AC Power

Let's recall one definition from the first lecture:

Definition 1

Power is energy per unit time.
Let's define *instantaneous power* as:

$$p(t) = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}W}{\mathrm{d}q}\frac{\mathrm{d}q}{\mathrm{d}t} = v(t)i(t) \tag{1}$$

In the AC steady state, the voltage and current are sinusoidal, i.e.:

$$v(t) = V_m \sin(\omega t + \varphi_v) \tag{2}$$

$$i(t) = I_m \sin(\omega t + \varphi_i) \tag{3}$$

so the instantaneous power is then:

$$p(t) = V_m I_m \sin(\omega t + \varphi_v)\sin(\omega t + \varphi_i) \tag{4}$$

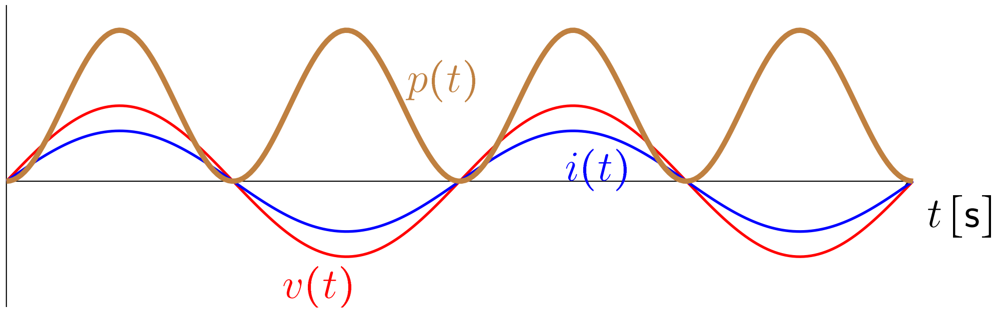
Employing the trigonometric identity:

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right] \tag{5}$$

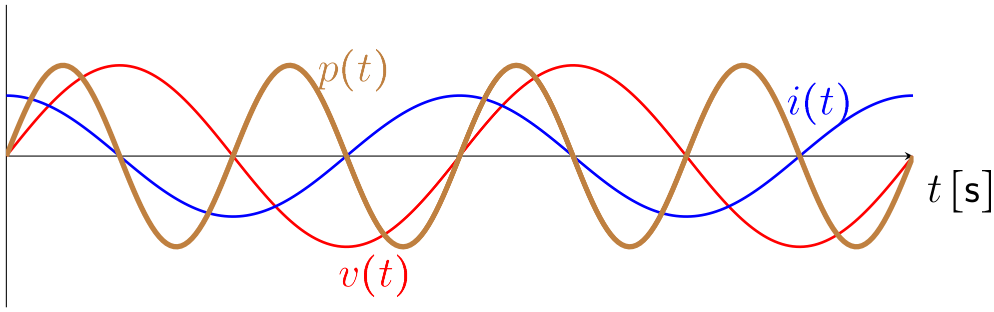
we can rewrite the equation 4 into the form

$$p(t) = V_m I_m \frac{1}{2}\left[\cos(\varphi_v - \varphi_i) - \cos(2\omega t + \varphi_v + \varphi_i)\right] \tag{6}$$

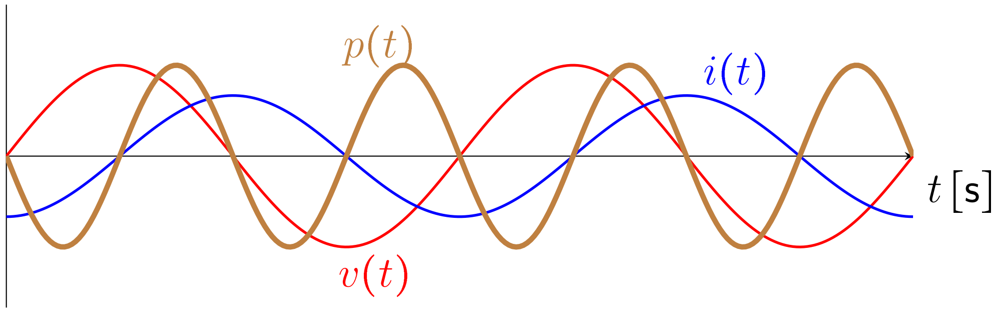
From a such modified form, it is obvious that the instantaneous power includes two terms – the constant term $\cos(\varphi_v - \varphi_i)$, independent on time, and periodic function of time $\cos(2\omega t + \varphi_v + \varphi_i)$. Let's compare the waveform of the instantaneous power for the resistor, capacitor, and inductor:



Obr. 1: Instantaneous power for a resistor



Obr. 2: Instantaneous power for a capacitor



Obr. 3: Instantaneous power for an inductor

Instantaneous power may be both positive or negative. A positive sign means that the passive circuit element receives energy; a negative sign means that it supplies energy back to the circuit. The resistor is not an accumulation element; therefore, it only absorbs energy (and converts it into heat), hence its instantaneous power $p(t) \geq 0$. The capacitor and inductor are storage elements – if the sign is positive, they charge and receive energy. If the instantaneous power sign is negative, they supply energy back to the circuit (discharge).

