$$a_n = \frac{sinx}{1^2} + \frac{sin2x}{2^2} + \cdots + \frac{sinnx}{n^2}, \text{证明}\{a_n\}\text{收敛}$$

$$a_{n+p} - a_n = \frac{sin(n+1)x}{(n+1)^2} + \frac{sin(n+2)x}{(n+2)^2} + \cdots + \frac{sin(n+p)x}{(n+p)^2}$$

$$|a_{n+p} - a_n| \le |\frac{sin(n+1)x}{(n+1)^2}| + |\frac{sin(n+2)x}{(n+2)^2}| + \cdots + |\frac{sin(n+p)x}{(n+p)^2}|$$

$$\le |\frac{1}{(n+1)^2}| + |\frac{1}{(n+2)^2}| + \cdots + |\frac{1}{(n+p)^2}|$$

$$\le |\frac{1}{(n+1)(n)}| + |\frac{1}{(n+2)(n+1)}| + \cdots + |\frac{1}{(n+p)(n+p-1))}|$$

$$\le (\frac{1}{n-1} - \frac{1}{n}) + (\frac{1}{n} - \frac{1}{n+1}) + \cdots + (\frac{1}{n+p-1} - \frac{1}{n+p})$$

$$= \frac{1}{n-1} - \frac{1}{n+p}$$

$$\le \frac{1}{n-1}$$

$\forall \epsilon > 0, \frac{1}{n-1} < \epsilon,$取 $\mathrm{N} = \frac{1}{\epsilon} + 1$

$\forall p \in \mathbb{N}, n > N, |a_{n+p} - a_n| < \epsilon$

根据柯西收敛定理，得证