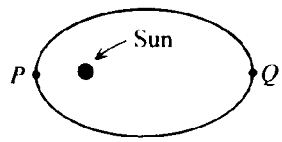
AP Physics Multiple Choice Practice – Gravitation

1. Each of five satellites makes a circular orbit about an object that is much more massive than any of the satellites. The mass and orbital radius of each satellite are given below. Which satellite has the greatest speed?

	Mass	Radius
(A)	½m	R
(B)	m	½R
(C)	m	R
(D)	m	2R

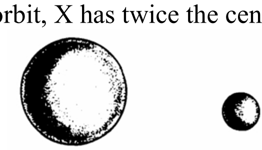


2. An asteroid moves in an elliptic orbit with the Sun at one focus as shown above. Which of the following quantities increases as the asteroid moves from point P in its orbit to point Q?
(A) Speed (B) Angular momentum (C) Kinetic energy (D) Potential energy

3. A person weighing 800 newtons on Earth travels to another planet with twice the mass and twice the radius of Earth. The person's weight on this other planet is most nearly
(A) 400 N (B) $\frac{800}{\sqrt{2}}$ N (C) $800\sqrt{2}$ (D) 1,600 N

4. Mars has a mass 1/10 that of Earth and a diameter 1/2 that of Earth. The acceleration of a falling body near the surface of Mars is most nearly
(A) 0.25 m/s^2 (B) 0.5 m/s^2 (C) 2 m/s^2 (D) 4 m/s^2

5. **Multiple correct:** If Spacecraft X has twice the mass of Spacecraft Y, then true statements about X and Y include which of the following? Select two answers.
(A) On Earth, X experiences twice the gravitational force that Y experiences.
(B) On the Moon, X has twice the weight of Y.
(C) The weight of the X on Earth will always be equal to the weight of Y on the Moon.
(D) When both are in the same circular orbit, X has twice the centripetal acceleration of Y



6. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?
(A) Acceleration (B) Kinetic energy (C) Displacement from the center of mass (D) Gravitational force

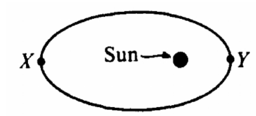
7. An object has a weight *W* when it is on the surface of a planet of radius *R*. What will be the gravitational force on the object after it has been moved to a distance of *4R* from the center of the planet?
(A) 16W (B) 4W (C)1/4W (D) 1/16 W

8. A new planet is discovered that has twice the Earth's mass and twice the Earth's radius. On the surface of this new planet, a person who weighs 500 N on Earth would experience a gravitational force of
(A) 125 N (B) 250 N (C) 500 N (D) 1000 N

9. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are the same. (B) Both are longer.
(C) The period of the mass on the spring is shorter, that of the pendulum is the same.
(D) The period of the pendulum is shorter; that of the mass on the spring is the same.

10. A satellite of mass m and speed v moves in a stable, circular orbit around a planet of mass M. What is the radius of the satellite's orbit?

(A) $\frac{Gv}{mM}$ (B) $\frac{GM}{v^2}$ (C) $\frac{GmM}{v}$ (D) $\frac{GmM}{v^2}$

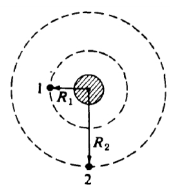


11. A satellite travels around the Sun in an elliptical orbit as shown above. As the satellite travels from point X to point Y. which of the following is true about its speed and angular momentum?

	Speed	Angular Momentum
(A)	Increases	Increases
(B)	Decreases	Decreases
(C)	Increases	Remains constant
(D)	Decreases	Remains constant

12. A newly discovered planet has a mass that is 4 times the mass of the Earth. The radius of the Earth is R_e. The gravitational field strength at the surface of the new planet is equal to that at the surface of the Earth if the radius of the new planet is equal to

(A) ½R_e (B) 2R_e (C) $\overline{R_e}$ (D) R_e^2



13. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations a_2 and a_1 of the two satellites are related by which of the following?