Just as it is usually more important for us to know that there is a voltage in the electrical outlet with an effective value of $V = 230\,\mathrm{V}$ than its time waveform, in the case of AC power, we are interested in the average value of the instantaneous power.

Average value of the second term in the equation 6 is zero, so:

$$P = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t = \frac{1}{2}V_m I_m \cos(\varphi_v - \varphi_i) \tag{7}$$

Obviously, on a resistor where the voltage and current are in phase, it is $P = \frac{1}{2}V_m I_m$. On the capacitor, the phase shift between voltage and current is $\frac{-\pi}{2}$, on inductor $\frac{\pi}{2}$, in both cases is the average power zero.

Definition 2

Active power, or average power

$$P = \frac{1}{2}V_m I_m \cos(\varphi_v - \varphi_i) \tag{8}$$

The unit of active power is Watt [W].
The active power is absorbed only by the resistor, on the capacitor, and the inductor $P = 0$.
Since the phase shift between voltage and current on the resistor is equal to 0, we can also calculate the active power using Ohm's law as:

$$P = \frac{1}{2}RI_m^2 = \frac{1}{2}\frac{V_m^2}{R} \tag{9}$$

If we know the RMS values, the active power is defined as

$$P = VI\cos(\varphi_v - \varphi_i) \tag{10}$$

In AC circuits flows current through the capacitor / inductor even though $P = 0$. In real circuits, this current, e.g., causes heating of supply wires. We must therefore be able to quantify the total energy per unit time that the source supplies to the circuit. It is quantified by:

Definition 3

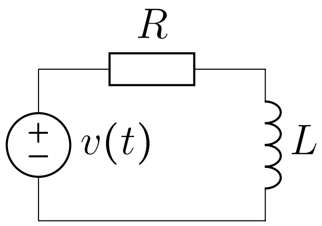
Apparent power

$$S = \frac{1}{2} V_m I_m = V I \tag{11}$$

The unit of apparent power is volt ampere [VA].

The [VA] has the same physical dimension as the Watt. We use it to distinguish what kind of power we are talking about.

Consider the following circuit:



Let e.g. $R = 1\,\mathrm{k}\Omega$, $L = 1\,\mathrm{H}$, $v(t) = 100\sin(1000t)\,\mathrm{V}$. How do we calculate AC powers here? First, we have to calculate the current, which flows from the source. Of course, we have to work with phasors and impedance.

$$\mathbf{I}_m = \frac{\mathbf{V}_m}{R + j\omega L} = \frac{100}{1000 + j\cdot 1000\cdot 1} = \frac{1}{10 + 10j} = \frac{1}{20} - \frac{1}{20}j = \frac{1}{10\sqrt{2}}\,e^{-j\frac{\pi}{4}} \tag{12}$$

Of course, we can substitute the calculated amplitude and phase into the equations 8 and 11. But can we use the calculated current phasor directly? What if we multiply the voltage and current phasors like we do in apparent power with magnitudes / RMS values?

$$\mathbf{V}\cdot\mathbf{I} = V\,e^{j\varphi_u}\cdot I\,e^{j\varphi_i} = V I\,e^{\varphi_v+\varphi_i} = V I\left[\cos(\varphi_v + \varphi_i) + j\sin(\varphi_v + \varphi_i)\right]$$

The first term of the equation is almost identical to the active power, except for the sign at the current phase shift. But we can change this by applying a complex conjugate operation *.

Definition 4

Complex power

$$\mathbf{S} = \frac{1}{2}\mathbf{V}_m\mathbf{I}_m^* = \mathbf{V}\mathbf{I}^* = VI\left[\cos(\varphi_v - \varphi_i) + j\sin(\varphi_v - \varphi_i)\right] = P + jQ \tag{13}$$

The unit of complex power is volt ampere [VA].

The real part of complex power is active power. Its module is apparent power. And the imaginary part is the still missing power, which describes the energy that flows between the reactive elements and the sources.

