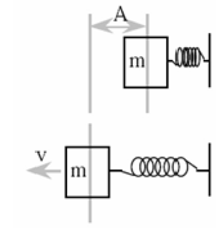
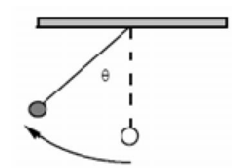
AP Physics Multiple Choice Practice – Work-Energy



1. A mass m attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the masses speed as it passes through its equilibrium position?
   $(A)\ 0 \quad (B)\ A\sqrt{\frac{k}{m}} \quad (C)\ A\sqrt{\frac{m}{k}} \quad (D)\ \frac{1}{A}\sqrt{\frac{k}{m}}$

2. A force F at an angle θ above the horizontal is used to pull a heavy suitcase of weight mg a distance d along a level floor at constant velocity. The coefficient of friction between the floor and the suitcase is μ. The work done by the frictional force is:
   (A) –Fd cos θ (B) – μ Fd cos θ (C) -μmgd (D) -μ mgd cos θ

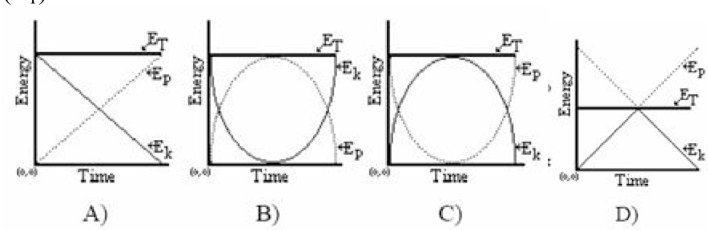
3. A 2 kg ball is attached to a 0.80 m string and whirled in a horizontal circle at a constant speed of 6 m/s. The work done on the ball during each revolution is:
   (A) 90 J (B) 72 J (D) 16 J (D) zero



4. A pendulum bob of mass m on a cord of length L is pulled sideways until the cord makes an angle θ with the vertical as shown in the figure to the right. The change in potential energy of the bob during the displacement is:
   (A) mgL (1–cos θ) (B) mgL (1–sin θ) (C) mgL sin θ
   (D) mgL cos θ

5. A softball player catches a ball of mass m, which is moving towards her with horizontal speed V. While bringing the ball to rest, her hand moved back a distance d. Assuming constant deceleration, the horizontal force exerted on the ball by the hand is
   (A) $mV^2/(2d)$ (B) mV^2/d (C) 2mV/d (D) mV/d

6. A pendulum is pulled to one side and released. It swings freely to the opposite side and stops. Which of the following might best represent graphs of kinetic energy (E_k), potential energy (E_p) and total mechanical energy (E_T)



Questions 7-8: A car of mass m slides across a patch of ice at a speed v with its brakes locked. It the hits dry pavement and skids to a stop in a distance d. The coefficient of kinetic friction between the tires and the dry road is μ.

7. If the car has a mass of 2m, it would have skidded a distance of
   (A) 0.5 d (B) d (C) 1.41 d (D) 2 d

8. If the car has a speed of 2v, it would have skidded a distance of
   (A) d (B) 1.41 d (C) 2 d (D) 4 d

9. A ball is thrown vertically upwards with a velocity v and an initial kinetic energy E_k. When half way to the top of its flight, it has a velocity and kinetic energy respectively of
   (A) $\frac{v}{2}, \frac{E_k}{2}$ (B) $\frac{v}{\sqrt{2}}, \frac{E_k}{2}$ (C) $\frac{v}{4}, \frac{E_k}{2}$ (D) $\frac{v}{2}, \frac{E_k}{\sqrt{2}}$

10. A football is kicked off the ground a distance of 50 yards downfield. Neglecting air resistance, which of the following statements would be INCORRECT when the football reaches the highest point?
(A) all of the balls original kinetic energy has been changed into potential energy
(B) the ball's horizontal velocity is the same as when it left the kickers foot
(C) the ball will have been in the air one-half of its total flight time
(D) the vertical component of the velocity is equal to zero

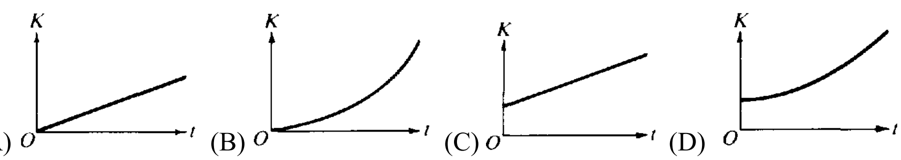
11. A mass m is attached to a spring with a spring constant k. If the mass is set into motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to equilibrium position?
(A) $v = \sqrt{\frac{kd}{m}}$ (B) $v = d\sqrt{\frac{k}{m}}$ (C) $v = \frac{kd}{mg}$ (D) $v^2 = \frac{mgd}{k}$

12. A fan blows the air and gives it kinetic energy. An hour after the fan has been turned off, what has happened to the kinetic energy of the air?
(A) it disappears (B) it turns into potential energy (C) it turns into thermal energy
(D) it turns into sound energy

13. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) they strike the ground more than half a second apart.
(D) they strike the ground with the same kinetic energy.

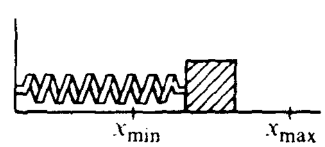
14. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(C) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
(D) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position

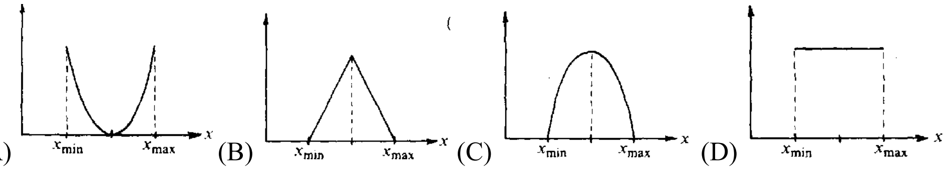
15. From the top of a high cliff, a ball is thrown horizontally with initial speed v_o. Which of the following graphs best represents the ball's kinetic energy K as a function of time t ?



(A) (B) (C) (D)

Questions 16-17: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.





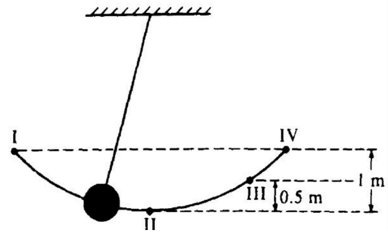
(A) (B) (C) (D)

16. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C (D) D

17. Which graph can represent the kinetic energy of the block as a function of x ?
    (A) A (B) B (C) C (D) D

Questions 18-19

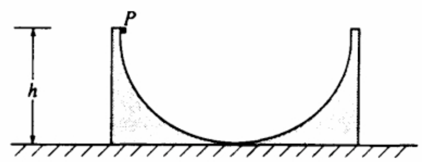
A ball swings freely back and forth in an arc from point I to point IV, as shown. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is I meter above II. Air resistance is negligible.



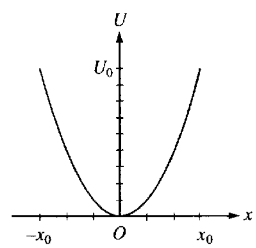
18. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?
    (A) At point II (B) At some point between II and III
    (C) At point III (D) At some point between III and IV

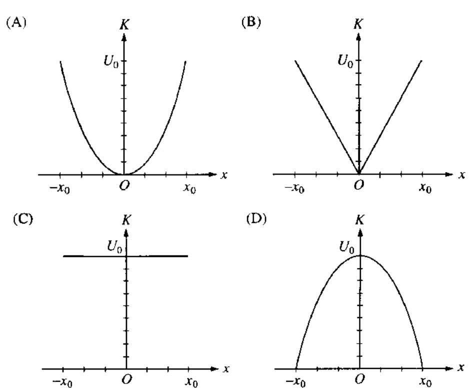
19. The speed of the ball at point II is most nearly
    (A) 3.0 m/s (B) 4.5 m/s (C) 9.8 m/s (D) 14 m/s

20. The figure shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?
    (A) It is equal to h/4 (B) It is equal to h/2
    (C) It is equal to h
    (D) It is between zero and h; the exact height depends on how much energy is lost to friction.



21. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?

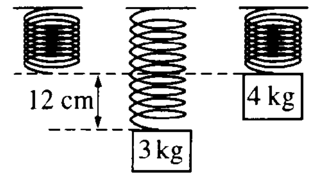




22. A child pushes horizontally on a box of mass m which moves with constant speed v across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

    (A) μmgv (B) mgv (C) $\mu mg/v$ (D) $\mu mg/v$

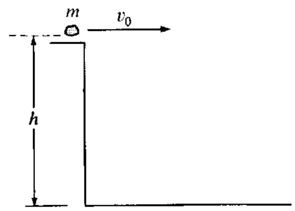
23. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?



(A) 18 cm (B) 24 cm
(C) 32 cm (D) 48 cm

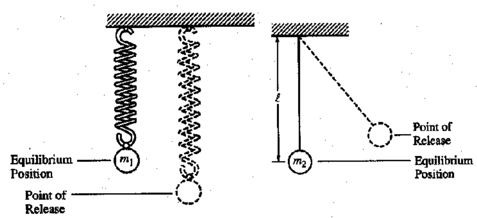
24. What is the kinetic energy of a satellite of mass m that orbits the Earth, of mass M, in a circular orbit of radius R?

(A) $\frac{1}{2}\frac{GMm}{R}$ (B) $\frac{1}{4}\frac{GMm}{R}$ (C) $\frac{1}{2}\frac{GMm}{R^2}$ (D) $\frac{GMm}{R^2}$



25. A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh (B) ½ mv_0^2 (C) ½ $mv_0^2 - mgh$ (D) ½ $mv_0^2 + mgh$



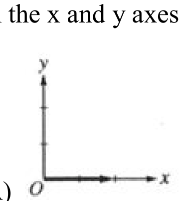
26. A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion Which of the following is true for both spheres?

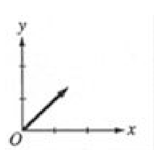
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position.
(B) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(C) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(D) The maximum total energy is attained only as the sphere passes through its equilibrium position.

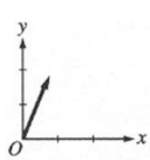
<u>Questions 27-28</u>

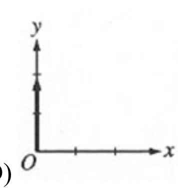
An object of mass m is initially at rest and free to move without friction in any direction in the xy-plane. A constant net force of magnitude F directed in the +x direction acts on the object for 1 s. Immediately thereafter a constant net force of the same magnitude F directed in the +y direction acts on the object for 1 s. After this, no forces act on the object.

27. Which of the following vectors could represent the velocity of the object at the end of 3 s, assuming the scales on the x and y axes are equal?

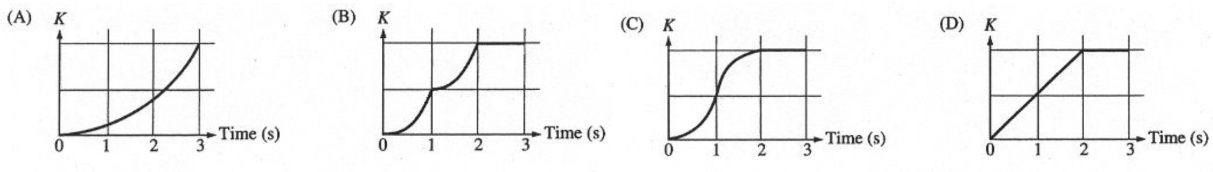
(A) 

(B) 

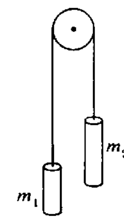
(C) 

(D) 

28. Which of the following graphs best represents the kinetic energy K of the object as a function of time?

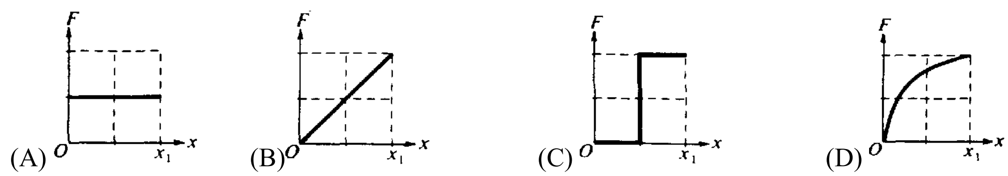


29. A system consists of two objects having masses m_1 and m_2 ($m_1 < m_2$). The objects are connected by a massless string, hung over a pulley as shown, and then released. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by

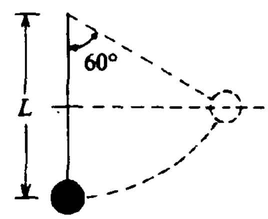


(A) $(m_2 - m_1)gh$ (B) m_2gh (C) $(m_1 + m_2)gh$ (D) $\frac{1}{2}(m_1 + m_2)gh$

30. The following graphs, all drawn to the same scale, represent the net force F as a function of displacement x for an object that moves along a straight line. Which graph represents the force that will cause the greatest change in the kinetic energy of the object from x = 0 to $x = x_1$?



31. A pendulum consists of a ball of mass m suspended at the end of a massless cord of length L as shown. The pendulum is drawn aside through an angle of 60° with the vertical and released. At the low point of its swing, the speed of the pendulum ball is

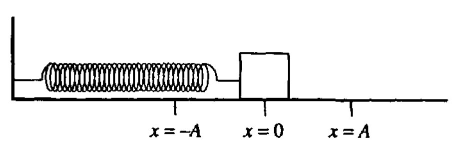


(A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) ½gL (D) gL

32. A rock is lifted for a certain time by a force F that is greater in magnitude than the rock's weight W. The change in kinetic energy of the rock during this time is equal to the

(A) work done by the net force (F - W)
(B) work done by F alone
(C) work done by W alone
(D) difference in the potential energy of the rock before and after this time.

33. A block on a horizontal frictionless plane is attached to a spring, as shown. The block oscillates along the x-axis with amplitude A. Which of the following statements about energy is correct?

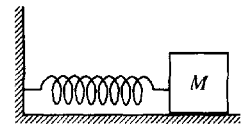


(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.

34. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

(A) h/4 (B) $\dfrac{h}{2\sqrt{2}}$ (C) h/2 (D) $\dfrac{h}{\sqrt{2}}$

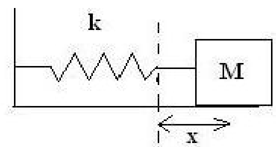
35. An ideal massless spring is fixed to the wall at one end, as shown. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is



(A) $\dfrac{Mgv_m}{2A}$ (B) $\dfrac{Mv_m^2}{2A}$ (C) $\dfrac{Mv_m^2}{A^2}$ (D) $\dfrac{Mv_m^2}{2A^2}$

36. A person pushes a block of mass M = 6.0 kg with a constant speed of 5.0 m/s straight up a flat surface inclined 30.0° above the horizontal. The coefficient of kinetic friction between the block and the surface is $\mu = 0.40$. What is the net force acting on the block?
(A) 0 N (B) 21 N (C) 30 N (D) 51 N

37. A block of mass *M* on a horizontal surface is connected to the end of a massless spring of spring constant *k* . The block is pulled a distance *x* from equilibrium and when released from rest, the block moves toward equilibrium. What coefficient of kinetic friction between the surface and the block would allow the block to return to equilibrium and stop?