(A) $a_2 = 4a_1$ (B) $a_2 = 2a_1$ (C) $a_2 = a_1/2$ (D) $a_2 = a_1/4$

14. A satellite moves in a stable circular orbit with speed v_o at a distance R from the center of a planet. For this satellite to move in a stable circular orbit a distance $2R$ from the center of the planet, the speed of the satellite must be

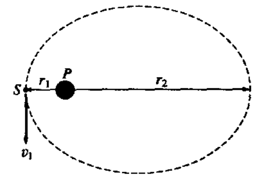
(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) $\sqrt{2v_0}$ (D) $2v_o$

15. If F_1 is the magnitude of the force exerted by the Earth on a satellite in orbit about the Earth and F_2 is the magnitude of the force exerted by the satellite on the Earth, then which of the following is true?

(A) F_1 is much greater than F_2. (B) F_1 is slightly greater than F_2.
(C) F_1 is equal to F_2. (D) F_2 is slightly greater than F_1

16. A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface. The radius of the new planet in terms of the radius R of Earth is

(A) ½R (B) $\frac{\sqrt{2}}{2}$ R (C) $\sqrt{2}$ R (D) 2R



17. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?

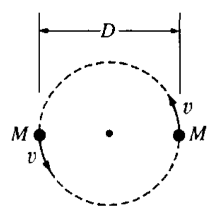
(A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $\frac{r_1+r_2}{2}v_1$ (D) $\frac{r_2-r_1}{r_1+r_2}v_1$

Questions 18-19

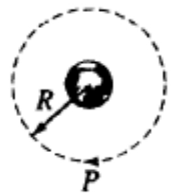
A ball is tossed straight up from the surface of a small, spherical asteroid with no atmosphere. The ball rises to a height equal to the asteroid's radius and then falls straight down toward the surface of the asteroid.

18. What forces, if any, act on the ball while it is on the way up?
(A) Only a decreasing gravitational force that acts downward
(B) Only a constant gravitational force that acts downward
(C) Both a constant gravitational force that acts downward and a decreasing force that acts upward
(C) No forces act on the ball.
19. The acceleration of the ball at the top of its path is
(A) equal to the acceleration at the surface of the asteroid
(B) equal to one-half the acceleration at the surface of the asteroid
(C) equal to one-fourth the acceleration at the surface of the asteroid
(D) zero

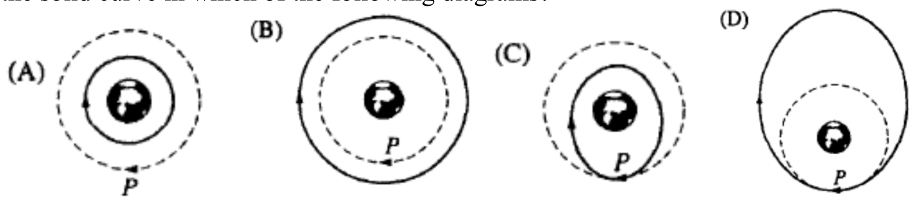
20. **Multiple Correct:** A satellite of mass M moves in a circular orbit of radius R with constant speed v. True statements about this satellite include which of the following? Select two answers.
(A) Its angular speed is v/R.
(B) The gravitational force does work on the satellite.
(C) The magnitude and direction of its centripetal acceleration is constant.
(D) Its tangential acceleration is zero.



21. Two identical stars, a fixed distance D apart, revolve in a circle about their mutual center of mass, as shown above. Each star has mass M and speed v. G is the universal gravitational constant. Which of the following is a correct relationship among these quantities?
(A) v^2 = GM/D (B) v^2 = GM/2D (C) $v^2 = GM/D^2$ (D) $v^2 = 2GM^2/D$



22. A spacecraft orbits Earth in a circular orbit of radius R, as shown above. When the spacecraft is at position P shown, a short burst of the ship's engines results in a small increase in its speed. The new orbit is best shown by the solid curve in which of the following diagrams?



