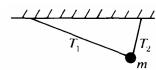
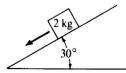
SECTION A – Linear Dynamics



1. A ball of mass m is suspended from two strings of unequal length as shown above. The magnitudes of the tensions T_1 and T_2 in the strings must satisfy which of the following relations?

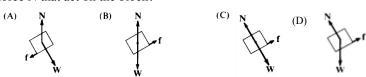
(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $T_1 < T_2$ (D) $T_1 + T_2 = mg$

Questions 2 - 3

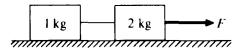


A 2-kg block slides down a 30° incline as shown above with an acceleration of 2 m/s².

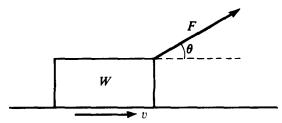
2. Which of the following diagrams best represents the gravitational force W. the frictional force f, and the normal force N that act on the block?



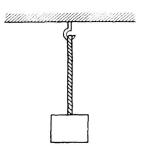
- 3. Which of the following correctly indicates the magnitudes of the forces acting up and down the incline?
 - (A) 20 N down the plane, 16 N up the plane
 - (B) 4 N down the plane, 4 N up the plane
 - (C) 0 N down the plane, 4 N up the plane
 - (D) 10 N down the plane, 6 N up the plane



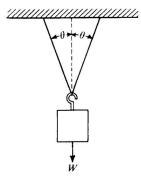
- 4. When the frictionless system shown above is accelerated by an applied force of magnitude the tension in the string between the blocks is (A) F (B) 2/3 F (C) ½ F (D) 1/3 F
- 5. A ball falls straight down through the air under the influence of gravity. There is a retarding force F on the ball with magnitude given by F = bv, where v is the speed of the ball and b is a positive constant. The ball reaches a terminal velocity after a time t. The magnitude of the acceleration at time t/2 is
 - (A) Increasing
 - (B) Decreasing
 - (C) 10 m/s/s
 - (D) Zero



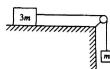
- 6. A block of weight W is pulled along a horizontal surface at constant speed v by a force F, which acts at an angle of θ with the horizontal, as shown above. The normal force exerted on the block by the surface has magnitude
 - (A) greater than W
 - (B) greater than zero but less than W
 - (C) equal to W
 - (D) zero



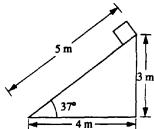
- 7. A uniform rope of weight 50 N hangs from a hook as shown above. A box of weight 100 N hangs from the rope. What is the tension in the rope?
 - (A) 75 N throughout the rope
- (B) 100 N throughout the rope
- (C) 150 N throughout the rope
- (D) It varies from 100 N at the bottom of the rope to 150 N at the top.



- 8. When an object of weight W is suspended from the center of a massless string as shown above, the tension at any point in the string is
 - (A) $2W\cos\theta$
- (B) ½Wcosθ
- (D) $W/(2cos\theta)$
- (E) $W/(cos\theta)$

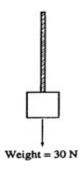


- 9. A block of mass 3m can move without friction on a horizontal table. This block is attached to another block of mass m by a cord that passes over a frictionless pulley, as shown above. If the masses of the cord and the pulley are negligible, what is the magnitude of the acceleration of the descending block?
 - (A) g/4
- (B) g/3
- (C) 2g/3
- (D) g



A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 N is placed at the top of the plane and allowed to slide down.

- 10. The magnitude of the normal force exerted on the block by the plane is
 - (A) greater than 20 N
 - (B) greater than zero but less than 20 N
 - (C) equal to 20 N
 - (D) zero
- 11. **Multiple correct:** Three forces act on an object. If the object is moving to the right in translational equilibrium, which of the following must be true? Select two answers.
 - (A) The vector sum of the three forces must equal zero.
 - (B) All three forces must be parallel.
 - (C) The magnitudes of the three forces must be equal.
 - (D) The object must be moving at a constant speed.
- 12. For which of the following motions of an object must the acceleration always be zero?
 - (A) Any motion in a straight line
 - (B) Simple harmonic motion
 - (C) Any motion at constant speed
 - (D) Any single object in motion with constant momentum



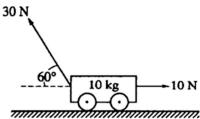
- 13. A rope of negligible mass supports a block that weighs 30 N, as shown above. The breaking strength of the rope is 50 N. The largest acceleration that can be given to the block by pulling up on it with the rope without breaking the rope is most nearly
 - (A) 6.7 m/s^2
- (B) 10 m/s^2
- (C) 16.7 m/s^2
- (D) 26.7 m/s^2

Questions 14-15

A horizontal, uniform board of weight 125 N and length 4 m is supported by vertical chains at each end. A person weighing 500 N is hanging from the board. The tension in the right chain is 250 N.

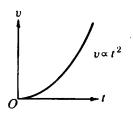
- 14. What is the tension in the left chain?
 - (A) 125 N
- (B) 250 N
- (C) 375 N
- (D) 625 N

- 15. Which of the following describes where the person is hanging?
 - (A) between the chains, but closer to the left-hand chain
 - (B) between the chains, but closer to the right-hand chain
 - (C) Right in the middle of the board
 - (D) directly below one of the chains



- 16. Multiple correct: The cart of mass 10 kg shown above moves without frictional loss on a level table. A 10 N force pulls on the cart horizontally to the right. At the same time, a 30 N force at an angle of 60° above the horizontal pulls on the cart to the left. Which of the following describes a manner in which this cart could be moving? Select two answers.
 - (A) moving left and speeding up
 - (B) moving left and slowing down
 - (C) moving right and speeding up
 - (D) moving right and slowing down
- 17. Two people are pulling on the ends of a rope. Each person pulls with a force of 100 N. The tension in the rope

- (A) 0 N (B) 50 N (C) 100 N (D) 200 N



18. The parabola above is a graph of speed v as a function of time t for an object. Which of the following graphs best represents the magnitude F of the net force exerted on the object as a function of time t?

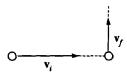




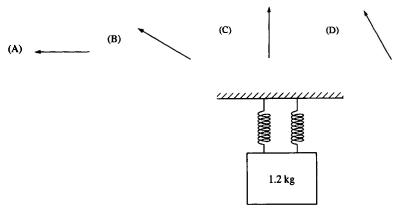




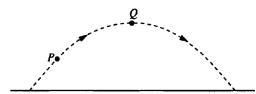




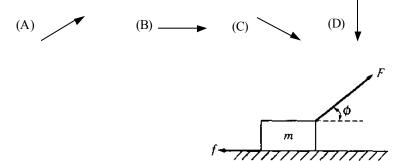
19. A ball initially moves horizontally with velocity v_i, as shown above. It is then struck by a stick. After leaving the stick, the ball moves vertically with a velocity v_f , which is smaller in magnitude than v_i . Which of the following vectors best represents the direction of the average force that the stick exerts on the ball?



- 20. Two massless springs, of spring constants k_1 and k_2 are hung from a horizontal support. A block of weight 12 N is suspended from the pair of springs, as shown above. When the block is in equilibrium, each spring is stretched an additional 24 cm. Thus, the equivalent spring constant of the two-spring system is 12 N / 24 cm = 0.5 N/cm. Which of the following statements is correct about k_1 and k_2 ?
 - (A) $k_1 = k_2 = 0.25$ N/cm
 - (B) $1/k_1 + 1/k_2 = 1/(0.5 \text{ N/cm})$
 - (C) $k_1 k_2 = 0.5$ N/cm
 - (D) $k_1 + k_2 = 0.5 \text{ N/cm}$

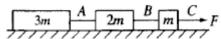


21. A ball is thrown and follows a parabolic path, as shown above. Air friction is negligible. Point Q is the highest point on the path. Which of the following best indicates the direction of the net force on the ball at point P?



- 22. A block of mass m is accelerated across a rough surface by a force of magnitude F that is exerted at an angle φ with the horizontal, as shown above. The frictional force on the block exerted by the surface has magnitude f. What is the acceleration of the block?
 - (A) F/m

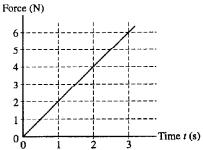
- (B) $(F\cos\phi)/m$ (C) (F-f)/m (D) $(F\cos\phi-f)/m$



23. Three blocks of masses 3m, 2m, ands are connected to strings A, B, and C as shown above. The blocks are pulled along a rough surface by a force of magnitude F exerted by string C. The coefficient of friction between each block and the surface is the same. Which string must be the strongest in order not to break?

(C) C

(D) They must all be the same strength.



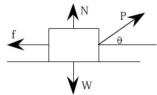
24. A block of mass 3 kg, initially at rest, is pulled along a frictionless, horizontal surface with a force shown as a function of time t by the graph above. The acceleration of the block at t = 2 s is

(A) $4/3 \text{ m/s}^2$

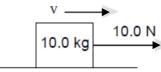
(B) 2 m/s^2

(C) 8 m/s^2

(D) 12 m/s^2



- 25. A student pulls a wooden box along a rough horizontal floor at constant speed by means of a force P as shown to the right. Which of the following must be true?
 - (A) P > f and N < W.
 - (B) P > f and N = W.
 - (C) P = f and N > W.
 - (D) P = f and N = W.



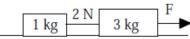
- 26. The 10.0 kg box shown in the figure to the right is sliding to the right along the floor. A horizontal force of 10.0 N is being applied to the right. The coefficient of kinetic friction between the box and the floor is 0.20. The box is moving with:
 - (A) acceleration to the left.
- (B) acceleration to the right.
- (C) constant speed and constant velocity. (D) constant speed but not constant velocity.
- 27. Assume the objects in the following diagrams have equal mass and the strings holding them in place are identical. In which case would the string be most likely to break?



В.