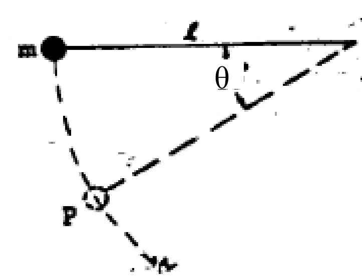
(A) $\dfrac{kx^2}{2Mg}$ (B) $\dfrac{kx}{Mg}$ (C) $\dfrac{kx}{2Mg}$ (D) $\dfrac{Mg}{2kx}$

38. An object is dropped from rest from a certain height. Air resistance is negligible. After falling a distance d, the object's kinetic energy is proportional to which of the following?
(A) $1 / d^2$ (B) 1 / d (C) $\sqrt{d}$ (D) d

39. An object is projected vertically upward from ground level. It rises to a maximum height H. If air resistance is negligible, which of the following must be true for the object when it is at a height H/2 ?
(A) Its speed is half of its initial speed.
(B) Its kinetic energy is half of its initial kinetic energy.
(C) Its potential energy is half of its initial potential energy.
(D) Its total mechanical energy is half of its initial value.

AP Physics Free Response Practice – Work Power Energy

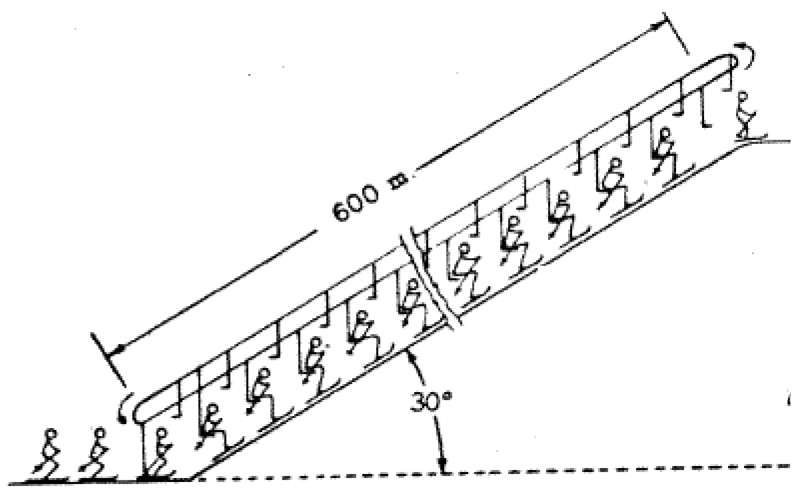
1974B1. A pendulum consisting of a small heavy ball of mass m at the end of a string of length L is released from a horizontal position. When the ball is at point P, the string forms an angle of θ with the horizontal as shown.

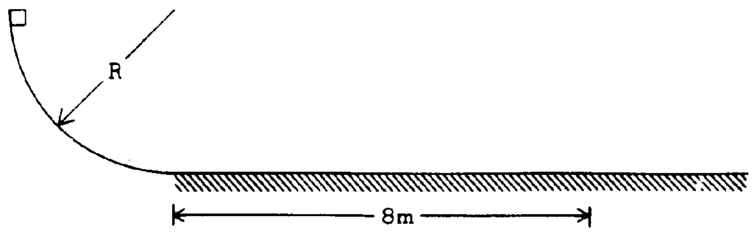


(a) In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

(b) Determine the speed of the ball at P.
(c) Determine the tension in the string when the ball is at P.

1974B7. A ski lift carries skiers along a 600 meter slope inclined at 30°. To lift a single rider, it is necessary to move 70 kg of mass to the top of the lift. Under maximum load conditions, six riders per minute arrive at the top. If 60 percent of the energy supplied by the motor goes to overcoming friction, what average power must the motor supply?



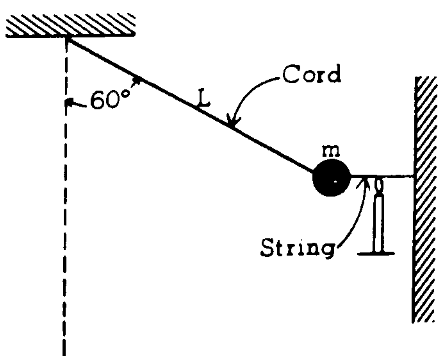


1975B1. A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume g = 10 meters per second^2.

a. Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.

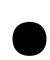
b. How much time elapses while the block is sliding horizontally?

c. Calculate the radius of the incline in meters.



1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.



b. Determine the tension in the cord before the string is burned.

c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

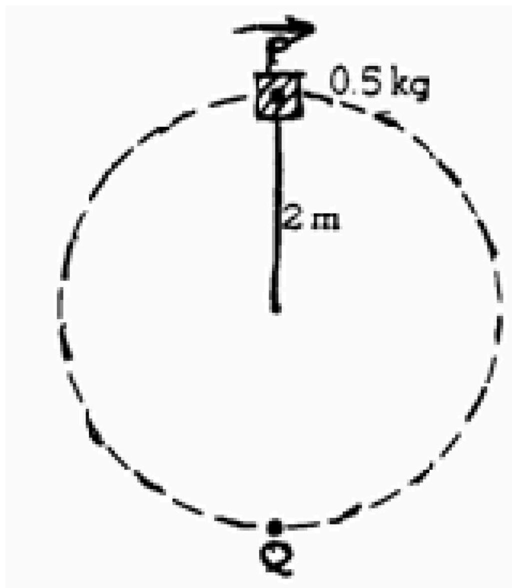
1977 B1. A block of mass 4 kilograms, which has an initial speed of 6 meters per second at time t = 0, slides on a horizontal surface.

a. Calculate the work W that must be done on the block to bring it to rest.

If a constant friction force of magnitude 8 newtons is exerted on the block by the surface, determine the following:

b. The speed v of the block as a function of the time t.

c. The distance x that the block slides as it comes to rest



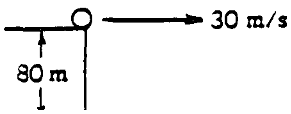
1978B1. A 0.5 kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume g = 10 meters per second squared.

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P.

(b) Calculate the speed of the object at point P.

(c) Calculate the increase in kinetic energy of the object as it moves from point P to point Q.

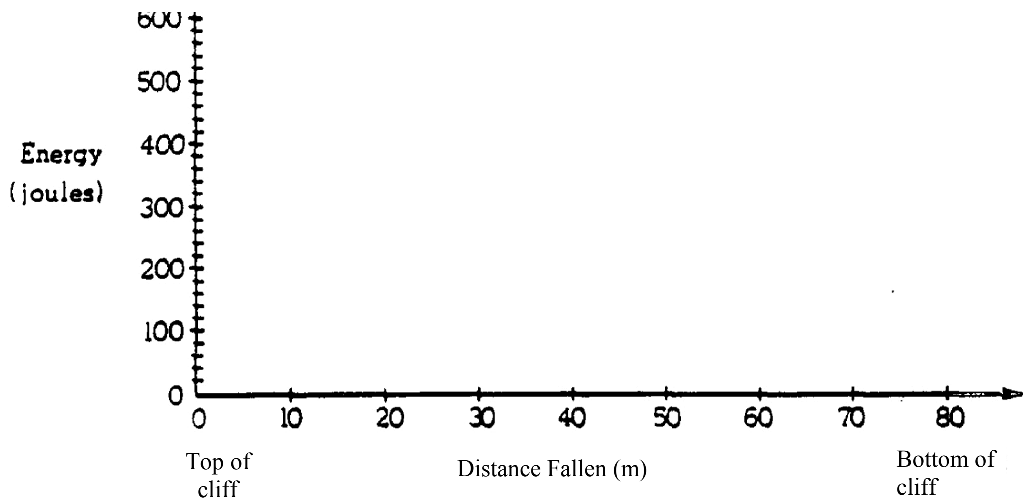
(d) Calculate the tension in the string as the object passes through point Q.



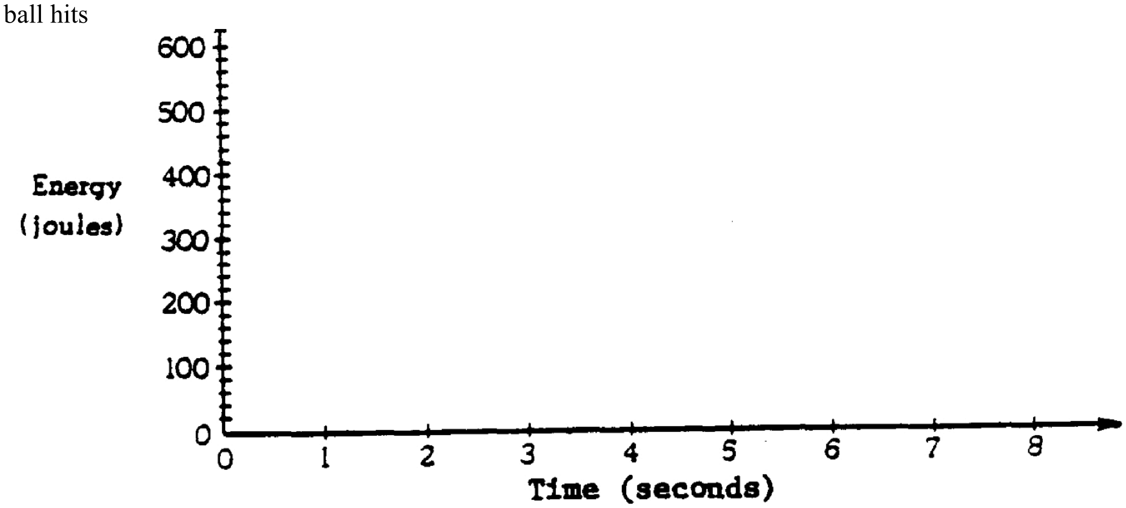
1979B1. From the top of a cliff 80 meters high, a ball of mass 0.4 kilogram is launched horizontally with a velocity of 30 meters per second at time t = 0 as shown above. The potential energy of the ball is zero at the bottom of the cliff. Use g = 10 meters per second squared.

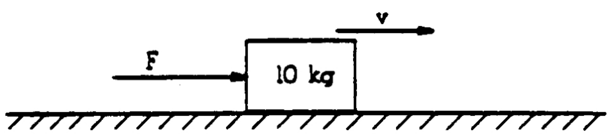
a. Calculate the potential, kinetic, and total energies of the ball at time t = 0.

b. On the axes below, sketch and label graphs of the potential, kinetic, and total energies of the ball as functions of the distance fallen from the top of the cliff



c. On the axes below sketch and label the kinetic and potential energies of the ball as functions of time until the ball hits





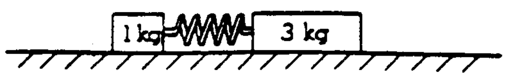
1981B1. A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time t = 0, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume g = 10 meters per second squared.

a. Calculate the force F necessary to keep the velocity constant.

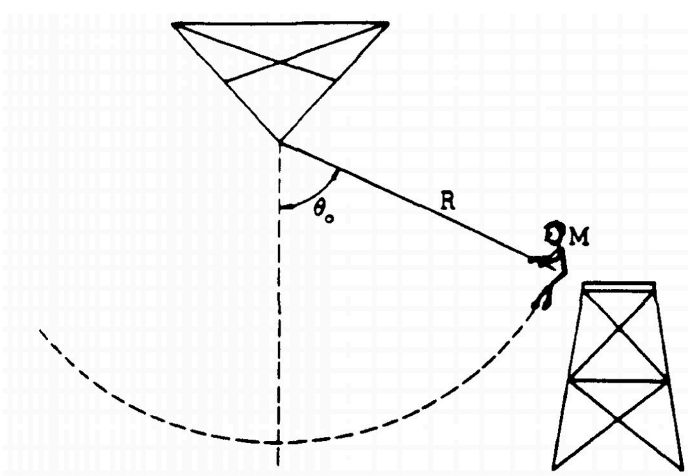
The force is now changed to a larger constant value F'. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.

b. Calculate the force F'.

c. Calculate the acceleration of the block.



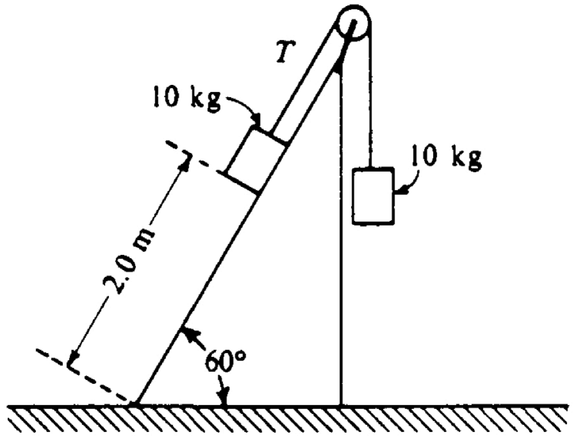
1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second. Determine the minimum work needed to compress the spring in this experiment.



1982B3. A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

a. Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and cos θ_o

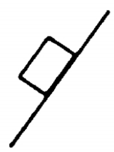
b. The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of cos θ_o.



1985B2. Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, $\sin 60° = 0.87$, and $\cos 60° = 0.50$.

a. What is the tension T in the string?

b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

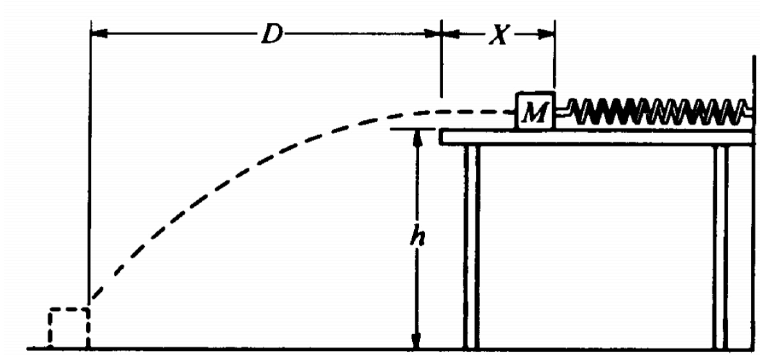


c. Determine the magnitude of the frictional force acting on the box on the plane.

The string is then cut and the left-hand box slides down the inclined plane.

d. Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

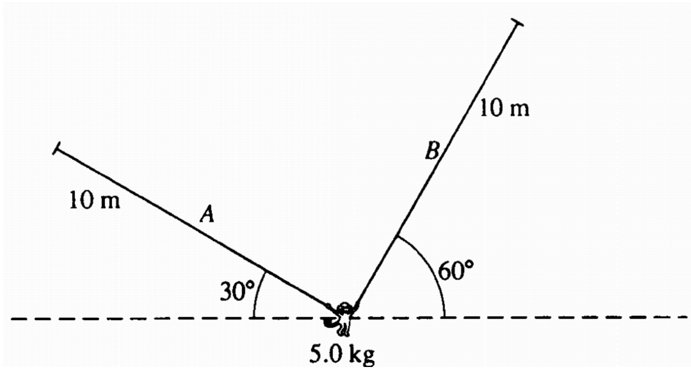
e. Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.



1986B2. One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance *h* above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible.

Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

a. The time elapsed from the instant the block leaves the table to the instant it strikes the floor
b. The horizontal component of the velocity of the block just before it hits the floor
c. The work done on the block by the spring
d. The spring constant



sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

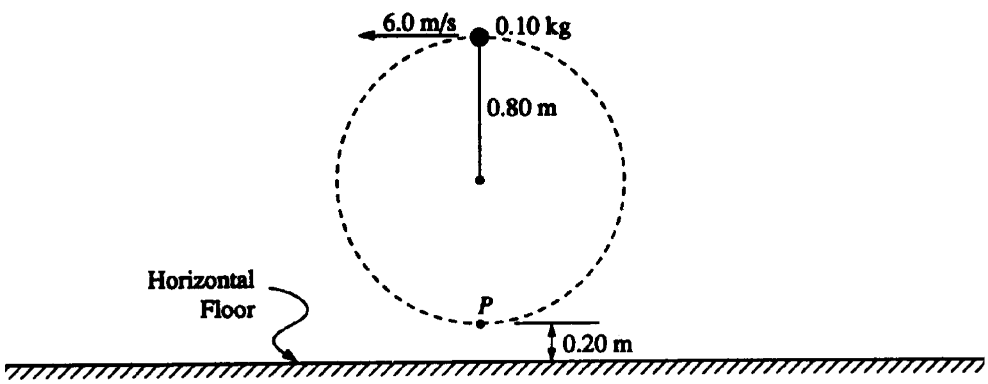
1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.