23. The escape speed for a rocket at Earth's surface is v_e . What would be the rocket's escape speed from the surface of a planet with twice Earth's mass and the same radius as Earth?

    (A) $2v_e$ (B) $\sqrt{2}v_e$ (C) v_e (D) $\dfrac{v_e}{\sqrt{2}}$

24. A hypothetical planet orbits a star with mass one-half the mass of our sun. The planet's orbital radius is the same as the Earth's. Approximately how many Earth years does it take for the planet to complete one orbit?
    (A) ½ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) 2

25. Two artificial satellites, 1 and 2, are put into circular orbit at the same altitude above Earth's surface. The mass of satellite 2 is twice the mass of satellite 1. If the period of satellite 1 is T, what is the period of satellite 2?
    (A) $T/2$ (B) T (C) $2T$ (D) $4T$

26. A planet has a radius one-half that of Earth and a mass one-fifth the Earth's mass. The gravitational acceleration at the surface of the planet is most nearly
    (A) 4.0 m/s^2 (B) 8.0 m/s^2 (C) 12.5 m/s^2 (D) 25 m/s^2

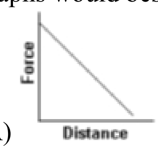
27. In the following statements, the word "weight" refers to the force a scale registers. If the Earth were to stop rotating, but not change shape,
    (A) the weight of an object at the equator would increase.
    (B) the weight of an object at the equator would decrease.
    (C) the weight of an object at the north pole would increase.
    (D) the weight of an object at the north pole would decrease.

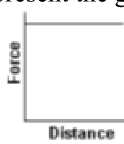
28. What happens to the force of gravitational attraction between two small objects if the mass of each object is doubled and the distance between their centers is doubled?
    (A) It is doubled (B) It is quadrupled (C) It is halved (D) It remains the same

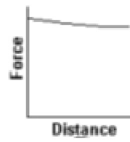
29. One object at the surface of the Moon experiences the same gravitational force as a second object at the surface of the Earth. Which of the following would be a reasonable conclusion?
    (A) both objects would fall at the same acceleration
    (B) the object on the Moon has the greater mass
    (C) the object on the Earth has the greater mass
    (D) both objects have identical masses

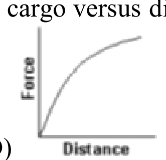
30. Consider an object that has a mass, m, and a weight, W, at the surface of the moon. If we assume the moon has a nearly uniform density, which of the following would be closest to the object's mass and weight at a distance halfway between Moon's center and its surface?
(A) ½ m & ½ W (B) ¼ m & ¼ W (C) 1 m & ½ W (D) 1 m & ¼ W

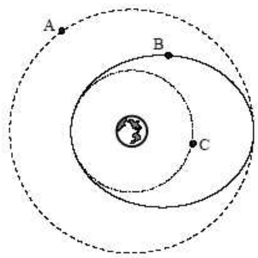
31. As a rocket blasts away from the earth with a cargo for the international space station, which of the following graphs would best represent the gravitational force on the cargo versus distance from the surface of the Earth?

(A) 

(B) 

(C) 

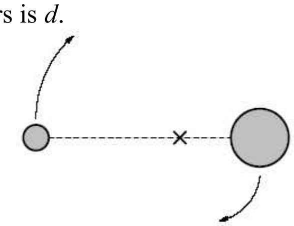
(D) 



32. Three equal mass satellites A, B, and C are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B, and L_C. Which of the following statements is correct?
(A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

<u>Questions 33-34</u>

Two stars orbit their common center of mass as shown in the diagram below. The masses of the two stars are $3M$ and M. The distance between the stars is d.