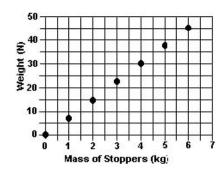
D. All would be equally likely to break



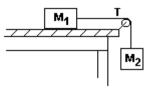
- 28. Two blocks of mass 1.0 kg and 3.0 kg are connected by a string which has a tension of 2.0 N. A force F acts in the direction shown to the right. Assuming friction is negligible, what is the value of F? (A) 2.0 N (B) 4.0 N (C) 6.0 N (D) 8.0 N
- 29. A 50-kg student stands on a scale in an elevator. At the instant the elevator has a downward acceleration of 1.0 m/s² and an upward velocity of 3.0 m/s, the scale reads approximately (A) 350 N (B) 450 N (C) 500 N (D) 550 N



- 30. A tractor-trailer truck is traveling down the road. The mass of the trailer is 4 times the mass of the tractor. If the tractor accelerates forward, the force that the trailer applies on the tractor is
 - (A) 4 times greater than the force of the tractor on the trailer.
 - (B) 2 times greater than the force of the tractor on the trailer.
 - (C) equal to the force of the tractor on the trailer.
 - (D) ¹/₄ the force of the tractor on the trailer.
- 31. A wooden box is first pulled across a horizontal steel plate as shown in the diagram A. The box is then pulled across the same steel plate while the plate is inclined as shown in diagram B. How does the force required to overcome friction in the inclined case (B) compare to the horizontal case (A)?
 - (A) the frictional force is the same in both cases
 - (B) the inclined case has a greater frictional force
 - (C) the inclined case has less frictional force
 - (D) the frictional force increases with angle until the angle is 90°, then drops to zero

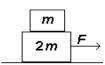


- 32. The graph at left shows the relationship between the mass of a number of rubber stoppers and their resulting weight on some far-off planet. The slope of the graph is a representation of the:
- (A) mass of a stopper
- (B) density of a stopper
- (C) acceleration due to gravity
- (D) number of stoppers for each unit of weight



- 33. Two masses, m_1 and m_2 , are connected by a cord and arranged as shown in the diagram with m_1 sliding along on a frictionless surface and m_2 hanging from a light frictionless pulley. What would be the mass of the falling mass, m_2 , if both the sliding mass, m_1 , and the tension, T, in the cord were known?
- (A) $\frac{m_1g-T}{g}$ (B) $\frac{1}{2}Tg$ (C) $\frac{m_1(T-g)}{\left(gm_1-T\right)}$ (D) $\frac{Tm_1}{\left(gm_1-T\right)}$

- 34. A mass is suspended from the roof of a lift (elevator) by means of a spring balance. The lift (elevator) is moving upwards and the readings of the spring balance are noted as follows:
 - Speeding up: R_U Constant speed: R_C Slowing down: R_D
 - Which of the following is a correct relationship between the readings?
- (A) $R_U > R_C$ (B) $R_U = R_D$ (C) $R_C < R_D$ (D) $R_C < R_D$



- 35. A small box of mass m is placed on top of a larger box of mass 2m as shown in the diagram at right. When a force F is applied to the large box, both boxes accelerate to the right with the same acceleration. If the coefficient of friction between all surfaces is μ , what would be the force accelerating the smaller mass?
 - (A) $\frac{F}{3} mg\mu$

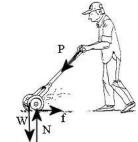
- (B) $F 3mg\mu$ (C) $F mg\mu$ (D) $\frac{F mg\mu}{3}$
- 36. A 6.0 kg block initially at rest is pushed against a wall by a 100 N force as shown. The coefficient of kinetic friction is 0.30 while the coefficient of static friction is 0.50. What is true of the friction acting on the block after a time of 1 second?



- (A) Static friction acts upward on the block.
- (B) Kinetic friction acts upward on the block
- (C) Kinetic friction acts downward on the block.
- (D) Static friction acts downward on the block.
- 37. A homeowner pushes a lawn mower across a horizontal patch of grass with a constant speed by applying a force P. The arrows in the diagram correctly indicate the directions but not necessarily the magnitudes of the various forces on the lawn mower. Which of the following relations among the various force magnitudes, W, f, N, P is



- (A) P > f and N > W
- (B) P < f and N = W
- (C) P > f and N < W
- (D) P = f and N > W



- 38. A mass, M, is at rest on a frictionless surface, connected to an ideal horizontal spring that is unstretched. A person extends the spring 30 cm from equilibrium and holds it at this location by applying a 10 N force. The spring is brought back to equilibrium and the mass connected to it is now doubled to 2M. If the spring is extended back 30 cm from equilibrium, what is the necessary force applied by the person to hold the mass stationary there?

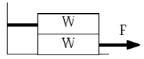
- (A) 20.0 N (B) 14.1 N (C) 10.0 N (D) 7.07 N
- 39. A crate of toys remains at rest on a sleigh as the sleigh is pulled up a hill with an increasing speed. The crate is not fastened down to the sleigh. What force is responsible for the crate's increase in speed up the hill?
 - (A) the contact force (normal force) of the ground on the sleigh
 - (B) the force of static friction of the sleigh on the crate
 - (C) the gravitational force acting on the sleigh
 - (D) no force is needed

40. A box slides to the right across a horizontal floor. A person called Ted exerts a force *T* to the right on the box. A person called Mario exerts a force *M* to the left, which is half as large as the force *T*. Given that there is friction *f* and the box accelerates to the right, rank the sizes of these three forces exerted on the box.

(A) f < M < T (B) M < f < T (C) M < T < f (D) f = M < T

41. A spaceman of mass 80 kg is sitting in a spacecraft near the surface of the Earth. The spacecraft is accelerating upward at five times the acceleration due to gravity. What is the force of the spaceman on the spacecraft?

(A) 4800 N (B) 4000 N (C) 3200 N (D) 800 N



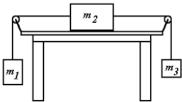
- 42. Two identical blocks of weight W are placed one on top of the other as shown in the diagram above. The upper block is tied to the wall. The lower block is pulled to the right with a force F. The coefficient of static friction between all surfaces in contact is μ . What is the largest force F that can be exerted before the lower block starts to slip?
 - (A) μW
- (B) 2µW
- (C) 3µW
- (D) $3\mu W/2$



43. A force F is used to hold a block of mass m on an incline as shown in the diagram (see above). The plane makes an angle of θ with the horizontal and F is perpendicular to the plane. The coefficient of friction between the plane and the block is μ . What is the minimum force, F, necessary to keep the block at rest?

(A) mgcos θ (B) mgsin θ (C) mgsin θ/μ (D) mg(sin $\theta - \mu$ cos θ)/ μ

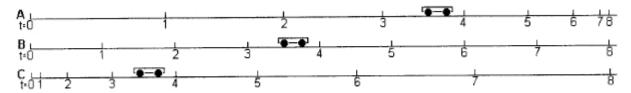
- 44. When the speed of a rear-drive car is increasing on a horizontal road, what is the direction of the frictional force on the tires?
 - (A) backward on the front tires and forward on the rear tires
 - (B) forward on the front tires and backward on the rear tires
 - (C) forward on all tires
 - (D) backward on all tires



- 45. Given the three masses as shown in the diagram above, if the coefficient of kinetic friction between the large mass (m_2) and the table is μ , what would be the upward acceleration of the small mass (m_3) ? The mass and friction of the cords and pulleys are small enough to produce a negligible effect on the system.
 - (A) $g(m_1 + m_2\mu)/(m_1 + m_2 + m_3)$ (B) $g\mu(m_1 + m_2 + m_3)/(m_1 m_2 m_3)$
 - $(C) \; g \mu (m_1 m_2 m_3) / \; (m_1 + m_2 + m_3) \quad (D) \; g (m_1 \mu m_2 m_3) / \; (m_1 + m_2 + m_3)$

Questions 47-48

Three identical laboratory carts A, B, and C are each subject to a constant force F_A , F_B , and F_C , respectively. One or more of these forces may be zero. The diagram below shows the position of each cart at each second of an 8.0 second interval.



47. Which car has the greatest average velocity during the interval?

(A) A (B) B (C) C (D) all three average velocities are equal

48. How does the magnitude of the force acting on each car compare?

(A) $F_A > F_B > F_C$ (B) $F_A = F_C > F_B$ (C) $F_A > F_C = F_B$ (D) $F_A = F_B > F_C$

49. A skydiver is falling at terminal velocity before opening her parachute. After opening her parachute, she falls at a much smaller terminal velocity. How does the total upward force before she opens her parachute compare to the total upward force after she opens her parachute?

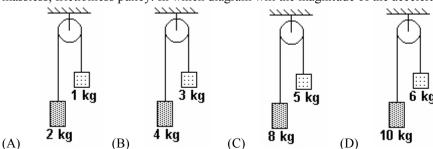
(A) The ratio of the forces is equal to the ratio of the velocities.

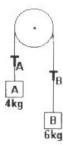
(B) The upward force with the parachute will depend on the size of the parachute.

(C) The upward force before the parachute will be greater because of the greater velocity.

(D) The upward force in both cases must be the same.

50. Each of the diagrams below represents two weights connected by a massless string which passes over a massless, frictionless pulley. In which diagram will the magnitude of the acceleration be the largest?

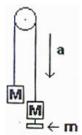




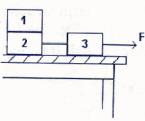
51. A simple Atwood's machine is shown in the diagram above. It is composed of a frictionless lightweight pulley with two cubes connected by a light string. If cube A has a mass of 4.0 kg and cube B has a mass of 6.0 kg, the system will move such that cube B accelerates downwards. What would be the tension in the two parts of the string between the pulley and the cubes?

(A) $T_A = 47 \text{ N}$; $T_B = 71 \text{ N}$ (B) $T_A = 47 \text{ N}$; $T_B = 47 \text{ N}$ (C) $T_A = 47 \text{ N}$; $T_B = 42 \text{ N}$

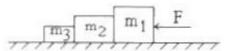
(D) $T_A = 39 \text{ N}$; $T_B = 39 \text{ N}$



- 52. A simple Atwood's machine remains motionless when equal masses M are placed on each end of the chord. When a small mass m is added to one side, the masses have an acceleration a. What is M? You may neglect friction and the mass of the cord and pulley. (A) $\frac{m(g-a)}{2a}$ (B) $\frac{2m(g-a)}{a}$ (C) $\frac{2m(g+a)}{a}$ (D) $\frac{m(g+a)}{2a}$



- 53. Block 1 is stacked on top of block 2. Block 2 is connected by a light cord to block 3, which is pulled along a frictionless surface with a force F as shown in the diagram. Block 1 is accelerated at the same rate as block 2 because of the frictional forces between the two blocks. If all three blocks have the same mass m, what is the minimum coefficient of static friction between block 1 and block 2?
- (A) 2F/mg (B) F/mg (C) 3F/2mg (D) F/3mg



- 54. Three blocks $(m_1, m_2, \text{ and } m_3)$ are sliding at a constant velocity across a rough surface as shown in the diagram above. The coefficient of kinetic friction between each block and the surface is μ . What would be the force of m_1 on m_2 ?

- (A) $(m_2 + m_3)g\mu$ (B) $F (m_2 m_3)g\mu$ (C) F (D) $m_1g\mu (m_2 + m_3)g\mu$

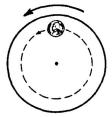
SECTION B - Circular Motion

- 1. **Multiple Correct:** A person stands on a merry-go-round which is rotating at constant angular speed. Which of the following are true about the frictional force exerted on the person by the merry-go-round? Select two answers.
 - (A) The force is greater in magnitude than the frictional force exerted on the person by the merry-go-round.
 - (B) The force is opposite in direction to the frictional force exerted on the merry-go-round by the person.
 - (C) The force is directed away from the center of the merry-go-round.
 - (D) The force is dependent on the person's mass.
- 2. A ball attached to a string is whirled around in a horizontal circle having a radius R. If the radius of the circle is changed to 4R and the same centripetal force is applied by the string, the new speed of the ball is which of the following?
 - (A) One-quarter the original speed
 - (B) One-half the original speed
 - (C) The same as the original speed
 - (D) Twice the original speed



View of Track from Above

- 3. A racing car is moving around the circular track of radius 300 meters shown above. At the instant when the car's velocity is directed due east, its acceleration is directed due south and has a magnitude of 3 meters per second squared. When viewed from above, the car is moving
 - (A) clockwise at 30 m/s
 - (B) clockwise at 10 m/s
 - (C) counterclockwise at 30 m/s
 - (D) counterclockwise at 10 m/s



View from Above

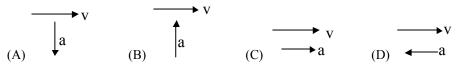
4. The horizontal turntable shown above rotates at a constant rate. As viewed from above, a coin on the turntable moves counterclockwise in a circle as shown. Which of the following vectors best represents the direction of the frictional force exerted on the coin by the turntable when the coin is in the position shown?



