Initially the capacitor was without the charge – it had zero voltage. Total source voltage was across the resistor R, through the circuit flowed electrical current $I = \frac{U}{R}$
    - Within the time Δt the electrical current, supply into the capacitor charge $\Delta q = I \Delta t$
    - To the charge is proportional the voltage $U_C = \frac{q}{C}$; the current decreases, new value is $I = \frac{U - U_C}{R}$
    - Within the same unit of time the less charge is delivered into the capacitor and the charge rate decrease – charging is not linear process
    - Hence it follows the differential equation $\mathrm{d}u_C(t) = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{1}{RC}\left[U - u_C(t)\right]\mathrm{d}t$
    - And its solution $u_C(t) = U\left(1 - e^{\frac{-t}{RC}}\right)$

- But presented procedure is not versatile process and with more complicated circuits it would be just very hardly applicable; so how to proceed?

  ☞ The solution is: **use circuit equations**

- Of course, we can't use circuit equations in DC, nor phasors in sinusoidal steady state (this is valid just in steady state, when transient is over), but integral-differential equations in time domain, or Laplace transform

Recall relationship of voltage and current on basic circuit elements:

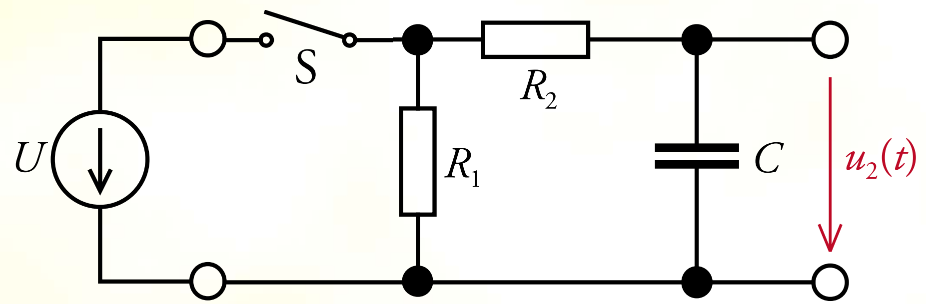
Resistor:	$u(t) = Ri(t)$	$i(t) = \dfrac{U(t)}{R}$
Capacitor:	$u(t) = \dfrac{1}{C}\displaystyle\int_0^t i(\tau)\,\mathrm{d}\tau + u_c(0)$	$i(t) = C\dfrac{\mathrm{d}u(t)}{\mathrm{d}t}$
Inductor:	$u(t) = L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t}$	$i(t) = \dfrac{1}{L}\displaystyle\int_0^t u(\tau)\mathrm{d}\tau + i_L(0)$

To find circuit equations, we can use KCL (nodal analysis), or KVL – mesh / loop analysis.

Comparing to DC / AC analysis, there are new terms – **initial conditions**, which represents an energy, stored in accumulative circuit element at $t = 0$

Initial conditions will be evaluated from steady state at $t < 0$

Example: In the circuit in figure at $t = 0$ the switcher „S" is switched on. When the transient response dies out, the switcher „S" is switched off. Determine initial condition in both cases and write circuit equations in time domain. The circuit was in the steady state when the switcher „S" was switched on.



$$U = 10\,\mathrm{V},\ R_1 = 500\,\Omega,\ R_2 = 1\,\mathrm{k}\Omega,\ C = 1\,\mu\mathrm{F}$$

1. At $t < 0$, when the switcher „S" was (for sufficiently long time) switched off, the capacitor was discharged on R_1 and R_2. Than the stored charge was 0, and so the voltage. ⇒ zero initial condition
2. We will write circuit equations **after** the switcher was switched on:
   Using **nodal analysis** we got one equation:

$$\frac{u_C(t) - U}{R_2} + C\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = 0 \longrightarrow \frac{u_C(t) - 10}{1000} + 10^{-6}\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = 0$$

In the left node we know the voltage, there is no need of an equation (*and we can't write it – unknown current which flows through voltage source*)

Mesh / loop analysis – there are 2 loops; complete description of the circuit has 2 equations, but if the current passing the resistor R_1 is not important for us, we may use just one loop, passing through resistor R_2 capacitor C and voltage source

$$R_1\,[i_1(t) - i_2(t)] - U = 0$$

$$R_1\,[i_2(t) - i_1(t)] + R_2 i_2(t) + \frac{1}{C}\int_0^t i_2(\tau)\,\mathrm{d}\tau + u_C(0) = 0$$

$$R_2 i_2(t) + \frac{1}{C}\int_0^t i_2(\tau)\,\mathrm{d}\tau + u_C(0) - U = 0$$

$$500\,[i_1(t) - i_2(t)] - 10 = 0$$

$$500\,[i_2(t) - i_1(t)] + 1000\,i_2(t) + \frac{1}{10^{-6}}\int_0^t i_2(\tau)\,\mathrm{d}\tau + 0 = 0$$

$$1000\,i_2(t) + \frac{1}{10^{-6}}\int_0^t i_2(\tau)\,\mathrm{d}\tau + 0 - 10 = 0$$

Solution of circuit equation:

First we will recall circuit equation which we got using nodal analysis:

$$\frac{u_C(t) - U}{R_2} + C\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = 0 \qquad\qquad \frac{u_C(t) - 10}{1000} + 10^{-6}\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = 0$$

- Preferable method of solution of differential circuit equations is method of variation of parameters:
  1. Move source(s) (and eventually initial conditions) on the right-hand side

     $$u_C(t) + R_2C\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = U$$

  2. First find general solution of a differential equation – with zero right-hand side

     $$u_C(t) + R_2C\,\frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = 0$$

  3. (Generally) derivative of voltage of order replace by n$^{\text{th}}$ power of λ (*of course, here just 1st power*)

     $$1 + R_2C\,\lambda = 0 \qquad \Rightarrow \qquad \lambda = \frac{-1}{R_2C} = \frac{-1}{\tau} = \frac{-1}{1000\cdot 10^{-6}} = -1000 \qquad \Rightarrow \qquad \tau = 1\,\mathrm{ms}$$

  4. General solution is the equation:

     $$u_C(t) = K\,e^{\lambda t} = K\,e^{-1000t}$$

     The time constant – *compare its inverse value with break frequency in frequency response of integrating RC circuit*

  5. Particular solution – steady state in the circuit, when the temporary response (transient) will die out with time (charging / discharging of a capacitors and inductors); in electrical circuits we can use with advantage any methods of circuit analysis – DC / AC, … according to the kind of circuit

In this circuit the capacitor is charged on the voltage U, so the particular solution is $u_{cp} = u_c(\infty) = U$

$$u_C(t) = K\,e^{\lambda t} + u_p(t) = K\,e^{-1000t} + 10$$

6. Finally, we need to compute constant of integration
if we set $t = 0$, we got the equation

$$u_C(0) = K\,e^{\lambda\cdot 0} + u_p(0) = K + u_p(0) \quad \Rightarrow \quad K = u_C(0) - u_p(0) = 0 - 10 = -10$$

7. So that we obtain the solution

$$u_C(t) = 10\left(1 - e^{-1000t}\right)$$

Note, that to find the solution of differential equation we use an ***initial condition****, even it is not a part of initial differential equation – but it is known value of solution at $t = 0$, which we have to know to find constant of integration*

- What will be different, if we will use mesh / loop analysis?

