设$f_1(x) = af(x)+bg(x), g_1(x) = cf(x)+dg(x)$
且$ad - bc \neq 0$

证明：$(f(x), g(x)) = (f_1(x), g_1(x))$

设$u(x) = (f(x), g(x)), v(x) = (f_1(x), g_1(x))$

$u(x)|f(x), u(x)|g(x) \Longrightarrow u(x)|af(x) + bg(x), u(x)|cf(x) + dg(x)$

即$ux(x)|f_1(x), u(x)|g_1(x) \Longrightarrow u(x)|(f_1(x), g_1(x)) = v(x)$

$f_1(x) = af(x) + bg(x) \Longrightarrow df_1(x) = adf(x) + bdg(x)$

$g_1(x) = cf(x) + dg(x) \Longrightarrow bg_1(x) = bcf(x) + bdg(x)$

$(ad - bc)f_1(x) = df_1(x) - bg_1(x)$

$$\Longrightarrow f(x) = \frac{d}{ad - bc}f_1(x) + \frac{b}{bc - ad}g_1(x)$$

同理$g(x) = \dfrac{a}{bc - ad}f_1(x) + \dfrac{c}{ad - bc}g_1(x)$

由前面的结论易知$v(x)|u(x)$

$\Longrightarrow u(x)|v(x), v(x)|u(x), u(x) = v(x)$

证毕