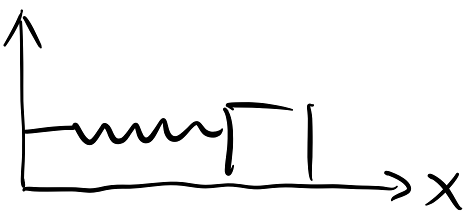
谐振运动. 阻尼与受迫振动

① Harmonic motion
② forced vibration / oscilation

(mechanical) vibration: the object reciprocates (往复) around the balancing position.

简谐运动 (harmonic motion)

① balancing position
② cos / sin change over time
③ no energy loss.

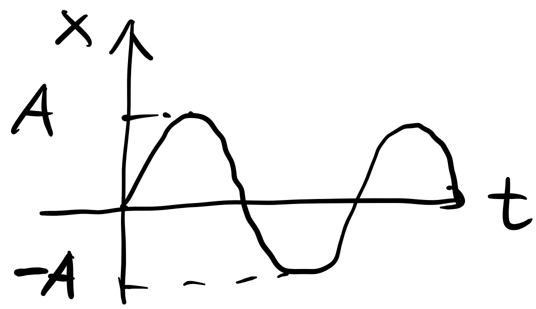


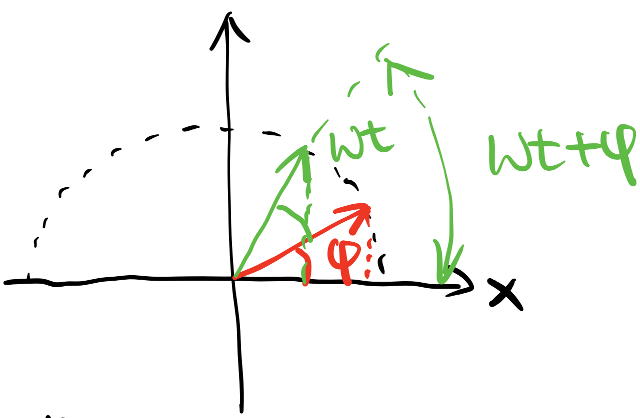
$x = A\cos(\omega t + \varphi)$

A — amplitude
ω — 圆频率 angular frequency
φ — initial phase

描述方法

① analytical method
② curve method
③ 旋转矢量法
rotation vector method





逆时针
counterclockwise

Phase difference

$x_1 = A_1\cos(\omega_1 t + \varphi_1)$ $x_2 = A_2\cos(\omega_2 t + \varphi_2)$

$$\Delta\varphi = (\omega_1 t+\varphi_1)-(\omega_2 t+\varphi_2)$$

$$\omega_1=\omega_2 \Rightarrow \Delta\varphi = \varphi_1-\varphi_2 \quad \begin{cases} \pm 2k\pi & \text{(in phase)} \\ \pm(2k+1)\pi & \text{(aniphase)} \end{cases}$$

$$x = A\cos(\omega t+\varphi)$$

Velocity & acceleration

$$v=\frac{dx}{dt} = -\omega A\sin(\omega t+\varphi) = \omega A\cos\left(\omega t+\varphi+\frac{\pi}{2}\right)$$

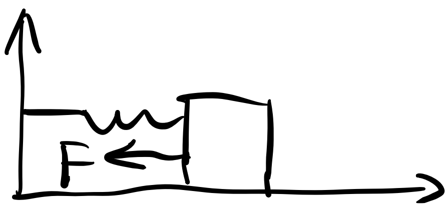
速度超前位移 $\frac{\pi}{2}$

$$a=\frac{dv}{dt} = -\omega^2 A\sin\left(\omega t+\varphi+\frac{\pi}{2}\right)$$

$$= -\omega^2 A\cos(\omega t+\varphi) = \omega^2 A\cos(\omega t+\varphi+\pi)$$

加速度超前位移 π.

$$a=-\omega^2 x$$



$$F=ma=-\underbrace{m\cdot\omega^2}_{K} x$$

$$= -Kx$$

保守力. $\xrightarrow{\text{做功}}$ 势能

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi) = \frac{1}{2}KA^2\sin^2(\omega t+\varphi)$$

$$E_p = \int F\,dx = \int -Kx\,dx = \left|-\frac{1}{2}Kx^2\right| = \frac{1}{2}Kx^2$$

$$= \frac{1}{2}KA^2\cos^2(\omega t+\varphi)$$

$$E = E_k+E_p = \frac{1}{2}KA^2$$

$$A=\sqrt{\frac{2E}{K}}$$

① mechanical energy conservation

② Kinematic $\rightleftharpoons$ potential

③ $A=\sqrt{\frac{2E}{K}}$

阻尼 damping (非主动力) ⟶ 只减缓变化，不改变变化

damped oscillation { 弹簧恢复力 recovery force ; resistance 阻力 }

定义: make the vibration slow down, even stop, but not enhance

$f = -\gamma \frac{dx}{dt}$ γ — damping coefficient

$ma = -kx \Rightarrow ma = -kx - \gamma \frac{dx}{dt}$ ← 阻力 $F = kv$ 与速度成正比 永远不为0