33. What is the value of the gravitational potential energy of the two star system?
(A) $-\frac{GM^2}{d}$ (B) $\frac{3GM^2}{d}$ (C) $-\frac{GM^2}{d^2}$ (D) $-\frac{3GM^2}{d}$

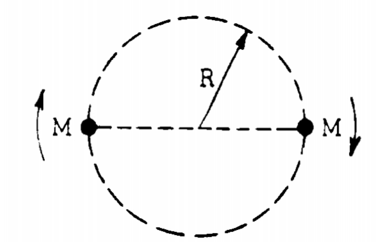
34. Determine the period of orbit for the star of mass $3M$.
(A) $\pi\sqrt{\frac{d^3}{GM}}$ (B) $\pi\sqrt{\frac{d^3}{3GM}}$ (C) $2\pi\sqrt{\frac{d^3}{GM}}$ (D) $\frac{\pi}{4}\sqrt{\frac{d^3}{GM}}$

35. Two iron spheres separated by some distance have a minute gravitational attraction, F. If the spheres are moved to one half their original separation and allowed to rust so that the mass of each sphere increases 41%, what would be the resulting gravitational force?
(A) $2F$ (B) $4F$ (C) $6F$ (D) $8F$

36. A ball thrown upward near the surface of the Earth with a velocity of 50 m/s will come to rest about 5 seconds later. If the ball were thrown up with the same velocity on Planet X, after 5 seconds it would be still moving upwards at nearly 31 m/s. The magnitude of the gravitational field near the surface of Planet X is what fraction of the gravitational field near the surface of the Earth?
(A) 0.16 (B) 0.39 (C) 0.53 (D) 0.63

37. Two artificial satellites I and II have circular orbits of radii R and $2R$, respectively, about the same planet. The orbital velocity of satellite I is v. What is the orbital velocity of satellite II?
(A) $\frac{v}{2}$ (B) $\frac{v}{\sqrt{2}}$ (C) $\sqrt{2}v$ (D) $2v$

AP Physics Free Response Practice – Gravitation



*1977M3. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

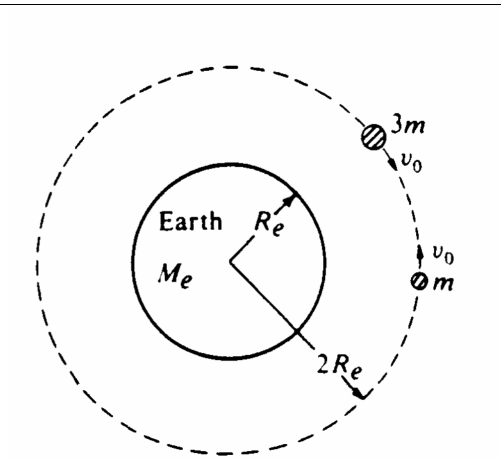
a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

b. Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.

d. Find the ratio of the speeds, v_{2M}/v_{M}.

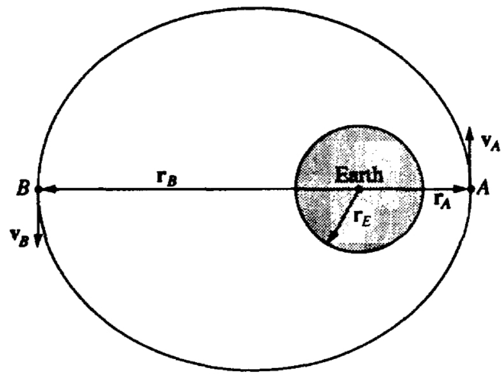


1984M2. Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



*1992M3. A spacecraft of mass 1,000 kilograms is in an elliptical orbit about the Earth, as shown above. At point A the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .

a. The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

b. The magnitude of the angular momentum of the spacecraft about the center of the Earth.

Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.

c. Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth. Determine each of the following.

d. The speed of the spacecraft if it is in a circular orbit around the Earth

e. The minimum speed of the spacecraft at point A if it is to escape completely from the Earth

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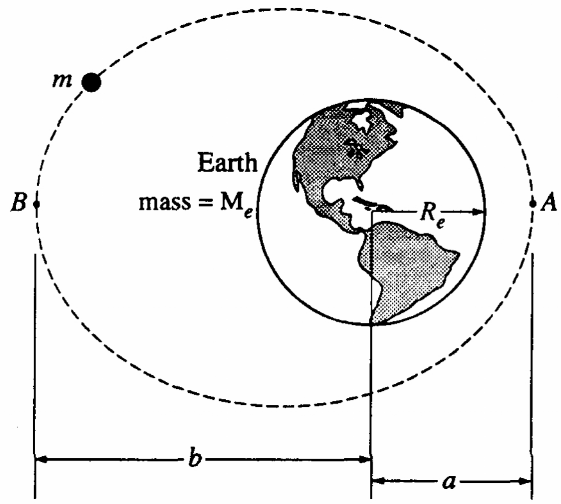
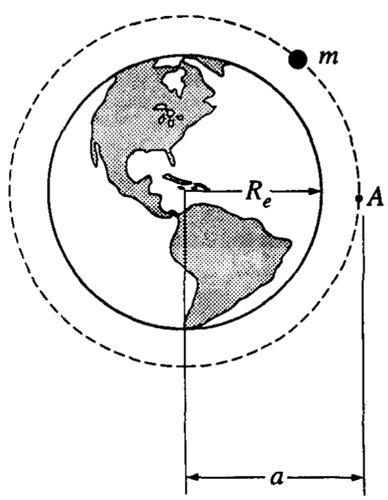


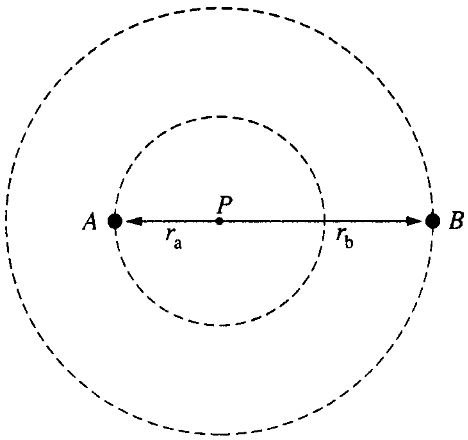
*1994M3 (modified) A satellite of mass *m* is in an elliptical orbit around the Earth, which has mass M_e and radius R_e. The orbit varies from a closest approach of distance *a* at point A to maximum distance of b from the center of the Earth at point B. At point A, the speed of the satellite is v_o. Assume that the gravitational potential energy $U_g = 0$ when masses are an infinite distance apart. Express your answers in terms of *a*, b, *m*, M_e, R_e, v_o, and G.

a. Determine the total energy of the satellite when it is at A.
b. What is the magnitude of the angular momentum of the satellite about the center of the Earth when it is at A?
c. Determine the velocity of the satellite as it passes point B in its orbit.



As the satellite passes point A, a rocket engine on the satellite is fired so that its orbit is changed to a circular orbit of radius a about the center of the Earth.

d. Determine the speed of the satellite for this circular orbit.
e. Determine the work done by the rocket engine to effect this change.



*1995M3 (modified) Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.
Determine expressions for the following three quantities in terms of r_a, r_b, T, and fundamental constants.

a. The centripetal acceleration of star A
b. The mass M_b of star B
c. The mass M_a of star A
d. Determine an expressions for the angular momentum of the system about the center of mass in terms of M_a, M_b, r_a, r_b, T, and fundamental constants.

2007M2. In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a. Calculate the radius of the GS orbit.
b. Calculate the mass of Mars.
c. Calculate the total mechanical energy of the GS in this orbit.
d. If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_________Greater than _________ Less than
Justify your answer.

e. In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

2001M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by

$$v = \sqrt{\frac{GM_J}{R}}$$

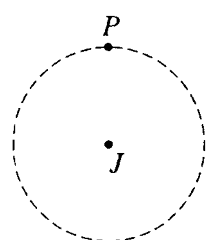
ii. The period of the orbit is given by

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

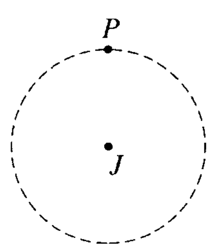
b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>faster</u> than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>slower</u> than the correct speed for a circular orbit of that radius.