- 5. In which of the following situations would an object be accelerated? Select two answers.
 - (A) It moves in a straight line at constant speed.
 - (B) It moves with uniform circular motion.
 - (C) It travels as a projectile in a gravitational field with negligible air resistance.
 - (D) It is at rest.



6. An automobile moves at constant speed down one hill and up another hill along the smoothly curved surface shown above. Which of the following diagrams best represents the directions of the velocity and the acceleration of the automobile at the instant that it is at the lowest position. as shown?



- 7. A car initially travels north and then turns to the left along a circular curve. This causes a package on the seat of the car to slide toward the right side of the car. Which of the following is true of the net force on the package while it is sliding?
 - (A) The force is directed away from the center of the circle.
 - (B) There is not enough force directed north to keep the package from sliding.
 - (C) There is not enough force tangential to the car's path to keep the package from sliding.
 - (D) There is not enough force directed toward the center of the circle to keep the package from sliding.
- A child has a toy tied to the end of a string and whirls the toy at constant speed in a horizontal circular path of radius R. The toy completes each revolution of its motion in a time period T. What is the magnitude of the acceleration of the toy?

(A) Zero (B) $\frac{4\pi^2 R}{T^2}$ (C) $\frac{\pi R}{T^2}$

(D) g



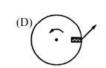
Top View

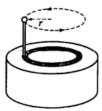
9. A compressed spring mounted on a disk can project a small ball. When the disk is not rotating, as shown in the top view above, the ball moves radially outward. The disk then rotates in a counterclockwise direction as seen from above, and the ball is projected outward at the instant the disk is in the position shown above. Which of the following best shows the subsequent path of the ball relative to the ground?



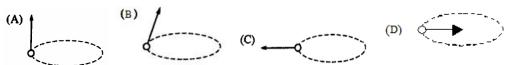




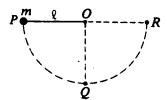




10. A steel ball supported by a stick rotates in a circle of radius *r*, as shown above. The direction of the net force acting on the ball when it is in the position shown is indicated by which of the following?



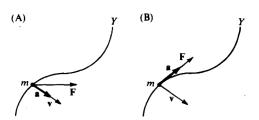
11. Inside a washing machine, the radius of the cylinder where the clothes sit is 0.50 m. In one of its settings the machine spins the cylinder at 2.0 revolutions per second. What is the acceleration of an item of clothing?
 (A) 0.080 m/s² (B) 1.6 m/s² (C) 8.0 m/s² (D) 79 m/s²

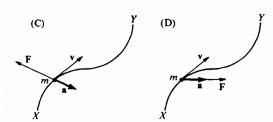


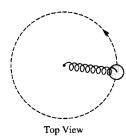
12. A ball of mass m is attached to the end of a string of length Q as shown above. The ball is released from rest from position P. where the string is horizontal. It swings through position Q. where the string is vertical, and then to position R. where the string is again horizontal. What are the directions of the acceleration vectors of the ball at positions Q and R?

Position Q
(A) Downward
(B) Downward
(C) Upward
Downward
(D) Upward
Downward
To the left

13. A mass m moves on a curved path from point X to point Y. Which of the following diagrams indicates a possible combination of the net force F on the mass, and the velocity v and acceleration a of the mass at the location shown?



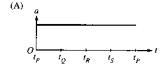


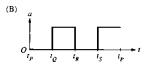


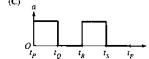
- 14. A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view above. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible. What is the centripetal force on the disc?
 - (A) 0.3 N
- (B) 3N
- (C) 10 N
- (D) 300 N

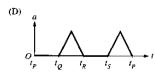


15. A figure of a dancer on a music box moves counterclockwise at constant speed around the path shown above. The path is such that the lengths of its segments, *PQ*, *QR*, *RS*, and *SP*, are equal. Arcs *QR* and *SP* are semicircles. Which of the following best represents the magnitude of the dancer's acceleration as a function of time t during one trip around the path, beginning at point *P*?





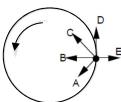




- 16. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
 - (A) vertically upward
 - (B) horizontally forward
 - (C) horizontally backward
 - (D) zero

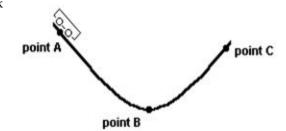


- 17. A car is traveling on a road in hilly terrain, see figure to the right. Assume the car has speed *v* and the tops and bottoms of the hills have radius of curvature *R*. The driver of the car is most likely to feel weightless:
- (A) at the top of a hill when $v > \sqrt{gR}$
- (B) at the bottom of a hill when $v > \sqrt{gR}$
- (C) going down a hill when $v = \sqrt{gR}$
- (D) at the top of a hill when v = gR



- 18. An object shown in the accompanying figure moves in uniform circular motion. Which arrow best depicts the net force acting on the object at the instant shown?
 - (A) A (B) B (C) C (D) D

- 19. Multiple Correct: A child whirls a ball at the end of a rope, in a uniform circular motion. Which of the following statements is true? Select two answers.
 - (A) The speed of the ball is constant
 - (B) The velocity is of the ball is constant
 - (C) The magnitude of the ball's acceleration is constant
 - (D) The net force on the ball is directed radially outwards
- 20. An astronaut in an orbiting space craft attaches a mass m to a string and whirls it around in uniform circular motion. The radius of the circle is R, the speed of the mass is ν , and the tension in the string is F. If the mass, radius, and speed were all to double the tension required to maintain uniform circular motion would be
 - (A) F
- (B) 2F
- (C) 4F
- (D) 8F
- 21. Assume the roller coaster cart rolls along the curved track from point A to point C under the influence of gravity. Assume the friction between the track and the cart is negligible. What would be the direction of the cart's acceleration at point B?



- (A) upward (B) downward
- (C) forward
- (D) backward

Questions 22 - 23

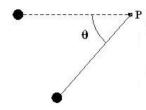
The diagram below is a snapshot of three cars all moving counterclockwise during a one lap race on an elliptical track.



- 22. Which car has had the lowest average speed during the race so far?
 - (A) car A
 - (B) car B
 - (C) car C
 - (D) all three cars have had the same average speed
- 23. Which car at the moment of the snapshot MUST have a net force acting on it?
 - (A) car A
 - (B) car B
 - (C) car C
 - (D) all three cars have net forces acting on them

24. A centripetal force of 5.0 newtons is applied to a rubber stopper moving at a constant speed in a horizontal circle. If the same force is applied, but the radius is made smaller, what happens to the speed, v, and the frequency, f, of the stopper?

- (A) *v* increases and *f* increases
- (B) v decreases and f decreases
- (C) v increases and f decreases
- (D) v decreases and f increases



25. Astronauts on the Moon perform an experiment with a simple pendulum that is released from the horizontal position at rest. At the moment shown in the diagram with $0^{\circ} < \theta < 90^{\circ}$, the total acceleration of the mass may be directed in which of the following ways?

- (A) straight to the right
- (B) straight upward
- (C) straight downward
- (D) straight along the connecting string toward point P (the pivot)

26. A 4.0 kg mass is attached to one end of a rope 2 m long. If the mass is swung in a vertical circle from the free end of the rope, what is the tension in the rope when the mass is at its highest point if it is moving with a speed of 5 m/s?

(A) 5.4 N (B) 10.8 N (C) 50 N (D) 65.4 N

27. A ball of mass m is fastened to a string. The ball swings at constant speed in a vertical circle of radius R with the other end of the string held fixed. Neglecting air resistance, what is the difference between the string's tension at the bottom of the circle and at the top of the circle?

(A) mg (B) 2mg (C) 4mg (D) 8mg

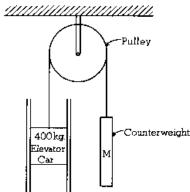
28. An object weighing 4 newtons swings on the end of a string as a simple pendulum. At the bottom of the swing, the tension in the string is 6 newtons. What is the magnitude of the centripetal acceleration of the object at the bottom of the swing?

- (A) 0.5 g
- (B) g
- (C) 1.5 g (D) 2.5 g

29. Riders in a carnival ride stand with their backs against the wall of a circular room of diameter 8.0 m. The room is spinning horizontally about an axis through its center at a rate of 45 rev/min when the floor drops so that it no longer provides any support for the riders. What is the minimum coefficient of static friction between the wall and the rider required so that the rider does not slide down the wall?

- (A) 0.056 (B) 0.11 (C) 0.53 (D) 8.9

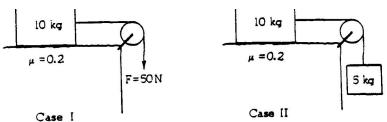
SECTION A – Linear Dynamics



- 1976B1. The two guide rails for the elevator shown above each exert a constant friction force of 100 newtons on the elevator car when the elevator car is moving upward with an acceleration of 2 meters per second squared. The pulley has negligible friction and mass. Assume $g = 10 \text{ m/sec}^2$.
- a. On the diagram below, draw and label all forces acting on the elevator car. Identify the source of each force.



- b. Calculate the tension in the cable lifting the 400-kilogram elevator car during an upward acceleration of 2 m/sec². (Assume g 10 m/sec².)
- c. Calculate the mass M the counterweight must have to raise the elevator car with an acceleration of 2 m/sec².

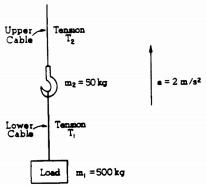


- 1979B2. A 10-kilogram block rests initially on a table as shown in cases I and II above. The coefficient of sliding friction between the block and the table is 0.2. The block is connected to a cord of negligible mass, which hangs over a massless frictionless pulley. In case I a force of 50 newtons is applied to the cord. In case II an object of mass 5 kilograms is hung on the bottom of the cord. Use g = 10 meters per second squared.
- a. Calculate the acceleration of the 10-kilogram block in case I.
- b. On the diagrams below, draw and label all the forces acting on each block in case II



5 kg

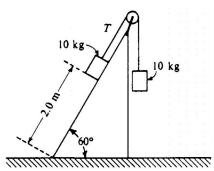
c. Calculate the acceleration of the 10-kilogram block in case II.



- 1982B2. A crane is used to hoist a load of mass $m_1 = 500$ kilograms. The load is suspended by a cable from a hook of mass $m_2 = 50$ kilograms, as shown in the diagram above. The load is lifted upward at a constant acceleration of 2 m/s².
- a. On the diagrams below draw and label the forces acting on the hook and the forces acting on the load as they accelerate upward



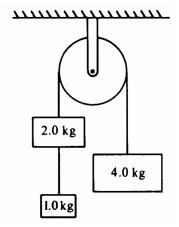
b. Determine the tension T_1 in the lower cable and the tension T_2 in the upper cable as the hook and load are accelerated upward at 2 m/s². Use g = 10 m/s².



- 1985B2 (modified) Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10 \text{ m/s}^2$, $\sin 60^{\circ} = 0.87$, and $\cos 60^{\circ} = 0.50$.
- a. What is the tension T in the string?
- b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

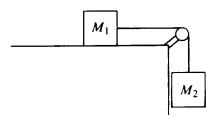


c. Determine the magnitude of the frictional force acting on the box on the plane.



1986B1. Three blocks of masses 1.0, 2.0, and 4.0 kilograms are connected by massless strings, one of which passes over a frictionless pulley of negligible mass, as shown above. Calculate each of the following.