The monkey releases vine A and swings on vine B. Neglect air resistance.

c. Determine the speed of the monkey as it passes through the lowest point of its first swing.
d. Determine the tension in vine B as the monkey passes through the lowest point of its first swing.

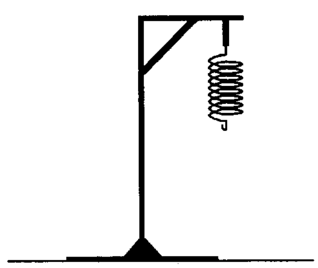


1992B1. A 0.10-kilogram solid rubber ball is attached to the end of an 0.80 meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

a. Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.
b. Determine the speed of the ball at point P, the lowest point of the circle.
c. Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.

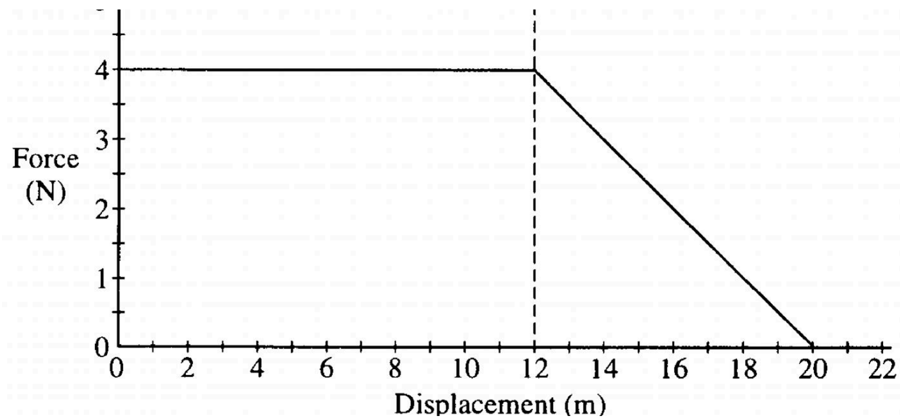
The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

d. Determine the horizontal distance that the ball travels before hitting the floor.



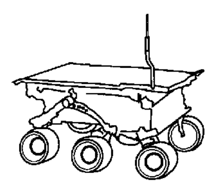
1996B2 (15 points) A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?
b. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

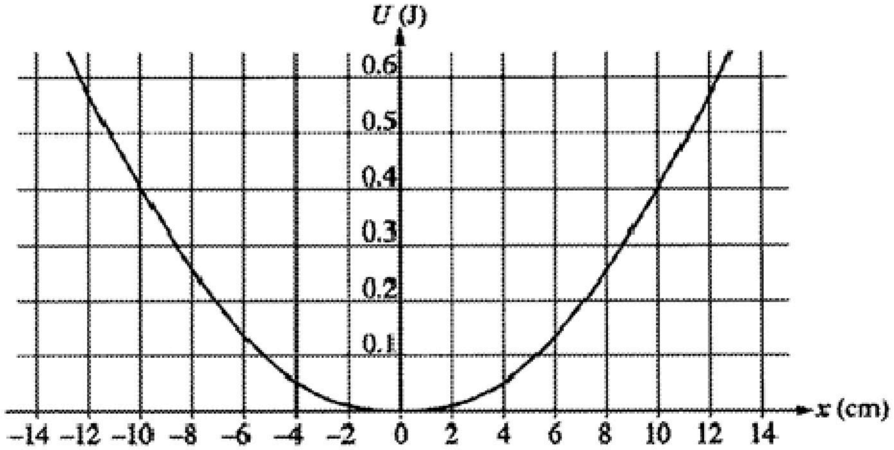
a. The acceleration of the particle when its displacement x is 6 m.
b. The time taken for the object to be displaced the first 12 m.
c. The amount of work done by the net force in displacing the object the first 12 m.
d. The speed of the object at displacement x = 12 m.
e. The final speed of the object at displacement x = 20 m.



1999B1. The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data	
Radius: 0.53 x Earth's radius	Mass of Sojourner vehicle:	11.5 kg
Mass: 0.11 x Earth's mass	Wheel diameter: 0.13 m	
	Stored energy available:	5.4×10^5 J
	Power required for driving under average conditions:	10 W
	Land speed:	6.7×10^{-3} m/s

a. Determine the acceleration due to gravity at the surface of Mars in terms of g, the acceleration due to gravity at the surface of Earth.
b. Calculate Sojourner's weight on the surface of Mars.
c. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
d. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
e. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
f. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.



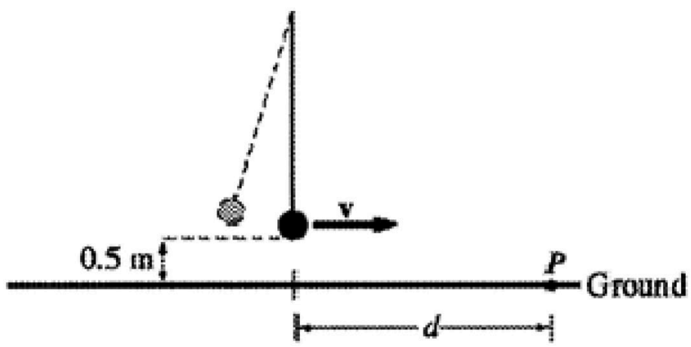
2002B2. A 3.0 kg object subject to a restoring force F is undergoing simple harmonic motion with a small amplitude. The potential energy U of the object as a function of distance x from its equilibrium position is shown above. This particular object has a total energy E: of 0.4 J.

(a) What is the object's potential energy when its displacement is +4 cm from its equilibrium position?

(b) What is the farthest the object moves along the x axis in the positive direction? Explain your reasoning.

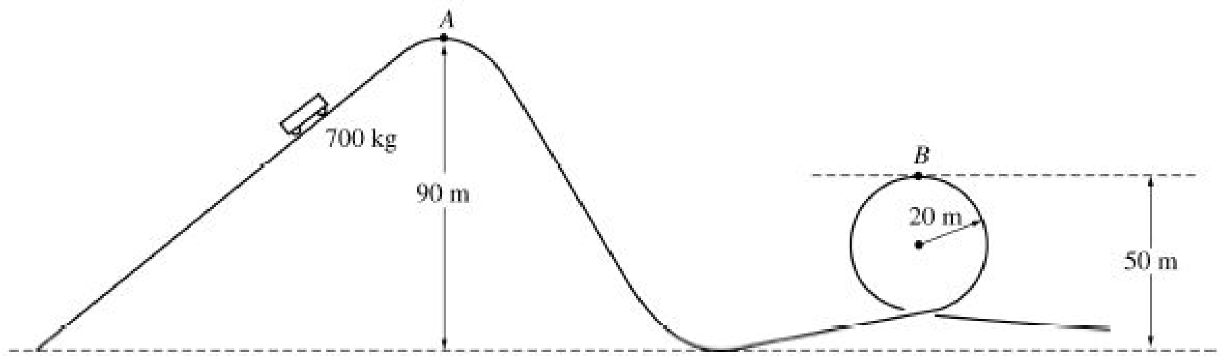
(c) Determine the object's kinetic energy when its displacement is – 7 cm.

(d) What is the object's speed at x = 0 ?



Note: Figure not drawn to scale.

(e) Suppose the object undergoes this motion because it is the bob of a simple pendulum as shown above. If the object breaks loose from the string at the instant the pendulum reaches its lowest point and hits the ground at point P shown, what is the horizontal distance d that it travels?



2004B1.
A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

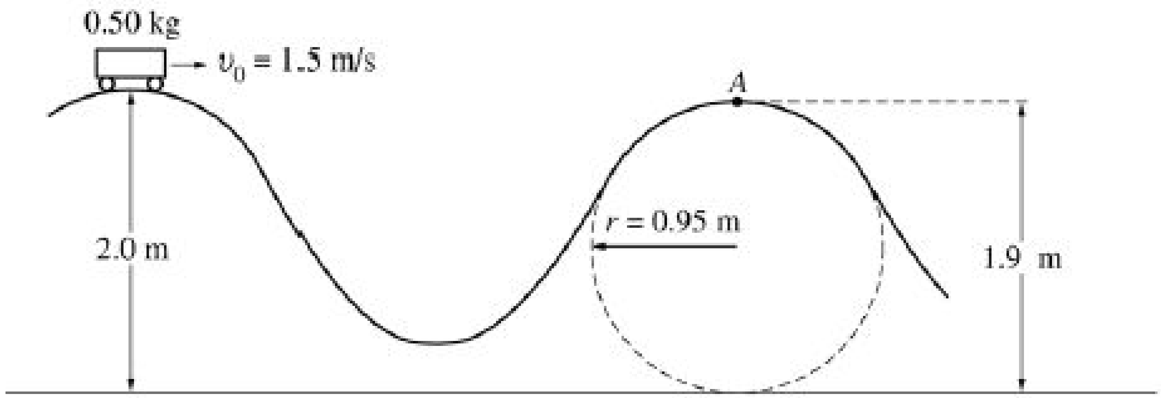
(a)
i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.
ii. Calculate the value v_{msx} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.

ii. Calculate the magnitude of all the forces identified in (c)

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.



B2004B1.
A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_o = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

(a) Calculate the speed of the car at point *A*.

(b) On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.

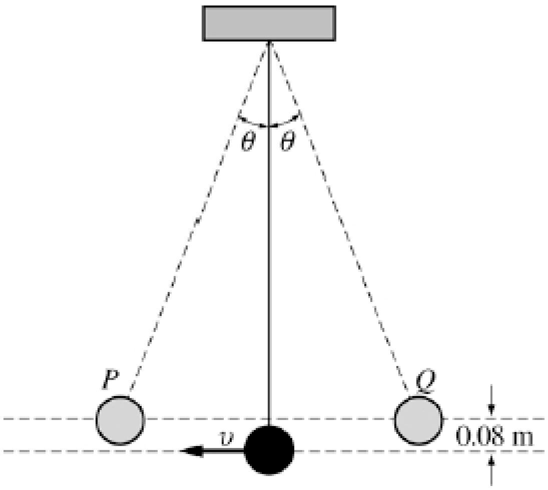
(c) Calculate the magnitude of the force of the track on the car at point *A*.

(d) In order to stop the car at point *A*, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

(e) Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

B2005B2

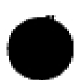
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



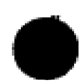
Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

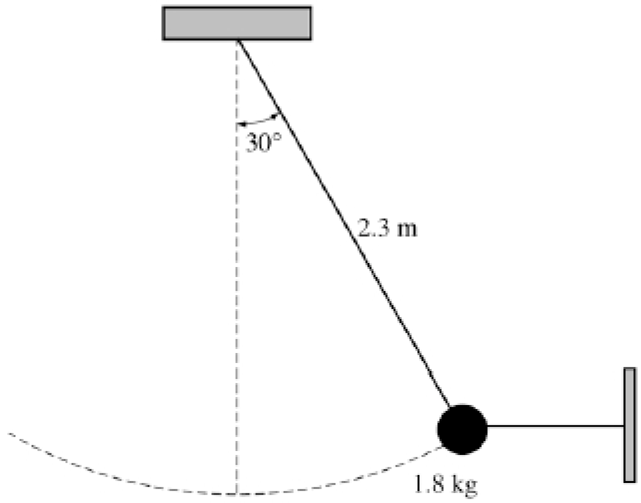


ii. When it is in motion at its lowest position

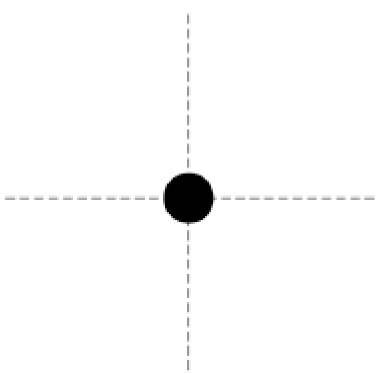


(b) Calculate the speed v of the bob at its lowest position.
(c) Calculate the tension in the string when the bob is passing through its lowest position.

2005B2.

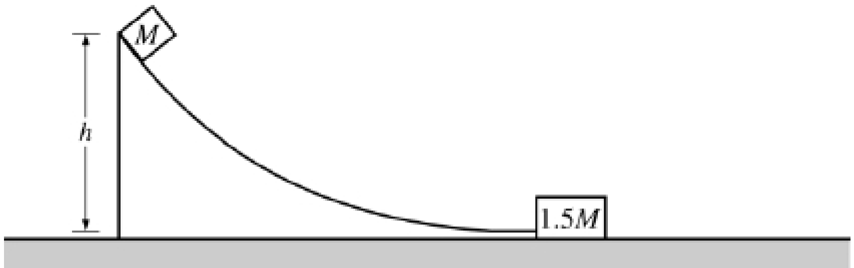


2. (10 points)
A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

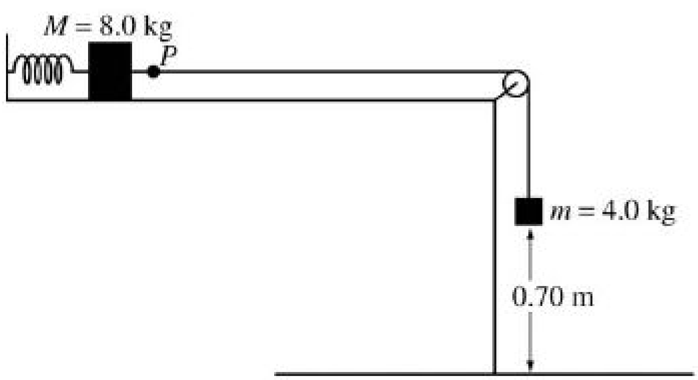
B2006B2



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.

Express your answers to the following questions in terms of the given quantities and fundamental constants.
(a) Determine the height h of the ramp from which the small block was released.
(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

2006B1

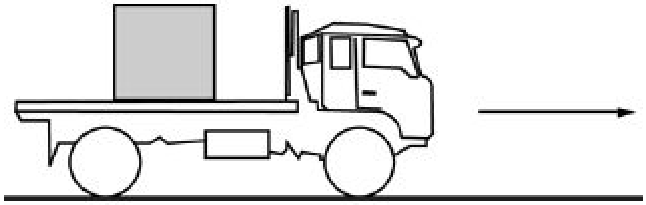


An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.
(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.
M = 8.0 kg $\qquad\qquad\qquad$ m = 4.0 kg

(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

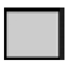
The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the maximum speed attained by the 8.0 kg block as it oscillates back and forth

B2008B2



A 4700 kg truck carrying a 900 kg crate is traveling at 25 m/s to the right along a straight, level highway, as shown above. The truck driver then applies the brakes, and as it slows down, the truck travels 55 m in the next 3.0 s. The crate does not slide on the back of the truck.

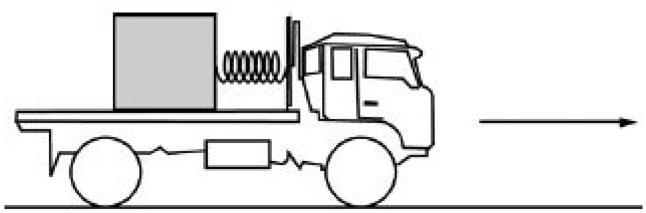
(a) Calculate the magnitude of the acceleration of the truck, assuming it is constant.
(b) On the diagram below, draw and label all the forces acting on the crate during braking.



(c)
i. Calculate the minimum coefficient of friction between the crate and truck that prevents the crate from sliding.

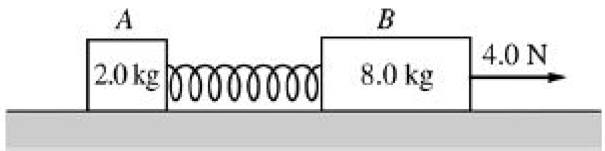
ii. Indicate whether this friction is static or kinetic.
____ Static ____Kinetic

Now assume the bed of the truck is frictionless, but there is a spring of spring constant 9200 N/m attaching the crate to the truck, as shown below. The truck is initially at rest.