$$R_2 i_2(t) + \frac{1}{C}\int_0^t i_2(\tau)\,\mathrm{d}\tau + u_C(0) - U = 0 \quad \left|\frac{\mathrm{d}}{\mathrm{d}t}\right. \qquad 1000\,i_2(t) + \frac{1}{10^{-6}}\int_0^t i_2(\tau)\,\mathrm{d}\tau + 0 - 10 = 0 \quad \left|\frac{\mathrm{d}}{\mathrm{d}t}\right.$$

$$R_2 C\frac{\mathrm{d}i_2(t)}{\mathrm{d}t} + i_2(t) = 0 \qquad 10^{-3}\,\frac{\mathrm{d}i_2(t)}{\mathrm{d}t} + i_2(t) = 0$$

$$R_2 C\lambda + 1 = 0 \qquad 10^{-3}\,\lambda + 1 = 0$$

$$\lambda = \frac{-1}{R_2 C} \qquad \lambda = -1000$$

$$i_2(t) = K e^{\lambda t} + i_p(t) = K e^{-1000t} + 0$$

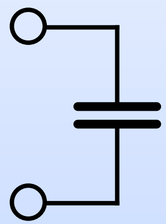
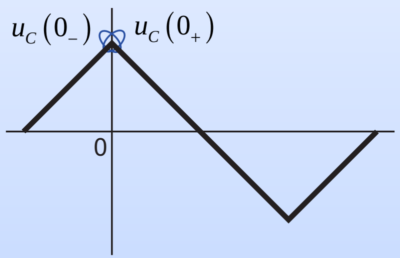
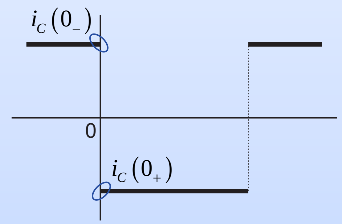
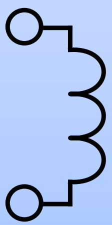
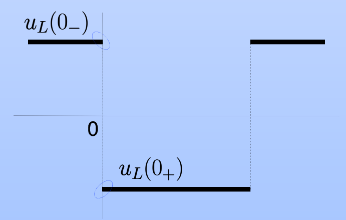
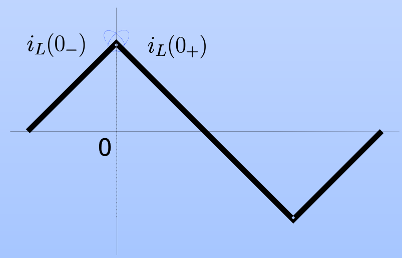
- We got the same general solution, but to find a constant of integration we have to know the value of initial condition – the current at $t = 0$

An energetic initial condition on capacitor is voltage (directly related to stored charge), not electric current

$\Rightarrow$ The voltage is continuous, initial condition, calculated in steady state at $t < 0$, is valid after the switch changed its state ($t = 0_+$)

$\Rightarrow$ An electric current passing the capacitor is not continuous, it is necessary to derive it from voltage

- $\Rightarrow$ **It is recommended to use nodal analysis in first order circuits when they contain a capacitor, the initial condition is voltage**
- $\Rightarrow$ **When the first order circuit contains inductor, it is recommended to use mesh / loop analysis, the initial condition is current passing an inductor**

	$q = Cu$ $i = \dfrac{\mathrm{d}q}{\mathrm{d}t}$ $i_C(t) = C\dfrac{\mathrm{d}u_C(t)}{\mathrm{d}t}$	must be $u_C(0_-) = u_C(0_+)$ 	may be $i_C(0_-) \neq i_C(0_+)$ 
	$\Phi_L = Li$ $u(t) = \dfrac{\mathrm{d}\Phi_L}{\mathrm{d}t}$ $u_L(t) = L\dfrac{\mathrm{d}i_L(t)}{\mathrm{d}t}$	may be $u_L(0_-) \neq u_L(0_+)$ 	must be $i_L(0_-) = i_L(0_+)$ 

recall

consequently

- In this circuit we can derive current initial condition from voltage initial condition:

$$i_C(0) = \frac{U - u_C(0)}{R_2} = \frac{10-0}{1000} = 0.01 \text{ A}$$

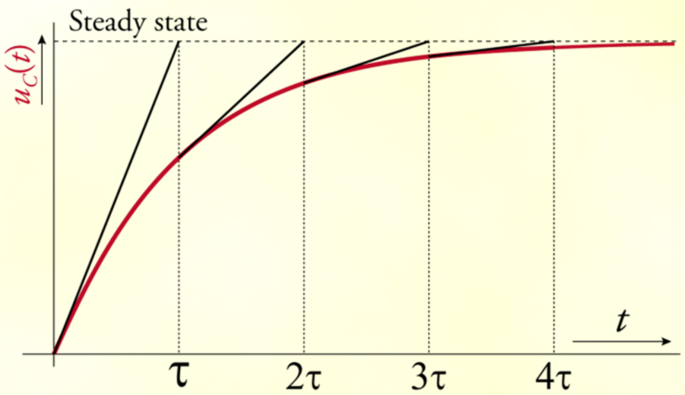
$$i_2(t) = 0.01\, e^{-1000t} \text{ A}$$

The meaning of the time constant:

In the RC circuit we define the time constant as: $\tau = \frac{-1}{\lambda} = RC$

In theory, the transient response will die out just with infinite time, but, in practice:

time	% of steady state
τ	63.21
3τ	95.02
5τ	99.33



- Usually, we consider the transient as finished just after time 3τ
- Time constant is a position on time axis of point of intersection of a tangent line to a voltage / current waveform with steady state value
- Time constant is one of a critical factors, which affects maximum allowed frequency of busses, amplifiers (see the relationship to the frequency response) and other circuits (airbag igniters, voltage converters, …)

Particular solution – sinusoidal steady state:

Compare solution of transient response in the RC circuit, where $R = 850\ \Omega$, $C = 1\ \mu\text{F}$ when the circuit is

a) Supplied by DC voltage source, $U = 90$ V
b) Supplied by AC voltage source, $u(t) = 90\sin(2\pi\, 5000\, t)$ V

Time constant of the circuit $\tau = RC = 850 \cdot 10^{-6} = 0.85\,\text{ms}$ $\Rightarrow$ $\lambda = \dfrac{-1}{0.00085} = -1176.47$

Complete solution $u_c(t) = Ke^{-1176.47t} + u_p(t)$

DC $u_p(t) = U$ $\qquad u_c(t) = \underline{\underline{90(1 - e^{-1176.47t})}}\ \text{V}$

AC
$$\widehat{\mathbf{U}}_C = \widehat{\mathbf{U}}\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \widehat{\mathbf{U}}\frac{1}{1 + j\omega RC} = 90\frac{1}{1 + j \cdot 2\pi \cdot 5000 \cdot 850 \cdot 10^{-6}} = 3.37\underline{/-1.53}$$