- a. The acceleration of the 4-kilogram block
- b. The tension in the string supporting the 4-kilogram block
- c. The tension in the string connected to the l-kilogram block



1987B1. In the system shown above, the block of mass M_1 is on a rough horizontal table. The string that attaches it to the block of mass M_2 passes over a frictionless pulley of negligible mass. The coefficient of kinetic friction μ_k between M_1 and the table is less than the coefficient of static friction μ_s

a. On the diagram below, draw and identify all the forces acting on the block of mass M₁.



b. In terms of M_1 and M_2 determine the minimum value of μ_s that will prevent the blocks from moving.

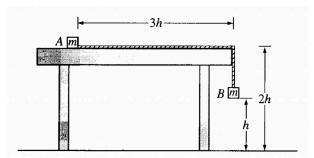
The blocks are set in motion by giving M_2 a momentary downward push. In terms of M_1 , M_2 , μ_k , and g, determine each of the following:

- c. The magnitude of the acceleration of M₁
- d. The tension in the string.

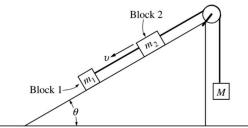
- 1988B1. A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of 5.2 m/s². Neglect air resistance on the package.
- a. On the diagram below, draw and label all of the forces acting on the package.



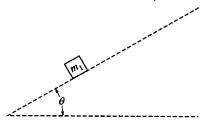
- b. Determine the tension in the rope.
- c. When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at 5.2 m/s². Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.



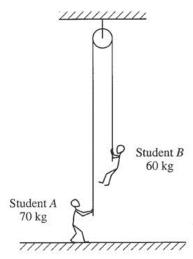
- 1998B1 Two small blocks, each of mass m, are connected by a string of constant length 4h and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance 2h above the floor. Block B is then released from rest at a distance h above the floor at time t = 0. Express all algebraic answers in terms of h, m, and g.
- a. Determine the acceleration of block B as it descends.
- b. Block B strikes the floor and does not bounce. Determine the time $t = t_1$ at which block B strikes the floor.
- c. Describe the motion of block A from time t = 0 to the time when block B strikes the floor.
- d. Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.
- e. Determine the distance between the landing points of the two blocks.



- 2000B2. Blocks 1 and 2 of masses m_1 and m_2 , respectively, are connected by a light string, as shown above. These blocks are further connected to a block of mass M by another light string that passes over a pulley of negligible mass and friction. Blocks 1 and 2 move with a constant velocity v down the inclined plane, which makes an angle θ with the horizontal. The kinetic frictional force on block 1 is f and that on block 2 is 2f.
- a. On the figure below, draw and label all the forces on block m₁.



- Express your answers to each of the following in terms of m_1 , m_2 , g, θ , and f.
- b. Determine the coefficient of kinetic friction between the inclined plane and block 1.
- c. Determine the value of the suspended mass *M* that allows blocks 1 and 2 to move with constant velocity down the plane.
- d. The string between blocks 1 and 2 is now cut. Determine the acceleration of block 1 while it is on the inclined plane.



2003B1 A rope of negligible mass passes over a pulley of negligible mass attached to the ceiling, as shown above. One end of the rope is held by Student A of mass 70 kg, who is at rest on the floor. The opposite end of the rope is held by Student B of mass 60 kg, who is suspended at rest above the floor. Use $g = 10 \text{ m/s}^2$.

a. On the dots below that represent the students, draw and label free-body diagrams showing the forces on Student A and on Student B.

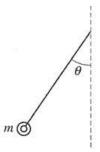
B

A

b. Calculate the magnitude of the force exerted by the floor on Student A.

Student B now climbs up the rope at a constant acceleration of 0.25 m/s² with respect to the floor.

- c. Calculate the tension in the rope while Student B is accelerating.
- d. As Student B is accelerating, is Student A pulled upward off the floor? Justify your answer.
- e. With what minimum acceleration must Student B climb up the rope to lift Student A upward off the floor?



- 2003Bb1 (modified) An airplane accelerates uniformly from rest. A physicist passenger holds up a thin string of negligible mass to which she has tied her ring, which has a mass m. She notices that as the plane accelerates down the runway, the string makes an angle θ with the vertical as shown above.
- a. In the space below, draw a free-body diagram of the ring, showing and labeling all the forces present.



The plane reaches a takeoff speed of 65 m/s after accelerating for a total of 30 s.

- b. Determine the minimum length of the runway needed.
- c. Determine the angle θ that the string makes with the vertical during the acceleration of the plane before it leaves the ground.



- *1996B2 (modified) A spring that can be assumed to be ideal hangs from a stand, as shown above. You wish to determine experimentally the spring constant k of the spring.
- a. i. What additional, commonly available equipment would you need?
 - ii. What measurements would you make?
 - iii. How would k be determined from these measurements?

Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.

- b. i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
 - ii. Explain how you would make the determination.



B2007B1. An empty sled of mass 25 kg slides down a muddy hill with a constant speed of 2.4 m/s. The slope of the hill is inclined at an angle of 15° with the horizontal as shown in the figure above.

- a. Calculate the time it takes the sled to go 21 m down the slope.
- b. On the dot below that represents the sled, draw/label a free-body diagram for the sled as it slides down the slope

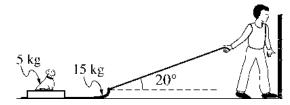


- c. Calculate the frictional force on the sled as it slides down the slope.
- d. Calculate the coefficient of friction between the sled and the muddy surface of the slope.
- e. The sled reaches the bottom of the slope and continues on the horizontal ground. Assume the same coefficient of friction.
 - i. In terms of velocity and acceleration, describe the motion of the sled as it travels on the horizontal ground.
 - ii. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel down the slope and across the horizontal ground. Clearly indicate with the symbol t_{ℓ} the time at which the sled leaves the slope.



B2007b1 (modified) A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20° above the horizontal, as shown in the figure. The coefficient of friction between the sled and the surface is 0.22.

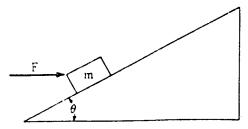
a. On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.



•

- b. Calculate the normal force of the surface on the system.
- c. Calculate the acceleration of the system.
- d. At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_t the time at which the dog rolls off.

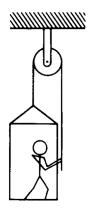




- 1981M1. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ .
- a. On the diagram of the block below, draw and label all the forces that act on the block as it slides up the plane.



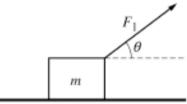
- b. Develop an expression in terms of m, θ , F, μ , and g, for the block's acceleration up the plane.
- c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?



- 1986M1. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10 \text{ m/ s}^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.
- a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is 2 m/s² upward.

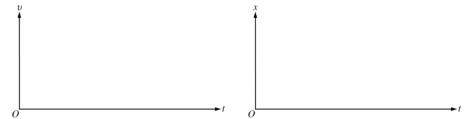
- b. What is the tension in the rope under these new conditions?
- c. Under these conditions, what is the force exerted by the platform on the person?



- 2007M1. A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1 . Express all algebraic answers in terms of m, F_1 , θ , a_1 , and fundamental constants.
- a. On the figure below, draw and label a free-body diagram showing all the forces on the block.



- b. Derive an expression for the normal force exerted by the surface on the block.
- c. Derive an expression for the coefficient of kinetic friction μ between the block and the surface.
- d. On the axes below, sketch graphs of the speed v and displacement x of the block as functions of time t if the block started from rest at x = 0 and t = 0.



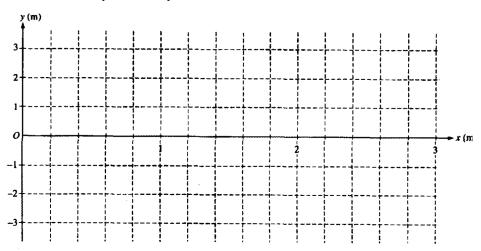
e. If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.

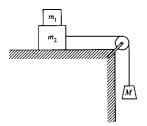


- 1996M2. A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5 m/s^2
- a. Determine the upward force exerted by the horizontal platform on the box as it is lowered.

At time t = 0, the forklift also begins to move forward with an acceleration of 2 m/s² while lowering the box as described above. The box does not slip or tip over.

- b. Determine the frictional force on the box.
- c. Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.
- d. Determine an equation for the path of the box that expresses y as a function of x (and <u>not</u> of t), assuming that, at time t = 0, the box has a horizontal position x = 0 and a vertical position y = 2 m above the ground, with zero velocity.
- e. On the axes below sketch the path taken by the box





1998M3. Block 1 of mass m₁ is placed on block 2 of mass m₂ which is then placed on a table. A string connecting block 2 to a hanging mass M passes over a pulley attached to one end of the table, as shown above. The mass and friction of the pulley are negligible. The coefficients of friction between blocks 1 and 2 and between block 2 and the tabletop are nonzero and are given in the following table.

	Coefficient Between Blocks 1 and 2	Coefficient Between Block 2 and the Tabletop
Static	μ_{s_1}	μ_{s2}
Kinetic	μ_{k1}	μ_{k2}

Express your answers in terms of the masses, coefficients of friction, and g, the acceleration due to gravity.

- a. Suppose that the value of M is small enough that the blocks remain at rest when released. For each of the following forces, determine the magnitude of the force and draw a vector on the block provided to indicate the direction of the force if it is nonzero.
 - i. The normal force N₁ exerted on block 1 by block 2



ii. The friction force f_1 exerted on block 1 by block 2



iii. The force T exerted on block 2 by the string



iv. The normal force N₂ exerted on block 2 by the tabletop



v. The friction force f_2 exerted on block 2 by the tabletop



- b. Determine the largest value of M for which the blocks can remain at rest.
- c. Now suppose that M is large enough that the hanging block descends when the blocks are released. Assume that blocks 1 and 2 are moving as a unit (no slippage). Determine the magnitude *a* of their acceleration.
- d. Now suppose that M is large enough that as the hanging block descends, block 1 is slipping on block 2. Determine each of the following.
 - i. The magnitude a₁ of the acceleration of block 1
 - ii. The magnitude a₂ of the acceleration of block 2

*2005M1 (modified) A ball of mass M is thrown vertically upward with an initial speed of v_o . It experiences a force of air resistance given by F = -kv, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_o , and fundamental constants.

a. Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

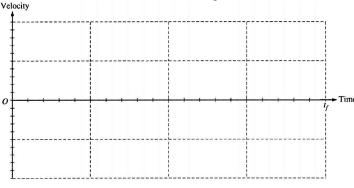
____ increases ____ decreases ____ remains the same Justify your answer.

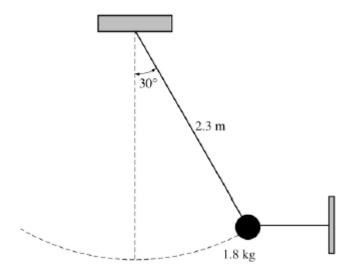
b. Determine the terminal speed of the ball as it moves downward.

c. Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____ longer to rise ____ longer to fall Justify your answer.

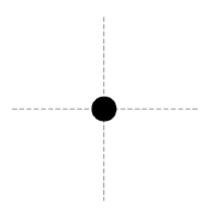
d. On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



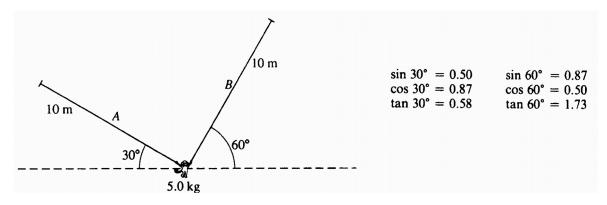


2005B2. A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



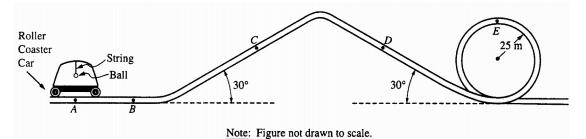
(b) Calculate the tension in the horizontal string.