Definition 5

Reactive power or quadrature power

$$Q = \mathrm{Im}\{\mathbf{S}\} = \frac{1}{2}V_m I_m \sin(\varphi_v - \varphi_i) = \frac{1}{2}vI\,\sin(\varphi_v - \varphi_i) \tag{14}$$

The unit of reactive power is volt-ampere reactive [var], to distinguish it from the previous two powers.
The reactive power of the inductor is positive, the reactive power of the capacitor is negative.

So now we can calculate the AC powers in our example:

$$\mathbf{S} = \frac{1}{2}\cdot 100 \cdot \frac{1}{10\sqrt{2}}e^{j\frac{\pi}{4}} = \frac{1}{2}\cdot 100 \cdot \frac{1}{10\sqrt{2}}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = 5 + 5j$$

$$P = \mathrm{Re}\{\mathbf{S}\} = 5\,\mathrm{W}$$

$$Q = \mathrm{Im}\{\mathbf{S}\} = 5\,\mathrm{var}$$

$$S = \sqrt{5^2 + 5^2} = 7.071\,\mathrm{VA}$$

Be careful to use symbols correctly – complex power is typed in bold, or in written form it has an accent $\widehat{S}$ and it is a complex number. Apparent power is not bold, it has no accent, and it is a real number.

Since apparent power is a modulus of complex power, we can define the following relationship between powers:

Definition 6

$$S = \sqrt{P^2 + Q^2} \tag{15}$$

Using impedance and Ohm's law, we can write (in terms of effective values) the complex power in the form:

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{Z}\mathbf{I}\mathbf{I}^* = \mathbf{Z}I^2 = (R + jX)\, I^2 \tag{16}$$

Let's look at this relationship closer. First of all, it follows that:

Definition 7

$$P = RI^2 = \frac{V^2}{R} \tag{17}$$

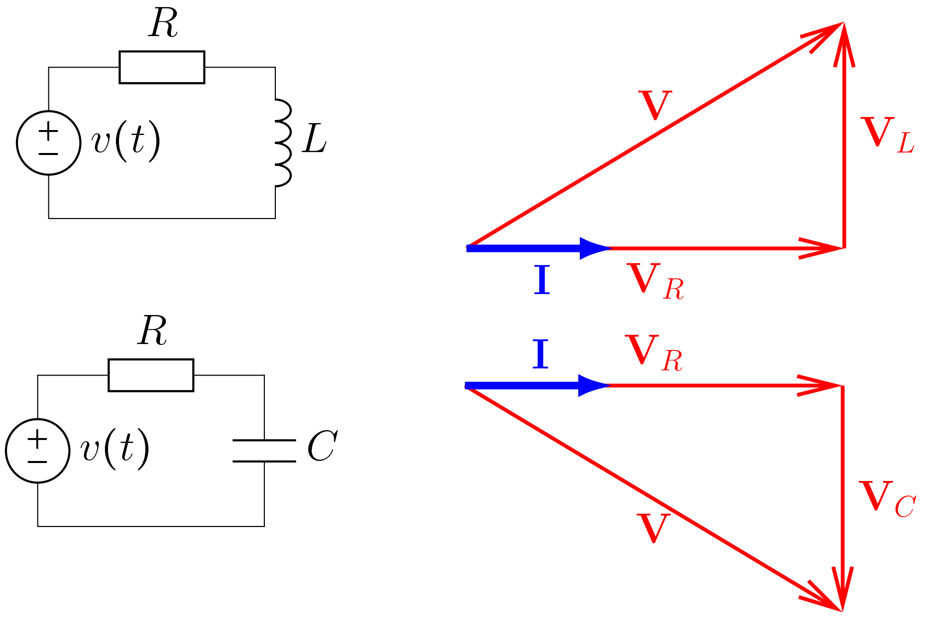
$$Q_C = \frac{-1}{\omega C} I^2 = -\omega C V_C^2 \tag{18}$$