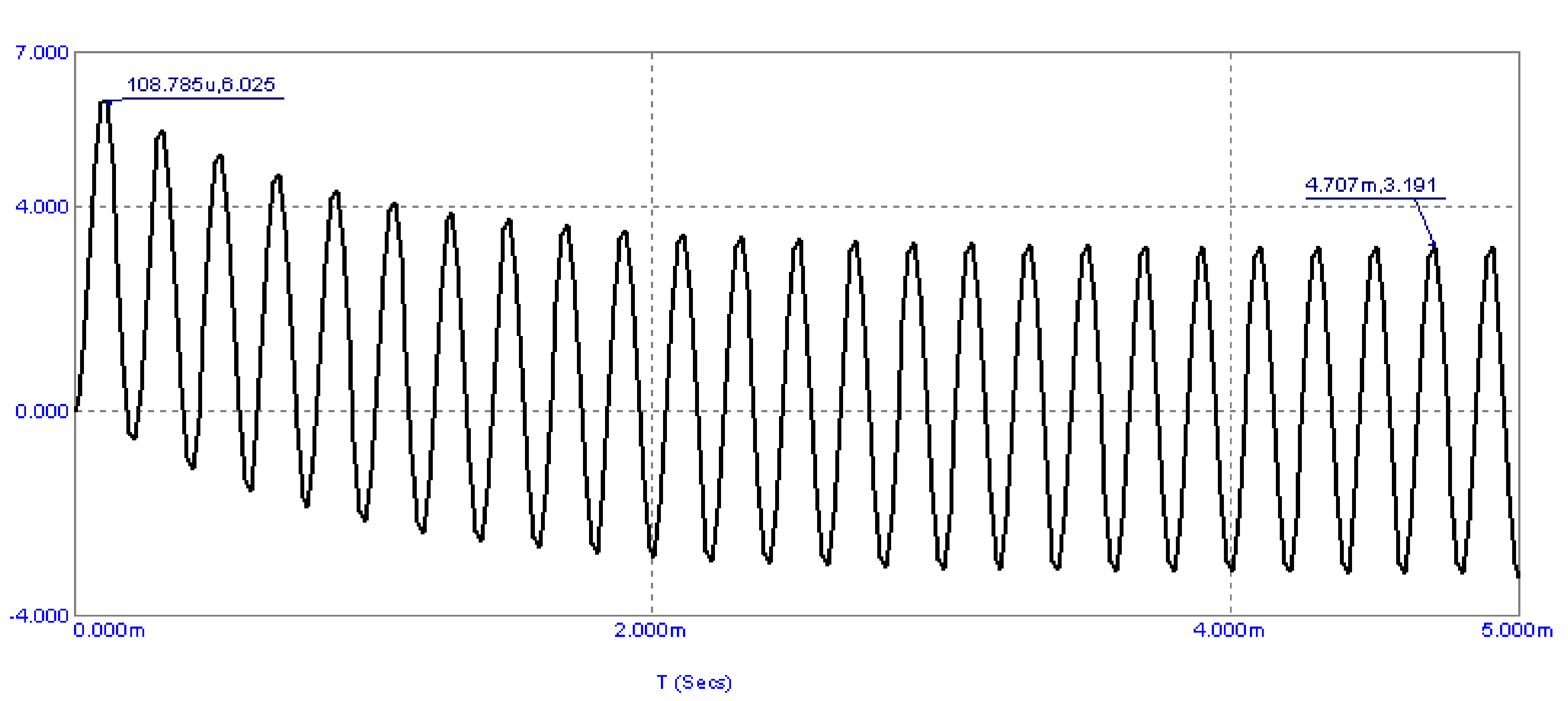
$\downarrow$

$$u_p(t) = 3.37\sin(31416t - 1.53)\ \ [\text{V}]$$

Now set $t = 0$

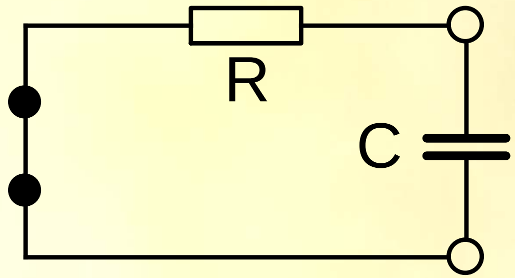
$$0 = K + 3.37\sin(-1.53) \qquad \Rightarrow \qquad K \doteq 3.37$$

$$u_c(t) = \underline{\underline{3.37e^{-1176.47t} + 3.37\sin(31416\,t - 1.53)}}\ \text{V}$$



- Maximum amplitude of the waveform is almost twice the value of the steady state amplitude
- **Exponential waveform is a response of the circuit just on connection of the source itself, not the waveform of the source; it is specific property of each circuit**, the waveform of the source affects just (steady state) amplitude, but not the time constant
- Amplitude of the exponential response depends on the initial properties of the circuit (if the capacitor was charged and the value of the source voltage at the moment of a source connection – in the case of AC source anything in the interval $\langle -U_m, U_m \rangle$
- In the case of AC source the amplitude of an exponential function is affected by initial phase shift of the source and its frequency

Disconnecting of the source



The capacitor is charged, initial condition (usually) nonzero

suppose, the previous transient dies out, the circuit is in steady state

DC $u_c(0) = U$ $\quad u_p = 0$

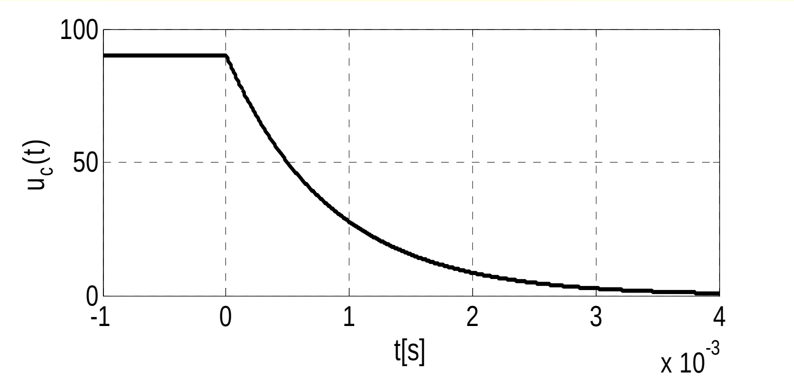
$$90 = K + 0 \qquad u_c(t) = \underline{90\, e^{-1176.47t}}\ \mathrm{V}$$

AC $t < 0$ $\quad u(t) = 3.37 \sin(31416t - 1.53)\ \mathrm{V}$

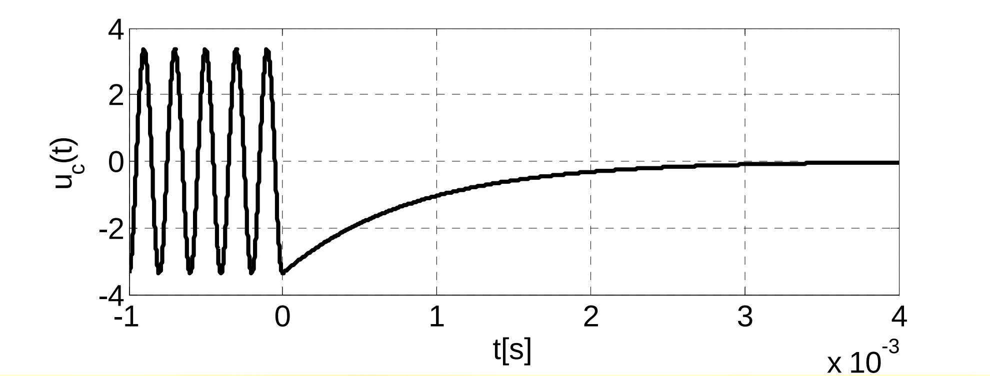
$t = 0$ $\quad u_c(0) = 3.37 \sin(-1.53) \doteq -3.37\,\mathrm{V}$