- 1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.
 - a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)



b. Determine the tension in vine B while the monkey is at rest.



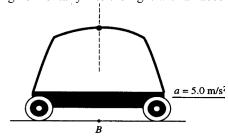
1995B3. Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

- a. Initially, the car is at rest at point A.
- i. On the diagram below, draw and label all the forces acting on the 0.10-kilogram ball.

ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a 30° incline, goes down a 30° incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. In each situation, assume that the ball has stopped swinging back and forth.

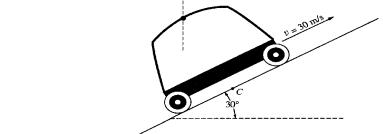
- 1) Determine the horizontal component T_h of the tension in the string in newtons and record your answer in the space provided.
- 2)Determine the vertical component T_v of the tension in the string in newtons and record your answer in the space provided.
- 3)Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.
- b. The car is at point B moving horizontally 2 to the right with an acceleration of 5.0 m/s.



 $T_h =$

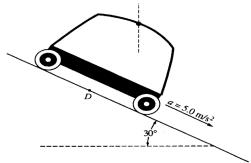
 $T_{v} =$

c. The car is at point C and is being pulled up the 30° incline with a constant speed of 30 m/s.



 $T_h =$

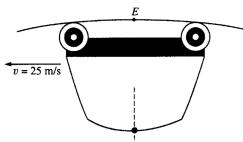
 $T_v =$



d. The car is at point D moving down the incline with an acceleration of 5.0 m/s^2 .

 $T_h =$

 $T_v =$

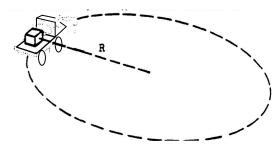


e. The car is at point E moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.

T_h =_____

 $T_v =$

SECTION B – Circular Motion



1977 B2. A box of mass M, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed v. The truck is on an unbanked circular roadway having radius of curvature R.

a. On the diagram provided above, indicate and clearly label all the force vectors acting on the box.

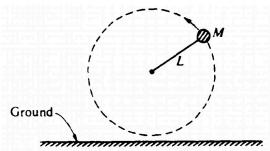
b. Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v, R, and g.



If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

c. On the diagram above indicate and clearly label the two forces acting on the box under these conditions

d. Which, if either, of the two forces acting on the box is greater in magnitude?



1984B1. A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

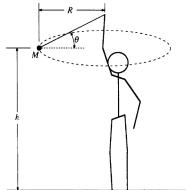
a. Determine the magnitude and direction of the net force on the ball when it is at the top.

b. Determine the speed v_0 of the ball at the top.

The string is then cut when the ball is at the top.

c. Determine the time it takes the ball to reach the ground.

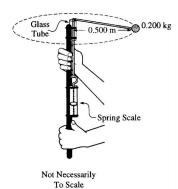
d. Determine the horizontal distance the ball travels before hitting the ground.



- 1989B1. An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_o and the circular path has radius R and is a height h above the ground. Neglect air friction.
- a. Determine the following, expressing all answers in terms of h, v_o , and g.
 - i. The time required for the object to hit the ground after the string breaks
 - ii. The horizontal distance the object travels from the time the string breaks until it hits the ground
 - iii. The speed of the object just before it hits the ground
- b. On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.



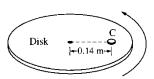
c. Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_o, & g.



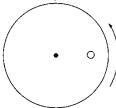
- 1997B2 (modified) To study circular motion, two students use the hand-held device shown above, which consists of a rod on which a spring scale is attached. A polished glass tube attached at the top serves as a guide for a light cord attached the spring scale. A ball of mass 0.200 kg is attached to the other end of the cord. One student swings the teal around at constant speed in a horizontal circle with a radius of 0.500 m. Assume friction and air resistance are negligible.
- a. Explain how the students, by using a timer and the information given above, can determine the speed of the ball as it is revolving.
- b. The speed of the ball is determined to be 3.7 m/s. Assuming that the cord is horizontal as it swings, calculate the expected tension in the cord.
- c. The actual tension in the cord as measured by the spring scale is 5.8 N. What is the percent difference between this measured value of the tension and the value calculated in part b.?
 - The students find that, despite their best efforts, they cannot swing the ball so that the cord remains exactly horizontal.
- d. i. On the picture of the ball below, draw vectors to represent the forces acting on the ball and identify the force that each vector represents.



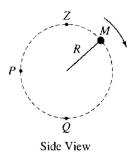
- ii. Explain why it is not possible for the ball to swing so that the cord remains exactly horizontal.
- iii. Calculate the angle that the cord makes with the horizontal.



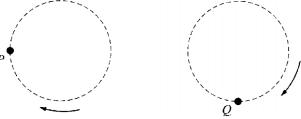
- 1999B5 A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.
- a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.



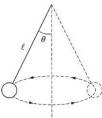
- b. Determine the linear speed of the coin.
- c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.
- d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c)? Explain your reasoning.



- 2001B1. A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.
- a. On the figures below, draw and label all the forces exerted on the ball when it is at points P and Q, respectively.



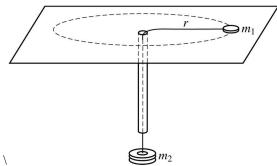
- b. Derive an expression for v_{min} the minimum speed the ball can have at point Z without leaving the circular path.
- c. The maximum tension the string can have without breaking is T_{max} Derive an expression for v_{max} , the maximum speed the ball can have at point Q without breaking the string.
- d. Suppose that the string breaks at the instant the ball is at point P. Describe the motion of the ball immediately after the string breaks.



- 2002B2B A ball attached to a string of length l swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and T is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.
- a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



- b. Determine the mass of the ball.
- c. Determine the speed of the ball.
- d. Determine the frequency of revolution of the ball.
- e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.



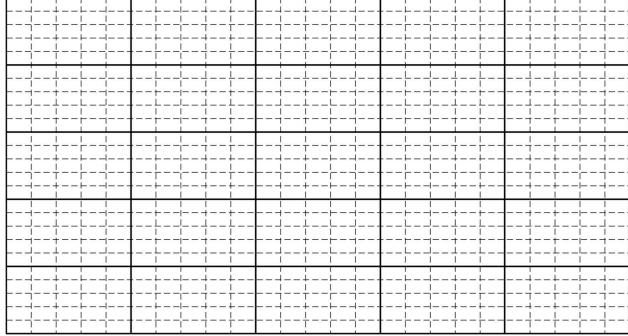
- 2009Bb1 An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.
- a. Derive the equation $P = 2\pi \sqrt{\frac{m_1 r}{m_2 g}}$ that relates P and m_2 .

The procedure is repeated, and the period P is determined for four different values of m_2 , where $m_1 = 0.012$ kg and r = 0.80 m. The data, which are presented below, can be used to compute an experimental value for g.

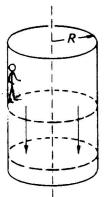
m_2 (kg)	0.020	0.040	0.060	0.080
<i>P</i> (s)	1.40	1.05	0.80	0.75

b. What quantities should be graphed to yield a straight line with a slope that could be used to determine g?

c. On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.



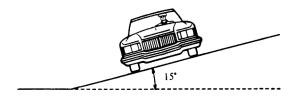
d. Use your graph to calculate the experimental value of g.



*1984M1 (modified) An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, The radius R of the cylinder is 5 meters, the frequency of the cylinder when rotating is $1/\pi$ revolutions per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.



- a. On the diagram above, draw and identify the forces on the rider when the system is rotating and the floor has dropped down.
- b. Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.
- c. Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.
- d. At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.

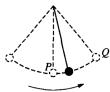


- 1988M1. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.
- a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.



- b. On the diagram above, in which the block represents the automobile, draw and label all of the forces on the automobile.
- c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.



1998B6 A heavy ball swings at the end of a string as shown above, with negligible air resistance. Point P is the lowest point reached by the ball in its motion, and point Q is one of the two highest points.

a. On the following diagrams draw and label vectors that could represent the velocity and acceleration of the ball at points P and Q. If a vector is zero, explicitly state this fact. The dashed lines indicate horizontal and vertical directions.



b. After several swings, the string breaks. The mass of the string and air resistance are negligible. On the following diagrams, sketch the path of the ball if the break occurs when the ball is at point P or point Q. In each case, briefly describe the motion of the ball after the break.



SECTION A – Linear Dynamics

	Solution	Answer
1.	As T_2 is more vertical, it is supporting more of the weight of the ball. The horizontal components of T_1 and T_2 are equal.	С
2.	Normal force is perpendicular to the incline, friction acts up, parallel to the incline (opposite the motion of the block), gravity acts straight down.	D
3.	The component of the weight down the plane is 20 N sin θ . The net force is 4 N, so the friction force up the plane must be 4 N less than 20 N.	D
4.	The force between objects is the applied force times the ratio of the mass behind the rope to the total mass being pulled. This can be derived from $a = F/m_{total}$ and $F_T = m_{behind \ the \ rope} a$	D
5.	Since the ball's speed is increasing from rest, the retarding force F is also increasing. The net force, which is the weight of the ball minus F , is thus decreasing. So the acceleration also must decrease. Time $t/2$ is before the constant-speed motion begins, so the acceleration has not yet decreased to zero.	В
6.	For vertical equilibrium, the weight equals the normal force plus the vertical component of F . This leads to the normal force being W – something. The block remains in contact with the surface, so the normal force does not reach zero.	В
7.	The bottom of the rope supports the box, while the top of the rope must support the rope itself and the box.	D
8.	The vertical components of the tension in the rope are two equal upward components of $T\cos\theta$, which support the weight. $\Sigma F_y = 0 = 2T\cos\theta - W$	D
9.	$\Sigma F_{\text{external}} = m_{\text{total}}a;$ mg is the only force acting from outside the system of masses so we have mg = (4m)a	A
10.	The weight component perpendicular to the plane is $20 \mathrm{N} \sin 37^{\circ}$. To get equilibrium perpendicular to the plane, the normal force must equal this weight component, which must be less than $20 \mathrm{N}$.	В
11.	(A) is the definition of translational equilibrium. Equilibrium means no net force and no acceleration, so (D) is also correct.	A,D
12.	Motion at constant speed includes, for example, motion in a circle, in which the direction of the velocity changes and thus acceleration exists. Constant momentum for a single object means, that the velocity doesn't change.	D
13.	$\Sigma F = ma$; $F_T - mg = ma$; Let $F_T = 50 \text{ N}$ (the maximum possible tension) and $m = W/g = 3 \text{ kg}$	A
14.	The sum of the tensions in the chains (250 N + T_{left}) must support the weight of the board and the person (125 N + 500 N)	С
15.	The board itself provides the same torque about the attachment point of both chains, but since the left chain provides a bigger force on the board, the person must be closer to the left chain in order to provide an equivalent torque on both chains by $\tau = Fd$.	A
16.	The horizontal component of the 30 N force is 15 N left. So the net force is 5 N left. So the acceleration is left. This could mean either A or D – when acceleration is opposite velocity, an object slows down.	A,D