2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

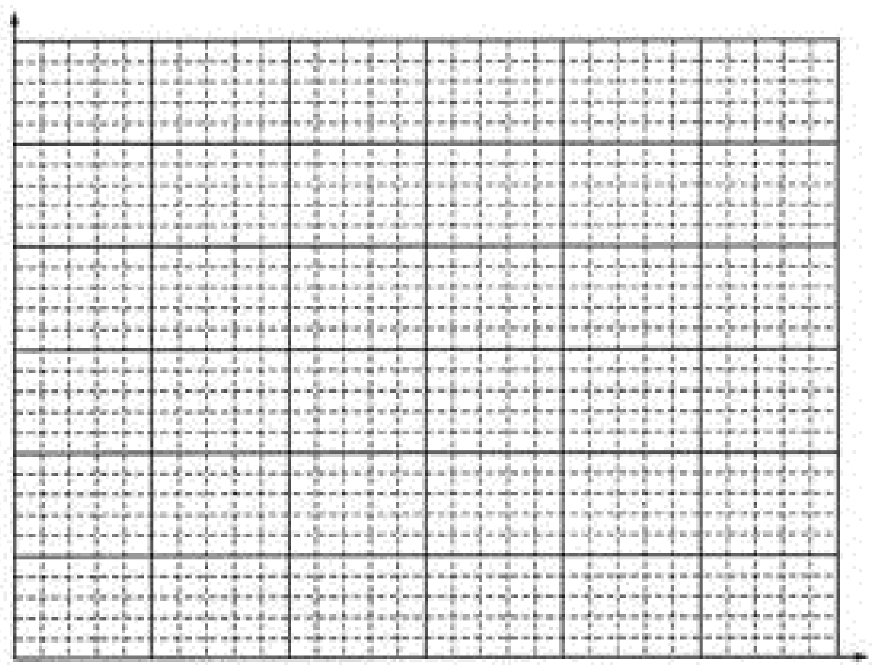
a. Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

ANSWERS - AP Physics Multiple Choice Practice – Gravitation

	Solution	Answer
1.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed. The smallest radius of orbit will be the fastest satellite.	B
2.	As a satellite moves farther away, it slows down, also decreasing its angular momentum and kinetic energy. The total energy remains the same in the absence of resistive or thrust forces. The potential energy becomes less negative, which is an increase.	D
3.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	A
4.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10	D
5.	The gravitational force on an object *is* the weight, and is proportional to the mass. In the same circular orbit, it is only the mass of the body being orbited and the radius of the orbit that contributes to the orbital speed and acceleration.	A,B
6.	Newton's third law	D
7.	Force is inversely proportional to distance between the centers squared. R × 4 = F ÷ 16	D
8.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	B
9.	A planet of the same size and twice the mass of Earth will have twice the acceleration due to gravity. The period of a mass on a spring has no dependence on g, while the period of a pendulum is inversely proportional to g.	D
10.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$	B
11.	Kepler's second law (Law of areas) is based on conservation of angular momentum, which remains constant. In order for angular momentum to remain constant, as the satellite approaches the sun, its speed increases.	C
12.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 4 = g × 4 and if the net effect is $g = g_{Earth}$ then r must be twice that of Earth.	B
13.	$a = g = \frac{GM}{r^2}$, if $R_2 = 2R_1$ then $a_2 = ¼\, a_1$	D

14. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. If r is doubled, v decreases by $\sqrt{2}$ B

15. Newton's third law C

16. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and if the net effect is g = g_{Earth} then r must be $\sqrt{2}$ times that of Earth C

17. From conservation of angular momentum $v_1r_1 = v_2r_2$ A

18. As the ball moves away, the force of gravity decreases due to the increasing distance. A

19. $g = \frac{GM}{r^2}$ At the top of its path, it has doubled its original distance from the center of the asteroid. C