$t \to \infty$ $\quad u_p = 0$

$$u_c(t) = \underline{\underline{-3.37\, e^{-1176.47t}}}\ \mathrm{V}$$



DC



AC

Complete response may be written as

$$\boxed{\text{Complete response} = \underbrace{\text{natural response}}_{\text{stored energy}} + \underbrace{\text{forced response}}_{\text{independent source}}}$$

$$\boxed{u_c(t) = \left[u_C(0_+) - u_p(0)\right] e^{\frac{-t}{\tau}} + u_p(t)}$$

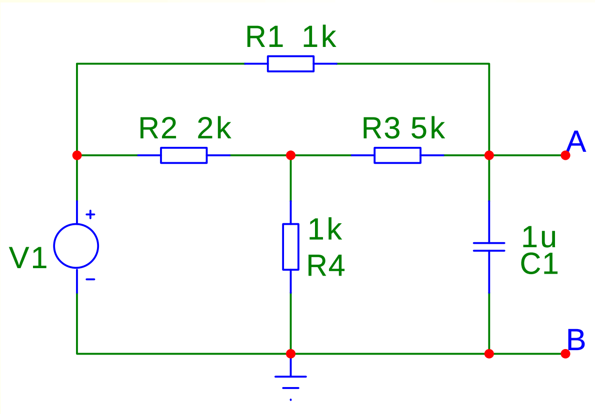
The voltage on the capacitor before the source was connected to (or disconnected from) the circuit

Steady state – DC, AC, or 0

The voltage of the steady state (forced response) at the time instant of connection / disconnection of the source – in AC it is related to the phase shift of the source

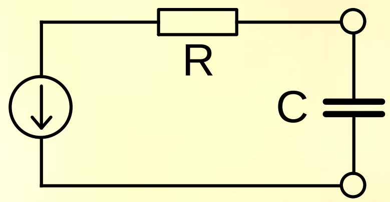
$$\boxed{\text{Complete response} = \text{transient response} + \text{steady state response}}$$

More complex circuit with one capacitor



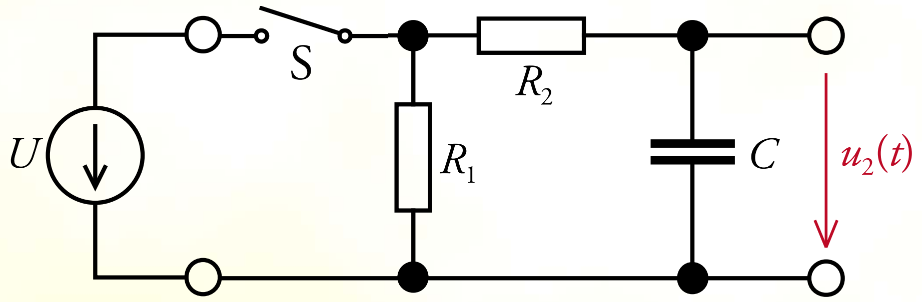
From the point of view of terminals of capacitor
the circuit is replace by Thévenin's equivalent circuit
the solution is then same like in simple RC circuit

$\Rightarrow$



Transient - Using Laplace transform:

Example: In the circuit in figure at $t = 0$ the switcher „S“ is switched on. When the transient response dies out, the switcher „S“ is switched off. Determine initial condition in both cases and write circuit equations in time domain. The circuit was in the steady state when the switcher „S“ was switched on.



$U = 10\,\mathrm{V},\ R_1 = 500\,\Omega,\ R_2 = 1\,\mathrm{k}\Omega,\ C = 1\,\mu\mathrm{F}$

- We will evaluate directly output voltage u_2

1. Initial condition – the capacitor was discharged $\Rightarrow u_C(0) = 0$
2. Since initial condition is zero, we can use directly voltage divider rule

$$U_2(p) = U_1(p)\frac{\frac{1}{pc}}{R_2 + \frac{1}{pc}} = U_1(p)\frac{1}{1 + pR_2C} = \frac{10}{p}\frac{1}{1 + 10^{-3}p} = \frac{10}{p}\frac{1000}{1000 + p}$$

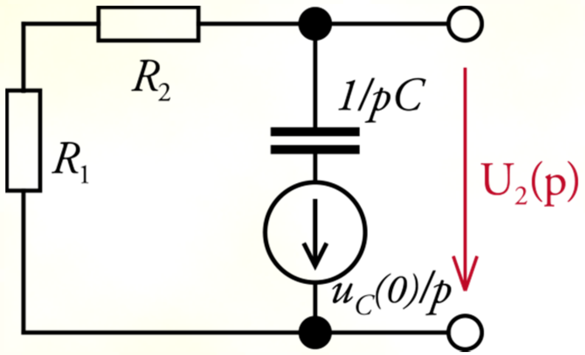
3. Apply partial fraction decomposition

$$U_2(p) = \frac{10}{p} + \frac{-10}{p + 1000}$$

4. Using table of transforms we find time waveform

$$u_2(t) = 10\left(1 - e^{-1000t}\right)$$

When we solve the case, when the switcher is switched-off, we must replace capacitor by combnation of operational impedance and voltage source:



$$U = 10\,\mathrm{V},\; R_1 = 500\,\Omega,\; R_2 = 1\,\mathrm{k}\Omega,\; C = 1\,\mu\mathrm{F}$$

1. Initial condition – the capacitor was charged to the source voltage $\Rightarrow\; u_C(0) = 10\,\mathrm{V}$
2. The output voltage is the same as on the series of both resistors – voltage divider

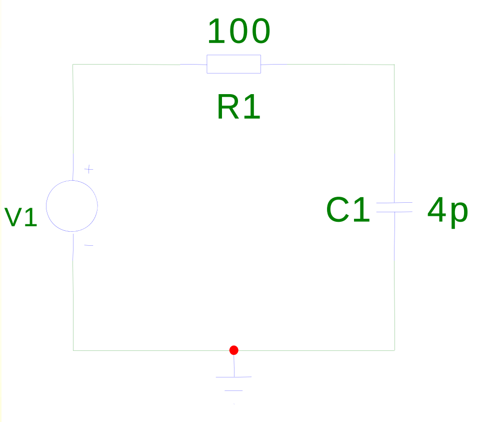
$$U_2(p) = \frac{u_C(0)}{p}\frac{R_1 + R_2}{R_1 + R_2 + \frac{1}{pC}} = \frac{u_C(0)}{p}\frac{(R_1 + R_2)pC}{(R_1 + R_2)pC + 1} = \frac{10}{p}\frac{1.5\cdot 10^{-3}p}{1.5\cdot 10^{-3}p + 1} = \frac{10}{p + 666.\bar{6}}$$

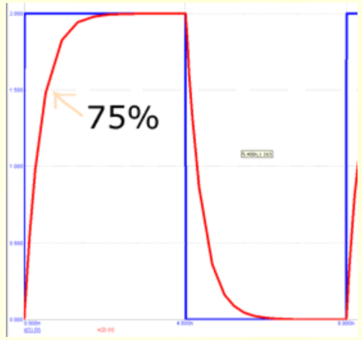
3. Using table of transforms we find time waveform

$$u_2(t) = 10\,e^{-666.\bar{6}t}$$

Example:

Circuit in the figure below is supplied from the rectangular voltage source with maximum voltage 2V. As the valid value of logic 1 will be considered voltage greater than 75% of maximum voltage. Let at the most 20 % of half period of the clock pulse is the allowed rising time. What is maximum possible clock frequency of the bus?





$$u(t) = U(1 - e^{-\frac{t}{\tau}}) \qquad \tau = RC = 100 \cdot 4 \cdot 10^{-12} = 400\,\mathrm{ps}$$

$$t = -\tau \ln(1 - \frac{u(t)}{U}) = -4 \cdot 10^{-10} \cdot \ln(1 - \frac{1.5}{2}) = 554\,\mathrm{ps}$$

$$f = \frac{1}{2 \cdot 5 \cdot t} = 180.38\,\mathrm{MHz}$$

Example:

The bus with speed 12 Mb/s has the voltage 1.8 V. The bus impedance is 90 Ω. Maximum allowed loading capacitance is 18 pF. Maximum allowed rising time is 10 ns (from 0,45 to 1,35 V). Satisfy the bus this specification?

$$\text{YES,}\quad \tau = 1.62\,\text{ns}$$

$$\text{from } 0.45\text{ V to } 1.35\text{ V it rises in } t = -1.65\cdot\ln\frac{1.35-1.8}{0.45-1.8} = 1.78\,\text{ns}$$

- Maximum allowed loading capacitance of high speed USB bus (240 MHz, 480 Mb/s) is 14 pF. Is the bus able to operate with voltage excitation?

  NO, time constant is too long

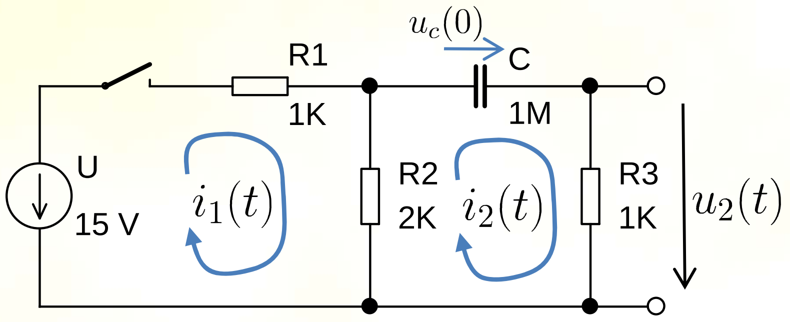
- Is the bus able to operate with current excitation 17.78 mA?

  YES, and it does – the capacitor is charged faster by current source

$$q = I \cdot t \qquad U = \frac{q}{C} = \frac{I \cdot t}{C}$$

$$t = \frac{CU}{I} = \frac{14\cdot 10^{-12}\cdot 0.9}{17.78\cdot 10^{-3}} = 0.7\,\text{ns}$$

Circuit, where we compute different circuit variable from voltage across capacitor



In circuit on the figure compute the waveform of voltage $u_2(t)$ when the switcher is switched on / off

Solution:

The waveform of the voltage across resistor R_3 is given by waveform of passing current $i_2(t)$ – the same current, passing the capacitor.

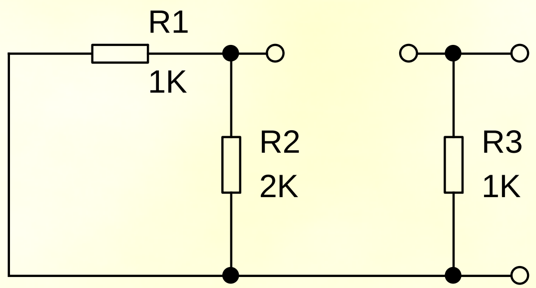
First, we compute the waveform of voltage across the capacitor.

Switching on:

$$u_c(0) = 0\,\mathrm{V}$$

$$U_i = u_p = U\frac{R_2}{R_1 + R_2} = 15\frac{2000}{1000 + 2000} = 10\,\mathrm{V}$$

$i_2(t) = 0$ ➡ no voltage drop across R_3, $U_{R2} = U_c$



$$R_i = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2} = 1000 + \frac{2000}{3 \cdot 10^6} = 1666.\overline{6}\,\Omega$$

$$\tau = R_i C = 1.\overline{6}\,\mathrm{ms}$$

$$u_c(t) = [0 - 10]\,e^{-600\,t} + 10$$

$$i_2(t) = C\frac{\mathrm{d}u_c(t)}{\mathrm{d}t} = 10^{-6}\left[-10 \cdot (-600)e^{-600\,t}\right] = 6e^{-600\,t}\,\mathrm{mA}$$

$$u_2(t) = R_3 \cdot i_2(t) = \underline{6\,e^{-600\,t}}\,\mathrm{V}$$

Switching off:

$$u_c(0) = U\,\frac{R_2}{R_1 + R_2} = 15\,\frac{2000}{1000 + 2000} = 10\,\mathrm{V}$$

$$u_p = 0\,\mathrm{V}$$

$$R_i = R_3 + R_2 = 1000 + 2000 = 3000\,\Omega$$

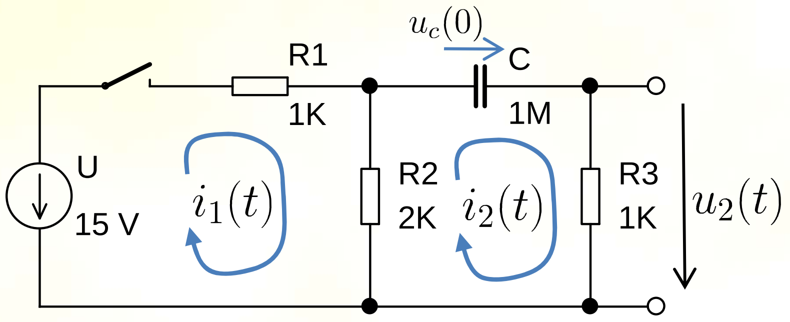
$$\tau = R_i C = 3\,\mathrm{ms}$$

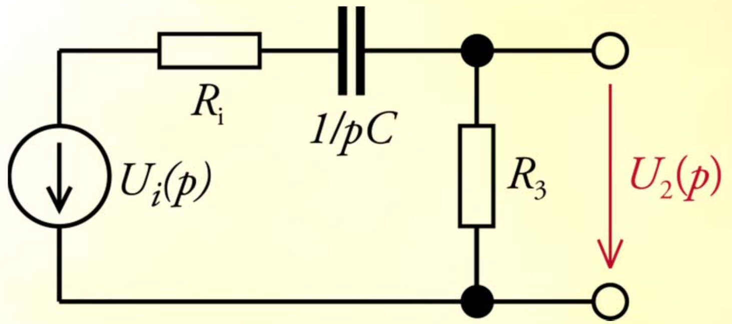
$$u_c(t) = [10 - 0]\,e^{-333.\overline{3}\,t} + 0$$

$$i_2(t) = C\frac{\mathrm{d}u_c(t)}{\mathrm{d}t} = 10^{-6}\left[10 \cdot (333.\overline{3})e^{-333.\overline{3}\,t}\right] = 3.\overline{3}\,e^{-333.\overline{3}\,t}\,\mathrm{mA}$$

$$u_2(t) = R_3 \cdot i_2(t) = \underline{\underline{3.\overline{3}\,e^{-333.\overline{3}\,t}}}\,\mathrm{V}$$