- 17. Consider that no part of the system is in motion, this means at each end of the rope, a person pulling with 100 N of force is reacted to with a tension in the rope of 100 N.
- 18. As v is proportional to t^2 and a is proportional to $\Delta v/t$, this means a should be proportional to t
- 19. The direction of the force is the same as the direction of the acceleration, which is proportional to $\Delta v = v_f + (-v_i)$
- 20. A force diagram will show that the forces provided by each spring add up to 12 N: $F_1 + F_2 = 12$ N. Each force is kx; each spring is stretched the same amount x = 24 cm. So $k_1x + k_2x = 12$ N; dividing both sides by x shows that $k_1 + k_2 = 0.5$ N/cm.
- 21. Net force is the gravitational force which acts downward D
- 22. $\Sigma F = ma = F\cos\phi f$
- 23. The string pulling all three masses (total 6m) must have the largest tension. String A is only pulling the block of mass 3m and string B is pulling a total mass of 5m.
- 24. At t = 2 s the force is 4 N. F = ma
- 25. Since P is at an upward angle, the normal force is decreased as P supports some of the weight. A Since a component of P balances the frictional force, P itself must be larger than f.
- 26. The force of friction = $\mu F_N = 0.2 \times 10 \text{ kg} \times 9.8 \text{ m/s}^2 = 19.6 \text{ N}$, which is greater than the applied force, which means the object is accelerating to the left, or slowing down
- 27. The upward component of the tension is $T_{up} = T\sin\theta$, where θ is the angle to the horizontal. This gives $T = T_{up}/\sin\theta$. Since the upward components are all equal to one half the weight, the rope at the smallest angle (and the smallest value of $\sin\theta$) will have the greatest tension, and most likely break
- 28. From the 1 kg block: F = ma giving $a = 2 \text{ m/s}^2$. For the system: $F = (4 \text{ kg})(2 \text{ m/s}^2)$
- 29. Elevator physics: F_N represents the scale reading. $\Sigma F = ma$; $F_N mg = ma$, or $F_N = m(g + a)$. B The velocity of the elevator is irrelevant.
- 30. Newton's third law C
- 31. The normal force is $mgcos\theta$. For a horizontal surface, $F_N = mg$. At any angle $F_N < mg$ and F_f is proportional to F_N .
- 32. Slope = $\Delta y/\Delta x$ = Weight/mass = acceleration due to gravity
- 33. Newton's second law applied to m_1 : $T = m_1 a$, or $a = T/m_1$, substitute this into Newton's second law for the hanging mass: $m_2 g T = m_2 a$
- 34. Elevator physics: R represents the scale reading. $\Sigma F = ma$; R mg = ma, or R = m(g + a). This ranks the value of R from largest to smallest as accelerating upward, constant speed, accelerating downward
- 35. $\Sigma F = \text{ma}$ for the whole system gives $F \mu(3m)g = (3m)a$ and solving for a gives $a = (F 3\mu mg)/3m$. For the top block, $F_m = ma = m[(F 3\mu mg)/3m]$
- 36. The normal force comes from the perpendicular component of the applied force which is Fcos θ = 50 N. The maximum value of static friction is then $\mu F_N = 25$ N. The upward component of the applied force is Fsin θ = 87 N. $\Sigma F_y = F_{up} mg = 87$ N-60 N > 25 N. Since the net force on the block is great than static friction can hold, the block will begin moving up the wall. Since it is in motion, kinetic friction is acting opposite the direction of the block's motion
- 37. Since P is at a downward angle, the normal force is increased. Since a component of P balances A the frictional force, P itself must be larger than f.

- 38. Since the force is applied horizontally, the mass has no effect.
- 39. The only force in the direction of the crate's acceleration is the force of friction from the sleigh B

C

- 40. Given that the box accelerates toward Ted, Ted's force must be greater than Mario's force plus the force of friction. Since Mario's force is ½ of Ted's force, the force of friction must be less than half of Ted's force.
- 41. $\Sigma F = ma$; F mg = m(5g) or F = 6mg
- 42. Between the lower block and the tabletop, there is a force of friction to the left of maximum magnitude $\mu(2W)$ as both blocks are pushing down on the tabletop. There is also a force of friction acting to the left on the upper surface of the lower block due to the upper block of maximum magnitude μW . The total maximum static frictional force holding the lower block in place is therefore $\mu(2W) + \mu W$
- 43. The normal force on the block can be found from $\Sigma F_y = 0 = F_N mg\cos\theta F$. The force of friction necessary to hold the block in place is $mg\sin\theta$. Setting the force of friction equal to $mg\sin\theta$ gives $\mu F_N = mg\sin\theta = \mu(F + mg\cos\theta)$
- 44. This is a tricky one. In order to move the car forward, the rear tires roll back against the ground, the force of friction pushing forward on the rear tires. The front tires, however, are not trying to roll on their own, rather they begin rolling due to the friction acting backward, increasing their rate of rotation
- 45. The external forces acting on the system of masses are the weights of block 1 (pulling the system to the left), the weight of block 3 (pulling the system to the right) and the force of friction on block 2 (pulling the system to the left with a magnitude $\mu F_N = \mu m_2 g$) $\Sigma F_{\text{external}} = m_{\text{total}} \text{a gives } (m_1 g \mu m_2 g m_3 g) = (m_1 + m_2 + m_3) \text{a}$
- 46. $F = \text{ma gives } 30 \text{ N} = (12 \text{ kg}) \text{a or an acceleration of } 2.5 \text{ m/s}^2$. The 5 kg block is accelerating due to the tension in the rope $F_T = \text{ma} = (5 \text{ kg})(2.5 \text{ m/s}^2) = 12.5 \text{ N}$.
- 47. As they are all at the same position after 8 seconds, they all have the same average velocity D
- 48. Car A decelerates with the same magnitude that C accelerates. Car B is moving at constant B speed, which means $F_B = 0$.
- 49. When falling with terminal velocity, the force of air resistance equals your weight, regardless of the speed.
- 50. For each case, $\Sigma F_{\text{external}} = m_{\text{total}} a \text{ gives } Mg mg = (M + m)a$, or $a = \frac{M m}{M + m} g$.
- 51. The two ends of the light string must have the same tension, eliminating choices A and C. If choice D was correct, both masses would be accelerating downward and T_A must be greater than the weight of block A.
- 52. $\Sigma F_{\text{external}} = m_{\text{total}} a \text{ gives } (M + m)g Mg = (2M + m)a$
- 53. As the entire system moves as one, F = (3m)a, or a = F/(3m). The force of friction acting on block 1 is the force moving block 1 and we have $\mu mg = m(F/(3m))$
- 54. Since the system is moving at constant velocity, m_1 is pushing m_2 and m_3 with a force equal to the force of friction acting on those two blocks, which is $\mu(F_{N2} + F_{N3})$

SECTION B - Circular Motion

- 1. Newton's third law and friction force B,D
- 2. $F = mv^2/r; \ v = \sqrt{\frac{Fr}{m}}; \text{ all other variables being constant, if r is quadrupled, } v \text{ is doubled}$
- 3. With acceleration south the car is at the top (north side) of the track as the acceleration points toward the center of the circular track. Moving east indicates the car is travelling clockwise. The magnitude of the acceleration is found from $a = v^2/r$
- 4. The frictional force acts as the centripetal force (toward the center)
- 5. Acceleration occurs when an object is changing speed and/or direction B,C
- 6. Velocity is tangential, acceleration points toward the center of the circular path B

D

- 7. To move in a circle, a force directed toward the center of the circle is required. While the package slides to the right in the car, it is actually moving in its original straight line path while the car turns from under it.
- 8. $a = v^2/r$ and $v = 2\pi r/T$ giving $a = 4\pi^2 r/T^2$
- 9. Once projected, the ball is no longer subject to a force and will travel in a straight line with a component of its velocity tangent to the circular path and a component outward due to the spring
- 10. The net force is inward. The normal force is counteracted by gravity.
- 11. $a = v^2/r$ where $v = 2\pi rf$ and f = 2.0 rev/sec
- 12. At Q the ball is in circular motion and the acceleration should point to the center of the circle.

 At R, the ball comes to rest and is subject to gravity as in free-fall.
- 13. The net force and the acceleration must point in the same direction. Velocity points tangent to the objects path.
- 14. The centripetal force is provided by the spring where $F_C = F_s = kx$
- 15. In the straight sections there is no acceleration, in the circular sections, there is a centripetal acceleration
- 16. Once the stone is stuck, it is moving in circular motion. At the bottom of the circle, the acceleration points toward the center of the circle at that point.
- 17. Feeling weightless is when the normal force goes to zero, which in only possible going over the top of the hill where mg (inward) F_N (outward) = mv^2/R . Setting F_N to zero gives a maximum speed of \sqrt{gR}
- 18. Centripetal force points toward the center of the circle B
- 19. While speed may be constant, the changing direction means velocity cannot be constant as velocity is a vector. In uniform circular motion, acceleration is constant.
- 20. $F = mv^2/r$. $F_{new} = (2m)(2v)^2/(2r) = 4(mv^2/r) = 4F$
- 21. Assuming the track is circular at the bottom, the acceleration points toward the center of the circular path
- 22. Average speed = (total *distance*)/(total time). Lowest average speed is the car that covered the

least distance

23. As all the cars are changing direction, there must be a net force to change the direction of their velocity vectors

D

24. $F = mv^2/r$; $v^2 = rF/m$, if r decreases, v will decrease with the same applied force. Also, $v = 2\pi rf$ so $4\pi^2 r^2 f = rF/m$, or $f = F/(4\pi^2 rm)$ and as r decreases, f increases.

D

25. There is a force acting downward (gravity) and a centripetal force acting toward the center of the circle (up and to the right). Adding these vectors cannot produce resultants in the directions of B, C or D

A

26. $\Sigma F = ma$; $mg + F_T = mv^2/r$ giving $F_T = mv^2/r - mg$

В

27. At the top of the circle, $\Sigma F = F_T + mg = mv^2/R$, giving $F_T = mv^2/R - mg$. At the bottom of the circle, $\Sigma F = F_T - mg = mv^2/R$, giving $F_T = mv^2/R + mg$ The difference is $(mv^2/R + mg) - (mv^2/R - mg)$

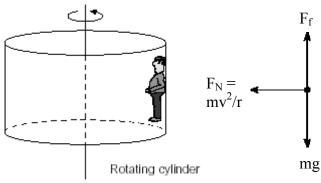
В

28. At the bottom of the swing, $\Sigma F = F_T - mg = ma_c$; since the tension is 1.5 times the weight of the object we can write 1.5mg - mg = ma_c, giving 0.5mg = ma_c

A

29.

В



 $F_f = mg$ to balance

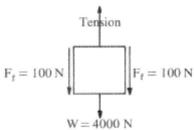
 $\mu F_N = \mu m v^2 / r = mg$, where $v = 2\pi r f$ which gives $\mu = g/(4\pi^2 r f^2)$

Be careful! f is given in rev/min (45 rev/min = 0.75 rev/sec) and 8.0 m is the ride's diameter

SECTION A – Linear Dynamics

1976B1

a



b. $\Sigma F = \text{ma}$; $T - W - 2F_f = 800 \text{ N}$; T = 5000 N

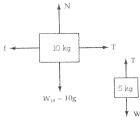
c. Looking at the FBD for the counterweight we have ΣF = ma; Mg – T = Ma M = T/(g – a) where T = 5000 N gives M = 625 kg



1979B2

a. $\Sigma F = ma$; 50 N – f = ma where f = μN and N = mg gives 50 N – $\mu mg = ma$; a = 3 m/s²

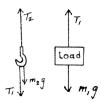
b.



c. ΣF = ma for each block gives $W_5 - T = m_5 a$ and $T - f = m_{10} a$. Adding the two equations gives $W_5 - f = (m_5 + m_{10})a$, or $a = 2 \text{ m/s}^2$

1982B2

a



b. T_1 is in internal system force and will cancel in combined equations. Using $\Sigma F_{\text{external}} = m_{\text{total}} a$ gives $T_2 - m_1 g - m_2 g = (m_1 + m_2) a$, solving yields $T_2 = 6600 \text{ N}$. Now using $\Sigma F = ma$ for the load gives $T_1 - m_1 g = m_1 a$ and $T_1 = 6000 \text{ N}$

- Note that the system is at rest. The only forces on the hanging block are gravity and the tension in the rope, which means the tension must equal the weight of the hanging block, or 100 N. You cannot use the block on the incline because friction is acting on that block and the amount of friction is unknown.
- b.