$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$

$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

$\frac{k}{m} = w_0^2$ w_0 natural frequency

$\frac{\gamma}{m} = 2\beta$ damping factor

$x = A\cos(wt+\varphi)$

$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + w_0^2 x = 0$

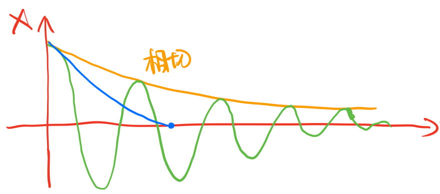
$x(t) = Ae^{\lambda t}$

$A\lambda^2 e^{\lambda t} + 2\beta A\lambda e^{\lambda t} + w_0^2 A e^{\lambda t} = 0$

$Ae^{\lambda t}\underbrace{(\lambda^2 + 2\beta\lambda + w_0^2)}$ $= 0$

$\lambda^2 + 2\beta\lambda + w_0^2 = 0$ $\lambda = \frac{-2\beta \pm \sqrt{4\beta^2 - 4w_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - w_0^2}$

Δ
① $\beta^2 - w_0^2 > 0$ 过阻尼 overdamping
② $\beta^2 - w_0^2 < 0$ 欠阻尼 underdamping
③ $\beta^2 - w_0^2 = 0$ 临界阻尼 critical damping



① $\beta^2 - w_0^2 < 0$ 欠阻尼 underdamping

$\Downarrow$

$\beta < w_0$

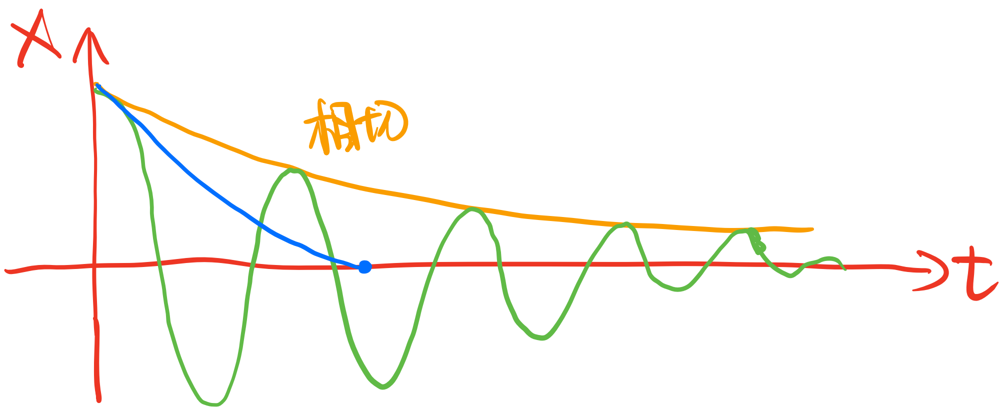
令 $\omega' = \sqrt{\omega_0^2 - \beta^2} \Rightarrow \lambda = -\beta \pm i\omega'$

$x(t) = A_1 e^{(-\beta + i\omega')t} + A_2 e^{(-\beta - i\omega')t}$

$= e^{-\beta t}(A_1 e^{i\omega' t} + A_2 e^{-i\omega' t})$

$e^{ix} = \cos x + i\sin x$ —— 欧拉公式

$x(t) = e^{-\beta t} A_0 \cos(\omega' t + \varphi)$



$T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} > \frac{2\pi}{\omega_0}$

$> T_0$

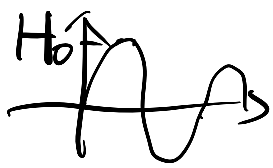
收敛

过阻尼 overdamping $\beta > \omega_0$

临界阻尼 critical damping $\beta = \omega_0$

受迫运动 forced oscillation / vibration. (可能 domine)

外力：驱动力 (driving force)



$H = H_0 \cos \omega t$

$m\frac{d^2x}{dt^2} = -kx - \gamma\frac{dx}{dt} + H_0 \cos \omega t$

$\frac{k}{m} = \omega_0^2 \quad \gamma/m = 2\beta \quad h = \frac{H_0}{m}$

$$x = A_0 e^{-\beta t}\cos\left(\sqrt{w_0^2-\beta^2}\,t + \varphi_0\right) + A\cos(wt+\varphi)$$

$t \to$ long time $\quad \to 0 \qquad$ steady state. 稳态

$$\begin{cases} A = \dfrac{h}{\left[(w_0^2-w^2)^2+4\beta^2 w^2\right]^{\frac{1}{2}}} \\ \varphi = \arctan \dfrac{-2\beta w}{w_0^2-w^2} \end{cases}$$

$w_0 = w \quad A\uparrow$

$\varphi \to -\frac{\pi}{2}$

共振. resonance

$$v = \frac{dx}{dt} = -Aw\sin(wt+\varphi) = wA\cos\left(wt+\varphi+\frac{\pi}{2}\right)$$

$$= wA\cos wt$$

$$P = F\cdot v = H_0\cos wt \cdot wA\cos wt = wAH_0\cos^2 wt$$

>0

电视. 收音机 transmit frequency = receive frequency