Laplace transform - switching on:





$$u_c(0) = 0\,\mathrm{V}$$

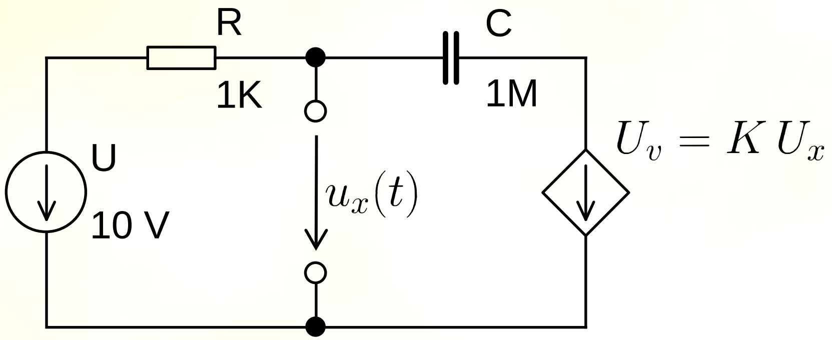
$$U_i = u_p = U\frac{R_2}{R_1+R_2} = 15\,\frac{2000}{1000+2000} = 10\,\mathrm{V}$$

$$R_i = \frac{R_1 \cdot R_2}{R_1+R_2} = \frac{2000}{3\cdot 10^6} = 666.\overline{6}\,\Omega$$

$$U_2(p) = U_i(p)\frac{R_3}{R_i+R_3+\frac{1}{pC}} = U_i(p)\frac{pR_3C}{p(R_i+R_3)C+1} = \frac{10}{p}\,\frac{10^{-3}p}{1.\overline{6}\cdot 10^{-3}p+1} = \frac{6}{p+600}$$

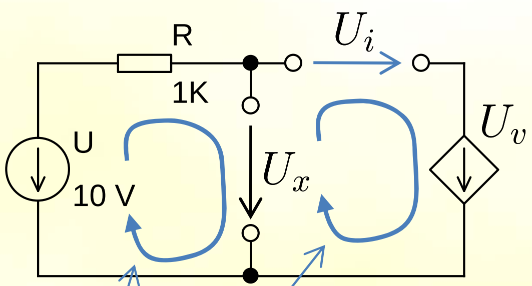
$$u_2(t) = \underline{\underline{6\,e^{-600\,t}}}\,\mathrm{V}$$

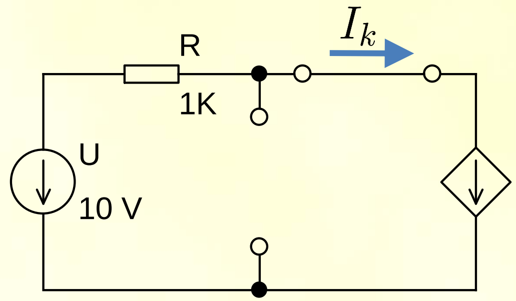
RC circuit with controlled source



In the circuit in the figure compute the waveform of voltage $u_x(t)$ after DC voltage source U is connected to the circuit

1. Thévenin's theorem





$$\left.\begin{aligned} U_i + K\,U_x - U_x &= 0 \\ -U + 0 + U_x &= 0 \end{aligned}\right\} \quad U_i = U(1-K)$$

$$I_k = \frac{U}{R} \quad \Rightarrow \quad R_i = \frac{U_i}{I_k} = \frac{U(1-K)}{\frac{U}{R}} = R(1-K)$$

$$\tau = R_i C = RC(1-K)$$

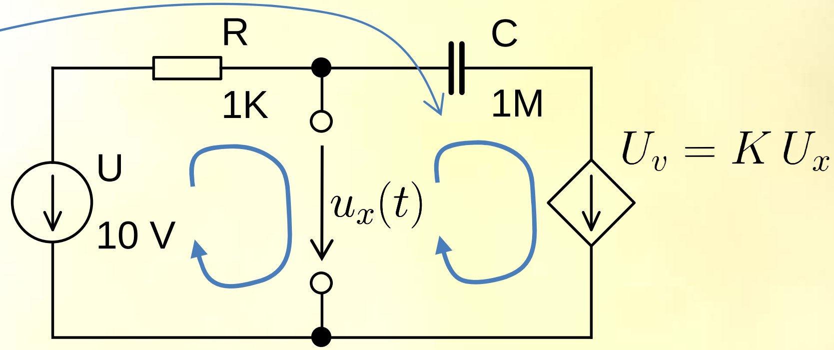
$$u_c(t) = [0 - U(1-K)]\, e^{\frac{-t}{RC(1-K)}} + U(1-K) = U(1-K)\left[1 - e^{\frac{-t}{RC(1-K)}}\right]$$

$$u_x(t) = u_c(t) + K\,u_x(t) \quad \Rightarrow \quad u_x(t)\cdot(1-K) = u_c(t) \quad \Rightarrow \quad \underline{\underline{u_x(t) = U\left(1 - e^{\frac{-t}{RC(1-K)}}\right)}}$$

2. Direct solution

$$u_c(0) = 0\,\mathrm{V}$$

$$u_c(0) + K\,u_x(0) - u_x(0) = 0 \qquad \Rightarrow \qquad u_x(0) = 0\,\mathrm{V}$$



We have to compute initial condition using circuit equations

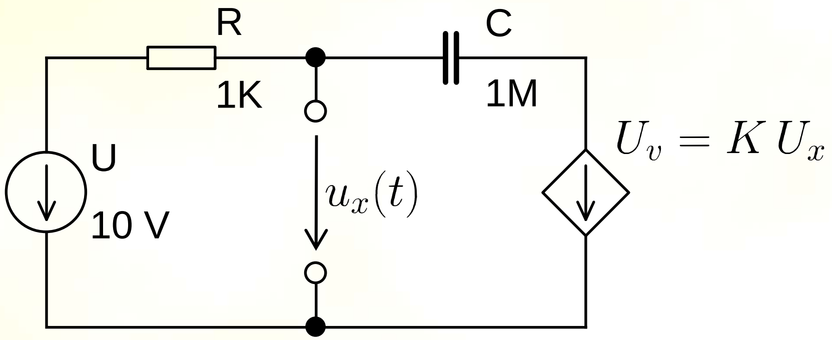
$$-U + R \cdot 0 + u_{xp} = 0 \qquad \Rightarrow \qquad u_{xp} = u_x(\infty) = U$$

$$\frac{u_x(t) - U}{R} + C\frac{\mathrm{d}}{\mathrm{d}t}\left[u_x(t) - K u_x(t)\right] = 0$$

$$RC(1-K)\lambda + 1 = 0 \qquad \Rightarrow \qquad \lambda = \frac{-1}{RC(1-K)}$$

$$u_x(t) = [0 - U]\, e^{\frac{-t}{RC(1-K)}} + U = \underline{\underline{U\left(1 - e^{\frac{-t}{RC(1-K)}}\right)}}$$

RC circuit with controlled source – Laplace transform



In the circuit in the figure compute the waveform of voltage $u_x(t)$ after DC voltage source U is connected to the circuit

$$u_C(0) = 0\,\mathrm{V}$$

Use nodal analysis

$$\frac{U_x(p) - U(p)}{R} + \frac{U_x(p) - K U_x(p)}{\frac{1}{pC}} = 0$$

Rewrite the equation so that we can evaluate $U_x(p)$

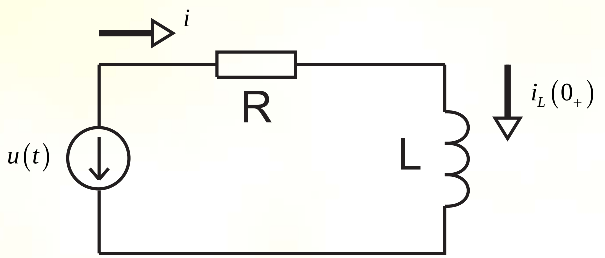
$$U_x(p)\left[\frac{1}{R} + pC\,(1-K)\right] = \frac{U(p)}{R}$$

Find appropriate form according to the table of Laplace transforms, use partial fraction decomposition

$$U_x(p) = \frac{U(p)}{1 + pRC(1-K)} = \frac{\frac{U}{pRC(1-K)}}{p + \frac{1}{RC(1-K)}} = \frac{U}{p} - \frac{U}{p + \frac{1}{RC(1-K)}}$$

$$\underline{\underline{u_x(t) = U\left(1 - e^{\frac{-t}{RC(1-K)}}\right)}}$$

1^{ST} ORDER CIRCUIT WITH INDUCTOR



- Energetic circuit equation – current which flowed through the circuit at $t = 0$.
  Even if we would compute voltage across inductor, it is more suitable to choose current as the initial condition – it is continuous.
- $\Rightarrow$ circuit equation – **mesh / loop analysis**

$$Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} - u(t) = 0$$

- To find the solution, method of variation of parameters is most suitable

$$\frac{L}{R}\frac{\mathrm{d}i(t)}{\mathrm{d}t} + i(t) = \frac{u(t)}{R}$$

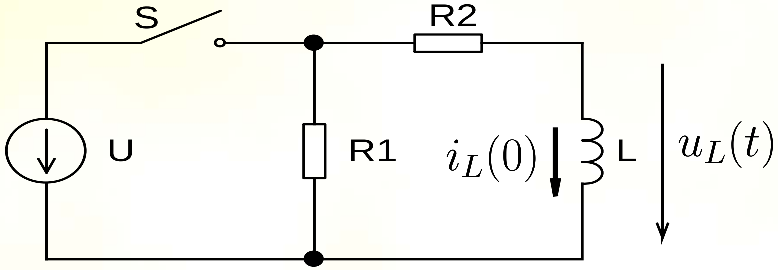
$$\frac{L}{R}\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = \frac{-R}{L} \qquad \Rightarrow \qquad \boxed{\tau = \frac{L}{R}}$$

$$i_{Lo}(t) = Ke^{\lambda t} = Ke^{-t\frac{R}{L}}$$

- Compute steady state in the circuit after transient dies away
- Setting the $t = 0$ compute integration constant K – *again, complete response has same form as in the case of capacitor*

$$\boxed{i_L(t) = \left[i_L(0_+) - i_p(0)\right] e^{\frac{-t}{\tau}} + i_p(t)}$$

Voltage across an inductor



$$R_1 = 1\,\mathrm{k}\Omega,\; R_2 = 1\,\Omega,\; L = 0.5\,\mathrm{H},\; U = 1\,\mathrm{V}$$

Energetic initial condition is current
$\Rightarrow$ First compute current, then voltage

Switcher is switched on:

Steady state at $t < 0$ and $t > 0$: $\quad i_L(0) = 0, \quad i_p = \dfrac{U}{R_2} = \dfrac{1}{1} = 1\,\mathrm{A}$

The equation and its solution: $\quad R_2\, i(t) + L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t} - U = 0 \quad \Rightarrow \quad R_2 + L\lambda = 0 \quad \Rightarrow \quad \lambda = \dfrac{-R_2}{L}$

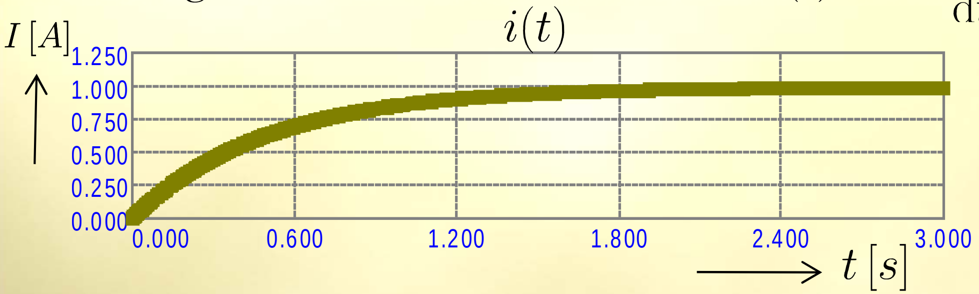
$$i(t) = Ke^{\lambda t} + i_p$$

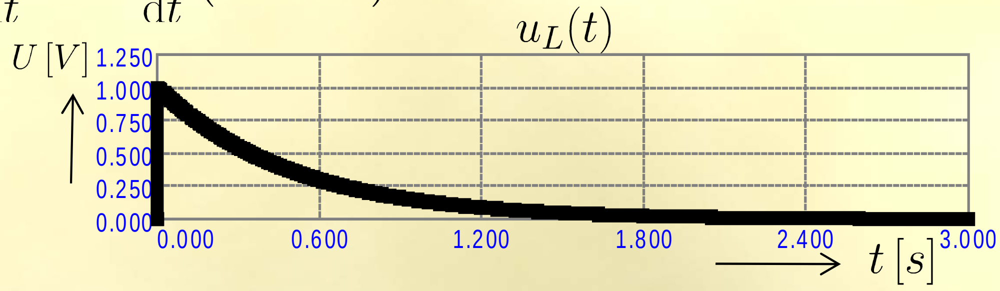
Time constant: $\quad \tau = \dfrac{L}{R_2} = \dfrac{0.5}{1} = 0.5\,\mathrm{s}$

$t = 0$: $\quad i_L(0) = K + i_p \quad \Rightarrow \quad K = i_L(0) - i_p$

Current waveform: $\quad i(t) = 1 - e^{-2t}$

Voltage waveform: $\quad u_L(t) = L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t} = L\dfrac{\mathrm{d}}{\mathrm{d}t}\left(1 - e^{-2t}\right) = e^{-2t}$





Switcher is switched off:

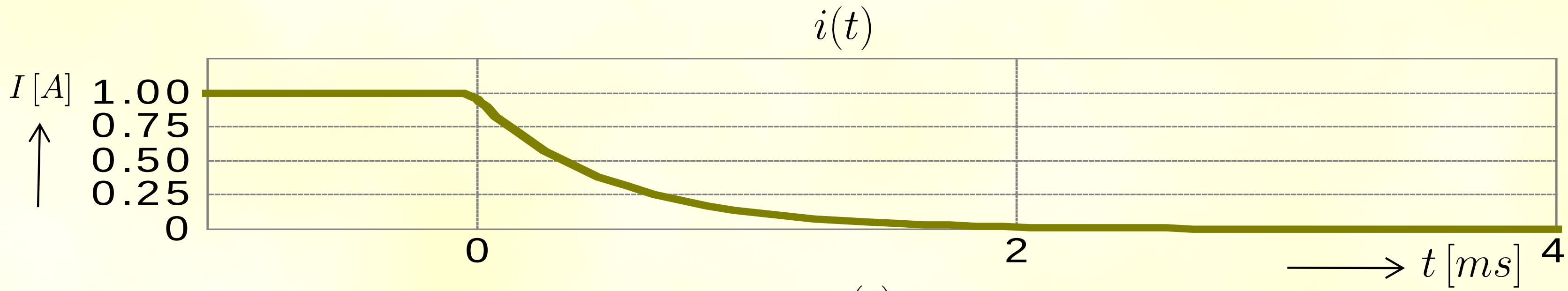
Steady states at $t < 0$ and $t > 0$: $i_L(0) = \dfrac{U}{R_2} = \dfrac{1}{1} = 1\,\text{A}, \quad i_p = 0$

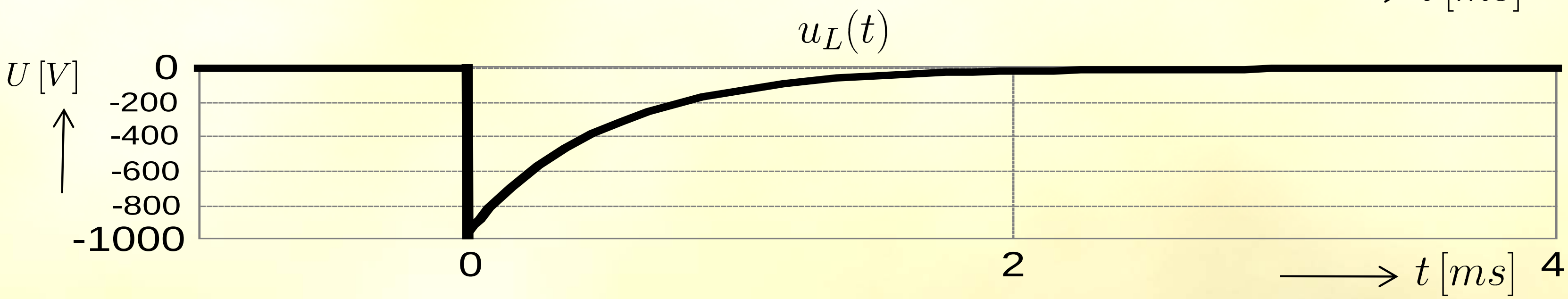
Equation and its solution: $(R_1+R_2)\, i(t) + L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t} = 0 \quad \Rightarrow \quad R_1+R_2+L\lambda = 0 \quad \Rightarrow \quad \lambda = -\dfrac{R_1+R_2}{L}$

Time constant: $\tau = \dfrac{L}{R_1+R_2} = \dfrac{0.5}{1001} = 499.5\,\mu\text{s}$

Current waveform: $i(t) = e^{-2002t}$

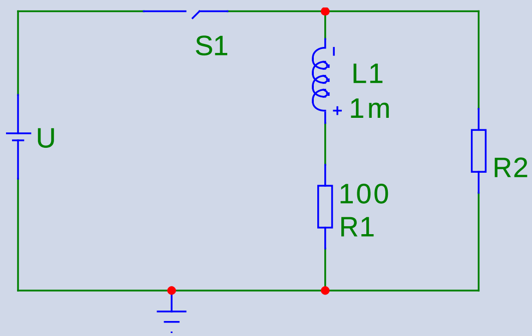
Voltage waveform: $u_L(t) = L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t} = L\dfrac{\mathrm{d}}{\mathrm{d}t}\left(e^{-2002t}\right) = -1001\, e^{-2002t}\,\text{V}$





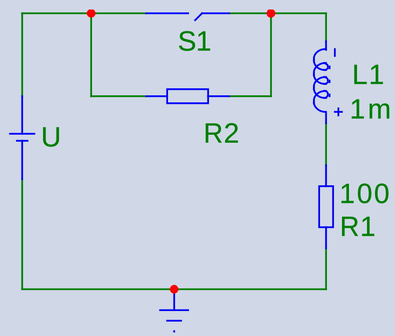
Current is continuous variable – voltage rise on such value, in order that the current passes arbitrary resistivity (theoretically even infinite voltage) ➡ **spark / arc if we just disconnect inductor from the source!!!**

Voltage across inductor – overvoltage protection



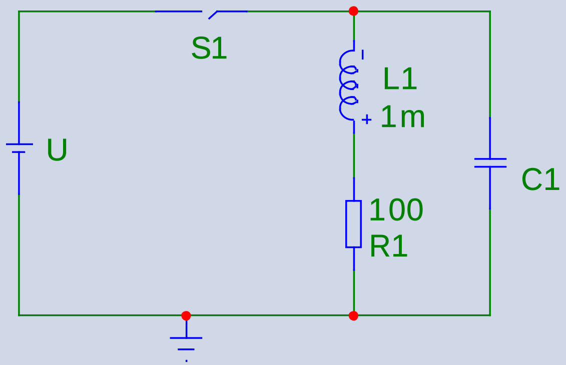
Resistor in parallel with the coil

R_2 all the time the circuit is switched on resistor dissipates power = unwanted losses; the larger the resistivity is, dissipated power is smaller, but at once the overvoltage when the source is disconnected is larger; large coils have large dissipated power on this resistor!!!



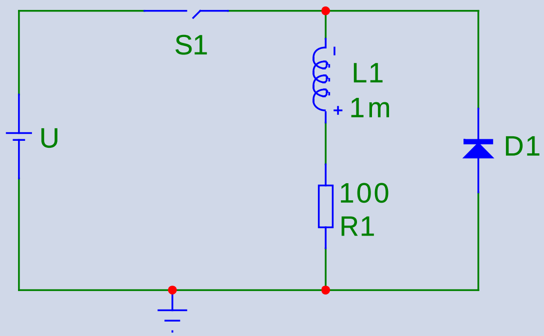
Resistor in parallel with the switcher

Such connection, of course, could not be used to safe disconnect the coil from the source; so it is applicable merely to reduce current passing the coil



Capacitor in parallel with the coil

Effective overvoltage protection, capacitor corrects power factor as well, accumulated energy dissipates on heat in the coil winding (R_1)



Diode in parallel with the coil

Applicable only to DC circuits – e.g. relay winding protection; correct selection of the diode is important (limit current), limiting current may be reduced by resistor

Application – Automobile Ignition Unit

Due to the ability of inductors to oppose rapid change in current and increase voltage as necessary a spark may be formed between contacts of switches, relays etc. In such cases sparks are undesirable, since it leads to contact wear and subsequent failures and then it is necessary to prevent sparks.

But in some cases such sparks are desired and they are essential function principle in some applications. Very important application is automobile ignition circuit.

The gasoline engine of an automobile requires that the fuel-air mixture be ignited at proper time instant. It is achieved by means of a spark plug, which consist of a pair of electrodes and a spark is formed between that electrodes due to large voltage (20 – 35 kV) applied to them.

Even the actual realization of ignition circuit can differ in details (capacitor or electronic switcher in controlling / switching part) and actual ignition inductor has two wirings (the low voltage high current primary and high voltage low current secondary) the basic principle is still the same – inductor that oppose rapid change in current and resulting jump of voltage.

Questions:

Why it is better to use two wirings with different number of turns (ratio approx. 80) and not just one?

Why in parallel to the switcher is connected capacitor? How it affects resulting voltage waveform?