 $\Sigma F = 0$; $f_s + \text{mg sin}\theta - T = 0$ gives $f_s = 13 \text{ N}$

1986B1

- $\Sigma F_{external} = m_{total} a; \ m_4 g m_1 g m_2 g = (m_4 + m_2 + m_1) a \ gives \ a = 1.4 \ m/s^2$
- b. For the 4 kg block:

$$\Sigma F = ma$$

$$\Sigma F = ma$$

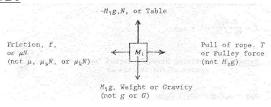
$$mg - T_4 = ma$$
 gives
 $T_4 = 33.6 N$

$$T_4 = 33.6 \text{ N}$$

- Similarly for the 1 kg block: $T_1 mg = ma$ gives $T_1 = 11.2$ N

1987B1

a.

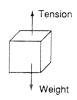


- $\Sigma F_{ext} = m_{tot}a$; Where the maximum force of static friction on mass M_1 is $\mu_s N$ and $N = M_1 g$; $M_2 g \mu_s M_1 g = 0$ gives $\mu_s = M_2/M_1$
- c/d. $\Sigma F_{ext} = m_{tot}a$ where we now have kinetic friction acting gives $M_2g \mu_k M_1g = (M_1 + M_2)a$ so $a = (M_2g - \mu_k M_1g)/(M_1 + M_2)$

 ΣF = ma for the hanging block gives $M_2g - T = M_2a$ and substituting a from above gives $T = \frac{M_1M_2g}{M_2+M_2}(1+\mu_k)$

1988B1

a.



- $\Sigma F = \text{ma gives } T \text{mg} = \text{ma and } T = 1050 \text{ N}$
- The helicopter and the package have the same initial velocity, 30 m/s upward. Use $d = v_i t + \frac{1}{2} a t^2$ $d_h = (+30 \text{ m/s})t + \frac{1}{2}(+5.2 \text{ m/s}^2)t^2$ and $d_p = (+30 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$. The difference between d_h and d_p is 30 m, but they began 5 m apart so the total distance is 35 m.

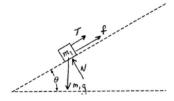
- a. $\Sigma F_{\text{ext}} = m_{\text{tot}} \text{a gives mg} = 2 \text{ma}, \text{ or a} = g/2$
- b. $d = v_0 t + \frac{1}{2} a t^2$; $h = 0 + \frac{1}{2} (g/2) t^2$ gives $t = 2 \left[\frac{\overline{h}}{g} \right]$
- c. Block A accelerates across the table with an acceleration equal to block B (g/2).
- d. Block A is still in motion, but with no more applied force, Block A will move at constant speed across the table.
- e. Since block B falls straight to the floor and stops, the distance between the landing points is equal to the horizontal distance block A lands from the edge of the table. The speed with which block A leaves the tabletop is the speed with which block B landed, which is found from $v = v_0 + at = \frac{g}{2} \left(2 \right) \left| \frac{\overline{h}}{g} \right| = \sqrt{hg}$ and the time for

block A to reach the floor is found from $2h = \frac{1}{2} gt^2$, which gives $t = 2 \int_{-\pi}^{\pi} \frac{1}{g}$.

The distance is now $d = vt = \sqrt{hg} \times 2 \left[\frac{\overline{h}}{g} = 2h \right]$

2000B2

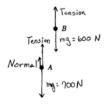
a.



- b. $f = \mu N$ where $N = m_1 g \cos \theta$ gives $\mu = \frac{f}{m_1 g \cos \theta}$
- c. constant velocity means $\Sigma F = 0$ where $\Sigma F_{\text{external}} = m_1 g \sin \theta + m_2 g \sin \theta f 2f Mg = 0$ solving for M gives $M = (m_1 + m_2) \sin \theta (3f)/g$
- d. Applying Newton's second law to block 1 gives $\Sigma F = m_1 g \sin \theta f = m_1 a$ which gives $a = g \sin \theta f / m_1$

2003B1

a.



- b. The tension in the rope is equal to the weight of student B: $T = m_B g = 600 \text{ N}$ $\Sigma F_A = T + N - m_A g = 0$ gives N = 100 N
- c. For the climbing student $\Sigma F = ma$; $T m_B g = m_B a$ gives T = 615 N
- d. For student A to be pulled off the floor, the tension must exceed the weight of the student, 700 N. No, the student is not pulled off the floor.
- e. Applying Newton's second law to student B with a tension of 700 N gives $\Sigma F = T m_B g = m_B a$ and solving gives $a = 1.67 \text{ m/s}^2$

2003Bb1

a.



- b. We can find the acceleration from $a = \Delta v/t = 2.17 \text{ m/s}^2$ and use $d = \frac{1}{2} at^2$ to find d = 975 m
- c. The x and y components of the tension are $T_x = T \sin \theta$ and $T_y = T \cos \theta$ (this is using the angle to the vertical) Relating these to the other variables gives $T \sin \theta = ma$ and $T \cos \theta = mg$. Dividing the two equations gives $\tan \theta = a/g = (2.17 \text{ m/s}^2)/(9.8 \text{ m/s}^2)$ and $\theta = 12.5^\circ$

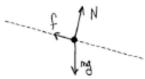
1996B2

- a. There are other methods, but answers are restricted to those taught to this point in the year.
 - i. A device to measure distance and a calibrated mass or force scale or sensor
 - ii. Hang the mass from the bottom of the spring and measure the spring extension (Δx) or pull on the spring with a known force and measure the resulting extension.
 - iii. Use hooke's law with the known force or weight of the known mass $F = k\Delta x$ or $mg = k\Delta x$ and solve for k
- b. Many methods are correct, for example, place the object held by the scale on an inclined plane and find the weight using $W\sin\theta = k\Delta x$. One could similarly use a pulley system to reduce the effort applied by the spring scale.

2007B1

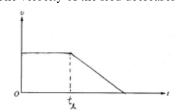
 \overline{a} . x = vt gives t = (21 m)/(2.4 m/s) = 8.75 s

b.



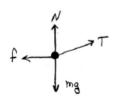
- c. $\Sigma F = 0$ if the sled moves at constant speed. This gives mg sin $\theta f = 0$, or $f = \text{mg sin } \theta = 63.4 \text{ N}$
- d. $f = \mu N$ where $N = mg \cos \theta$ so $\mu = f/N = (mg \sin \theta)/(mg \cos \theta) = \tan \theta = 0.27$
- e. i. The velocity of the sled decreases while its acceleration remains constant

ii.



<u>2007B1B</u>

a

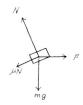


- b. $\Sigma F_y = 0$; N + T sin θ mg = 0 gives N = mg T sin θ = 177 N
- c. $f = \mu N = 38.9 \text{ N}$ and $\Sigma F_x = \text{ma}$; $T \cos \theta f = \text{ma}$ yields $a = 0.64 \text{ m/s}^2$

d.

1981M1

a.



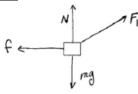
- b. F can be resolved into two components: F sin θ acting into the incline and F cos θ acting up the incline. The normal force is then calculated with $\Sigma F = 0$; N F sin θ mg cos θ = 0 and f = μ N Putting this together gives ΣF = ma; F cos θ mg sin θ μ (F sin θ + mg cos θ) = ma, solve for a
- c. for constant velocity, a = 0 in the above equation becomes $F \cos \theta mg \sin \theta \mu(F \sin \theta + mg \cos \theta) = 0$ solving for F gives $F = mg \left(\frac{\mu \cos \theta + \sin \theta}{\cos \theta \mu \sin \theta} \right)$ In order that F remain positive (acting to the right), the denominator must remain positive. That is $\cos \theta > \mu \sin \theta$, or $\tan \theta < 1/\mu$

1986M1

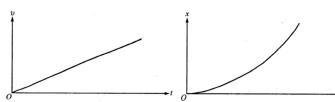
- a. Combining the person and the platform into one object, held up by two sides of the rope we have ΣF = ma; 2T = (80 kg + 20 kg)g giving T = 500 N
- b. Similarly, $\Sigma F = \text{ma}$; $2T 1000 \text{ N} = (100 \text{ kg})(2 \text{ m/s}^2)$ giving T = 600 N
- c. For the person only: $\Sigma F = ma$; N + 600 N mg = ma gives N = 360 N

2007M1

a.



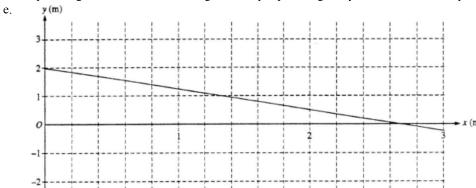
- b. $\Sigma F_v = 0$; $N + F_1 \sin \theta mg = 0$ gives $N = mg F_1 \sin \theta$
- c. $\Sigma F_x = ma$; $F_1 \cos \theta \mu N = ma_1$. Substituting N from above gives $\mu = (F_1 \cos \theta ma_1)/(mg F_1 \sin \theta)$
- d.



e. The condition for the block losing contact is when the normal force goes to zero, which means friction is zero as well. $\Sigma F_x = F_{max} \cos \theta = ma_{max}$ and $\Sigma F_y = F_{max} \sin \theta - mg = 0$ giving $F_{max} = mg/(\sin \theta)$ and $a_{max} = (F_{max} \cos \theta)/m$ which results in $a_{max} = g \cot \theta$

1996M2

- a. $\Sigma F = ma$; using downward as the positive direction, $mg N = ma_y$ gives $N = m(g a_y) = 2490 \text{ N}$
- b. Friction is the only horizontal force exerted; $\Sigma F = f = ma_x = 600 \text{ N}$
- c. At the minimum coefficient of friction, static friction will be at its maximum value $f = \mu N$, giving $\mu = f/N = (600 \text{ N})/(2490 \text{ N}) = 0.24$
- d. $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = 2 \text{ m} + \frac{1}{2}(-1.5 \text{ m/s}^2)t^2$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 = \frac{1}{2}(2 \text{ m/s}^2)t^2$, solving for t^2 in the x equation gives $t^2 = x$. Substituting into the y equation gives y as a function of x: y = 2 0.75x

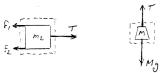


1998M3

- a. i. $N_1 = m_{12}$ $N_1 = m_{13}$
 - ii. m_{Γ} $f_{I} = 0$
 - iii. $T = M_{\mathcal{E}}$
 - iv. $N_2 = (m_1 + m_2)g$
 - V. $f_2 = Mg$
- b. The maximum friction force on the blocks on the table is $f_{2max} = \mu_{s2}N_2 = \mu_{s2}(m_1 + m_2)g$ which is balanced by the weight of the hanging mass: $Mg = \mu_{s2}(m_1 + m_2)g$ giving $M = \mu_{s2}(m_1 + m_2)g$
- c. $F_2 \xrightarrow{M_1} T$ M_3

For the hanging block: Mg – T = Ma; For the two blocks on the plane: T – $f_2 = (m_1 + m_2)a$ Combining these equations (by adding them to eliminate T) and solving for a gives $a = \left| \frac{M - \mu_{k2}(m_1 + m_2)}{M + m_1 + m_2} \right| g$ d. i. $f_1 = \mu_{k1} m_1 g = m_1 a_1$ giving $a_1 = \mu_{k1} g$

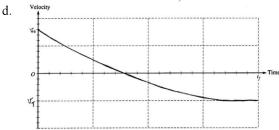
ii.