(d) If the truck and crate have the same acceleration, calculate the extension of the spring as the truck accelerates from rest to 25 m s in 10 s.
(e) At some later time, the truck is moving at a constant speed of 25 m/s and the crate is in equilibrium. Indicate whether the extension of the spring is greater than, less than, or the same as in part (d) when the truck was accelerating.
___ Greater ___ Less ___ The same
Explain your reasoning.

2008B2

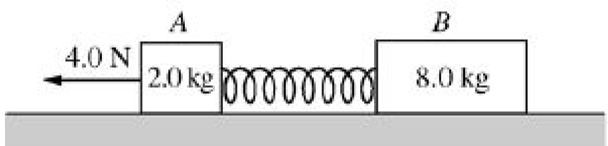


Block *A* of mass 2.0 kg and block *B* of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.

(b**)** Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c**)** Is the magnitude of the acceleration greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

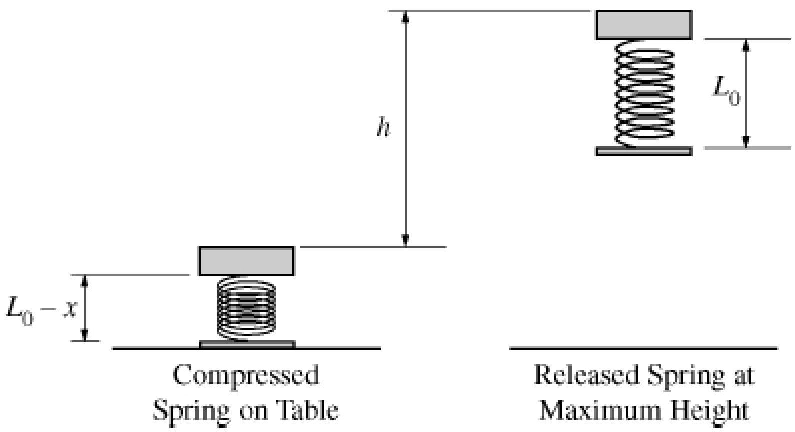
(d) Is the amount the spring has stretched greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m s to the left. Block *A* then hits and sticks to a wall. Calculate the maximum compression of the spring.

2009B1



In an experiment, students are to calculate the spring constant k of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance x from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height h above the point of maximum compression. The students repeat the experiment several times, measuring h with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is m. The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped to it.

(a) Derive an expression for the height h in terms of m, x, k, and fundamental constants.

With the spring compressed a distance $x = 0.020$ m in each trial, the students obtained the following data for different values of m.

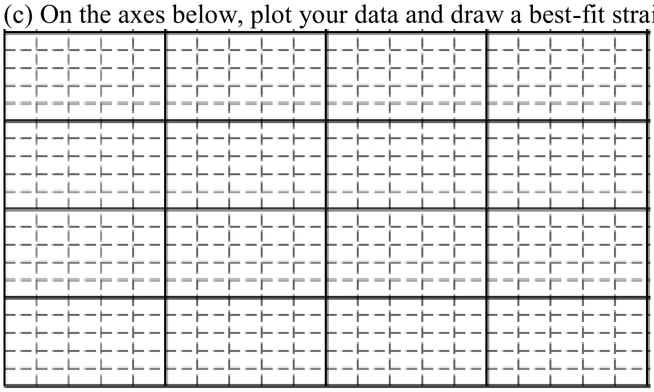
	m (kg)	h (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

(b)

i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant k ?

ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.

(c) On the axes below, plot your data and draw a best-fit straight line. Label the axes and indicate the scale.



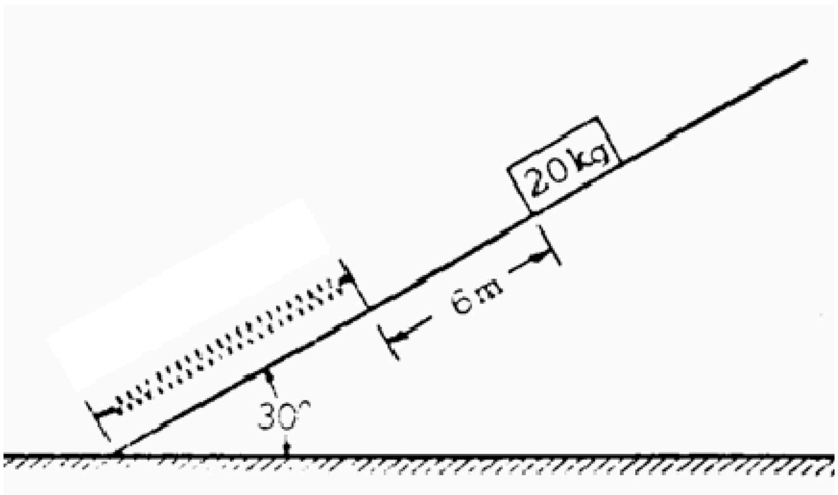
(d) Using your best-fit line, calculate the numerical value of the spring constant.

(e) Describe a procedure for measuring the height h in the experiment, given that the toy is only momentarily at that maximum height.

C1973M2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.

a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.

b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?

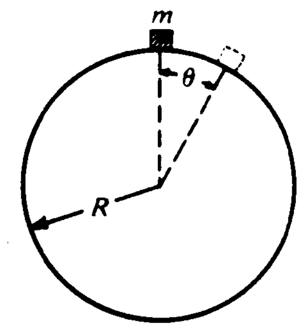


C1982M1. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30º with the horizontal and strikes a spring of unknown spring constant as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved.

a. Determine the speed of the block just before it hits the spring.

b. Determine the spring constant given that the distance the spring compresses along the incline is 3m when the block comes to rest.

c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

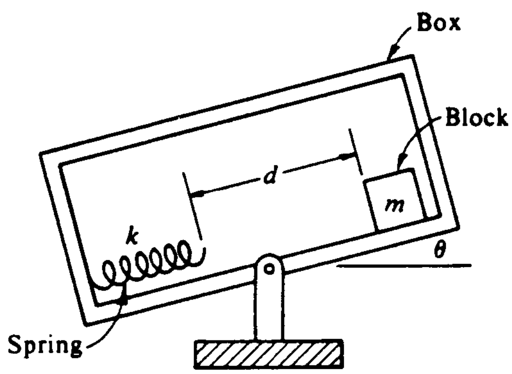


C1983M3. A particle of mass m slides down a fixed, frictionless sphere of radius *R.* starting from rest at the top.

a. In terms of m, g, *R.* and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass



C1985M2. An apparatus to determine coefficients of friction is shown above. At the angle θ shown with the horizontal, the block of mass m just starts to slide. The box then continues to slide a distance d at which point it hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities and fundamental constants, derive an expression for each of the following.

a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

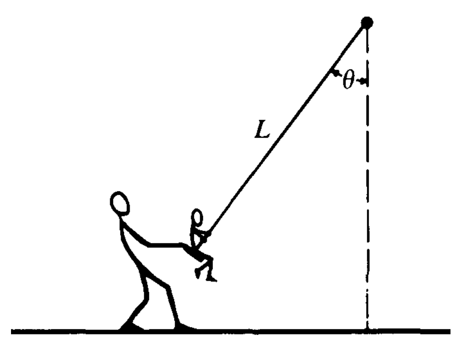
c. μ_k, the coefficient of kinetic friction.

C1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

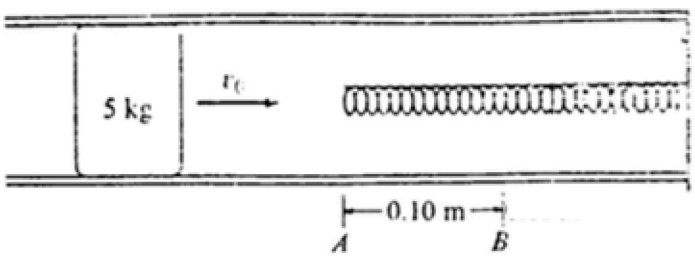
a) The tension in the rope

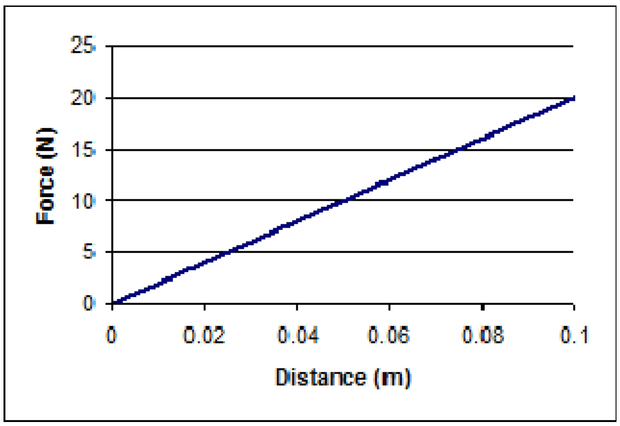
b) The horizontal force exerted by the adult.

c) The adult releases the swing from rest. In terms of W and θ determine the tension in the rope as the swing passes through its lowest point



C1988M2.



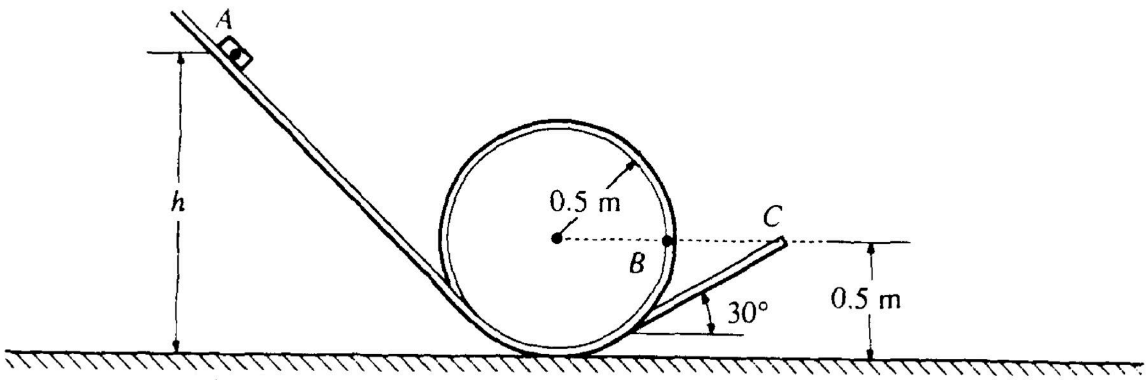


A 5-kilogram object initially slides with speed v_o in a hollow frictionless pipe. The end of the pipe contains a spring as shown. The object makes contact with the spring at point A and moves 0.1 meter before coming to rest at point B. The graph shows the magnitude of the force exerted on the object by the spring as a function of the objects distance from point A.

a. Calculate the spring constant for the spring.

b. Calculate the decrease in kinetic energy of the object as it moves from point A to point B.

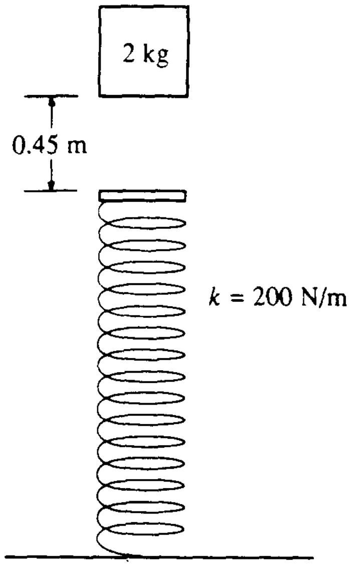
c. Calculate the initial speed v_o of the object



C1989M1. A 0.1 kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 m/s at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

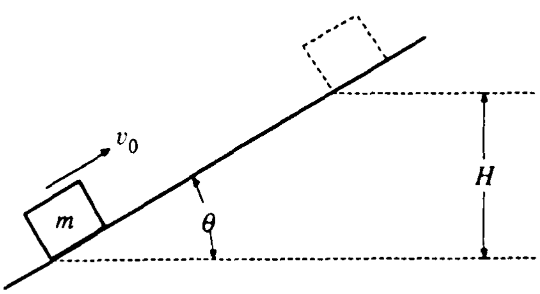
a. Determine the height h .
b. On the figure below, draw and label all the forces acting on the block when it is at point B. which is 0.5 meter above the ground.

c. Determine the magnitude of the velocity of the block when it is at point B.
d. Determine the magnitude of the force exerted by the track on the block when it is at point B.
e. Determine the maximum height above the ground attained by the block after it leaves the track.
f. Another track that has the same configuration, but is **NOT** frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 m/s, the height h must be 2 meters. Determine the work done by the frictional force.



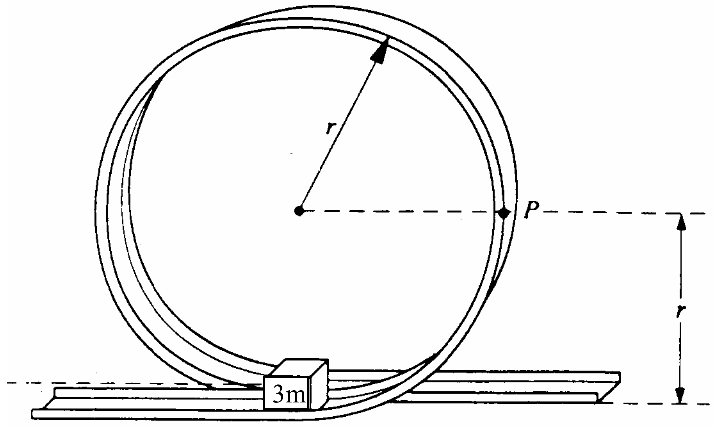
C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Is the speed of the block a maximum at the equilibrium position, explain.



C1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, the box slides a distance $d = h_2 / \sin\theta$ along the length of the ramp as it reaches a new maximum height h_2. Determine the new maximum height h_2 in terms of the given quantities.



C1991M1. A small block of mass 3m moving at speed $v_0 / 3$ enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o , r, and g.

a. The kinetic energy of the block and bullet when they reach point P on the loop

b. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times

c The new required entry speed v_o' at the bottom of the loop such that the conditions in part b apply.

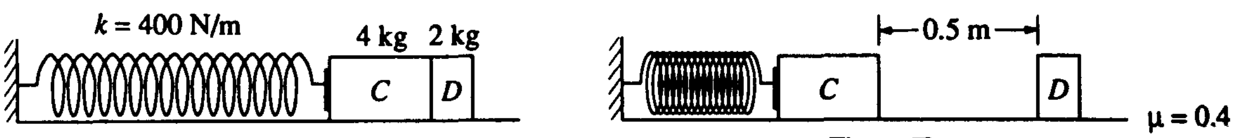


Figure I Figure II

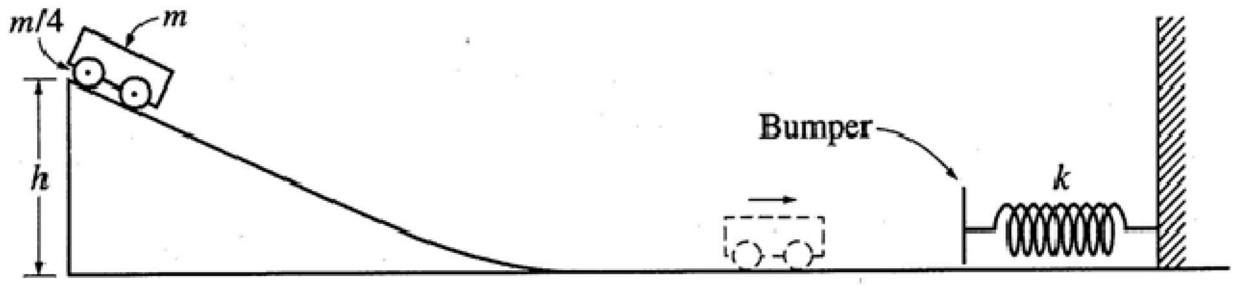
C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a rough horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right at 3 m/s. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

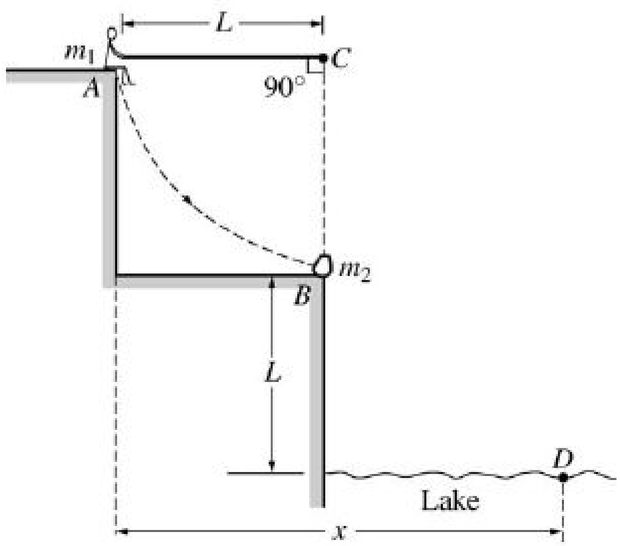
c. The horizontal distance the combined blocks move after leaving the spring before coming to rest



C2002M2. The cart shown above has a mass 2m. The cart is released from rest and slides from the top of an inclined frictionless plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the speed of the cart when it reaches the bottom of the incline.
b. After sliding down the incline and across the frictionless horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

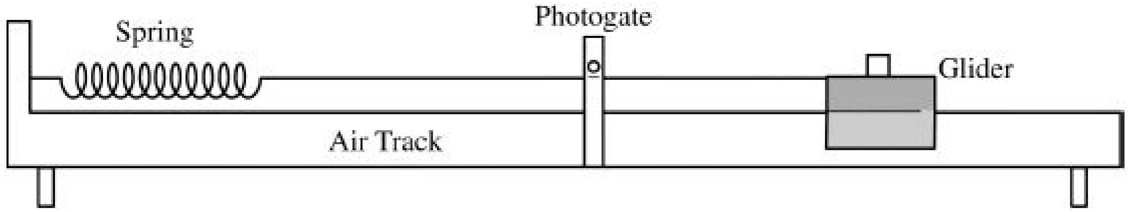
C2004M1



A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) After the person hits and grabs the rock, the speed of the combined masses is determined to be v'. In terms of v' and the given quantities, determine the total horizontal displacement x of the person from position A until the person and object land in the water at point D.

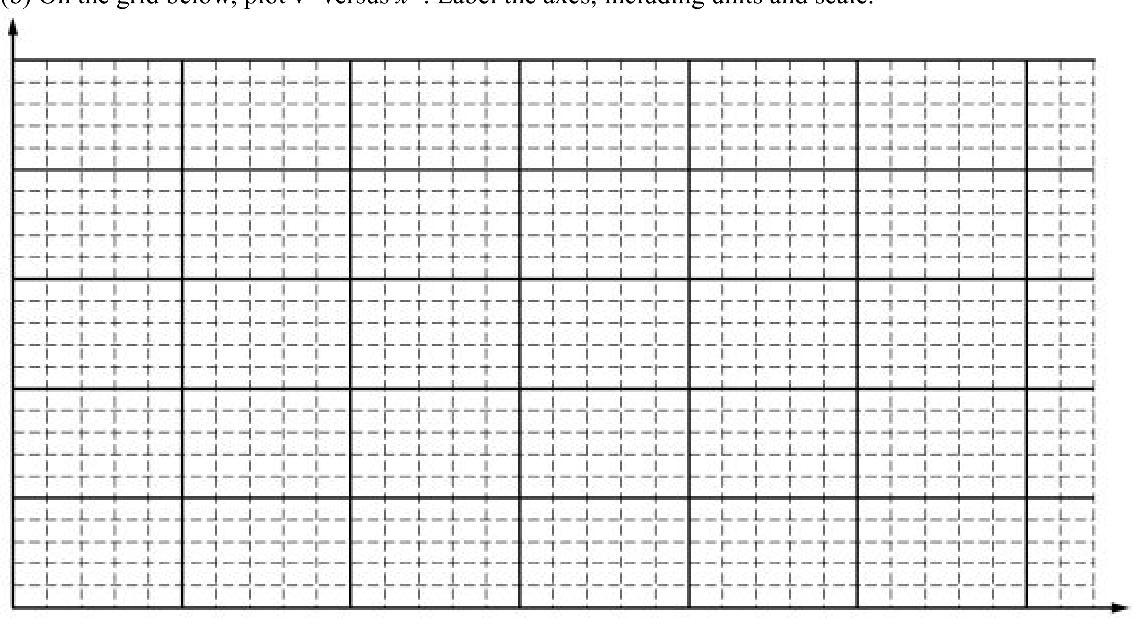
C2007M3.



The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass *m*. The glider is pulled to stretch the spring an amount *x* from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time *t* that it takes the small block on top of the glider to pass through. Information about the distance *x* and the speed *v* of the glider as it passes through the photogate are given below.

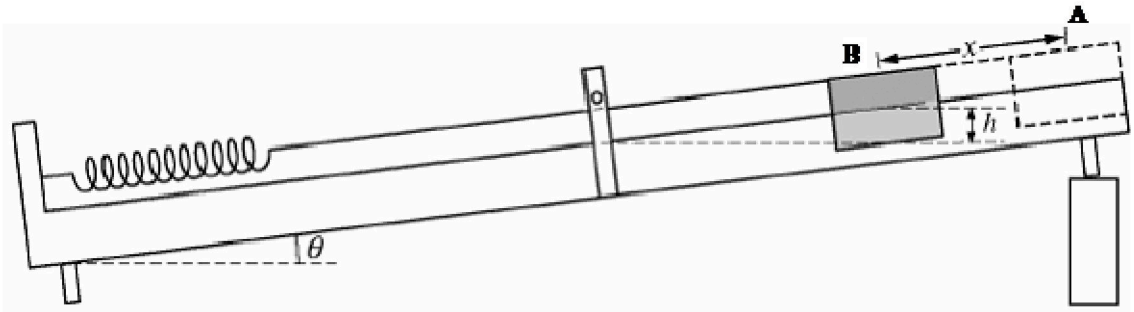
Trial #	Extension of the Spring x (m)	Speed of Glider v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.
(b) On the grid below, plot v^2 versus x^2. Label the axes, including units and scale.



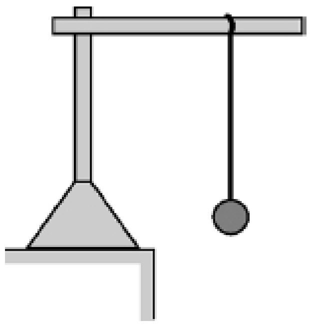
(c)
i. Draw a best-fit straight line through the data.

ii. Use the best-fit line to obtain the mass m of the glider.
(d) The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height h above the photogate. The experiment is repeated with a variety of values of x.



Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation starting from position A and ending at position B

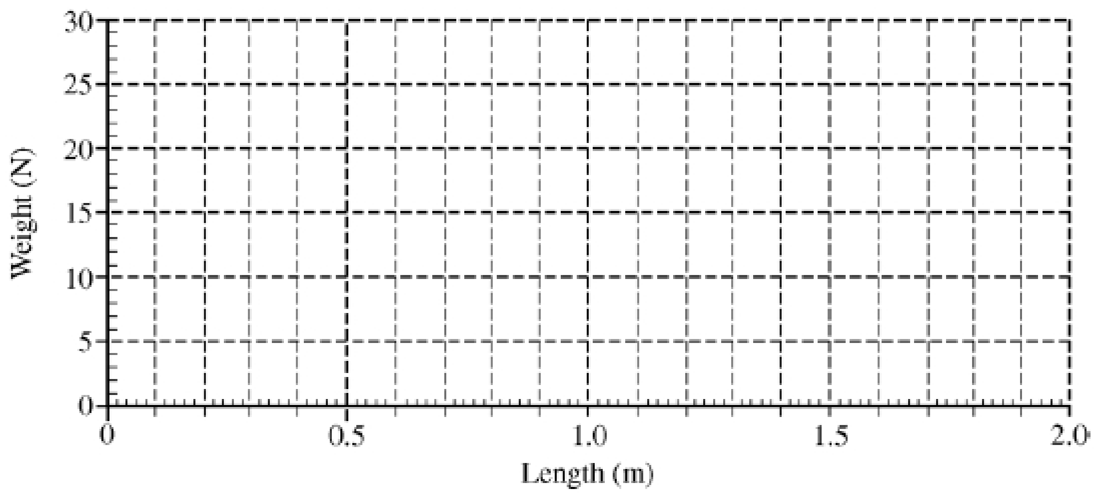
C2008M3



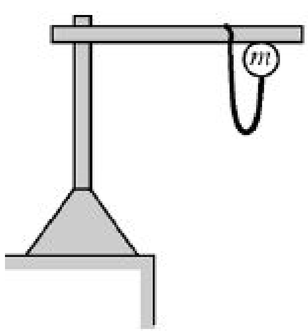
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



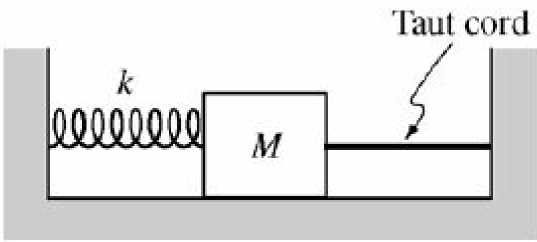
The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass m of the object.

(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

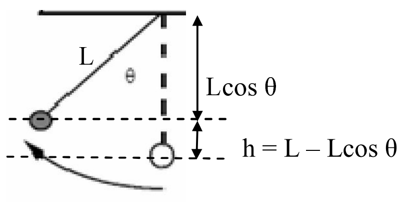
(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

<u>ANSWERS - AP Physics Multiple Choice Practice – Work-Energy</u>

	Solution	Answer
1.	Conservation of Energy, $U_{sp} = K$, $\frac{1}{2} kA^2 = \frac{1}{2} mv^2$ solve for v	B
2.	Constant velocity → $F_{net}=0$, $f_k = Fx = F\cos\theta$ $W_{fk} = - f_k d = - F\cos\theta\, d$	A
3.	In a circle moving at a constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle	D
4.	 The potential energy at the first position will be the amount “lost” as the ball falls and this will be the change in potential. $U=mgh = mg(L-L\cos\theta)$	A
5.	The work done by the stopping force equals the loss of kinetic energy. $-W=\Delta K$ $-Fd = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$ $v_f = 0$ $F = mv^2/2d$	A
6.	This is a conservative situation so the total energy should stay same the whole time. It should also start with max potential and min kinetic, which only occurs in choice C	C
7.	Stopping distance is a work-energy relationship. Work done by friction to stop = loss of kinetic $-f_k\, d = -\frac{1}{2} mv_i^2$ $\mu_k mg = \frac{1}{2} mv_i^2$ The mass cancels in the relationship above so changing mass doesn’t change the distance	B
8.	Same relationship as above … double the v gives 4x the distance	D
9.	Half way up you have gained half of the height so you gained ½ of potential energy. Therefore you must have lost ½ of the initial kinetic energy so $E_2 = (E_k/2)$. Subbing into this relationship $E_2 = (E_k/2)$ $\frac{1}{2} mv_2^2 = \frac{1}{2} m v^2 / 2$ $v_2^2 = v^2 / 2$ …. Sqrt both sides gives answer	B
10.	At the top, the ball is still moving (v_x) so would still possess some kinetic energy	A
11.	Same as question #1 with different variables used	B
12.	Total energy is always conserved so as the air molecules slow and lose their kinetic energy, there is a heat flow which increases internal (or thermal) energy	C
13.	Eliminating obviously wrong choices only leaves A as an option. The answer is A because since the first ball has a head start on the second ball it is moving at a faster rate of speed at all times. When both are moving in the air together for equal time periods the first faster rock will gain more distance than the slower one which will widen the gap between them.	A
14.	For a mass on a spring, the max U occurs when the mass stops and has no K while the max K occurs when the mass is moving fast and has no U. Since energy is conserved it is transferred from one to the other so both maximums are equal	C

15. Since the ball is thrown with initial velocity it must start with some initial K. As the mass falls it gains velocity directly proportional to the time (V=Vi+at) but the K at any time is equal to 1/2 mv^2 which gives a parabolic relationship to how the K changes over time. D

16. Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates D

17. The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the K gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph. C

18. Point IV is the endpoint where the ball would stop and have all U and no K. Point II is the minimum height where the ball has all K and no U. Since point III is halfway to the max U point half the energy would be U and half would be K C

19. Apply energy conservation using points IV and II. $U_4 = K_2$ $mgh = ½ mv^2$ B

20. Since the track is rough there is friction and some mechanical energy will be lost as the block slides which means it cannot reach the same height on the other side. The extent of energy lost depends on the surface factors and cannot be determined without more information D

21. As the object oscillates its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D D

22. To push the box at a constant speed, the child would need to use a force equal to friction so $F=f_k=\mu mg$. The rate of work (W/t) is the power. Power is given by P=Fv → µmgv A

23. Two steps. I) use hookes law in the first situation with the 3 kg mass to find the spring constant (k). $F_{sp}=k\Delta x$, $mg=k\Delta x$, k = 30/.12 = 250. II) Now do energy conservation with the second scenario (note that the initial height of drop will be the same as the stretch Δx). C
U top = U_{sp} bottom, $mgh = ½ k \Delta x^2$, $(4)(10)(\Delta x) = ½ (250) (\Delta x^2)$

24. In a circular orbit, the velocity of a satellite is given by $v = \sqrt{\dfrac{Gm_e}{r}}$ with $m_e = M$. Kinetic energy of the satellite is given by $K = ½ m v^2$. Plug in v from above to get answer A

25. Projectile. V_x doesn't matter $V_{iy} = 0$. Using $d = v_{iy}t + ½ at^2$ we get the answer D

26. A is true; both will be moving the fastest when they move through equilibrium. A

27. X and Y directions are independent and both start with the same velocity of zero in each direction. The same force is applied in each direction for the same amount of time so each should gain the same velocity in each respective direction. B

28. Kinetic energy is not a vector and the total resultant velocity should be used to determine the KE. For the 1st second the object gains speed at a uniform rate in the x direction and since KE is proportional to v^2 we should get a parabola. However, when the 2nd second starts the new gains in velocity occur only in the y direction and are at smaller values so the gains essentially start over their parabolic trend as shown in graph B B

29. As the system moves, m_2 loses energy over distance h and m_1 gains energy over the same distance h but some of this energy is converted to KE so there is a net loss of U. Simply subtract the $U_2 - U_1$ to find this loss — A

30. In a force vs. displacement graph, the area under the line gives the work done by the force and the work done will be the change in the K so the largest area is the most K change — D

31. Use energy conservation, U top = K bottom. As in problem #6 (in this document), the initial height is given by L – Lcos θ, with cos 60 = .5 so the initial height is ½ L. — A

32. Use application of the net work energy theorem which says … $W_{net} = \Delta K$. The net work is the work done by the net force which gives you the answer — A

33. There is no U_{sp} at position x=0 since there is no Δx here so this is the minimum U location — A

34. Using energy conservation in the first situation presented K=U gives the initial velocity as $v = \sqrt{2gh}$. The gun will fire at this velocity regardless of the angle. In the second scenario, the ball starts with the same initial energy but at the top will have both KE and PE so will be at a lower height. The velocity at the top will be equal to the v_x at the beginning — C

35. Use energy conservation $K=U_{sp}$ $\frac{1}{2} m{v_m}^2 = \frac{1}{2} k \Delta x^2$, with Δx=A, solve for k — D

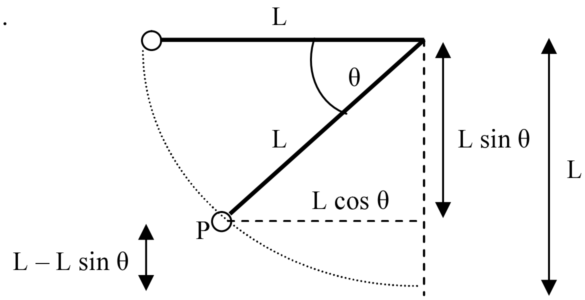
36. Based on net work version of work energy theorem. $W_{net} = \Delta K$, we see that since there is a constant speed, the ΔK would be zero, so the net work would be zero requiring the net force to also be zero. — A

37. As the block slides back to equilibrium, we want all of the initial spring energy to be dissipated by work of friction so there in no kinetic energy at equilibrium where all of the spring energy is now gone. So set work of friction = initial spring energy and solve for μ. The distance traveled while it comes to rest is the same as the initial spring stretch, d = x.
$\frac{1}{2} kx^2 = \mu mg(x)$ — C

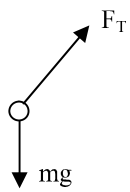
38. V at any given time is given by $v = v_i + at$, with $v_i = 0$ gives $v = at$, — D
V at any given distance is found by $v^2 = v_i^2 + 2ad$, with $v_i = 0$ gives $v^2 = 2ad$
This question asks for the relationship to distance.
The kinetic energy is given by $K = \frac{1}{2} m v^2$ and since $v^2 = 2ad$ we see a linear direct relationship of kinetic energy to distance (2*d → 2*K)
Another way of thinking about this is in relation to energy conservation. The total of $mgh+1/2mv^2$ must remain constant so for a given change in (h) the $\frac{1}{2} mv^2$ term would have to increase or decrease directly proportionally in order to maintain energy conservation.

39. Similar to the discussion above. Energy is conserved so the term $mgh + \frac{1}{2} mv^2$ must remain constant. As the object rises it loses K and gains U. Since the height is H/2 it has gained half of the total potential energy it will end up with which means it must have lost half of its kinetic energy, so its K is half of what it was when it was first shot. — B

1974B1.



(a) FBD



(b) Apply conservation of energy from top to point P

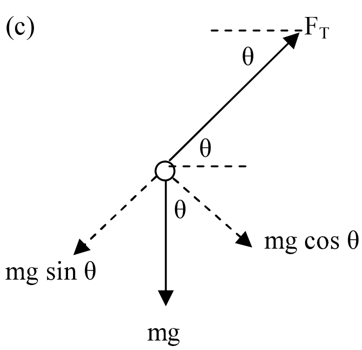
$U_{top} = U_p + K_p$

$mgh = mgh_p + ½ m v_p^2$

$gL = g(L – L\sin θ) + ½ v_p^2$

$$v = \sqrt{2gL\sin\theta}$$

(c)



$F_{NET(C)} = m v^2 / r$

$F_T – mg \sin θ = m v^2 / r$

$F_T – mg \sin θ = m (2gL\sin θ) / L$

$F_T = 2mg \sin θ + mg \sin θ$

$F_T = 3mg \sin θ$

1974B7.

6 riders per minute is equivalent to 6x(70kg)*9.8 = 4116 N of lifting force in 60 seconds

Work to lift riders = work to overcome gravity over the vertical displacement (600 sin 30)
Work lift = Fd = 4116N (300m) = 1.23×10^6 J

P lift = W / t = 1.23×10^6 J / 60 sec = 20580 W

But this is only 40% of the necessary power. ➔ 0.40 (total power) = 20580 W

Total power needed = 51450 W

1975B1.

(a) $F_{net} = ma$ $\quad - f_k = ma$ $\quad - 8 = 2a$ $\quad a = -4 \text{ m/s}^2$

(b) $v_f^2 = v_i^2 + 2ad$ $\quad (0)^2 = v_i^2 + 2(-4)(8)$ $\quad v_i = 8 \text{ m/s}$

$v_f = v_i + at$ $\quad t = 2 \text{ sec}$

(c) Apply energy conservation top to bottom

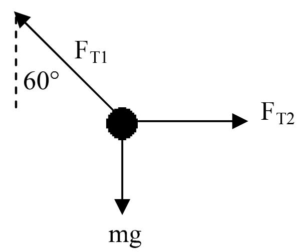
$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$(10)(R) = ½ (8)^2$ $\quad R = 3.2 \text{ m}$

1975 B7

(a)

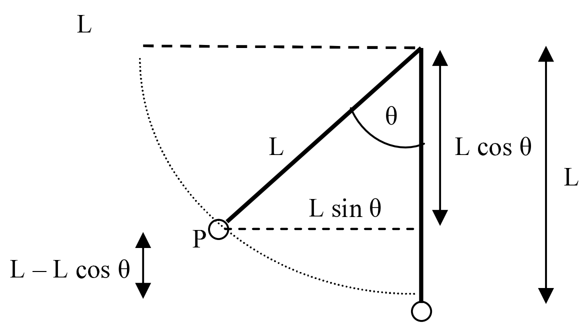


(b) $F_{NET(Y)} = 0$

$F_{T1} \cos \theta = mg$

$F_{T1} = mg / \cos(60) = 2mg$

(c) When string is cut it swing from top to bottom, similar to diagram for 1974B1 with θ moved as shown below



$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

1977B1.

(a) Apply work–energy theorem

$W_{NC} = \Delta ME$

$W_{fk} = \Delta K \quad (K_f - K_i)$

$W = - K_i$

$W = - ½ mv_i2 \qquad - ½ (4)(6)^2 \qquad = - 72$ J

(b) $F_{net} = ma$

$- f_k = m\ a \qquad\qquad v = v_i + at$

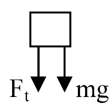
$a = - (8)/4 = - 2\ m/s^2 \qquad\qquad v = (6) + (-2)\ t$

(c) $W_{fk} = - f_k\ d$

$- 72J = - (8)\ d \qquad\qquad d = 9$ m

1978B1,

(a)



(b) Apply $F_{net(C)} = mv^2 / r$ .towards center as + direction

$(F_t + mg) = mv^2/r \qquad (20+0.5(10))=(0.5)v^2 / 2 \qquad v = 10$ m/s

(c) As the object moves from P to Q, it loses U and gains K. The gain in K is equal to the loss in U.

$\Delta U = mg\Delta h = (0.5)(10)(4) = 20$ J

(d) First determine the speed at the bottom using energy.

$K_{top} + K_{gain} = K_{bottom} \qquad ½ mv_{top}^2 + 20\ J = ½ mv_{bot}^2 \qquad v_{bot} = 13.42$ m/s

At the bottom, Ft acts up (towards center) and mg acts down (away from center)

Apply $F_{net(C)} = mv^2 / r$ .towards center as + direction

$(F_t - mg) = mv^2/r \qquad (F_t - 0.5(10)) = (0.5)(13.42)^2 / 2 \qquad F_t = 50$ N

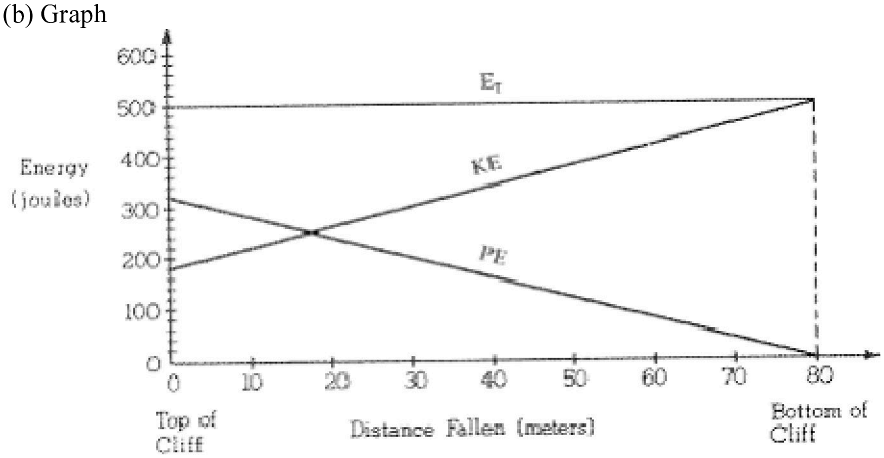
1979B1.

(a) $U = mgh = 320$ J

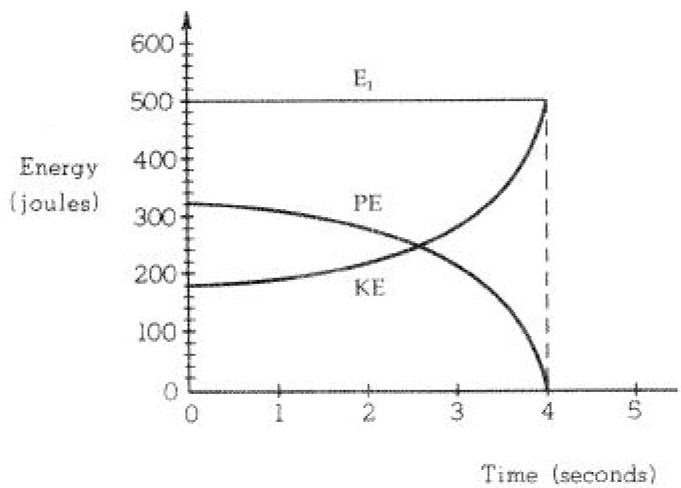
$K = ½ m v^2 = 180$ J

Total = U + K = 500 J

(b) Graph



(c) First determine the time at which the ball hits the ground, using $d_y = 0 + ½ g t^2$, to find it hits at 4 seconds.



1981B1.

(a) constant velocity means $F_{net} = 0$, $F - f_k = ma$ $F - \mu_k mg = 0$ $F - (0.2)(10)(10) = 0$
$F = 20$ N

(b) A change in K would require net work to be done. By the work–energy theorem:
$W_{net} = \Delta K$
$F_{net}\, d = 60$ J
$F_{net}\,(4\text{m}) = 60$ $F_{net} = 15$ N
$F' - f_k = 15$
$F' - 20 = 15$ $F = 35$ N

(c) $F_{net} = ma$
$(15) = (10)\, a$ $a = 1.5$ m/s^2

1981B2.

The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring
$W = ½\, m\, v^2 = ½\,(3)\,(10)^2 = 150$ J

1982B3.

Same geometry as in problem 1975B7.

(a) Apply energy conservation top to bottom
$U_{top} = K_{bot}$
$mgh = ½\, m\, v^2$
$mg\,(R - R\cos\theta) = ½\, m\, v^2$
$v = \sqrt{2g(R - R\cos\theta)}$

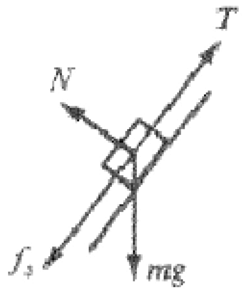
(b) Use $F_{NET(C)} = mv^2 / r$
$F_t - mg = m\,(2g(R\text{-}R\cos\theta)) / R$
$1.5\, mg - mg = 2mg(1 - 1\cos\theta)$
$.5 = 2(1 - \cos\theta)$

$2\cos\theta = 1.5 \quad \rightarrow \quad \cos\theta = ¾$

1985B2.

(a) The tension in the string can be found easily by isolating the 10 kg mass. Only two forces act on this mass, the Tension upwards and the weight down (mg) . Since the systems is at rest, T = mg = 100 N

(b) FBD



(c) Apply $F_{net} = 0$ along the plane. $T - f_s - mg \sin \theta = 0$ $(100\text{ N}) - f_s - (10)(10)(\sin 60)$
$f_s = 13$ N

(d) Loss of mechanical energy = Work done by friction while sliding
First find kinetic friction force Perpendicular to plane $F_{net} = 0$ $F_n = mg \cos \theta$
$F_k = \mu_k F_n = \mu_k mg \cos \theta$

$W_{fk} = f_k d = \mu_k mg \cos \theta (d) = (0.15)(10)(10)(\cos(60)) = 15\text{J}$ converted to thermal energy

(e) Using work-energy theorem . The U at the start – loss of energy from friction = K left over
$U - W_{fk} = K$
$mgh - W_{fk} = K$
$mg(d \sin 60) - 15 = K$
$(10)(10)(2) \sin 60 - 15 = K$ $K = 158$ J

1986B2.

(a) Use projectile methods to find the time. $d_y = v_{iy}t + \frac{1}{2} a t^2$ $\quad$ $h = 0 + g t_2 / 2$

$$t = \sqrt{\frac{2h}{g}}$$

(b) v_x at ground is the same as v_x top $\quad$ $V_x = d_x / t$ $\quad$ $v_x = \frac{D}{\sqrt{\frac{2h}{g}}}$

multiply top and bottom by reciprocal to rationalize

$$v_x = D\sqrt{\frac{g}{2h}}$$

(c) The work done by the spring to move the block is equal to the amount of K gained by it = K_f

$W = K_f = \frac{1}{2} m v^2 = (\frac{1}{2} M (D^2 / (2h/g)) = MD^2 g / 4h$

(d) Apply energy conservation $\quad$ $U_{sp} = K$

$\frac{1}{2} k\Delta^2 = \frac{1}{2} mv^2$ (plug in V from part b) $\quad$ $v_x = \frac{MD^2 g}{2hX^2}$

If using $F=k\Delta$ you have to plug use F_{avg} for the force

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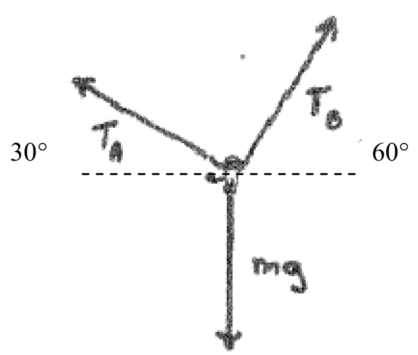
1991B1.

(a) FBD



(b) SIMULTANEOUS EQUATIONS

$F_{net(X)} = 0$ $\quad$ $F_{net(Y)} = 0$

$T_a \cos 30 = T_b \cos 60$ $\quad$ $T_a \sin 30 + T_b \sin 60 - mg = 0$

..Solve above for T_b and plug into Fnet(y) eqn and solve

$T_a = 24$ N $\quad$ $T_b = 42$ N

(c) Using energy conservation with similar diagram as 1974B1 geometry

$U_{top} = U_p + K_p$

$mgh = \frac{1}{2} m v^2$

$g(L - L\sin\theta) = \frac{1}{2} v^2$

$(10)(10 - 10\sin 60) = \frac{1}{2} v^2$ $\quad$ $v = 5.2$ m/s

(d) $F_{net(C)} = mv^2/r$

$F_t - mg = mv^2 / r$ $\quad$ $F_t = m(g + v^2/r)$ $\quad$ $F_t = (5)(9.8+(5.2)^2/10) = 62$ N

1992B1.

(a) K + U $\frac{1}{2}$ m v^2 + mgh $\frac{1}{2}(0.1)(6)^2 + (0.1)(9.8)(1.8) = 3.6$ J

(b) Apply energy conservation using ground as h=0

$E_{top} = E_p$

3.6 J = K + U

$3.6 = \frac{1}{2} m v^2 + mgh$

$3.6 = \frac{1}{2}(0.1)(v^2) + (0.1)(9.8)(.2)$ v = 8.2 m/s

(c) Apply net centripetal force with direction towards center as +

i) Top of circle = Ft points down and Fg points down

$F_{net(c)} = mv^2/r$

$Ft + mg = mv^2/r$ $F_t = mv^2/r - mg$ $(0.1)(6)^2/(0.8) - (.1)(9.8)$ $F_t = 3.5$ N

ii) Bottom of circle = Ft points up and Fg points down

$F_{net(c)} = mv^2/r$

$F_t - mg = mv^2/r$ $F_t = mv^2/r + mg$ $(0.1)(8.2)^2/(0.8) + (0.1)(9.8)$ $F_t = 9.5$ N

(d) Ball moves as a projectile.

First find time of fall in y direction

$d_y = v_{iy}t + \frac{1}{2} a t^2$

$(-0.2) = 0 + \frac{1}{2}(-9.8) t^2$

t = .2 sec

Then find range in x direction

$d_x = v_x t$

$d_x = (8.2)(0.2)$

$d_x = 1.6$ m

1996B2.

(a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k\,\Delta x$ and solve for k

(b) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan\theta$ Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} - \mu_s$ mg, Giving $mg = F_{spring} / \mu$

Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid dynamics unit.

1997B1.

(a) The force is constant, so simple $F_{net} = ma$ is sufficient. $(4) = (0.2)\ a$ $a = 20\ m/s^2$

(b) Use $d = v_i t + \frac{1}{2} a t^2$ $12 = (0) + \frac{1}{2}(20) t^2$ $t = 1.1$ sec

(c) $W = Fd$ $W = (4\ N)(12\ m) = 48\ J$

(d) Using work energy theorem $W = \Delta K$ $(K_i = 0)$ $W = Kf - Ki$
$W = \frac{1}{2} m v_f^2$

Alternatively, use $v_f^2 = v_i^2 + 2\ a\ d$ $48J = \frac{1}{2}(0.2)(v_f^2)$ $v_f = 22\ m/s$

(e) The area under the triangle will give the extra work for the last 8 m
$\frac{1}{2}(8)(4) = 16J$ + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem $W = \frac{1}{2} m v_f^2$ $64\ J = \frac{1}{2}(0.2)\ v_f^2$ $v_f = 25.3\ m/s$

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

1999B1.

(a) Plug into $g = GM_{planet} / r_{planet}^2$ lookup earth mass and radius
$g_{mars} = 3.822\ m/s^2$ to get it in terms of g_{earth} divide by 9.8 $g_{mars} = 0.39\ g_{earth}$

(b) Since on the surface, simply plug into $F_g = mg = (11.5)(3.8) = 44$ N

(c) On the incline, $F_n = mg \cos\theta = (44)\cos(20) = 41$ N

(d) moving at constant velocity → $F_{net} = 0$

(e) $W = P\ t$ $(5.4 \times 10^5\ J) = (10\ W)\ t$ $t = 54000$ sec
$d = v\ t$ $(6.7 \times 10^{-3})(54000\ s)$ $d = 362$ m

(f) $P = Fv$ $(10) = F\ (6.7 \times 10^{-3})$ $F_{push} = 1492.54$ N total pushing force used
* (.0001) use for drag
→ $F_{drag} = 0.15$ N

2002B2.

(a) From graph U = 0.05 J

(b) Since the total energy is 0.4 J, the farthest position would be when all of that energy was potential spring energy. From the graph, when all of the spring potential is 0.4 J, the displacement is 10 cm

(c) At –7 cm we read the potential energy off the graph as 0.18 J. Now we use energy conservation.
$ME = U_{sp} + K$ $0.4J = 0.18\ J + K$ → $K = 0.22$ J

(d) At x=0 all of the energy is kinetic energy $K = \frac{1}{2} m v^2$ $0.4 = \frac{1}{2}(3)\ v^2$ $v = 0.5\ m/s$

(e) The object moves as a horizontally launched projectile when it leaves.

First find time of fall in y direction
$d_y = v_{iy} t + \frac{1}{2} a t^2$
$(-0.5) = 0 + \frac{1}{2}(-9.8)\ t^2$
$t = 0.3$ sec

Then find range in x direction
$d_x = v_x\ t$
$d_x = (0.5)(0.3)$
$d_x = 0.15$ m

2004B1.

(a) i) fastest speed would be the lowest position which is the bottom of the first hill where you get all sick and puke your brains out.

ii) Applying energy conservation from the top of the hill where we assume the velocity is approximately zero we have

$U_{top} = K_{bottom}$

$mgh = \frac{1}{2} m v^2$ $\quad$ $(9.8)(90) = \frac{1}{2} v^2$ $\quad$ $v = 42$ m/s

(b) Again applying energy conservation from the top to position B

$U_{top} = K_b + U_b$

$mgh = \frac{1}{2} m v_B^{\ 2} + mgh_B$

$(9.8)(90) = \frac{1}{2} v_B^2 + (9.8)(50)$ $\quad$ $v_B = 28$ m/s

(c) i) FBD

ii) $mg = (700)(9.8) = 6860$ N

$F_{net(C)} = mv^2 / r$

$F_n + mg = mv^2 / r$

$F_n = mv^2/r - mg \quad = m (v^2/r - g) \quad = (700)(28^2/20 - 9.8) = 20580$ N

(d) The friction will remove some of the energy so there will not be as much Kinetic energy at the top of the loop. In order to bring the KE back up to its original value to maintain the original speed, we would need less PE at that location. A lower height of the loop would reduce the PE and compensate to allow the same KE as before. To actually modify the track, you could flatten the inclines on either side of the loop to lower the height at B.

B2004B1.

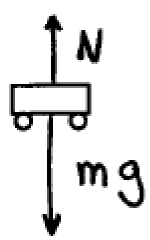
(a) set position A as the h=0 location so that the PE=0 there.

Applying energy conservation with have

$U_{top} + K_{top} = K_A$ $\quad$ $mgh + \frac{1}{2} m v^2 = \frac{1}{2} m v_A^{\ 2}$

$(9.8)(0.1) + \frac{1}{2} (1.5)^2 = \frac{1}{2} v_A^2$ $\quad$ $v_A = 2.05$ m/s

(b) FBD



(c) $F_{net(C)} = mv^2 / r$

$mg - F_N = mv^2 / r$

$Fn = mg - mv^2/r \quad = m (g - v^2/r) = (0.5)(9.8 - 2.05^2 / 0.95) \quad = 2.7$ N

(c) To stop the cart at point A, all of the kinetic energy that would have existed here needs to be removed by the work of friction which does negative work to remove the energy.

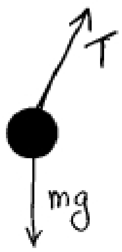
$W_{fk} = - K_A$

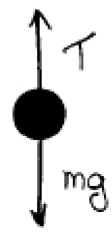
$W_{fk} = - \frac{1}{2} m v_A^{\ 2} = - \frac{1}{2} (0.5)(2.05^2) = - 1.1$ J

(d) The car is rolling over a hill at point A and when F_n becomes zero the car just barely loses contact with the track. Based on the equation from part (c) the larger the quantity (mv^2 / r) the more likely the car is to lose contact with the track (since more centripetal force would be required to keep it there) .to increase this quantity either the velocity could be increased or the radius could be decreased. To increase the velocity of the car, make the initial hill higher to increase the initial energy. To decrease the radius, simply shorten the hill length.

B2005B2.

FBD i) ii)





(b) Apply energy conservation?

$U_{top} = K_{bottom}$

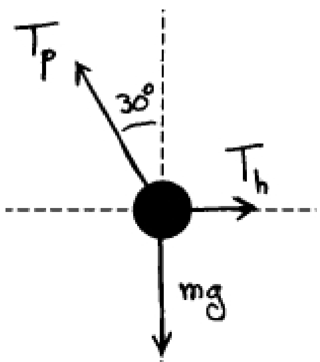
$mgh = ½\, m\, v^2$ $\quad (9.8)(.08) – ½\, v^2$ $\quad v = 1.3$ m/s

(c) $F_{net(c)} = mv^2/r$

$F_t – mg = mv^2/r$ $\quad F_t = mv^2/r + mg$ $\quad (0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $\quad F_t = 0.93$ N

2005B2.

(a) FBD



(b) Apply

$F_{net(X)} = 0$

$T_P \cos 30 = mg$

$T_P = 20.37$ N

$F_{net(Y)} = 0$

$T_P \sin 30 = T_H$

$T_H = 10.18$ N

(c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$

$g(L – L \cos \theta) = ½\, v^2$

$(10)\,(2.3 – 2.3 \cos 30) = ½\, v^2$ $\quad v_{bottom} = 2.5$ m/s

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B2006B2.

(a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$ $\quad Mgh = ½\,(M)\,(3.5v_o)^2$ $\quad h = 6.125\, v_o^2 / g$

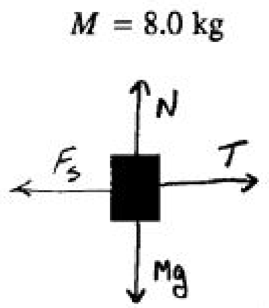
(b) $W_{NC} = \Delta K\ (K_f - K_i)$ $\quad K_f = 0$

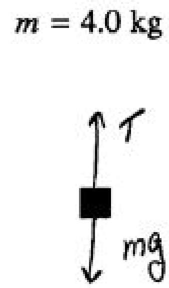
$– f_k\, d = 0 – ½\,(1.5M)(2v_o)^2$

$\mu_k\,(1.5\, M)\, g\, (d) = 3Mv_o^2$ $\quad \mu_k = 2v_o^2 / gD$

2006B1.

(a) FBD





(b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

(c) Tension in string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k\,(0.05)$ $k = 780$ N/m

(d) 4 kg mass is in free fall. $D = v_i t + \frac{1}{2} g t^2$ $-0.7 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.38$ sec

(e) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $\frac{1}{2} k \Delta x^2 = \frac{1}{2} mv^2$ $\frac{1}{2}(780)(0.05) = \frac{1}{2}(8)v^2$ $v = 0.49$ m/s

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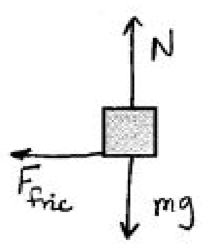
B2008B2.

(a) $d = v_i t + \frac{1}{2} a t^2$ $(55) = (25)(3) + \frac{1}{2} a (3)^2$ $a = -4.4$ m/s^2

(b) FBD



(c) using the diagram above and understanding that the static friction is actually responsible for decelerating the box to match the deceleration of the truck, we apply F_{net}
$F_{net} = ma$
$-f_s = -\mu_s F_n = ma$ $-\mu_s\, mg = ma$ $-\mu_s = a/g$ $-\mu_s = -4.4 / 9.8$ $\mu_s = 0.45$

Static friction applied to keep the box at rest relative to the truck bed.

(d) Use the given info to find the acceleration of the truck $a = \Delta v / t = 25/10 = 2.5$ m/s^2
To keep up with the trucks acceleration, the crate must be accelerated by the spring force, apply F_{net}
$F_{net} = ma$ $F_{sp} = ma$ $k\Delta x = ma$ $(9200)(\Delta x) = (900)(2.5)$ $\Delta x = 0.24$ m

(e) If the truck is moving at a constant speed the net force is zero. Since the only force acting directly on the crate is the spring force, the spring force must also become zero therefore the Δx would be zero and is **LESS** than before. Keep in mind the crate will stay on the frictionless truck bed because its inertia will keep it moving forward with the truck (remember you don't necessarily need forces to keep things moving)

2008B2.

(a) In a connected system, we must first find the acceleration of the system as a whole. The spring is internal when looking at the whole system and can be ignored.

$F_{net} = ma$ $\quad$ $(4) = (10)\,a$ $\quad$ $a = 0.4\ m/s^2$ → the acceleration of the whole system and also of each individual block when looked at separate

Now we look at just the 2 kg block, which has only the spring force acting on its FBD horizontal direction.

$F_{net} = ma$ $\quad$ $F_{sp} = (2)(.4)$ $\quad$ $F_{sp} = 0.8\ N$

(b) Use $F_{sp} = k\Delta x$ $\quad$ $0.8 = (80)\,\Delta x$ $\quad$ $\Delta x = 0.01\ m$

(c) Since the same force is acting on the same total mass and $F_{net} = ma$, the acceleration is the same

(d) The spring stretch will be MORE. This can be shown mathematically by looking at either block. Since the 8 kg block has only the spring force on its FBD we will look at that one.

$F_{sp} = ma$ $\quad$ $k\Delta x = ma$ $\quad$ $(80)(\Delta x) = (8)(0.4)$ $\quad$ $\Delta x = 0.04\ m$

(e) When the block A hits the wall it instantly stops, then block B will begin to compress the spring and transfer its kinetic energy into spring potential energy. Looking at block B energy conservation:

$K_b = U_{sp}$ $\quad$ $\frac{1}{2} m v_b^2 = \frac{1}{2} k \Delta x^2$ $\quad$ $(8)(0.5)^2 = (80)\Delta x^2$ $\quad$ $\Delta x = 0.16\ m$

2009B1.

(a) Apply energy conservation. All of the spring potential becomes gravitational potential

$U_{sp} = U$

$\frac{1}{2} k \Delta x^2 = mgh$ $\qquad$ $\frac{1}{2} k x^2 = mgh$ $\qquad$ $h = kx^2 / 2mg$

(b) You need to make a graph that is of the form y = m x, with the slope having "k" as part of it and the y and x values changing with each other. Other constants can also be included in the slope as well to make the y and x variables simpler. h is dependent on the different masses used so we will make h our y value and use m as part of our x value. Rearrange the given equation so that is it of the form y = mx with h being y and mass related to x.

We get $\qquad$ y = m x

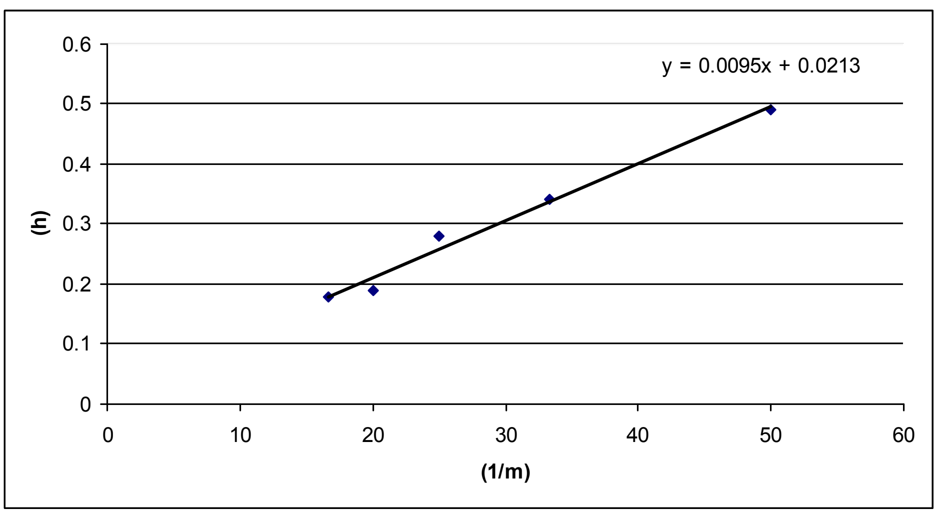
$$h = \left(\frac{kx^2}{2g}\right)\frac{1}{m}$$

so we use h as y and the value 1/m as x and graph it.

(note: we lumped all the things that do not change together as the constant slope term. Once we get a value for the slope, we can set it equal to this term and solve for k)

1/m	m (kg)	h (m)
50	0.020	0.49
33.33	0.030	0.34
25	0.040	0.28
20	0.050	0.19
16.67	0.060	0.18
X values		Y values

(c) Graph



(d) The slope of the best fit line is 0.01

We set this slope equal to the slope term in our equation, plug in the other known values and then solve it for k

$$0.01 = \left(\frac{kx^2}{2g}\right)$$

$$0.01 = \left(\frac{k(0.02)^2}{2(9.8)}\right)$$

Solving gives us $k = 490$ N/m

(e) - Use a stopwatch, or better, a precise laser time measurement system (such as a photogate), to determine the time it takes the toy to leave the ground and raise to the max height (same as time it takes to fall back down as well). Since its in free fall, use the down trip with $v_i=0$ and apply $d = \frac{1}{2} g t^2$ to find the height.

- Or, videotape it up against a metric scale using a high speed camera and slow motion to find the max h.

C1973M2

(a) Apply work-energy theorem
$W_{nc} = \Delta ME$
$W_{fk} = \Delta K$ $(K_f - K_i)$ $K_f = 0$
$- f_k d = - ½ mv_i^2$ $- f_k (0.12) = - ½ (0.030) (500)^2$ $f_k = 31250$ N

(b) Find find acceleration $- f_k = ma$ $- (31250) = (0.03) a$ $a = - 1.04\text{x}10^6$ m/s^2

Then use kinematics $v_f = v_i + at$ $0 = 500 + (- 1.04\text{x}10^6) t$ $t = 4.8\text{x}10^{-4}$ sec

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C1982M1
(a) Apply energy conservation, set the top of the spring as h=0, therefore H at start = $L\sin\theta = 6 \sin 30 = 3$ m

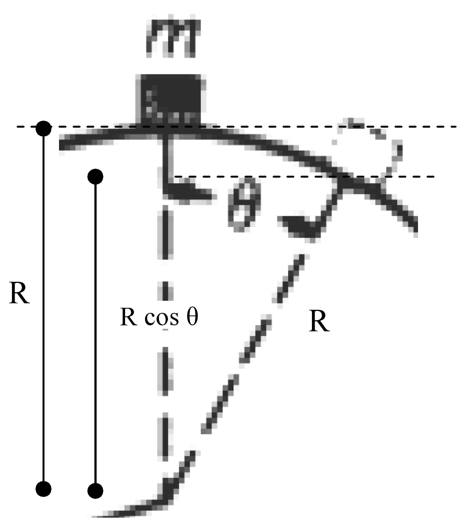
$U_{top} = K_{bot}$ $mgh = ½ mv^2$ $(9.8)(3) = ½ (v^2)$ $v = 7.67$ m/s

(b) Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.
$h = L \sin\theta + \Delta x \sin\theta$

$U_{top} = U_{sp}(\text{bot})$
$mgh = ½ k \Delta x^2$
$mg(L \sin\theta + \Delta x \sin\theta) = ½ k \Delta x^2$
$(20)(9.8)(6 \sin 30 + 3 \sin 30) = ½ k (3)^2$ $k = 196$ N/m

(c) The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

C1983M3.



$h = R - R\cos\theta = R(1 - \cos\theta)$

i) $K_2 = U_{top}$
$K_2 = mg(R(1 - \cos\theta))$

ii) From, $K = ½ m v^2 = mgR(1 - \cos\theta)$ ∴ $v^2 = 2gR(1 - \cos\theta)$

Then $a_c = v^2 / R = 2g(1 - \cos\theta)$

C1985M1

(a) We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} - f_k = 0$ $mg \sin\theta = \mu_s(F_n)$
$mg \sin\theta = \mu_s mg \cos\theta$ $\mu_s = \tan\theta$

(b) Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so h= (d+x)sin θ

$U_f - U_i + U_{sp(F)} - U_{sp(i)}$

$0 - mgh + \frac{1}{2} k\Delta x^2 - 0$ $\frac{1}{2} kx^2 - mg(d+x)\sin\theta$

(c) To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k

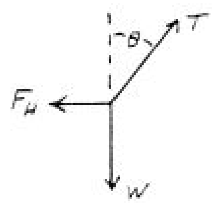
$W_{NC} = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$ $-f_k(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$-\mu_k mg\cos\theta(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$\mu_k = [mg(d+x)\sin\theta - \frac{1}{2} kx^2] / [mg(d+x)\cos\theta]$

C1987M1

(a)



$F_{net(y)} = 0$

$T\cos\theta - W = 0$ $T = W / \cos\theta$

(b) Apply SIMULTANEOUS EQUATIONS

$F_{net(y)} = 0$ $F_{net(x)} = 0$

$T\cos\theta - W = 0$ $T\sin\theta - F_h = 0$

Sub T into X equation to get F_h $F_h = W\tan\theta$

(c) Using the same geometry diagram as solution 1975B7 solve for the velocity at the bottom using energy conservation

$U_{top} = K_{bot}$

$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos\theta)}$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(T - W) = m(2gL(1-\cos\theta)) / L$

$T = W + 2mg - 2mg\cos\theta$

$T = W + 2W - 2W\cos\theta = W(3 - 2\cos\theta)$

C1988M2

(a) The graph is one of force vs Δx so the slope of this graph is the spring constant. Slope = 200 N/m

(b) Since there is no friction, energy is conserved and the decrease in kinetic energy will be equal to the gain in spring potential $|\Delta K| = \Delta U_{sp(f)} = \frac{1}{2} k\Delta x^2 = \frac{1}{2}(200)(0.1)^2 = 1J$.
Note: This is the same as the area under the line since the area would be the work done by the conservative spring force and the work done by a conservative force is equal to the amount of energy transferred.

(c) Using energy conservation. $K_i = U_{sp(f)}$ $\frac{1}{2} mv_o^2 = 1$ J $\frac{1}{2}(5)v_o^2 = 1$ $v_o = 0.63$ m/s

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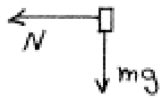
C1989M1

(a) Apply energy conservation from point A to point C setting point C as h=0 location
(note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

$U_A = K_C$ $\quad$ $mgh_a = ½ m v_c^2$ $\quad$ $(0.1)(9.8)(h_a) = ½ (0.1)(4)^2$ $\quad$ $h_a = 0.816m$
$h = h_a + 0.5$ m $= 1.32$ m

(b)



(c) Since the height at B and the height at C are the same, they would have to have the same velocities $v_b = 4$ m/s

(d) $F_{net(c)} = mv^2 / r$ $\quad$ $F_n = (0.1)(4)^2/(0.5) = 3.2$ N

(e) Using projectile methods .V. $_{iy} = 4\sin 30 = 2$ m/s $\quad$ Then $\quad v_{fy}^2 = v_{iy}^2 + 2\,a\,d_y$
$(0) = (2)^2 + 2(-9.8)(d_y)$ $\quad$ $d_y = 0.2$
$h_{max} = d_y$ + initial height $= 0.7$ m

Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

(f) Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $\quad$ $U_{new} - U_{old} = mgh_{net} - mgh_{old}$ $\quad$ $(0.1)(9.8)(2-1.32) = 0.67$ J lost.

C1989M3

(a) Apply energy conservation from start to top of spring using h=0 as top of spring.
$U = K$ $\quad$ $mgh = ½ m v^2$ $\quad$ $(9.8)(0.45) = ½ v^2$ $\quad$ $v = 3$ m/s

(b) At equilibrium the forces are balances $F_{net} = 0$ $\quad$ $F_{sp} = mg = (2)(9.8) = 19.6$ N

(c) Using the force from part b, $F_{sp} = k\,\Delta x$ $\quad$ $19.6 = 200\,\Delta x$ $\quad$ $\Delta x = 0.098$ m

(d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the start position is now increased by the amount of Δx found in part c $\quad$ $h_{new} = h + \Delta x = 0.45 + 0.098 = 0.548$ m)
$U_{top} = U_{sp} + K$
$mgh = ½\,k\,\Delta x^2 + ½\,mv^2$ $\quad$ $(2)(9.8)(0.548) = ½\,(200)(0.098)^2 + ½\,(2)(v^2)$ $\quad$ $v = 3.13$ m/s

(e) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

C1990M2

(a) Energy conservation, $K_{bot} = U_{top}$ $\quad$ $½\,m\,v^2 = mgh$ $\quad$ $½\,(v_o^2) = gh$ $\quad$ $h = v_o^2 / 2g$

(b) Work-Energy theorem. $W_{nc} = \Delta K + \Delta U$ $\quad$ $(U_i = 0, K_f = 0)$
$-f_k d = (mgh - 0) + (0 - ½\,m\,v_o^2)$ $\quad$ $-(\mu_k\,mg\cos\theta)\,h_2 / \sin\theta = mgh_2 - ½\,mv_o^2$

$\mu\,mg\cos\theta\,h_2 / \sin\theta + mgh_2 = ½\,mv_o^2$ $\quad$ $h_2\,(\mu\,g\cos\theta / \sin\theta + g) = ½\,v_o^2$
$h_2 = v_o^2 / (2g(\mu\cot\theta + 1))$

C1991M1

(a) Apply energy conservation.

$K_{bottom} = U_p + K_p$ $\qquad$ $\frac{1}{2} m v_{bot}^2 = mgh_p + K_p$

$\frac{1}{2} 3m (v_o/3)^2 = 3mg(r) + K_p$ $\qquad$ $K_p = mv_o^2/6 - 3mgr$

(b) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set $F_n=0$ at the top of the loop so that only mg is acting down on the block. Then apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\qquad$ $3mg = 3m\, v^2 /r$ $\qquad$ $v = \sqrt{rg}$

(c) Energy conservation, top of loop to bottom of loop

$U_{top} + K_{top} = K_{bot}$

$mgh + \frac{1}{2} m\, v_{top}^2 = \frac{1}{2} m\, v_{bot}^2$ $\qquad$ $g(2r) + \frac{1}{2} (\sqrt{rg})^2 = \frac{1}{2} (v_o')^2$ $\qquad$ $v_o' = \sqrt{5gr}$

C1993M1

- since there is friction on the surface the whole time, this is not an energy conservation problem, use work-energy.

(a) $U_{sp} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2} (400)(0.5)^2 = 50$ J

(b) Using work-energy $\qquad$ $W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)} - U_{sp(i)}) + (K_f - K_i)$

$-f_k d = (\,0 - 50J\,) + (\,\frac{1}{2} m\, v_f^2 - 0)$

$-\mu mg\ \ d = \frac{1}{2} m v_f^2 - 50$

$-(0.4)(4)(9.8)(0.5) = \frac{1}{2} (4)(v_c^2) - 50$ $\qquad$ $v_c = 4.59$ m/s

(c) $W_{nc} = (K_f - K_i)$

$-f_k d = (0 - \frac{1}{2} m\, v_i^2\,)$ $\qquad$ $-\mu mg\ d = -\frac{1}{2} m\, v_i^2$ $\qquad$ $(0.4)(6)(9.8)\ d = \frac{1}{2} (6)(3)^2$ $\qquad$ $d = 1.15$ m

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C2002M2

(a) Energy conservation, potential top = kinetic bottom $\qquad$ $v = \sqrt{2gh}$

(b) Energy conservation, potential top = spring potential $\qquad$ $U=U_{sp}$ $\qquad$ $(2m)gh = \frac{1}{2} k\, x_m^2$

$$x_m = 2\sqrt{\frac{mgh}{k}}$$

C2004M1

(a) Energy conservation with position B set as h=0. $U_a = K_b$ $\quad$ $v_b = \sqrt{2gL}$

(b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = mv_b^2 / r$ $\qquad$ $F_t - mg = m(2gL) / L$ $\qquad$ $F_t = 3mg$

(c) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + \frac{1}{2} g\, t^2$ $\qquad$ $L = 0 + gt^2 / 2$

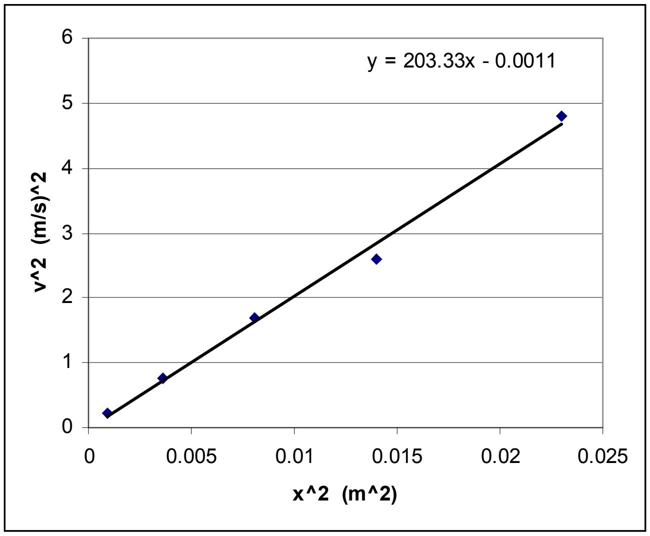
$t = \sqrt{\frac{2L}{g}}$ $\quad$ Then use $v_x = d_x / t$ $\quad$ $d_x = v'\sqrt{\frac{2L}{g}}$ $\quad$ total distance $\quad$ $x = v'\sqrt{\frac{2L}{g}} + L$

total distance includes the initial horizontal displacement L so it is added to the range

C2007M3

(a) Spring potential energy is converted into kinetic energy $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

(b) (c) i)



ii) using the equation above and rearrange to the form $y = m\,x$ with v^2 as y and x^2 as x

$y = m \quad x$

$v^2 = (k/m)\, x^2$

Slope = 200 = k /m

200 = (40) / m

m = 0.2 kg

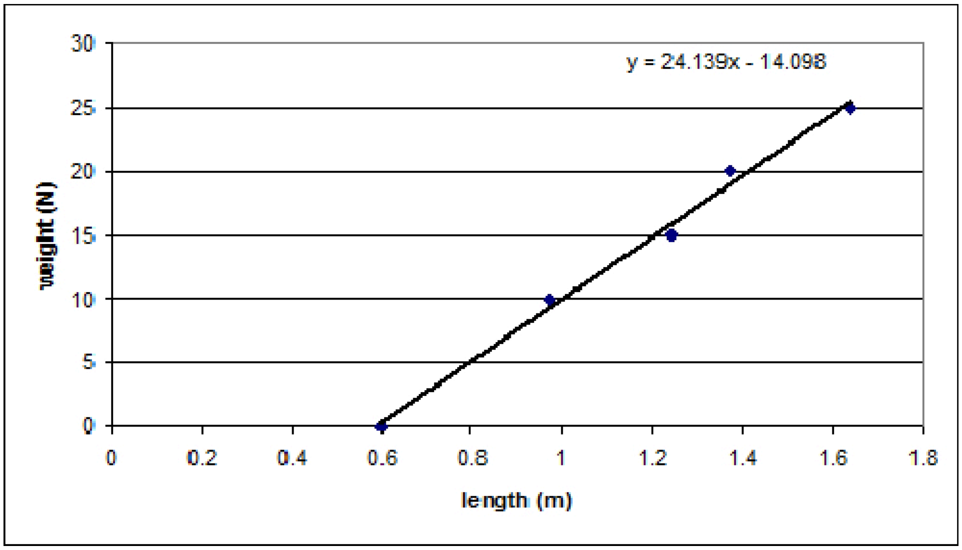
(d) Now you start with spring potential and gravitational potential and convert to kinetic. Note that at position A the height of the glider is given by h + the y component of the stretch distance x. $h_{initial} = h + x \sin\theta$

$U + U_{sp} = K$

$mgh + \frac{1}{2} k x^2 = \frac{1}{2} m v^2 \qquad mg(h + x \sin\theta) + \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

C2008M3

(a)



(b) The slope of the line is $F / \Delta x$ which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

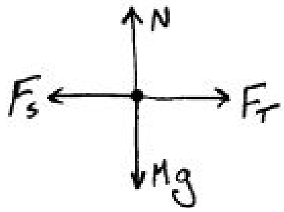
$mgh = \frac{1}{2} k \Delta x^2$ $m(9.8)(1.5) = \frac{1}{2}(24)(0.9)^2$ m = 0.66 kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $F_{sp} = (0.66)(9.8)$ $F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $6.5 = (24) \Delta x$ $\Delta x = 0.27$ m

Supplemental

(a)



(b) $F_{net} = 0$ $\quad$ $F_t = F_{sp} = k\Delta x$ $\quad$ $\Delta x = F_t / k$

(c) Using energy conservation $\quad$ $U_{sp} = U_{sp} + K$ $\quad$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} k\Delta x_2^2 + \frac{1}{2} mv^2$
$k(\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\quad$ $\frac{3}{4} k(F_t/k)^2 = Mv^2$ $\quad$ $v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$

(d) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get
$F_{net} = ma$ $\quad$ $F_{sp} - f_k = ma$
From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ $\quad$ ➔ $a = F_t / m - \mu_k g$