2023 年山东大学数学分析真题

(15 分)　$a_n > 0, \lim\limits_{n\to\infty} sup\sqrt[n]{a_n} <= 1$

证明：$\forall l > 1, \lim\limits_{n\to\infty} \dfrac{a_n}{l^n} = 0$

我们仅需证明，对$\forall l > 1, \forall \epsilon > 0, \exists N, \forall n > N, \frac{a_n}{l^n} < \epsilon$

$$\lim_{n\to\infty} sup\sqrt[n]{a_n} <= 1$$

$$\Longrightarrow \exists N_1, \forall n > N_1, \sqrt[n]{a_n} < 1 + \frac{l-1}{2}$$

显然 $\quad 0 < 1 + \frac{l-1}{2} < l$

$$\Longrightarrow \lim_{n\to\infty} (\frac{1+\frac{l-1}{2}}{l})^n = 0$$

$$\Longrightarrow \exists N_2, \forall n > N_2, (\frac{1+\frac{l-1}{2}}{l})^n < \epsilon$$

取$N = max\{N_1, N_2\}, \forall n > N,$

$$\frac{a_n}{l^n} = (\frac{\sqrt[n]{a_n}}{l})^n \leq (\frac{1+\frac{l-1}{2}}{l})^n < \epsilon$$

证毕