For the two blocks: $Mg - T = Ma_2$ and $T - f_1 - f_2 = m_2a_2$. Eliminating T and substituting values for friction gives $a_2 = \left| \frac{M - \mu_{k1} m_1 - \mu_{k2} (m_1 + m_2)}{M + m_2} \right| g$

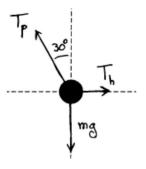
2005M1

- a. The magnitude of the acceleration decreases as the ball moves upward. Since the velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.
- b. At terminal speed $\Sigma F = 0$. $\Sigma F = -Mg + kv_T$ giving $v_T = Mg/k$
- c. It takes longer for the ball to fall. Friction is acting on the ball on the way up and on the way down, where it begins from rest. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.



2005B2.

(a)



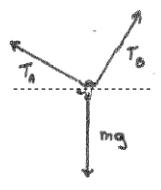
(b) Apply

 $\begin{aligned} F_{net(X)} &= 0 \\ T_P \cos 30 &= mg \\ T_P &= 20.37 \text{ N} \end{aligned}$

 $\begin{aligned} F_{net(Y)} &= 0 \\ T_P \sin 30 &= T_H \\ T_H &= 10.18 \ N \end{aligned}$

1991B1.

a)



(b) SIMULTANEOUS EQUATIONS

$$F_{\text{net}(X)} = 0$$
 $F_{\text{net}(Y)} = 0$ $F_{\text{a}} \cos 30 = T_{\text{b}} \cos 60$ $F_{\text{a}} \sin 30 + T_{\text{b}} \sin 60 - \text{mg} = 0$

.... Solve above for T_b and plug into Fnet(y) eqn and solve

$$T_a = 24 \text{ N}$$
 $T_b = 42 \text{ N}$

a) i)

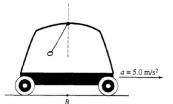


ii) T = mg = 1 N

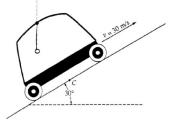
b) The horizontal component of the tension supplies the horizontal acceleration.

$$T_h = ma = 0.5 \text{ N}$$

The vertical component of the tension is equal to the weight of the ball, as in (a) ii. $T_v = 1 \text{ N}$



c) Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball. $T_h = zero$, $T_v = 1N$



d) The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:

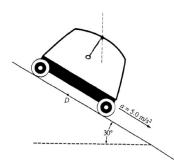
 $T_h = ma \cos \theta$, where θ is the angle between the acceleration and horizontal

$$T_h = (0.10 \text{ kg})(5.0 \text{ m/s}^2) \cos 30^\circ, T_h = 0.43 \text{ N}$$

The vertical component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration.

Applying Newton's second law.
$$Tv = mg - ma \sin \theta$$

$$Tv = (0.10 \text{ kg})(10 \text{ m/s}^2) - (0.10 \text{ kg})(5.0 \text{ m/s}^2) \sin 30^\circ$$
, $T_v = 0.75 \text{ N}$

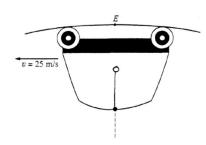


e) Since there is no horizontal acceleration, there is no horizontal component of the tension. $T_h = zero$

Assuming for the moment that the string is hanging downward, the centripetal is the difference between the gravitational force and the tension. Applying Newton's second law.

$$mv^2/r = mg - T_v$$
, Solving for the vertical component of tension:

 $T_v = -1.5$ N i.e. the string is actually pulling down on the ball.



SECTION B - Circular Motion

1977B2

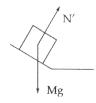
a.

1 = normal force; 2 = friction; 3 = weight



b. Friction, $f \le \mu N$ where N = Mg. Friction provides the necessary centripetal force so we have $f = Mv^2/R$ $Mv^2/R \le \mu Mg$, or $\mu \ge v^2/Rg$

c.



d. from the diagram below, a component of the normal force N' balances gravity so N' must be greater than mg



1984B1

a. At the top of the path, tension and gravity apply forces downward, toward the center of the circle. $\Sigma F = T + mg = 2Mg + Mg = 3Mg$

b. In the circular path, $F = mv^2/r$ which gives $3Mg = mv_0^2/L$ and $v_0 = \sqrt{3Lg}$

c. The ball is moving horizontally ($v_{0y} = 0$) from a height of 2L so this gives $2L = \frac{1}{2}$ gt² or $t = 2 \frac{L}{g}$

d. $x = v_0 t = \sqrt{3Lg} \times 2 \left[\frac{L}{g} \right] = 2\sqrt{3}L$

1989B1

a. i. $v_{iy} = 0$ so we have $h = \frac{1}{2} gt^2$ which gives $t = \int \frac{\overline{2h}}{g}$

ii.
$$x = v_0 t = v_0 \int_{-1}^{2h} \frac{\overline{h}}{g}$$

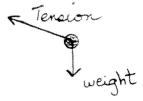
iii. $v_x = v_0$; $v_y = v_{iy} + gt = \sqrt{2gh}$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gh}$

b.



c. Horizontal forces: T cos $\theta = Mv_0^2/R$; Vertical forces: T sin $\theta = Mg$. Squaring and adding the equations gives $T = M \left[\frac{1}{g^2 + \frac{v_0^4}{R^2}} \right]$

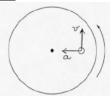
- a. The circumference of the path, d, can be calculated from the given radius. Use the timer to obtain the period of revolution, t, by timing a number of revolutions and dividing the total time by that number of revolutions. Calculate the speed using v = d/t.
- b. If the cord is horizontal, $T = mv^2/r = 5.5 \text{ N}$
- c. $(5.5 \text{ N} 5.8 \text{ N})/(5.8 \text{ N}) \times 100 = -5.2\%$
- d. i



- ii. The cord cannot be horizontal because the tension must have a vertical component to balance the weight of the ball.
- iii. Resolving tension into components gives T sin $\theta = mg$ and T cos $\theta = mv^2/r$ which gives $\theta = 21^\circ$

1999B5

а



- b. $v = circumference/period = 2\pi R/T = 2\pi (0.14 \text{ m})/(1.5 \text{ s}) = 0.6 \text{ m/s}$
- c. The coin will slip when static friction has reached its maximum value of $\mu_s N = \mu_s mg = mv^2/r$ which gives $v = \sqrt{\mu_s gr} = 0.83$ m/s
- d. It would not affect the answer to part (c) as the mass cancelled out of the equation for the speed of the coin.

2001B1

a.





- b. The minimum speed occurs when gravity alone supplies the necessary centripetal force at the top of the circle (i.e. tension is zero and is not required). Therefore we have $Mg = Mv_{min}^2/R$ which gives $v_{min} = \sqrt{Rg}$
- c. At the bottom of the swing $\Sigma F = \text{ma becomes } T Mg = Mv^2/R$ which gives $T_{\text{max}} Mg = Mv_{\text{max}}^2/R$ and solving for v_{max} gives $v_{\text{max}} = \int_{-M}^{R} (T_{\text{max}} Mg)$
- d. At point P the ball is moving straight up. If the string breaks at that point, the ball would continue to move straight up, slowing down until it reaches a maximum height and fall straight back to the ground.

2002B2B

a.



b. $\Sigma F_v = 0$; T cos $\theta - mg = 0$ gives $m = (T \cos \theta)/g$

c. The centripetal force is supplied by the horizontal component of the tension: $F_C = T \sin \theta = mv^2/r$. Substituting the value of m found in part b. and the radius as $(l \sin \theta)$ gives $v = \sqrt{gl \sin \theta \tan \theta}$

d. substituting the answer above into $v = 2\pi rf$ gives $f = \frac{1}{2\pi} \int \frac{g}{l\cos\theta}$

e. The initial velocity of the ball is horizontal and the subsequent trajectory is parabolic.

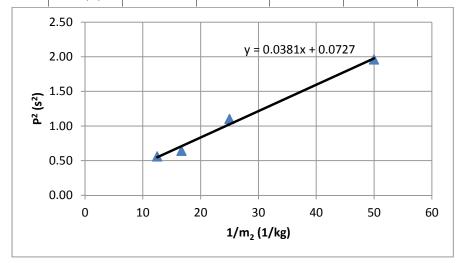
2009B1B

a. The centripetal force is provided by the weight of the hanging mass: F_C = m₂g = m₁v²/r and v is related to the period of the motion v = 2πr/P. This gives m₂g = m₁v²/r = m₁/r = m₁/r (4π²r²/P²) and thus P² = 4π² (m₁r/m₂g)
 b. The quantities that may be graphed to give a straight line are P² and 1/m₂, which will yield a straight line with a

b. The quantities that may be graphed to give a straight line are P^2 and $1/m_2$, which will yield a straight line with a slope of $4\pi^2 \left(\frac{m_1 r}{g}\right)$

c.

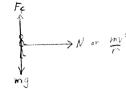
$1/m_2 (kg^{-1})$	50	25	16.7	12.5
m_2 (kg)	0.020	0.040	0.060	0.080
<i>P</i> (s)	1.40	1.05	0.80	0.75
P^2 (s ²)	1.96	1.10	0.64	0.56



d. Using the slope of the line (0.038 kg/s^2) in the equation from part b. gives $g = 9.97 \text{ m/s}^2$

1984M1

a.



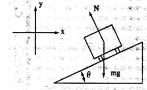
b. $F = mv^2/r$ where $v = 2\pi rf = 2\pi r(1/\pi) = 2r = 10$ m/s giving F = 1000 N provided by the normal force

c. $\Sigma F_v = 0$ so the upward force provided by friction equals the weight of the rider = mg = 490 N

d. Since the frictional force is proportional to the normal force and equal to the weight of the rider, m will cancel from the equation, meaning a rider with twice the mass, or any different mass, will not slide down the wall as mass is irrelevant for this condition.

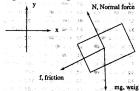
1988M1

а



Toward the center of the turn we have $\Sigma F = N \sin \theta = mv^2/r$ and vertically $N \cos \theta = mg$. Dividing the two expressions gives us $\tan \theta = v^2/rg$ and v = 16 m/s

b.

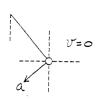


c. $\Sigma F_y = N \cos \theta - f \sin \theta - mg = 0$ and $\Sigma F_x = N \sin \theta + f \cos \theta = mv^2/r$ solve for N and f and substitute into $f = \mu N$ gives $\mu_{min} = 0.32$

1998B6

a i

ii



b. i.



The horizontal velocity is constant, the vertical motion is in free fall and the path is parabolic

ii.



The ball falls straight down in free fall