$$Q_L = \omega L I^2 = \frac{V_L^2}{\omega L} \tag{19}$$

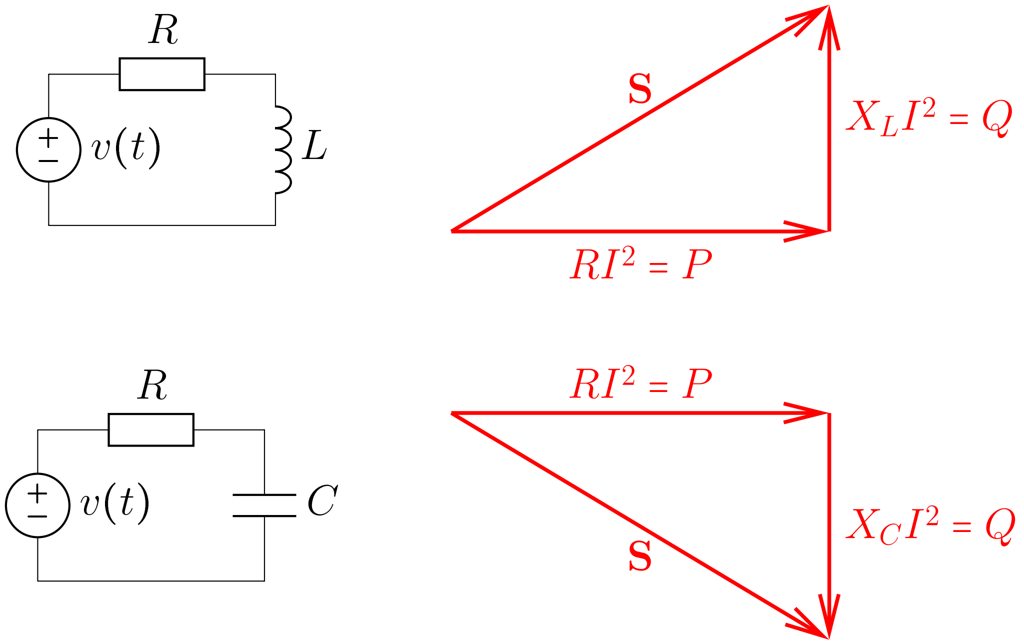
Suppose we know the current flowing through the resistor, or inductor, or capacitor, or know the voltage on these elements. In that case, we can use the equation 17 for active power, the equation 18 for the reactive power of the capacitor, and 19 for the reactive power of the inductor. The given equations are valid for the RMS value. In the maximum values scale, we multiply the result by $\frac{1}{2}$.

Furthermore, the definition of 7 implies the sign of reactive power – the sign of reactive power corresponds to the sign of reactance and is therefore related to the phase shift of current to voltage. If the current leads the voltage, the sign is negative, and vice versa.

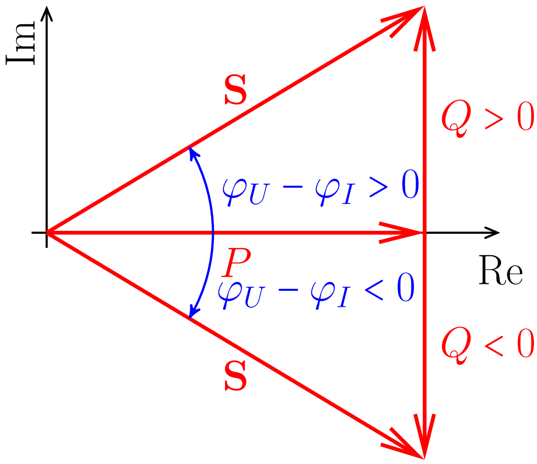
Finally, we can derive from the equation 16 a graphical representation of relationships among different power quantities, called power triangle. The term RI is the voltage across the resistor, and jXI is the voltage across the reactance element. For voltage phasors, we can draw phasor diagrams:



Similarly, we can draw in the complex plane according to the equation 16 AC power quantities:



We draw this triangle from a complex power – see Fig. 4. In the complex plane, the active power represents the real part of the complex power $\mathbf{S}$. It lays on the real axis. The reactive power represents the imaginary part of the complex power $\mathbf{S}$. For an inductor, Q is positive and lies in the first quadrant. For a capacitor, Q is negative and lies in the fourth quadrant. The complex power $\mathbf{S}$ completes the triangle. The angle that makes the complex power with the real axis represents the phase shift between voltage and current – as well as the impedance angle.

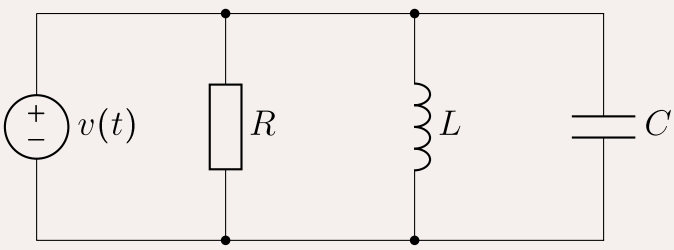


Obr. 4: Power triangle

When to use which formula?

Example 1

In the circuit in the figure, calculate the AC powers supplied by the voltage source to the circuit.



$v(t) = 100\sin(1000t)$V, $R = 1\,\mathrm{k}\Omega$, $L = 2\,\mathrm{H}$, $C = 1\,\mu\mathrm{F}$.