20. Angular speed (in radians per second) is v/R. Since the satellite is not changing speed, there is no tangential acceleration and v^2/r is constant. A,D

21. The radius of each orbit is ½ D, while the distance between them is D. This gives B

$$\frac{GMM}{D^2} = \frac{Mv^2}{D/2}$$

22. An burst of the ships engine produces an increase in the satellite's energy. Now the satellite is moving at too large a speed for a circular orbit. The point at which the burst occurs must remain part of the ship's orbit, eliminating choices A and B. The Earth is no longer at the focus of the ellipse in choice E. D

23. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is B

$\frac{1}{2}mv^2 = \left|-\frac{GMm}{r}\right|$ which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$

24. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 7 = g × 7 and r × 2 = g ÷ 4, so the net effect is g × 7/4 C

25. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed or period. B

26. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 5 = g ÷ 5 and r ÷ 2 = g × 4, so the net effect is g × 4/5 B

27. Part of the gravitational force acting on an object at the equator is providing the necessary centripetal force to move the object in a circle. If the rotation of the earth were to stop, this part of the gravitational force is no longer required and the "full" value of this force will hold the object to the Earth. A

28. $F = \frac{GMm}{r^2}$. F is proportional to each mass and inversely proportional to the distance between their centers squared. If each mass is doubled, F is quadrupled. If r is doubled F is quartered. D

29. Since the acceleration due to gravity is less on the surface of the moon, to have the same gravitational force as a second object on the Earth requires the object on the Moon to have a larger mass. B

30. The mass of an object will not change based on its location. As one digs into a sphere of uniform density, the acceleration due to gravity (and the weight of the object) varies directly with distance from the center of the sphere. C

31. $F = \frac{GMm}{r^2}$ so F is proportional to $1/r^2$. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration due to gravity is only slightly smaller in orbit compared to the surface of the Earth. C

32. The angular momentum of each satellite is conserved independently so we can compare the orbits at any location. Looking at the common point between orbit A and B shows that satellite A is moving faster at that point than satellite B, showing $L_A > L_B$. A similar analysis at the common point between B and C shows $L_B > L_C$ A

33. $U = -\frac{GMm}{r}$ D

34. Since they are orbiting their center of mass, the larger mass has a radius of orbit of $\frac{1}{4}d$. The speed can be found from $\frac{G(3M)M}{d^2} = \frac{(3M)v^2}{d/4}$ which gives $v = \sqrt{\frac{GM}{4d}} = \frac{2\pi(d/4)}{T}$ A

35. $F = \frac{GMm}{r^2}$; If r ÷ 2, F × 4. If each mass is multiplied by 1.41, F is doubled (1.41×1.41) D

36. $g = \Delta v/t = (31\text{ m/s} - 50\text{ m/s})/(5\text{ s}) = -3.8\text{ m/s}^2$ B

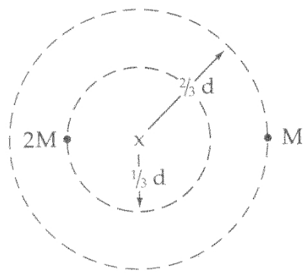
37. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. B

1977M3

a. $F_g = F_c$ gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b. E = PE + KE = $-\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$

c.



d. $F_{g2} = F_{g1} = F_c$

$\frac{(2M){v_2}^2}{{}^1/_3 d} = \frac{M{v_1}^2}{{}^2/_3 d}$ gives $v_2/v_1 = ½$

1984M2

a. $F_g = F_c$ gives $\frac{GM_e m}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives $(3m)v_0 - mv_0 = (4m)v'$ giving $v' = ½\ v_0$

c. E = PE + KE = $-\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_e m}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_e m}{4R_e}$

1992M3

a. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2 = -8.1 \times 10^9$ J

b. L = mvr = 8.5×10^{13} kg-m^2/s

c. Angular momentum is conserved so $mv_a r_a = mv_b r_b$ giving $v_b = 2.4 \times 10^3$ m/s