In this case, the voltage source is connected in parallel to the circuit elements, so we know their voltage. Therefore, we can calculate the powers directly from the equations 17, 18, 19, and 15 – version with element voltage.

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\cdot\frac{100^2}{1000} = 5\,\mathrm{W}$$

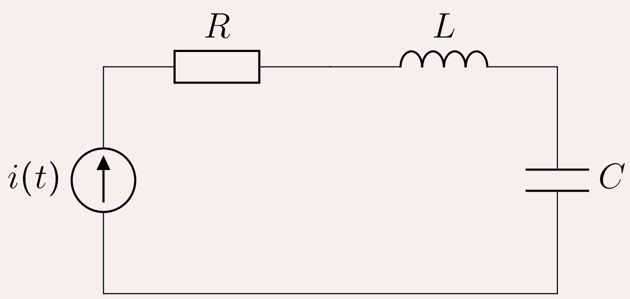
$$Q = \frac{1}{2}\frac{V_m^2}{\omega L} - \omega C U^2 = \frac{1}{2}\cdot 100^2 \cdot \left(\frac{1}{1000\cdot 2} - 1000\cdot 10^{-6}\right) = -2.5\,\mathrm{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{5^2 + (-2.5)^2} = 5.59\,\mathrm{VA}$$

Because in the assignment is the voltage waveform, which is in the maximum values scale, we must substitute the term $\frac{1}{2}$ into the equations.

Example 2

In the circuit in the figure, calculate the AC powers supplied by the current source to the circuit.



$i(t) = 100\sin(1000t)$ mA, $R = 1\,\mathrm{k\Omega}$, $L = 2\,\mathrm{H}$, $C = 1\,\mathrm{\mu F}$.

In this case, the current source is connected in series to the circuit elements, so we know their current. Therefore, we can calculate the powers directly from the equations 17, 18, 19, and 15 – version with element current.

$$P = \frac{1}{2}RI_m^2 = \frac{1}{2}\cdot 1000\cdot 0.1^2 = 5\,\mathrm{W}$$

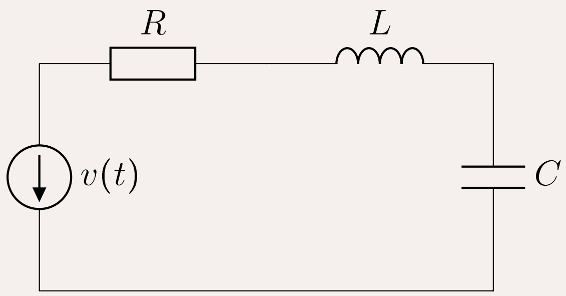
$$Q = \frac{1}{2}\omega L I_m^2 - \frac{1}{\omega C}I^2 = \frac{1}{2}\cdot 0.1^2\left(1000\cdot 2 - \frac{1}{1000\cdot 10^{-6}}\right) = 5\,\mathrm{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{5^2 + 5^2} = 7.071\,\mathrm{VA}$$

Because in the assignment is the voltage waveform, which is in the maximum values scale, we must substitute the term $\frac{1}{2}$ into the equations.

Example 3

In the circuit in the figure, calculate the AC powers supplied by the voltage source to the circuit.



$v(t) = 100\sin(1000t)$ V, $R = 1$ kΩ, $L = 2$ H, $C = 1$ μF.

In this case, the voltage source is connected in series to the circuit elements. In this circuit, we do not know the voltage on the individual elements (we would have to calculate it, e.g., using a voltage divider rules), nor the current. For that reason, we must first calculate the current.

$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\mathbf{V}\cdot j\omega C}{j\omega CR - \omega^2 LC + 1} =$$

$$= \frac{100\cdot j\cdot 1000\cdot 10^{-6}}{j\cdot 1000\cdot 10^{-6}\cdot 1000 - 1000^2\cdot 2\cdot 10^{-6} + 1} = 0.05 - 0.05j = \frac{0.1}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

Now we can choose which equations to use. As in the previous example, we can use equations 17, 18, 19, and 15 – the current version. So the first calculation option is:

$$P = \frac{1}{2}RI_m^2 = \frac{1}{2}\cdot 1000\cdot\left(\frac{0.1}{\sqrt{2}}\right)^2 = 2.5\,\mathrm{W}$$

$$Q = \frac{1}{2}\omega LI_m^2 - \frac{1}{\omega C}I^2 = \frac{1}{2}\cdot\left(1000\cdot 2 - \frac{1}{1000\cdot 10^{-6}}\right)\cdot\left(\frac{0.1}{\sqrt{2}}\right)^2 = 2.5\,\mathrm{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{2.5^2 + 2.5^2} = 3.54\,\mathrm{VA}$$

We know the voltage of the source, and the current flowing from the source, so we can also use complex power 13:

$$\mathbf{S} = \frac{1}{2} \cdot \mathbf{V}_m \mathbf{I}_m^* = \frac{1}{2} \cdot 100 \cdot \frac{0.1}{\sqrt{2}} e^{j\frac{\pi}{4}} = 2.5 + 2.5j$$

$$P = \mathrm{Re}\{\mathbf{S}\} = 2.5\,\mathrm{W}$$

$$Q = \mathrm{Im}\{\mathbf{S}\} = 2.5\,\mathrm{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{2.5^2 + 2.5^2} = 3.54\,\mathrm{VA}$$

or we can substitute into the equations 8, 14, and 11 voltage, current and the phase shift:

$$P = \frac{1}{2} V_m I_m \cos(\varphi_V - \varphi_I) = \frac{1}{2} \cdot 100 \cdot \frac{0.1}{\sqrt{2}} \cdot \cos(0 - \frac{-\pi}{4}) = 2.5\,\mathrm{W}$$

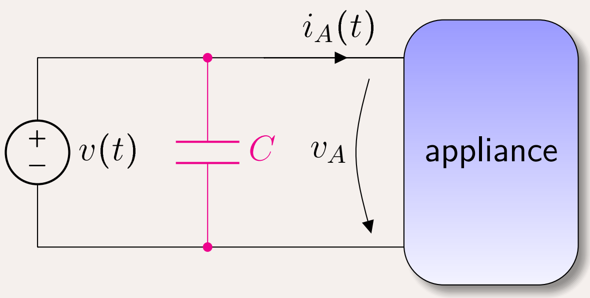
$$Q = \frac{1}{2} V_m I_m \sin(\varphi_V - \varphi_I) = \frac{1}{2} \cdot 100 \cdot \frac{0.1}{\sqrt{2}} \cdot \sin(0 - \frac{-\pi}{4}) = 2.5\,\mathrm{var}$$

$$S = \frac{1}{2} V_m I_m = \frac{1}{2} \cdot 100 \cdot \frac{0.1}{\sqrt{2}} = 3.54\,\mathrm{VA}$$

Because in the assignment is the voltage waveform, which is in the maximum values scale, we must substitute the term $\frac{1}{2}$ into the equations.

Example 4

In the circuit in the figure, calculate the AC powers supplied by the source to the circuit.



$v_A(t) = 325.36 \sin\left(\omega t + \frac{\pi}{4}\right)$ V, $i_A(t) = 0.46 \sin(\omega t)$A, $f = 50$ Hz, $C = 1\,\mu$F.

In this case, we have to combine two different procedures. We separately calculate the power absorbed by the appliance. We know the terminal voltage and the current flowing. We can therefore use the fundamental relationship for active and reactive power 8, and 14:

$$P_A = \frac{1}{2} V_{A_m} I_{A_m} \cos(\varphi_V - \varphi_I) = \frac{1}{2} \cdot 325.26 \cdot 0.46 \cdot \cos\left(\frac{-\pi}{4} - 0\right) = 52.915\,\mathrm{W}$$

$$Q_A = \frac{1}{2} V_{A_m} I_{A_m} \sin(\varphi_V - \varphi_I) = \frac{1}{2} \cdot 325.26 \cdot 0.46 \cdot \sin\left(\frac{-\pi}{4} - 0\right) = 52.915\,\mathrm{var}$$

We do not calculate the apparent power yet. First, we calculate the reactive power of the capacitor – we know the voltage on it, so we can use the equation 18.

$$Q_C = -\frac{1}{2} \omega C V_m^2 = -\frac{1}{2} \cdot 314.159 \cdot 10^{-6} \cdot 325.36^2 = -16.628\,\mathrm{var}$$

Just as we can add, for example, the wattage of light bulbs connected in a chandelier, we can add the capacitor's reactive power and the appliance's reactive power as well. Is therefore

$$Q = Q_A + Q_C = 52.915 - 16.628 = 36.286\,\mathrm{var}$$

Now we can calculate the apparent power. For the appliance and the capacitor itself, it did not make sense to calculate the apparent power. It does not apply $\cancel{S = S_A + S_C}$, since apparent power is not a linear function of P and Q. The apparent power is $S = \sqrt{P^2 + (Q_A + Q_C)^2}$, whereas $S_A = \sqrt{P_A^2 + Q_A^2}$, and $S_C = Q_C$. Is therefore

$$S = \sqrt{52.915^2 + 36.286^2} = 64.16\,\mathrm{VA}$$

Definition 8

Power factor – symbol pf, or λ:

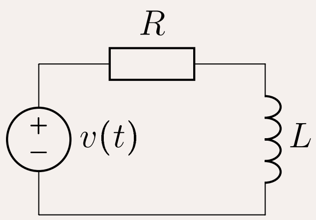
$$\lambda = \frac{P}{S} = \cos(\varphi_V - \varphi_I) \tag{20}$$

is a dimensionless quantity that indicates what portion of the energy supplied by a source has an active power. They are usually multiplied by 100, and the unit is then $[\%]$

Reactive elements – capacitors and inductors – do not perform any useful work (heat source, light, kinetic energy...). Unlike resistors, which only absorb energy, these elements store energy and then release it back to the circuit. If $\lambda = 100\,\%$, then all the power supplied from the source is used for useful work. If $\lambda = 0\,\%$, then the circuit does no useful work, although current flows from the source. The practical conductors have nonzero resistance, so this current only heats the supply wires and causes energy losses on the lines. For that reason, in power engineering, it is a very important quantity. With a low power factor, losses in the Czech energy network could be in the order of many hundreds of MW, which is why it is artificially increased – compensated for most appliances.

Example 5

The appliance consists of a series connection of inductor L and resistor R. Compensate for the circuit's power factor. $L = 0.42\,\mathrm{H}$, $R = 68\,\Omega$, $V = 230\,\mathrm{V}$ (RMS), $f = 50\,\mathrm{Hz}$



Note it is a model of a fluorescent lamp. For a more detailed description of the function, see classic slides.

First we calculate the current in the circuit.

$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L} = \frac{230}{68 + j \cdot 100 \cdot \pi \cdot 0.42} = 0.71 - 1.377j = 1.55e^{-j1.095}$$

We can now calculate the complex power

$$\mathbf{S} = \mathbf{V} \cdot \mathbf{I}^* = 230 \cdot 1.55e^{j1.095} = 356.5e^{1.095j} = 163.37 + 316.71j\,\mathrm{VA}$$

The voltage was assigned in the RMS scale, so here we do not multiply by $\frac{1}{2}$.

$$P = \mathrm{Re}\{\mathbf{S}\} = 163.37\,\mathrm{W}$$

$$Q = \mathrm{Im}\{\mathbf{S}\} = 316.71\,\mathrm{var}$$

$$S = |\mathbf{S}| = 356.5\,\text{VA}$$

The circuit has power factor

$$\lambda = \frac{P}{S} = \frac{163.7}{356.5} = 0.458 = 45.8\,\%$$

How to increase the power factor of a circuit to 100 %? Such a power factor has a circuit where $Q = 0\,\text{var}$. In the example 4 it was a similar circuit. The appliance had a reactive power of Q_A, a parallel connected capacitor Q_C, and both reactive powers were added. Since the reactive power of our RL fluorescent lamp is now positive, we will again use a capacitor connected in parallel (since it has a negative reactive power). The power factor $\lambda = 100\,\%$ is obtained if $Q_C + Q_A = 0$. Thus $Q_C = -316.71\,\text{var} = -V^2 \cdot \omega C$ a

$$C = \frac{-Q_C}{V^2 \cdot \omega} = \frac{316.71}{230^2 \cdot 100 \cdot \pi} = 19.07\,\mu\text{F}$$

The impedance of the circuit is

$$\mathbf{Z} = \frac{\frac{1}{j\omega C} \cdot (R + j\omega L)}{\frac{1}{j\omega C} + (R + j\omega L)} \doteq 324\,\Omega$$

So the impedance is a real number – it must be, otherwise would be $Q \neq 0$, and it is greater than the resistance R. From the source now flows the current $I = \frac{230}{324} = 0.71\,\text{A}$, so 45.8 % of the original value. Therefore, the current is more than twice as low due to compensation, and the heat loss in the practical conductors is more than four times less. So we can also write

$$\lambda = \frac{I_{comp}}{I} \tag{21}$$

where I_{comp} is the current drawn by the fully (100%) compensated circuit, and I is the current before compensation.

Note that we do not compensate the power factor to 100 %, because it is a so-called current resonance (we will discuss it later). Typically, the compensation is only at 95 %.

Compensation at 95 % means, that $\cos\varphi = 0.95$, where $\varphi = \varphi_V - \varphi_I$. Reactive power in such a case is not 0, but $Q = S \sin\varphi$ (see power triangle).

Since $P = S\cos\varphi = S\cdot 0.95$, the $S = \frac{P}{0.95}$. In our fluorescent lamp example, after compensation at 95 %, the apparent power $S = \frac{163.37}{0.95} = 171.97$ var. Next we need to calculate the phase shift $\varphi = \arccos 0.95 = 0.3176$ rad. Then the reactive power that will have the compensated circuit is $Q_{comp} = 171.97\cdot\sin 0.3176 = 53.7$ var. The $|Q_C| = Q - Q_{comp} = 316.71 - 53.7 = 263.71$. Furthermore, the procedure is the same. $C = \frac{-Q_C}{V^2\cdot\omega} = \frac{263.71}{230^2\cdot 100\cdot\pi} = 15.83$ µF.

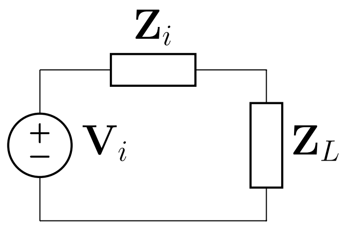
Definition 9

Maximum average power transfer:

$$\mathbf{Z}_i = \mathbf{Z}_L^* \tag{22}$$

If we have a voltage source with internal impedance $\mathbf{Z}_i$, then the source supplies to the load $\mathbf{Z}_L$ the maximum possible active power, if the condition 22 is met.

Let's have the circuit:



The load voltage $\mathbf{V}_L = \mathbf{V}_i \frac{\mathbf{Z}_L}{\mathbf{Z}_i + \mathbf{Z}_L}$. Circuit current $\mathbf{I} = \frac{\mathbf{V}_i}{\mathbf{Z}_i + \mathbf{Z}_L}$. The active power delivered to the load is a real part of the complex power:

$$P_L = \mathrm{Re}\{\mathbf{V}_L \mathbf{I}_L^*\} = \mathrm{Re}\left\{\mathbf{V}_i \frac{\mathbf{Z}_L}{\mathbf{Z}_i + \mathbf{Z}_L}\cdot\left(\frac{\mathbf{V}_i}{\mathbf{Z}_i + \mathbf{Z}_L}\right)^*\right\} =$$

$$= \mathrm{Re}\left\{\mathbf{V}_i\mathbf{V}_i^* \frac{\mathbf{Z}_L}{(\mathbf{Z}_i + \mathbf{Z}_L)\cdot(\mathbf{Z}_i + \mathbf{Z}_L)^*}\right\} =$$

$$= \mathrm{Re}\left\{V_i^2 \frac{R_L + jX_L}{(R_i + R_L)^2 + (X_i + X_L)^2}\right\} = V_i^2 \frac{R_L}{(R_i + R_L)^2 + (X_i + X_L)^2} \tag{23}$$

When editing this equation were used relations $\mathbf{V}_i\mathbf{V}_i^* = V_i\,e^{j\varphi}\,V_i\,e^{-j\varphi} = V_i^2\,e^{j(\varphi-\varphi)} = V_i^2$, $\mathbf{Z}_L = R_L + jX_L$, and $\mathbf{Z}_i = R_i + jX_i$.

Regarding the reactive component, P_L in the function 23 has maximum value, if $Q_i + Q_L = 0$, or $X_i + X_L = 0$. Regarding the resistance, we look for the extreme of the function according to the variable R_L. We solve the equation $\frac{\partial P_L}{\partial R_L} = 0$:

$$\frac{\partial P_L}{\partial R_L} = V_i^2 \frac{(R_i + R_L)^2 - 2R_L(R_i + R_L)}{(R_i + R_L)^4} = V_i^2 \frac{R_i^2 - R_L^2}{(R_i + R_L)^4} \stackrel{!}{=} 0 \tag{24}$$

That is how we derive the equation 22.

Let's go back for a moment to the power factor compensation. When compensating for 100 %, the reactive power in the circuit was Q = 0 var. In the task of maximum average power transfer, the total reactive power Q = 0 var as well. Is it also power factor compensation? In the example 5, the circuit impedance after power factor compensation was real, but 4.76 times greater than the resistor's resistance. The current drawn from the source decreased, which was the goal of compensation. When the reactive components (L and C) are connected in series, the whole circuit's impedance is real again but equal to $2R_L$ – the smallest possible impedance. In such a case, through the circuit flows the largest possible current. But it is the exact opposite of why power factor compensation is done. In the lecture devoted to resonance, we will see that this is the so-called voltage resonance. Thus:

1. Power factor compensation – parallel connection of L and C – the current resonance – minimizes the current drawn from the source.
2. Maximum average power transfer – series connection of L and C – the voltage resonance – maximizes the current drawn from the source.