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM/R}$ = 5.8×10^3 m/s

e. Escape occurs when E = PE + KE = 0 giving $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0\ and\ v_{esc} = \sqrt{2GM/R}$ = 8.2×10^3 m/s

1994M3

a. E = PE + KE = $-\frac{GM_e m}{a} + \frac{1}{2}m{v_0}^2$

b. L = mvr = $mv_0 a$

c. Conservation of angular momentum gives $mv_0a = mv_bb$, or $v_b = v_0a/b$

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_e/a}$

e. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2}m\left(\frac{GM_e}{a} - {v_0}^2\right)$

1995M3

a. $v = \frac{2\pi r}{T}$ $and\ a = \frac{v^2}{r} = \frac{4\pi^2 r_a}{T^2}$

b. The centripetal force on star A is due to the gravitational force exerted by star B.
$M_a a_a = \frac{GM_a M_b}{(r_a+r_b)^2}$ and substituting part (a) gives $M_b = \frac{4\pi^2 r_a(r_a+r_b)^2}{GT^2}$

c. The same calculations can be performed with the roles of star A and star B switched.
$M_a = \frac{4\pi^2 r_b(r_a+r_b)^2}{GT^2}$

d. $L_{total} = M_a v_a r_a + M_b v_b r_b = M_a \frac{2\pi r_a}{T} r_a + M_b \frac{2\pi r_b}{T} r_b = \frac{2\pi}{T}(M_a r_a{}^2 + M_b r_b{}^2)$

2007M2

a. v = 2πR/T gives R = 3.83 × 10^6 m

b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ M = \frac{v^2 R}{G}$ = 6.64 × 10^23 kg

c. $E = PE + KE = -\frac{GMm}{R} + \frac{1}{2}mv^2 = -5.38 \times 10^9$ J

d. From Kepler's third law r^3/T^2 = constant so if r decreases, then T must also.

e. Conservation of angular momentum gives $mv_1r_1 = mv_2r_2$ so $v_2 = r_1v_1/r_2$, but the distances *above the surface* are given so the radius of Mars must be added to the given distances before plugging them in for each r. This gives $v_2 = 3.34 \times 10^3$ m/s.

2001M2

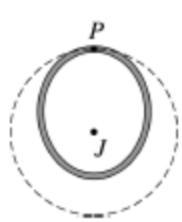
a. i. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_J/R}$

ii. v = d/T = 2πR/T giving $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM_J/R}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$

b. Plugging numerical values into a.ii. above gives R = 1.59 × 10^8 m

c. i.

ii.



2005M2

a. $F = \frac{GM_S m}{R^2}$

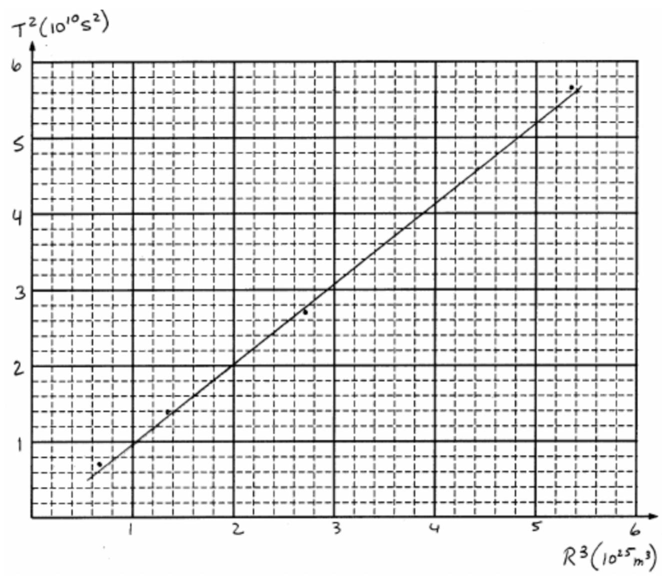
b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

e.



f. From part c. we have an expression for the slope of the line.
Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg