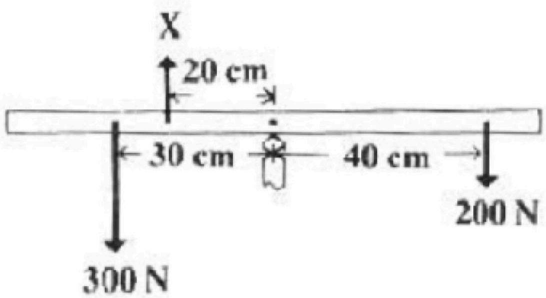
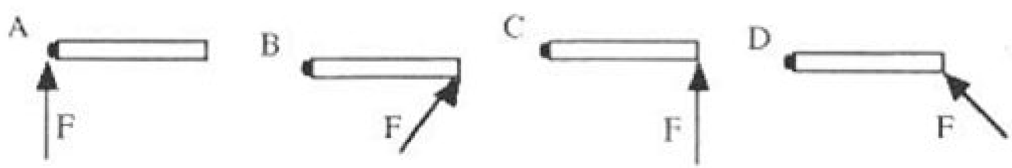
AP Physics Multiple Choice Practice – Torque

1. A uniform meterstick of mass 0.20 kg is pivoted at the 40 cm mark. Where should one hang a mass of 0.50 kg to balance the stick?
   (A) 16 cm (B) 36 cm (C) 44 cm (D) 46 cm

2. A uniform meterstick is balanced at its midpoint with several forces applied as shown below. If the stick is in equilibrium, the magnitude of the force X in newtons (N) is
   (A) 50 N (B) 100 N (C) 200 N (D) 300 N

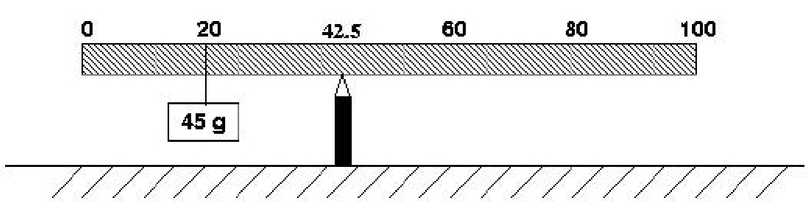


3. A door (seen from above in the figures below) has hinges on the left hand side. Which force produces the largest torque? The magnitudes of all forces are equal.



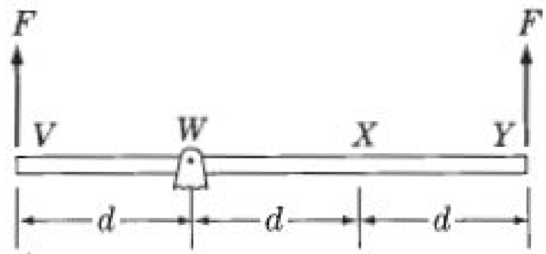
4. A meterstick is supported at each side by a spring scale. A heavy mass is then hung on the meterstick so that the spring scale on the left hand side reads four times the value of the spring scale on the right hand side. If the mass of the meterstick is negligible compared to the hanging mass, how far from the right hand side is the large mass hanging.
   (A) 25 cm (B) 67 cm (C) 75 cm (D) 80 cm

5. A uniform meter stick has a 45.0 *g* mass placed at the 20 *cm* mark as shown in the figure. If a pivot is placed at the 42.5 *cm* mark and the meter stick remains horizontal in static equilibrium, what is the mass of the meter stick?



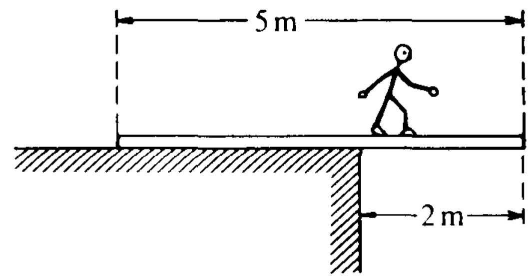
   (A) 45.0 *g* (B) 72.0 *g* (C) 120.0 *g* (D) 135.0 *g*

6. A massless rigid rod of length $3d$ is pivoted at a fixed point W, and two forces each of magnitude F are applied vertically upward as shown. A third vertical force of magnitude F may be applied, either upward or downward, at one of the labeled points. With the proper choice of direction at each point, the rod can be in equilibrium if the third force of magnitude F is applied at point

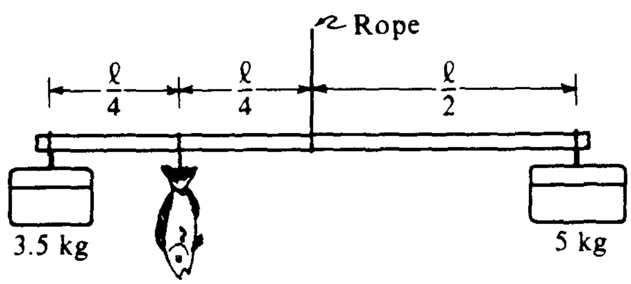


(A) Y only (B) V or X only (C) V or Y only (D) V, W, or X

7. A 5-meter uniform plank of mass 100 kilograms rests on the top of a building with 2 meters extended over the edge as shown. How far can a 50-kilogram person venture past the edge of the building on the plank before the plank just begins to tip?
(A) 0.5 m (B) 1 m (C) 1.5 m (D) 2 m

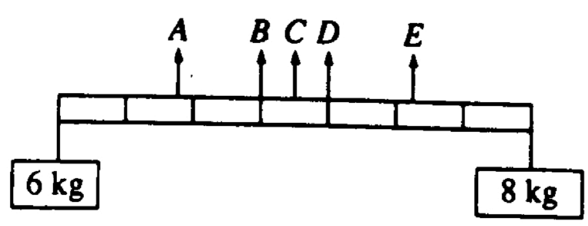


8. To weigh a fish, a person hangs a tackle box of mass 3.5 kilograms and a cooler of mass 5 kilograms from the ends of a uniform rigid pole that is suspended by a rope attached to its center. The system balances when the fish hangs at a point 1/4 of the rod's length from the tackle box. What is the mass of the fish?

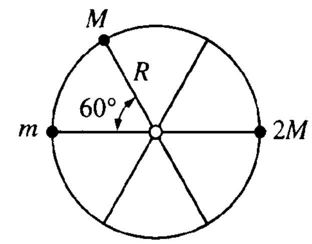


(A) 1.5 kg (B) 2 kg (C) 3 kg (D) 6 kg

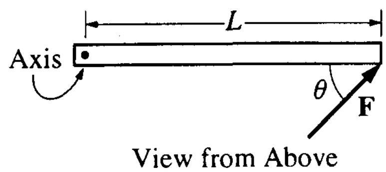
9. Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 cm long and has marks every 10 cm, as shown. If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?
(A) A (B) B (C) C (D) D



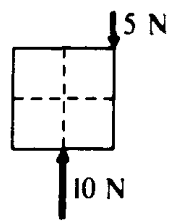
10. A wheel of radius R and negligible mass is mounted on a horizontal frictionless axle so that the wheel is in a vertical plane. Three small objects having masses m, M, and $2M$, respectively, are mounted on the rim of the wheel, as shown. If the system is in static equilibrium, what is the value of m in terms of M?
(A) $M/2$ (B) M (C) $3M/2$ (D) $5M/2$



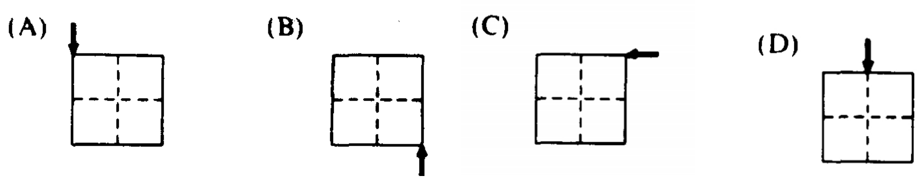
11. A rod on a horizontal tabletop is pivoted at one end and is free to rotate without friction about a vertical axis, as shown. A force **F** is applied at the other end, at an angle θ to the rod. If **F** were to be applied perpendicular to the rod, at what distance from the axis should it be applied in order to produce the same torque?

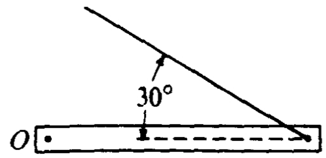


(A) L sin θ (B) L cos θ (C) L (D) L tan θ

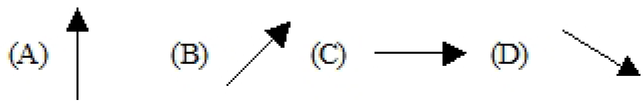


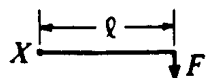
12. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?



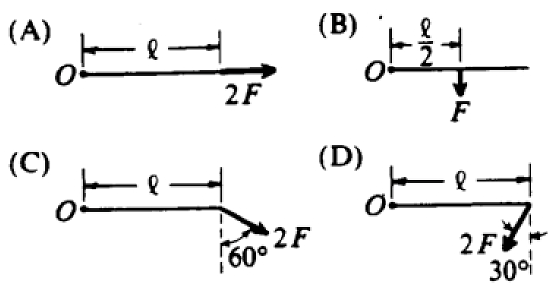


13. A uniform rigid bar of weight W is supported in a horizontal orientation as shown by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?

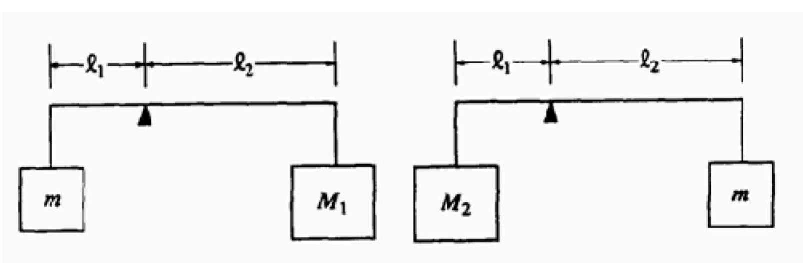




14. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram? (All forces lie in the plane of the paper.)

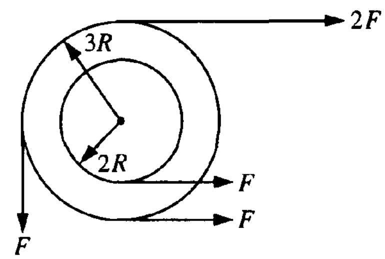


15. A rod of length L and of negligible mass is pivoted at a point that is off-center with lengths shown in the figure below. The figures show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

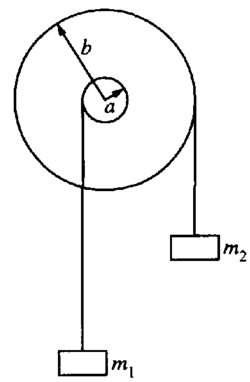


(A) $\sqrt{M_1 M_2}$ (B) $\frac{1}{2}(M_1 + M_2)$ (C) $M_1 M_2$ (D) $\frac{1}{2}M_1M_2$

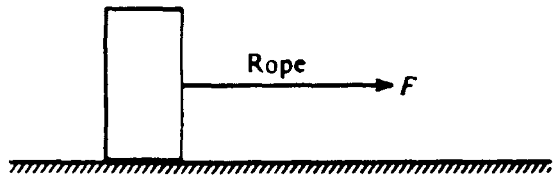
16. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown. The magnitude of the net torque on the system about the axis is
(A) zero (B) 2FR (C) 5FR (D) 14FR



17. For the wheel-and-axle system shown, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$
(B) $am_1 = bm_2$
(C) $am_2 = bm_1$
(D) $a^2m_1 = b^2m_2$

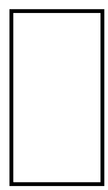


18. A meterstick of negligible mass is placed on a fulcrum at the 0.60 m mark, with a 2.0 kg mass hung at the 0 m mark and a 1.0 kg mass hung at the 1.0 m mark. The meterstick is released from rest in a horizontal position. Immediately after release, the magnitude of the net torque on the meterstick about the fulcrum is most nearly
(A) 2.0 N•m (B) 8.0 N•m (C) 10 N•m (D) 16 N•m

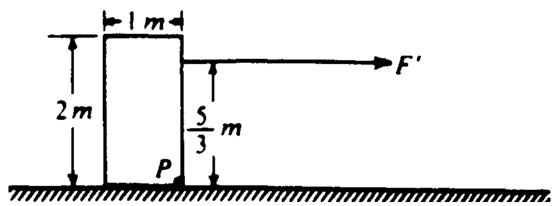


1983B1. A box of uniform density weighing 100 newtons moves in a straight line with constant speed along a horizontal surface. The coefficient of sliding friction is 0.4 and a rope exerts a force F in the direction of motion as shown above.

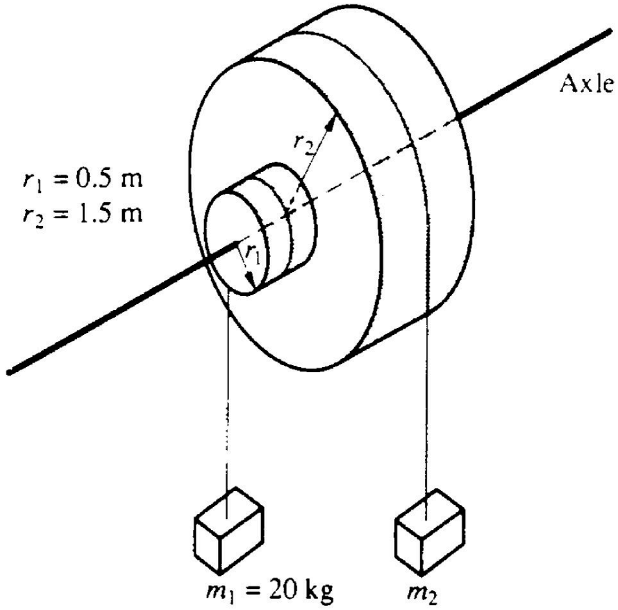
a. On the diagram below, draw and identify all the forces on the box.



b. Calculate the force F exerted by the rope that keeps the box moving with constant speed.

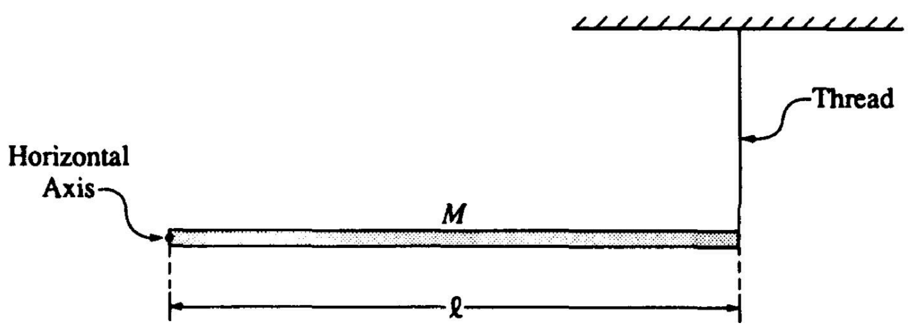


c. A horizontal force F', applied at a height 5/3 meters above the surface as shown in the diagram above, is just sufficient to cause the box to begin to tip forward about an axis through point P. The box is 1 meter wide and 2 meters high. Calculate the force F'.



C1991M2. Two masses. m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above with $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

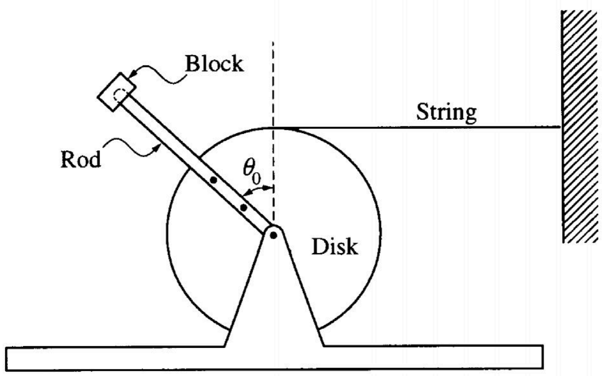
a. Determine m_2 such that the system will remain in equilibrium.



C1993M3. A long, uniform rod of mass M and length l is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. Express the answers to all parts of this question in terms of M, L and g.

a. Determine the magnitude and direction of the force exerted on the rod by the axis.

b. If the breaking strength of the thread is 2Mg, determine the maximum distance, r, measured from the hinge axis, that a box of mass 4M could be placed without breaking the thread

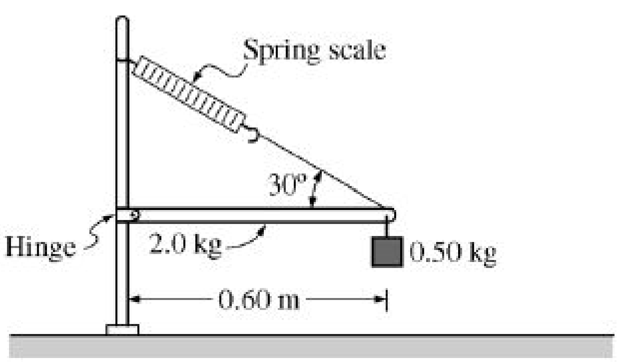


C1999M3. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk. A block is attached to the end of the rod. Properties of the rod, and block are as follows.

Rod: mass = m, length = 2R
Block: mass = 2m
Disk: radius = R

The system is held in equilibrium with the rod at an angle θ_o to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Determine the tension in the string in terms of m, θ_o, and g.

C2008M2.



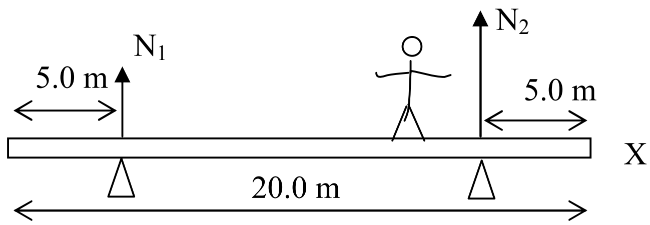
The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

(b) Calculate the reading on the spring scale.
(c) Calculate the magnitude of the force exerted by the hinge on the rod

Supplemental Problem

The diagram below shows a beam of length 20.0 m and mass 40.0 kg resting on two supports placed at 5.0 m from each end.



A person of mass 50.0 kg stands on the beam between the supports. The reaction forces at the supports are shown.

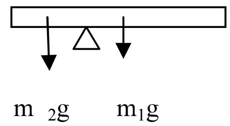
(a) State the value of $N_1 + N_2$

(b) The person now moves toward the X end of the beam to the position where the beam just begins to tip and reaction force N_1 becomes zero as the beam starts to leave the left support. Determine the distance of the girl from the end X when the beam is about to tip.

ANSWERS - AP Physics Multiple Choice Practice – Torque

Solution	Answer

1. Mass of stick m_1=0.20 kg at midpoint, Total length L=1.0 m, Pivot at 0.40 m, attached mass m_2=0.50 kg. **B**

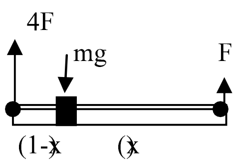


Applying rotational equilibrium $\tau_{net}=0$
$(m_1g)\ r_1 = (m_2g)\ r_2$
(0.2) (0.1 m) = (0.5)(x)
x = 0.04 m (measured away from 40 cm mark)
→ gives a position on the stick of 36 cm

2. As above, apply rotational equilibrium **A**
+ (300) (30cm) – X (20 cm) – (200) (40 cm) = 0

3. Torque = $(Fr)_p$ Choices A and E make zero torque, Of the remaining choices, each has momentarm = r but choice C has the full value of F to create torque (perpendicular) while the others would only use a component of F to make less torque **C**

4. Applying rotational equilibrium, using location of unknown mass as pivot **D**
4F (1 – x) = (F) (x)
4F = 5Fx
x = 4/5 = 0.80 m measured from right side



5. Applying rotational equilibrium (g cancels on each side) **D**
$(m_1g)\ r_1 = (m_2g)\ r_2$
(45) (22.5 cm) = (m) (7.5 cm) → m = 135 g

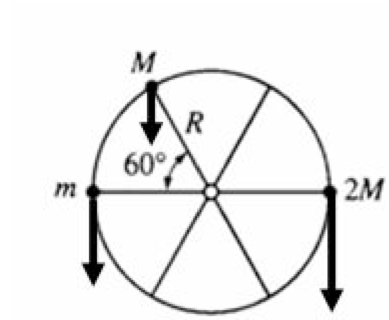
6. On the left of the pivot τ = Fd, on the right side of the pivot τ = F(2d). So we either have to add 1(Fd) to the left side to balance out the torque or remove 1(Fd) on the right side to balance out torque. Putting an upwards force on the left side at V gives (2Fd) on the left to balance torques, or putting a downwards force on the right side at X give a total of Fd on the right also causing a balance **B**

7. Apply rotational equilibrium using the corner of the building as the pivot point Weight of plank (acting at midpoint) provides torque on left and weight of man provides torque on right **B**
$(m_1g)\ r_1 = (m_2g)\ r_2$
(100 kg) (0.5m) = (50 kg) (r) → r = 1m

8. Apply rotational equilibrium using the rope as the pivot point **C**
(3.5)(9.8)(L/2) + m(9.8)(L/4) – (5)(9.8)(L/2) = 0 → m = 3 kg

9. To balance the torques on each side, we obviously need to be closer to the heavier mass **D**
Trying point D as a pivot point we have:
$(m_1g)\ r_1$?=? $(m_2g)\ r_2$
(6kg) (40 cm) ?=? (8kg) (30 cm) and we see it works

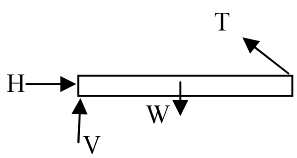
10. Applying rotational equilibrium at the center pivot we get: C
$+mg(R) + Mg(R\cos 60°) - 2Mg(R) = 0$.
Using $\cos 60 = ½$ we arrive at the answer 3M/2



11. Finding the torque in the current configuration we have: A
$(F\sin\theta)(L) = FL \sin\theta$.
To get the same torque with F applied perpendicular we would have to change the L to get $F (L \sin\theta)$

12. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed. A

13. FBD B



Since the rope is at an angle it has x and y components of force.
Therefore, H would have to exist to counteract Tx
Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar

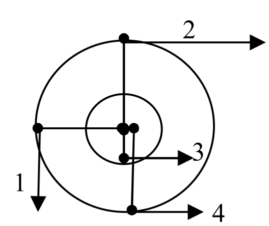
14. In the given diagram the torque is = FL. C
Finding the torque of all the choices reveals C as correct
$(2F\sin 60)(L) = 2F ½ L = FL$

15. Applying rotational equilibrium to each diagram gives A

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$
$L_1 = M_1(L_2) / m$
(sub this L_1) into the Diagram 2 eqn, and solve.

DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$
$M_2 (L_1) = m (L_2)$

16. Find the torques of each using proper signs and add up. B
$+ (1) - (2) + (3) + (4)$
$+F(3R) - (2F)(3R) + F(2R) + F(3R) = 2FR$

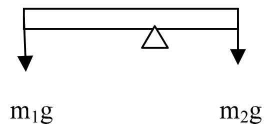


17. Simply apply rotational equilibrium B
$(m_1g)\ r_1 = (m_2g)\ r_2$
$m_1a = m_2b$

18. Question says meterstick has no mass, so ignore that force. Pivot placed at 0.60 m. Based on the applied masses, this meterstick would have a net torque and rotate. Find the net Torque as follows B

$\tau_{net} = +(m_1 g)\ r_1 - (m_2 g)\ r_2$

$+ (2)(10\ \text{m/s}^2)(0.6\ \text{m}) - (1)(10\ \text{m/s}^2)(0.4\ \text{m})$



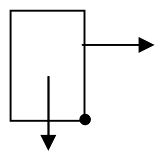
1983B1.

a) FBD. F_n pointing up, F_g pointing down, f_k applied to base of box pointing left

b) Constant speed → a=0.

Fnet = 0 F − fk = 0 F − uFn = 0 F − (0.4)(100)=0 F = 40 N

c) The force F' occurs at the limit point of tipping which is when the torque trying to tip it (caused by F') is equal to the torque trying to stop it from tipping (from the weight) using the tipping pivot point of the bottom right corner of the box.



(F')(5/3 m) = Fg (0.5m)
(F') (5/3) = (100)(0.5)
F' = 30 N

C1991M2.

Apply rotational equilibrium with the center as the pivot

$(m_1g)\ r_1 = (m_2g)\ r_2$ $(20)(9.8)(0.5) = m_2(9.8)(1.5)$ $m_2 = 6.67$ kg

C1993M3

(a) There is no H support force at the hinge since there are no other horizontal forces acting, so there is only vertical support for V. The tension in the thread T acts upwards and the weight of the rod acts at the midpoint. Apply rotational equilibrium using the hinge axis as the pivot:

+(T)(L) − (Mg)(L/2) = 0 T = Mg/2

Then using Fnet(y) = 0 V + T − Mg = 0 V + Mg/2 − Mg =0 V = Mg / 2

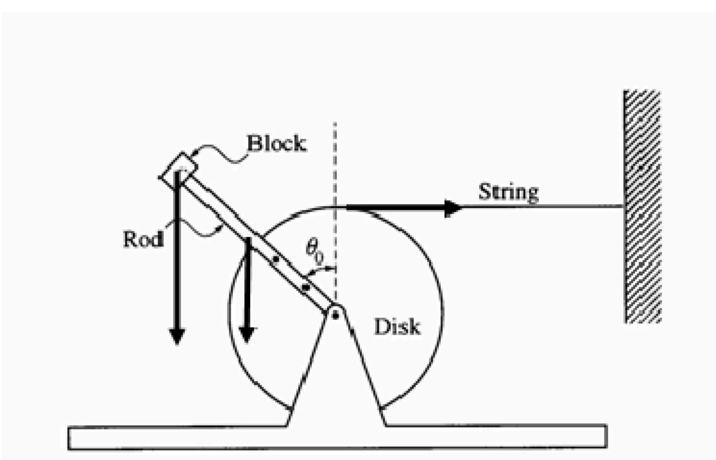
(b) Apply rotational equilibrium using the hinge axis as the pivot and r as the unknown distance of the box

Thread torque − Box torque - Rod Torque= 0

(2Mg)(L) − (4Mg)(r) − (Mg)(L/2) = 0

2L − 4r − L/2 = 0 r = 3/8 L

C1999M3



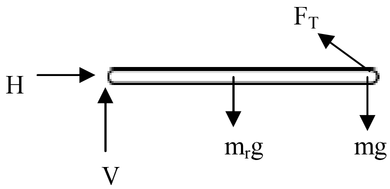
Apply rotational equilibrium using the center of the disk as the pivot

$(m_bg)(2R\sin\theta_o) + (m_rg)(R\sin\theta_o) - T(R) = 0$
$(2mg)(2R\sin\theta_o) + (mg)(R\sin\theta_o) - T(R) = 0$

$T = 5mg(\sin\theta_o)$

C2008M2

a) FBD



b) Apply rotational equilibrium using the hinge as the pivot

$+(F_T\sin30)(0.6) - (mg)(0.6) - (m_rg)(0.3) = 0$

$+(F_T\sin30)(0.6) - (0.5)(9.8)(0.6) - (2)(9.8)(0.3)=0$ $\qquad F_T = 29.4$ N

c) Apply Fnet(x), Fnet(y) = 0 to find H and V $\qquad$ V=9.8N, H=25.46N

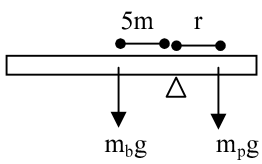
combining H and V $\qquad F_{hinge} = 27.28$ N

Supplemental

(a) Simple application of Fnet(y) = 0 $\qquad N_1 + N_2 - m_bg - m_pg = 0$

$N_1 + N_2 = (40)(9.8) + (50)(9.8) \quad = 882$ N

(b)



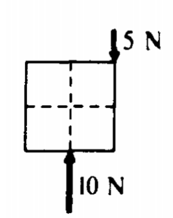
Apply rotational equilibrium

$(m_bg)\ r_1 = (m_pg)\ r_2$

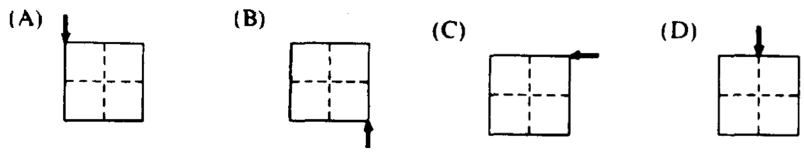
(40) (5m) = (50) (r) $\qquad$ r = 4m from hinge

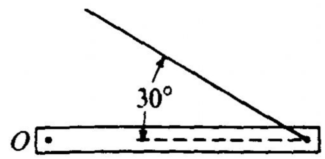
➔ 1 m from point X

SECTION A – Torque and Statics

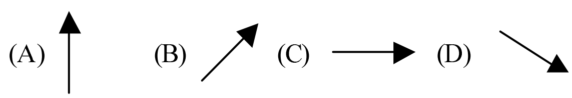


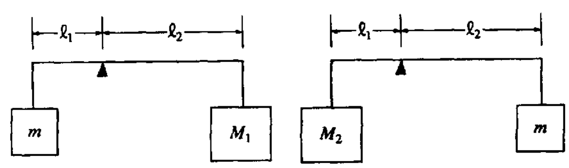
1. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown above. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?





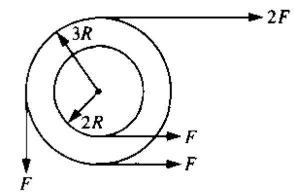
2. A uniform rigid bar of weight W is supported in a horizontal orientation as shown above by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?



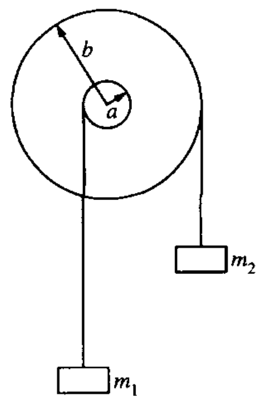


3. A rod of negligible mass is pivoted at a point that is off-center, so that length l_1 is different from length l_2. The figures above show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

(A) $M_1 + M_2$ (B) $\frac{1}{2}(M_1 + M_2)$ (C) $M_1 M_2$ (D) $\sqrt{M_1 M_2}$



4. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown above. The magnitude of the net torque on the system about the axis is
(A) FR (B) 2FR (C) 5FR (D) 14FR

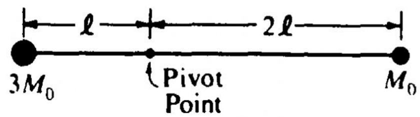


5. For the wheel-and-axle system shown above, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$ (B) $am_1 = bm_2$ (C) $am_2 = bm_1$ (D) $a^2m_1 = b^2m^2$

SECTION B – Rotational Kinematics and Dynamics

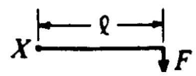
1. A uniform stick has length L. The moment of inertia about the center of the stick is I_0. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

   (A) $I_0 + \frac{1}{4}ML^2$ (B) $I_0 + \frac{1}{2}ML^2$ (C) $I_0 + \frac{3}{4}ML^2$ (D) $I_0 + ML^2$

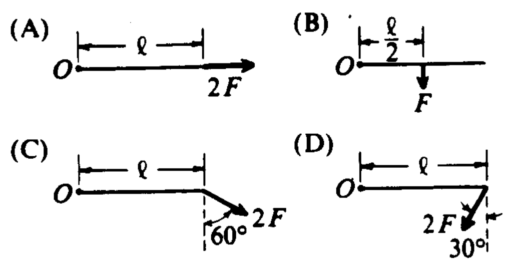


2. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude

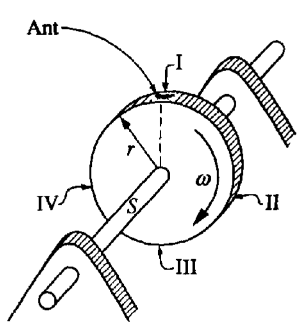
   (A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$



3. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram above? (All forces lie in the plane of the paper.)



Questions 4-5



An ant of mass m clings to the rim of a flywheel of radius r, as shown above. The flywheel rotates clockwise on a horizontal shaft S with constant angular velocity ω. As the wheel rotates, the ant revolves past the stationary points I, II, III, and IV. The ant can adhere to the wheel with a force much greater than its own weight.

4. It will be most difficult for the ant to adhere to the wheel as it revolves past which of the four points?
   (A) I (B) II (C) III (D) IV

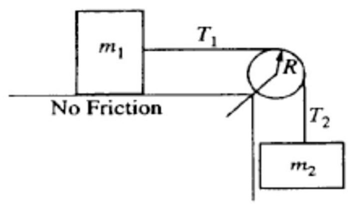
5. What is the magnitude of the minimum adhesion force necessary for the ant to stay on the flywheel at point III?
   (A) mg (B) $m\omega^2 r$ (C) $m\omega^2 r - mg$ (D) $m\omega^2 r + mg$

6. A turntable that is initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?
(A) $\sqrt{2\alpha}$ (B) $\sqrt{2\pi\alpha}$ (C) $\sqrt{4\pi\alpha}$ (D) $4\pi\alpha$

<u>Questions 7-8</u>
A wheel with rotational inertia I is mounted on a fixed, frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

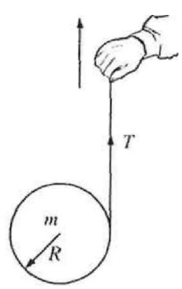
11. What is the average net torque on the wheel during this time interval?
(A) $\frac{\omega_f}{T}$ (B) $\frac{I\omega_f^2}{T}$ (C) $\frac{I\omega_f}{T^2}$ (D) $\frac{I\omega_f}{T}$

12. What is the average power input to the wheel during this time interval?
(A) $\frac{I\omega_f}{2T}$ (B) $\frac{I\omega_f^2}{2T}$ (C) $\frac{I\omega_f^2}{2T^2}$ (D) $\frac{I^2\omega_f}{2T^2}$



9. Two blocks are joined by a light string that passes over the pulley shown above, which has radius R and moment of inertia I about its center. T_1 and T_2 are the tensions in the string on either side of the pulley and α is the angular acceleration of the pulley. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?
(A) $m_2gR = I\alpha$ (B) $T_2R = I\alpha$ (C) $(T_2 - T_1)R = I\alpha$ (D) $(m_2 - m_1)gR = I\alpha$

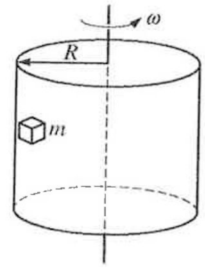
<u>Questions 10-11</u>



A solid cylinder of mass m and radius R has a string wound around it. A person holding the string pulls it vertically upward, as shown above, such that the cylinder is suspended in midair for a brief time interval Δt and its center of mass does not move. The tension in the string is T, and the rotational inertia of the cylinder about its axis is $\frac{1}{2}MR^2$

10. the net force on the cylinder during the time interval Δt is
(A) mg (B) $T - mgR$ (C) $mgR - T$ (D) zero

11. The linear acceleration of the person's hand during the time interval Δt is
(A) $\frac{T \square mg}{m}$ (B) $2g$ (C) $\frac{g}{2}$ (D) $\frac{T}{m}$



12. A block of mass *m* is placed against the inner wall of a hollow cylinder of radius *R* that rotates about a vertical axis with a constant angular velocity *ω,* as shown above. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction *μ* between the mass and the wall must satisfy which of the following inequalities?

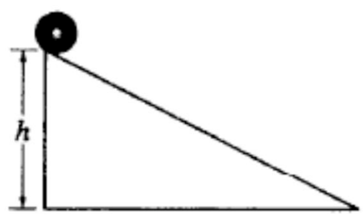
    (A) $\mu \,\square\, \dfrac{g}{\omega^2 R}$ (B) $\mu \,\square\, \dfrac{\omega^2 R}{g}$ (C) $\mu \,\square\, \dfrac{g}{\omega^2 R}$ (D) $\mu \,\square\, \dfrac{\omega^2 R}{g}$

SECTION C — Rolling

1. A bowling ball of mass M and radius R, whose moment of inertia about its center is (2/5)MR^2, rolls without slipping along a level surface at speed *v*. The maximum vertical height to which it can roll if it ascends an incline is

    (A) $\dfrac{v^2}{5g}$ (B) $\dfrac{2v^2}{5g}$ (C) $\dfrac{v^2}{2g}$ (D) $\dfrac{7v^2}{10g}$

Questions 2-3



A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown above.

2. If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?

    (A) $\sqrt{2gh}$ (B) $\dfrac{2Mghr^2}{I}$ (C) $\sqrt{\dfrac{2Mghr^2}{I}}$ (D) $\sqrt{\dfrac{2Mghr^2}{I+Mr^2}}$

3. If the plane has friction so that the sphere rolls without slipping, what is the speed v_{cm} of the center of mass at the bottom of the incline?

    (A) $\sqrt{2gh}$ (B) $\dfrac{2Mghr^2}{I}$ (C) $\sqrt{\dfrac{2Mghr^2}{I}}$ (D) $\sqrt{\dfrac{2Mghr^2}{I+Mr^2}}$

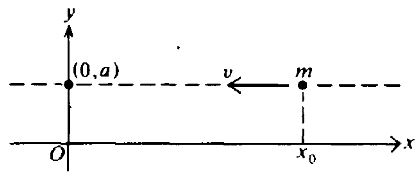
4. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is

    (A) vertically upward (B) horizontally forward (C) horizontally backward
    (D) upward and forward, at approximately 45° to the horizontal

SECTION D – Angular Momentum

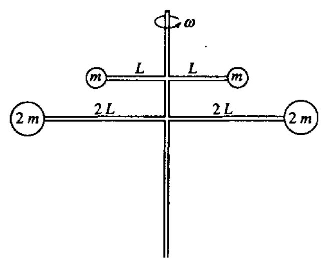
1. An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

	Angular Momentum	Kinetic Energy
(A)	Increases	Increases
(B)	Increases	Remains Constant
(C)	Remains Constant	Increases
(D)	Remains Constant	Remains Constant



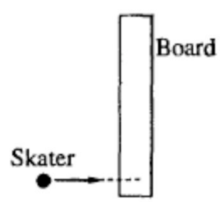
2. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_0, the magnitude of the angular momentum of the particle with respect to the origin of the system is

(A) zero (B) mva (C) mvx_o (D) $mv\sqrt{x_0^{\,2}+a^2}$



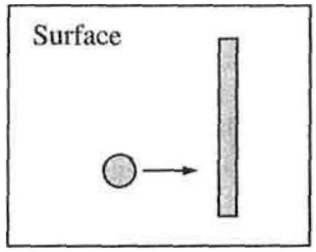
3. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?

(A) 2/1 (B) 1/2 (C) 1/4 (D) 1/8



Top View

4. A long board is free to slide on a sheet of frictionless ice. As shown in the top view above, a skater skates to the board and hops onto one end, causing the board to slide and rotate. In this situation, which of the following occurs?
(A) Linear momentum is converted to angular momentum.
(B) Rotational kinetic energy is conserved.
(C) Translational kinetic energy is conserved.
(D) Linear momentum and angular momentum are both conserved.



Top View

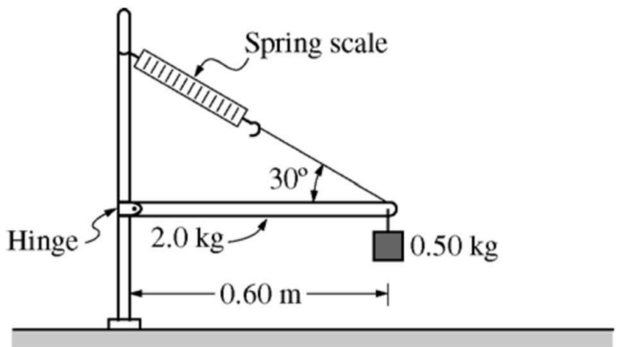
5. **Multiple Correct.** A disk sliding on a horizontal surface that has negligible friction collides with a rod that is free to move and rotate on the surface, as shown in the top view above. Which of the following quantities must be the same for the disk-rod system before and after the collision? Select two answers.

    I. Linear momentum
    II. Angular momentum
    III. Kinetic energy

    (A) Linear Momentum
    (B) Angular Momentum
    (C) Kinetic Energy
    (D) Mechanical Energy

WARNING: These are AP Physics C Free Response Practice – Use with caution!

SECTION A – Torque and Statics



2008M2. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

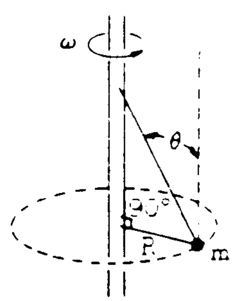
b. Calculate the reading on the spring scale.

The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length.

c. Calculate the rotational inertia of the rod-block system about the hinge.

d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

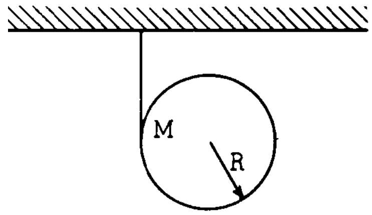
SECTION B – Rotational Kinematics and Dynamics



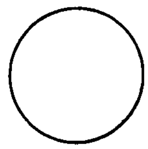
1973M3. A ball of mass m is attached by two strings to a vertical rod. as shown above. The entire system rotates at constant angular velocity ω about the axis of the rod.

a. Assuming ω is large enough to keep both strings taut, find the force each string exerts on the ball in terms of ω, m, g, R, and θ.

b. Find the minimum angular velocity, ω_{min} for which the lower string barely remains taut.



1976M2. A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $\frac{1}{2}MR^2$.



a. On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.
b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.
c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

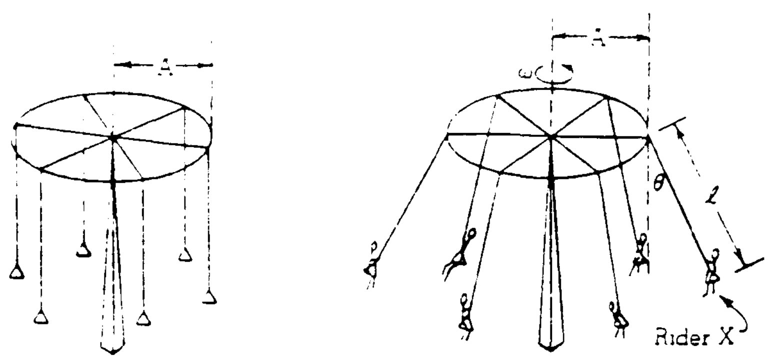
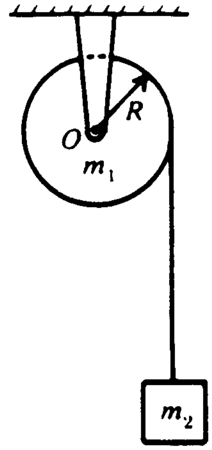


Figure I Figure II

1978M1. An amusement park ride consists of a ring of radius A from which hang ropes of length ℓ with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

a. In the space below, draw and label all the forces acting on rider X (represented by the point below) under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

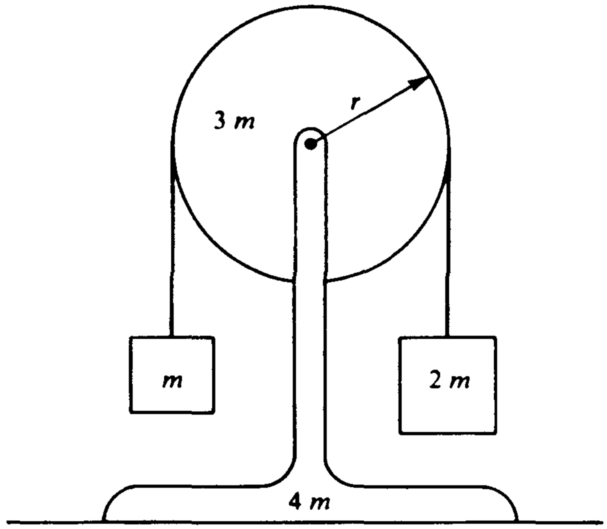
b. Derive an expression for ω in terms of A, ℓ, θ and the acceleration of gravity g.
c. Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, ℓ, θ, and the speed v of each rider.



1983M2. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = ½m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.

a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.

b. In terms of m_1, m_2, R. and g, determine each of the following.
   i. The acceleration of the block
   ii. The tension in the cord



1985M3. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$.
Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

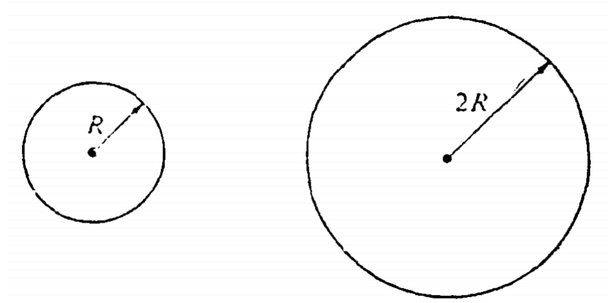
a. On the diagrams below, draw and label all the forces acting on each block.

b. Use the symbols identified in part a. to write each of the following.
   i. The equations of translational motion (Newton's second law) for each of the two blocks
   ii. The analogous equation for the rotational motion of the pulley

c. Solve the equations in part b. for the acceleration of the two blocks.

d. Determine the tension in the segment of the cord attached to the block of mass m.

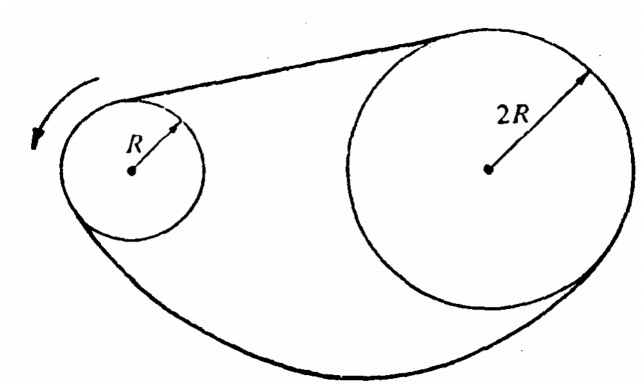
e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.



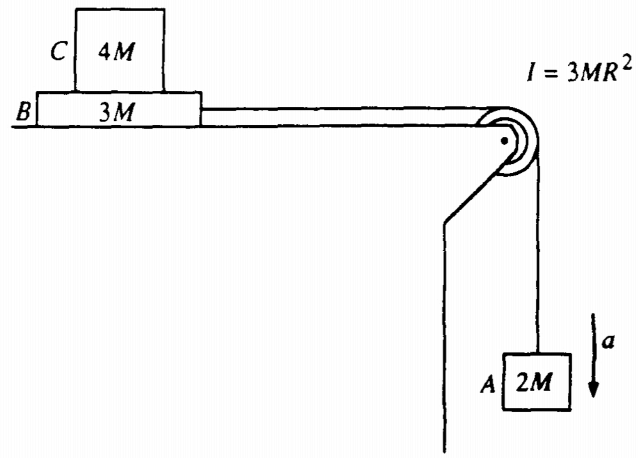
1988M3. The two uniform disks shown above have equal mass, and each can rotate on frictionless bearings about a fixed axis through its center. The smaller disk has a radius R and moment of inertia I about its axis. The larger disk has a radius 2R

a. Determine the moment of inertia of the larger disk about its axis in terms of I.

The two disks are then linked as shown below by a light chain that cannot slip. They are at rest when, at time t = 0, a student applies a torque to the smaller disk, and it rotates counterclockwise with constant angular acceleration α. Assume that the mass of the chain and the tension in the lower part of the chain, are negligible. In terms of I, R, α, and t, determine each of the following:



b. The angular acceleration of the larger disk
c. The tension in the upper part of the chain
d. The torque that the student applied to the smaller disk
e. The rotational kinetic energy of the smaller disk as a function of time



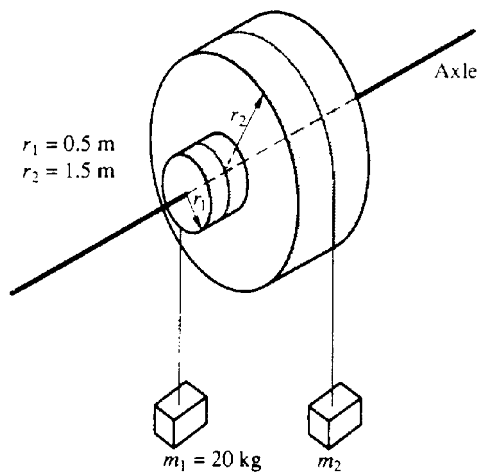
1989M2. Block A of mass 2M hangs from a cord that passes over a pulley and is connected to block B of mass 3M that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass 4M is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the

a. tension T_v in the vertical section of the cord
b. tension T_h in the horizontal section of the cord

If a = 2 meters per second squared, determine the

c. coefficient of kinetic friction between blocks B and C
d. acceleration of block C

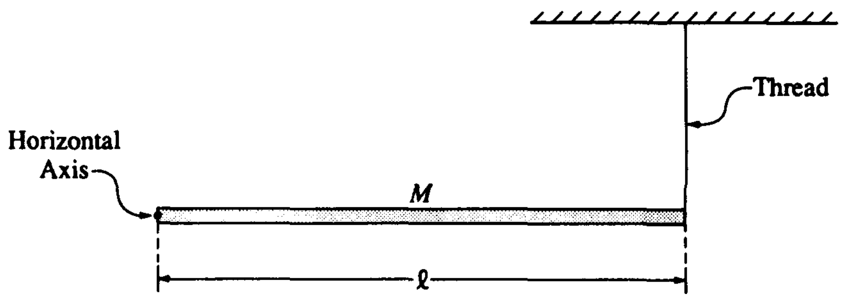


1991M2. Two masses. m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is I = 45 kg•m^2
Also r_1 = 0.5 meter, r_2 = 1.5 meters, and m_1 = 20 kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.
c. Determine the tension in the cable supporting m_1
d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.



1993M3. A long, uniform rod of mass M and length ℓ is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of M, ℓ and g.

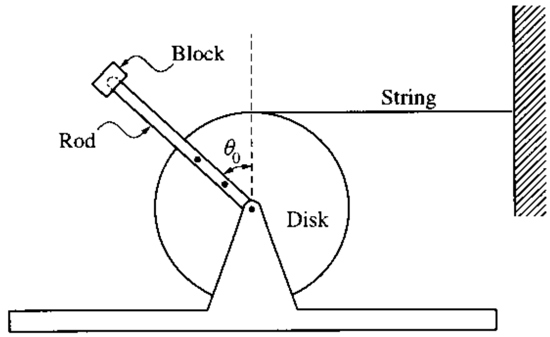
a. Determine the magnitude and direction of the force exerted on the rod by the axis.

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

b. The angular acceleration of the rod about the axis
c. The translational acceleration of the center of mass of the rod
d. The force exerted on the end of the rod by the axis

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod has swung.



1999M3 As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass = 3m, radius = R, moment of inertia about center $I_D = 1.5mR^2$

Rod: mass = m, length = 2R, moment of inertia about one end $I_R = 4/3(mR^2)$

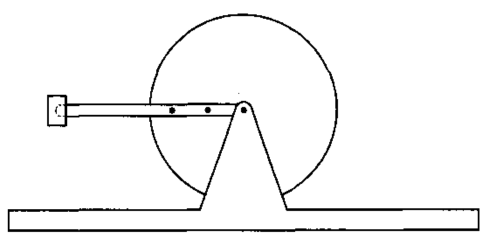
Block: mass = 2m

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

a. Determine the tension in the string.

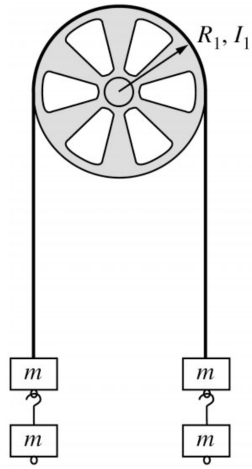
The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.
i. The magnitude of the angular acceleration of the disk
ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.



2000M3. A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley. Determine the answers to parts a. and b. in terms of m, R_1, I_1, and fundamental constants.

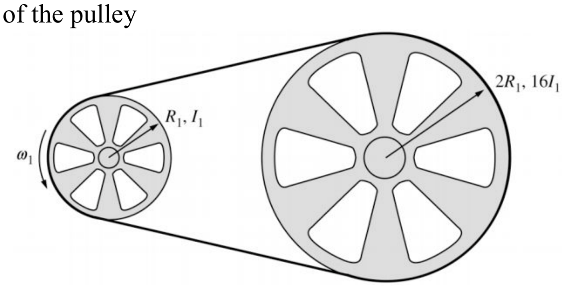
a. Determine the tension T in the cord.

b. One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration g/3 . Determine the following.

i. The tension T_3 in the section of cord supporting the three blocks on the left

ii. The tension T_1 in the section of cord supporting the single block on the right

iii. The rotational inertia I_1 of the pulley

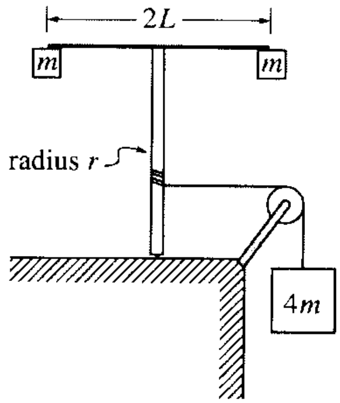


c. The blocks are now removed and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_I, and ω_I.

i. The angular speed ω_2 of the larger pulley

ii. The angular momentum L_2 of the larger pulley

iii. The total kinetic energy of the system

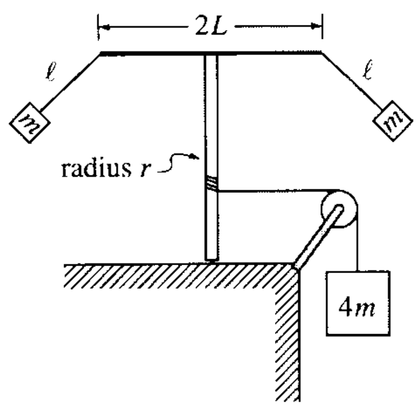


Experiment A

2001M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.
b. Determine the downward acceleration of the large block.
c. When the large block has descended a distance *D,* how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD*? Check the appropriate space below and justify your answer.

Greater than 4mgD ______ Equal to 4mgD ____ Less than 4mgD _____



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? Check the appropriate space below and justify your answer.

Greater before ___ Equal to before ___ Less than before ___

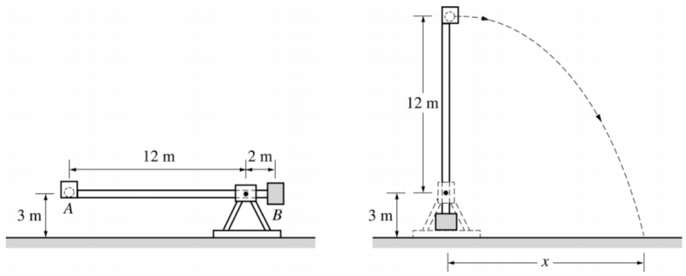


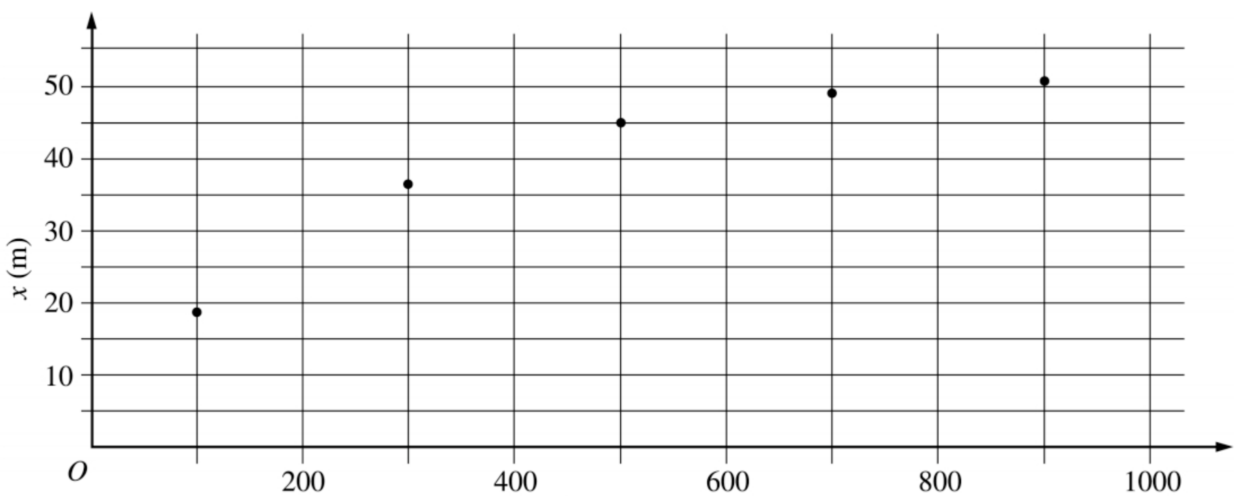
Figure 1 Figure 2

2003M3. Some physics students build a catapult, as shown above. The supporting platform is fixed firmly to the ground. The projectile, of mass 10 kg, is placed in cup A at one end of the rotating arm. A counterweight bucket B that is to be loaded with various masses greater than 10 kg is located at the other end of the arm. The arm is released from the horizontal position, shown in Figure 1, and begins rotating. There is a mechanism (not shown) that stops the arm in the vertical position, allowing the projectile to be launched with a horizontal velocity as shown in Figure 2.

a. The students load five different masses in the counterweight bucket, release the catapult, and measure the resulting distance x traveled by the 10 kg projectile, recording the following data.

Mass (kg)	100	300	500	700	900
x (m)	18	37	45	48	51

i. The data are plotted on the axes below. Sketch a best-fit curve for these data points.



ii. Using your best-fit curve, determine the distance x traveled by the projectile if 250 kg is placed in the counterweight bucket.

b. The students assume that the mass of the rotating arm, the cup, and the counterweight bucket can be neglected. With this assumption, they develop a theoretical model for x as a function of the counterweight mass using the relationship $x = v_x t$, where v, is the horizontal velocity of the projectile as it leaves the cup and t is the time after launch.

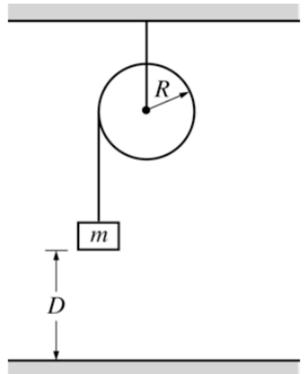
i. How many seconds after leaving the cup will the projectile strike the ground?

ii. Derive the equation that describes the gravitational potential energy of the system relative to the ground when in the position shown in Figure 1, assuming the mass in the counterweight bucket is M.

iii. Derive the equation for the velocity of the projectile as it leaves the cup, as shown in Figure 2.

c. i. Complete the theoretical model by writing the relationship for x as a function of the counterweight mass using the results from b. i and b. iii.

ii. Compare the experimental and theoretical values of x for a counterweight bucket mass of 300 kg. Offer a reason for any difference.



2004M2. A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

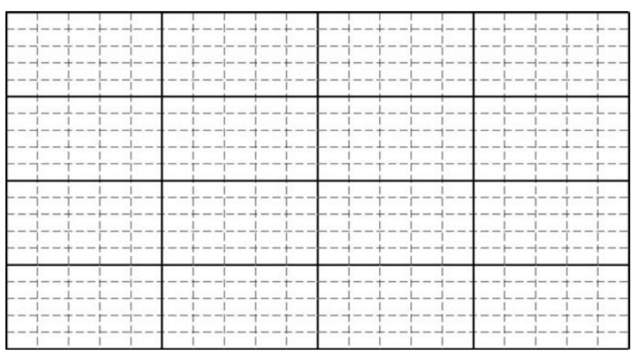
a. Calculate the linear acceleration a of the falling block in terms of the given quantities.

b. The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in b. i., label the axes, and draw the best-fit line to the data.

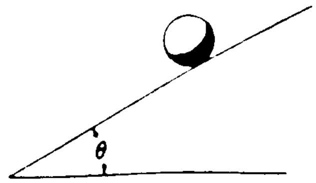


iii. Use your graph to calculate the magnitude of the acceleration.

c. Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

d. The value of acceleration found in b.iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

SECTION C – Rolling



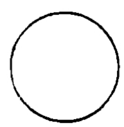
1974M2. The moment of inertia of a uniform solid sphere (mass M, radius R) about a diameter is $2MR^2/5$. The sphere is placed on an inclined plane (angle θ) as shown above and released from rest.

a. Determine the minimum coefficient of friction μ between the sphere and plane with which the sphere will roll down the incline without slipping

b. If μ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part a.? Explain your answer.

1977M2. A uniform cylinder of mass M, and radius R is initially at rest on a rough horizontal surface. The moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$. A string, which is wrapped around the cylinder, is pulled upwards with a force T whose magnitude is 0.6Mg and whose direction is maintained vertically upward at all times. In consequence, the cylinder both accelerates horizontally and slips. The coefficient of kinetic friction is 0.5.

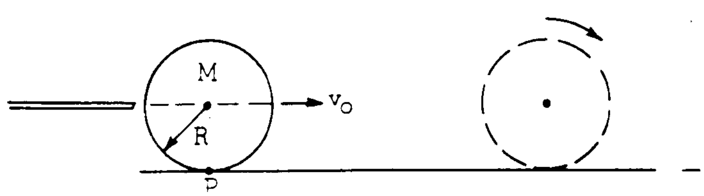
a. On the diagram below, draw vectors that represent each of the forces acting on the cylinder identify and clearly label each force.



b. Determine the linear acceleration a of the center of the cylinder.

c. Calculate the angular acceleration α of the cylinder.

d. Your results should show that a and αR are not equal. Explain.



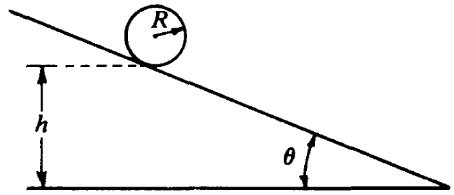
1980M3. A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c = 2\ MR^2/5$
The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_o as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.

a. Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

b. Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

c. Determine the time at which the ball begins to roll without slipping.

d. When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

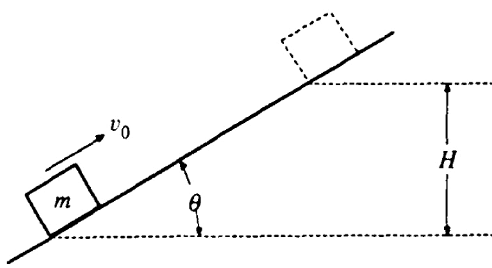


1986M2. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height *h* above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R. *h,* g, and θ.

a. Determine the following for the sphere when it is at the bottom of the plane:
   i. Its translational kinetic energy
   ii. Its rotational kinetic energy
b. Determine the following for the sphere when it is on the plane.
   i. Its linear acceleration
   ii. The magnitude of the frictional force acting on it

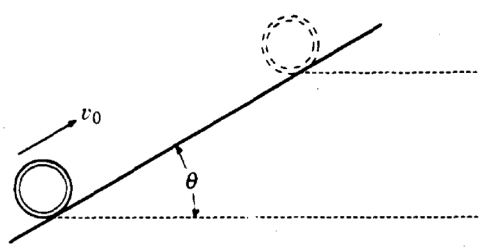
The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?
d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.



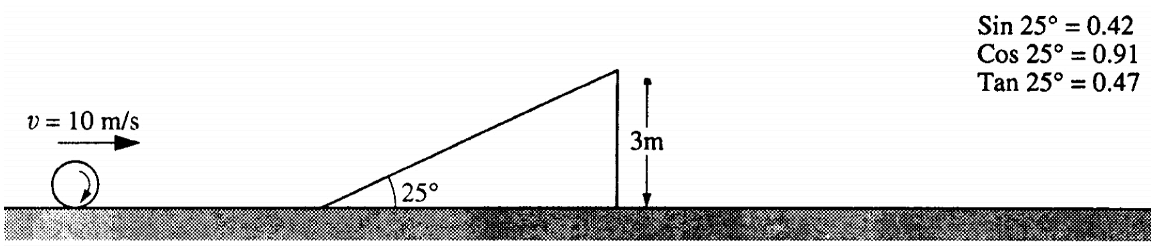
1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_o in the position shown.

c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities.
d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.



Note: Diagram not drawn to scale.

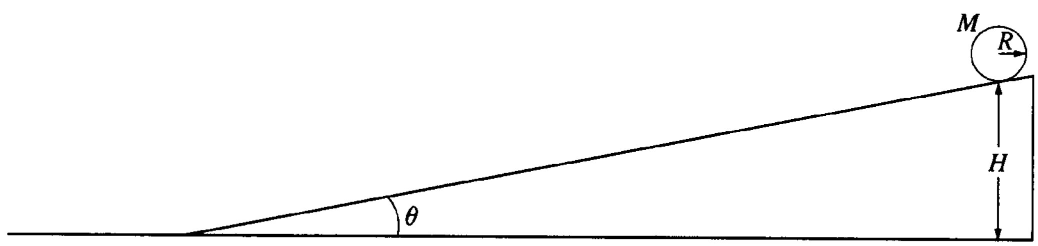
1994M2. A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass m of 25 kilograms, and a radius r of 0.2 meter. The moment of inertia of the sphere about its center of mass is $I = 2mr^2/5$. The sphere approaches a 25° incline of height 3 meters as shown above and rolls up the incline without slipping.

a. Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b. i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.
ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

c. Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

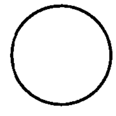
d. Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in b. Explain briefly.



1997M3. A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



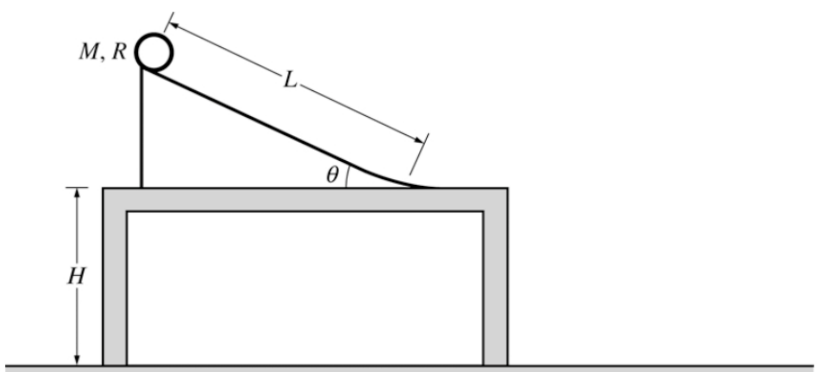
c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is (2/3)g sinθ.

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction μ is now made less than the value determined in part d., so that the cylinder both rotates and slips.
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part a. Justify your answer.
ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

2002M2. The cart shown above is made of a block of mass m and four solid rubber tires each of mass m/4 and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia ½ ML^2, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the total rotational inertia of all four tires.
b. Determine the speed of the cart when it reaches the bottom of the incline.
c. After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.
d. Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part c.. Give a reasonable explanation for this decrease.

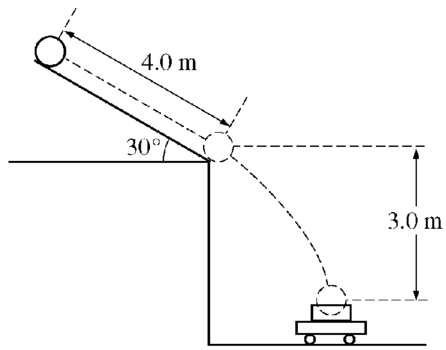


2006M3. A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.
b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.
c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.
d. Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

Less than____ The same as____ Greater than____

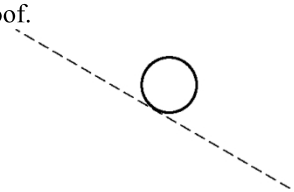
Briefly justify your response.



Note: Figure not drawn to scale.

2010M2. A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $2MR^2/5$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.

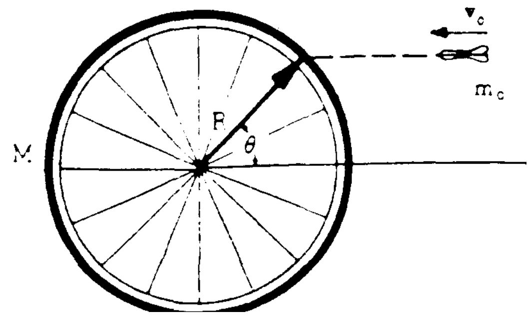


b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part a. to assist in your solution, use the space below. Do NOT add anything to the figure in part a.

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

SECTION D – Angular Momentum



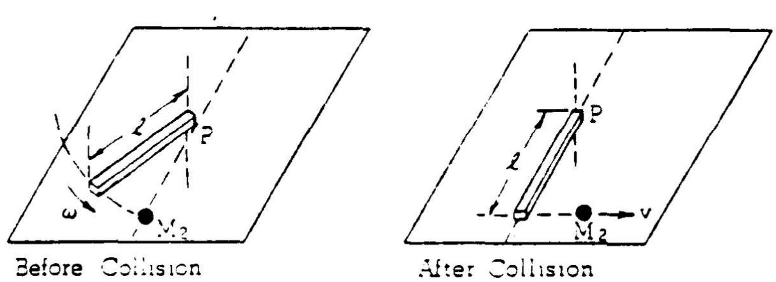
View From Above

1975M2. A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_o is thrown with velocity v_o as shown above and sticks in the tire.

a. If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

b. In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$



1978M2. A system consists of a mass M_2 and a uniform rod of mass M_1 and length l. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, l, and ω.

b. Determine the linear momentum of this system just before the collision in terms of M_1, l, and ω.

c. Determine the linear momentum of this system just after the collision in terms of M_1 l, and ω.

d. What is responsible for the change in the linear momentum of this system during the collision?

e. Why is the angular momentum of this system about point P conserved during the collision?

Views From Above

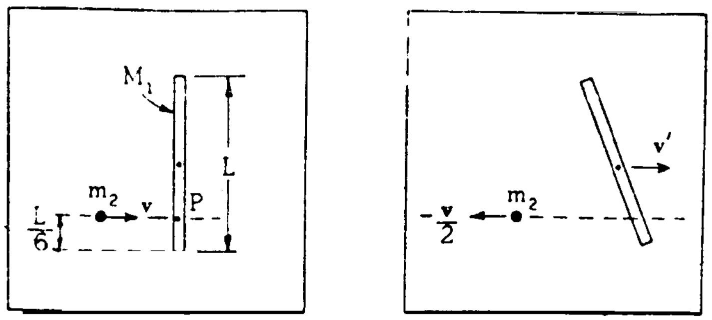


Figure I: Before

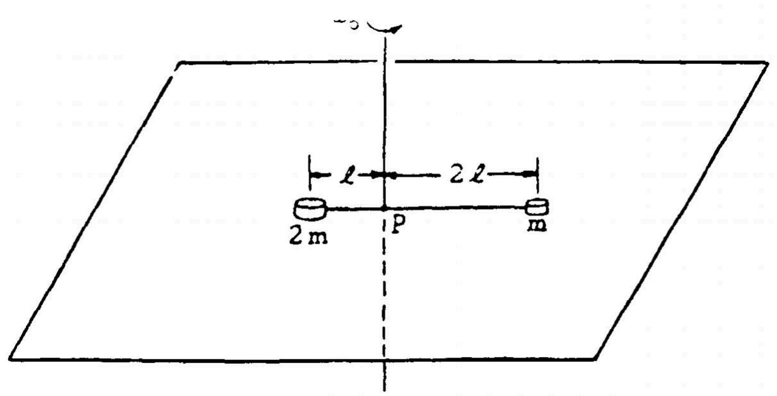
Figure II: After

1981M3. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $M_1L^2/12$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity $-½\mathbf{v}$ as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity **v**' of the center of mass of this rod after the collision.

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

c. Determine the change in kinetic energy of the system resulting from the collision.

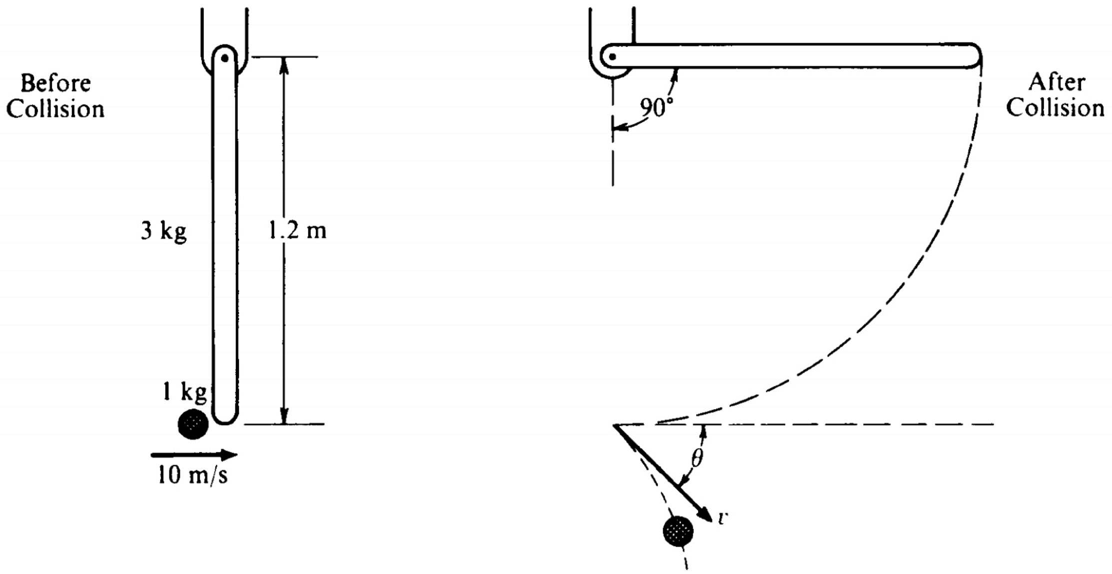


1982M3. A system consists of two small disks, of masses m and 2m, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time t = 0, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction. Develop expressions for the following quantities in terms of μ m, l, g, and ω_o

a. The initial angular momentum of the system about the axis through P

b. The frictional torque acting on the system about the axis through P

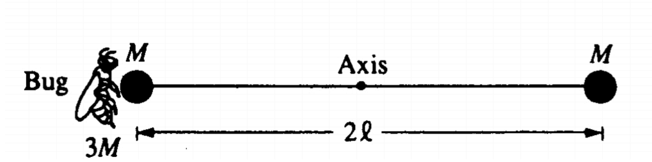
c. The time T at which the system will come to rest.



Note: You may use $g = 10\ m/s^2$.

1987M3. A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.

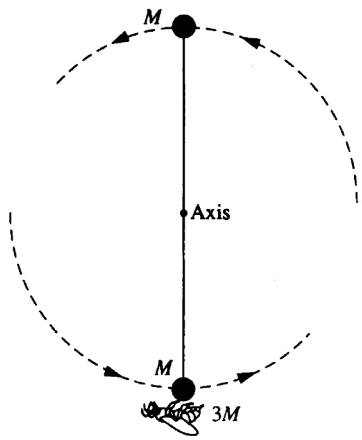
a. Determine the angular velocity of the bar immediately after the collision.
b. Determine the speed v of the 1-kilogram object immediately after the collision.
c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.
d. Determine the angle θ.



1992M2. Two identical spheres, each of mass M and negligible radius, are fastened to opposite ends of a rod of negligible mass and length $2l$. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass 3M, lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of M, l, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

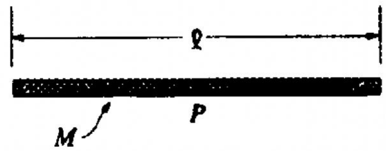


The rod-spheres-bug system swings about the axis. At the instant that the rod is vertical, as shown above, determine each of the following.

c. The angular speed of the bug

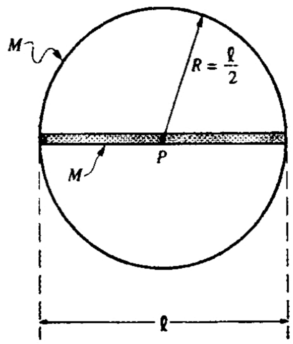
d. The angular momentum of the system

e. The magnitude and direction of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere



1996M3. Consider a thin uniform rod of mass M and length l, as shown above.

a. Show that the rotational inertia of the rod about an axis through its center and perpendicular to its length is $Ml^2/12$.



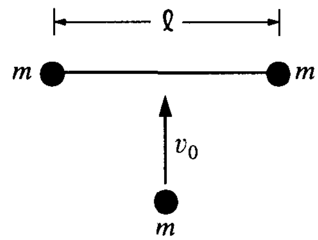
The rod is now glued to a thin hoop of mass M and radius R/2 to form a rigid assembly, as shown above. The centers of the rod and the hoop coincide at point P. The assembly is mounted on a horizontal axle through point P and perpendicular to the page.

b. What is the rotational inertia of the rod-hoop assembly about the axle?

Several turns of string are wrapped tightly around the circumference of the hoop. The system is at rest when a cat, also of mass M, grabs the free end of the string and hangs vertically from it without swinging as it unwinds, causing the rod-hoop assembly to rotate. Neglect friction and the mass of the string.

c. Determine the tension T in the string.
d. Determine the angular acceleration a of the rod-hoop assembly.
e. Determine the linear acceleration of the cat.
f. After descending a distance $H = 5l/3$, the cat lets go of the string. At that instant, what is the angular momentum of the cat about point P?

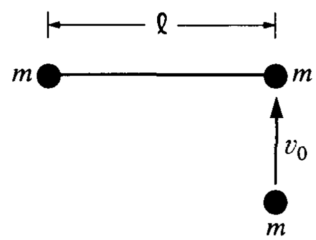
1998M2. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length l and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0 . Express your answers in terms of m, v_0 l. and fundamental constants.



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision.

ii. Determine the change in kinetic energy as a result of the collision.



b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly.

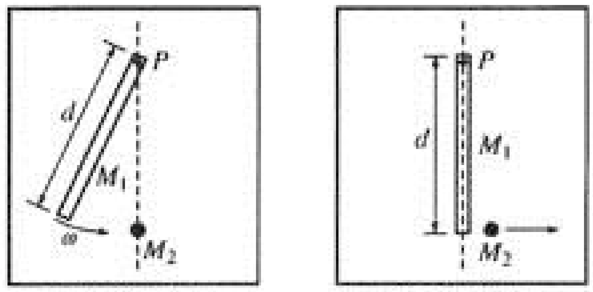
i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.)

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision.

iii. Determine the speed of the center of mass immediately after the collision.

iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

v. Determine the change in kinetic energy as a result of the collision.



Before Collision After Collision

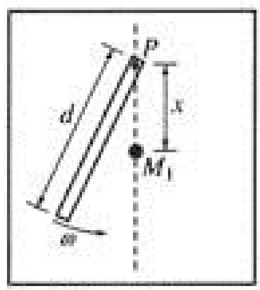
TOP VIEWS

2005M3. A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

a. Derive an expression for the angular momentum of the rod about point P before the collision.

b. Derive an expression for the speed v of the ball after the collision.

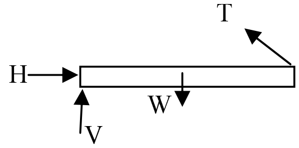
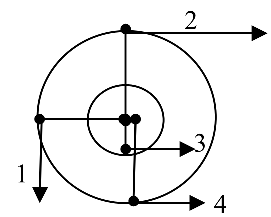
c. Assuming that this collision is elastic, calculate the numerical value of the ratio M_1 / M_2



Before Collision

d. A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

SECTION A – Torque and Statics

Solution	Answer
1. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed	A
2. FBD  Since the rope is at an angle it has x and y components of force. Therefore, H would have to exist to counteract T_x. Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to choose a pivot point at the right end of the bar	B
3. Applying rotational equilibrium to each diagram gives DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$ $L_1 = M_1(L_2)/m$ (sub this L_1) into the Diagram 2 eq, and solve. DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$ $M_2(L_1) = m(L_2)$	D
4. Find the torques of each using proper signs and add up + (1) – (2) + (3) + (4) +F(3R) – (2F)(3R) + F(2R) +F(3R) = 2FR 	B
5. Simply apply rotational equilibrium $(m_1g)\ r_1 = (m_2g)\ r_2$ $m_1a = m_2b$	B

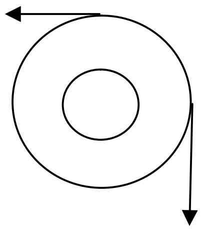
SECTION B – Rotational Kinematics and Dynamics

1. $I_{tot} = \Sigma I = I_0 + I_M = I_0 + M(L)^2$ — A
2. $\Sigma\tau = I\alpha$ where $\Sigma\tau = (3M_0)(l) - (M_0)(2l) = M_0 l$ and $I = (3M_0)(l)^2 + (M_0)(2l)^2 = 7M_0 l^2$ — A
3. $\tau_X = Fl$; $\tau_O = F_O L_O \sin\theta$, solve for the correct combination of F_O and L_O — C
4. Just as the tension in a rope is greatest at the bottom of a vertical circle, the force needed to maintain circular motion in any vertical circle is greatest at the bottom as the applied force must balance the weight of the object and additionally provide the necessary centripetal force — C
5. $\Sigma F_{bottom} = F_{adhesion} - mg = F_{centripetal} = m\omega^2 r$ — D
6. For one complete revolution $\theta = 2\pi$; $\omega^2 = \omega_0^2 + 2\alpha\theta$ — C

7. $\tau = \Delta L/\Delta t = (I\omega_f - 0)/T$ D

8. $P_{avg} = \tau\omega_{avg} = (I\omega_f/T)(\tfrac{1}{2}\omega_f)$ or $P_{avg} = \Delta K/T$ B

9. C



$\Sigma\tau = T_2R - T_1R = I\alpha$

10. If the cylinder is suspended in mid air (i.e. the linear acceleration is zero) then $\Sigma F = 0$ D

11. $\Sigma\tau = TR = I\alpha = \tfrac{1}{2}MR^2\alpha$ which gives $\alpha = 2T/MR$ and since $\Sigma F = 0$ then $T = Mg$ so $\alpha = 2g/R$ the acceleration of the person's hand is equal to the linear acceleration of the string around the rim of the cylinder $a = \alpha R = 2g$ B

12. In order that the mass not slide down $f = \mu F_N \geq mg$ and $F_N = m\omega^2 R$ solving for μ gives $\mu \geq g/\omega^2 R$ A

SECTION C – Rolling

1. $K_{tot} = K_{rot} + K_{trans} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}Mv^2 = \tfrac{1}{2}(2/5)MR^2\omega^2 + \tfrac{1}{2}Mv^2 = (1/5)Mv^2 + \tfrac{1}{2}Mv^2 = (7/10)Mv^2 = MgH$, solving gives $H = 7v^2/10g$ D

2. $Mgh = K_{tot} = K_{rot} + K_{trans}$, however without friction, there is no torque to cause the sphere to rotate so $K_{rot} = 0$ and $Mgh = \tfrac{1}{2}Mv^2$ A

3. $Mgh = K_{tot} = K_{rot} + K_{trans} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}Mv^2$; substituting v/r for ω gives $Mgh = (\tfrac{1}{2}I/r^2 + \tfrac{1}{2}M)v^2$ and solving for v gives $v^2 = 2Mgh/(I/r^2 + M)$, multiplying by r^2/r^2 gives desired answer D

4. The first movement of the point of contact of a rolling object is vertically upward as there is no side to side (sliding) motion for the point in contact A

SECTION D — Angular Momentum

1. $L_i = L_f$ so $I_i\omega_i = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ — C
   The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater

2. $L = mvr_\perp$ where $r_\perp$ is the perpendicular line joining the origin and the line along which the particle is moving — B

3. $L = I\omega$ and since ω is uniform the ratio $L_{upper}/L_{lower} = I_{upper}/I_{lower} = 2mL^2/2(2m)(2L)^2 = 1/8$ — D

4. Since it is a perfectly inelastic (sticking) collision, KE is not conserved. As there are no external forces or torques, both linear and angular momentum are conserved — D

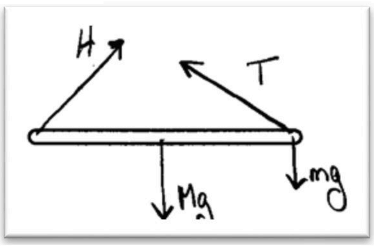
5. As there are no external forces or torques both linear and angular momentum are conserved. As the type of collision is not specified, we cannot say kinetic or mechanical energy *must* be the same. — A/B

WARNING: These are AP Physics C Free Response Practice – Rotation – ANSWERS
Use with caution!

SECTION A – Torque and Statics

2008M2

a.



b. $\Sigma\tau = 0$
About the hinge: $TL \sin 30° - mgL - Mg(L/2) = 0$ gives $T = 29$ N

c. $I_{total} = I_{rod} + I_{block}$ where $I_{rod,end} = I_{cm} + MD^2 = ML^2/12 + M(L/2)^2 = ML^2/3$
$I_{total} = ML^2/3 + mL^2 = 0.42$ kg-m^2

d. $\Sigma\tau = I\alpha$
$mgL + MgL/2 = I\alpha$ gives $\alpha = 21$ rad/s^2

SECTION B – Rotational Kinematics and Dynamics

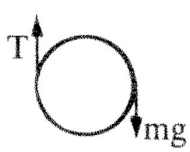
1973M3

a. Define a coordinate system with the x-axis directed to the vertical rod and the y-axis directed upwards and perpendicular to the first. Let T_1 be the tension in the horizontal string. Let T_2 be the tension in the string tilted upwards.
Applying Newton's Second Law: $\Sigma F_x = T_1 + T_2 \sin\theta = m\omega^2 R$; $\Sigma F_y = T_2 \cos\theta - mg = 0$
Solving yields: $T_2 = mg/\cos\theta$ and $T_1 = m(\omega^2 R - g \tan\theta)$

b. Let $T_1 = 0$ and solving for ω gives $\omega = (g \tan\theta/R)^{1/2}$

1976M2

a.



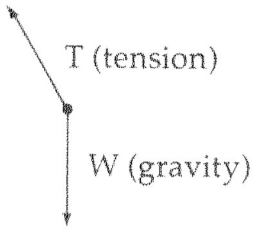
b. $\Sigma\tau = I\alpha$ (about center of mass) (one could also choose about the point at which the tape comes off the cylinder)
$TR = ½ MR^2 \times (a/R)$
$T = ½ Ma$

$\Sigma F = ma$
$Mg - T = Ma$
$Mg = 3Ma/2$
$a = 2g/3$

c. As there are no horizontal forces, the cylinder moves straight down.

1978M1

a.



b. $\Sigma F = ma$; $T \cos\theta = mg$ and $T \sin\theta = m\omega^2 r = m\omega^2(A + \ell \sin\theta)$

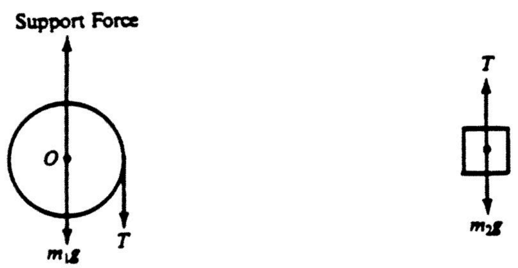
$$\omega = \sqrt{\frac{g\,tan\theta}{A + l\,sin\theta}}$$

c. $W = \Delta E = \Delta K + \Delta U = \frac{1}{2} mv^2 + mg\ell(1 - \cos\theta)$ for each rider

$W = 6(\frac{1}{2} mv^2 + mg\ell(1 - \cos\theta))$

1983M2

a.



b. i./ii. On the disk: $\Sigma\tau = I\alpha = TR = \frac{1}{2} m_1R^2\alpha$

For the block $a = \alpha R$ so $\alpha = a/R$ and $\Sigma F = m_2g - T = m_2a$

Solving yields

$$a = \frac{2m_2g}{2m_2 + m_1}\ and\ T = \frac{m_1m_2g}{2m_2 + m_1}$$

1985M3

a.

b. i. $\Sigma F = ma$; $T_1 - mg = ma$ and $2mg - T_2 = 2ma$
ii. $\Sigma\tau = I\alpha$; $(T_2 - T_1)r = I\alpha$

c. $\alpha = a/r$
Combining equations from b.i. gives $T_2 - T_1 = mg - 3ma$
Substituting for $(T_2 - T_1)$ into torque equation gives $a = 2g/9$

d. $T_1 = m(g + a) = 11mg/9$

e. $F_N = 7mg + T_1 + T_2$ (the table counters all the downward forces on the *apparatus*)
$T_2 = 2m(g - a) = 14mg/9$
$F_N = 88mg/9$

1988M3

a. I is proportional to mR^2; masses are equal and R becomes 2R
$I_{2R} = 4I$

b. The disks are coupled by the chain along their rims, which means the linear motion of the rims have the same displacement, velocity and acceleration.
$v_R = v_{2R}$; $R\omega_R = 2R\omega_{2R}$; $R\alpha t = 2R\alpha_{2R}t$ gives $\alpha_{2R} = \alpha/2$

c. $\tau_{2R} = T(2R) = I_{2R}\alpha_{2R} = (4I)(\alpha/2) = 2I\alpha$ giving $T = I\alpha/R$

d. $\Sigma\tau = \tau_{student} - TR = I\alpha$
$\tau_{student} = I\alpha + TR = I\alpha + (I\alpha/R)R = 2I\alpha$

e. $K = ½ I\omega^2 = ½ I(\alpha t)^2$

1989M2

a. $\Sigma F = ma$; $2Mg - T_v = 2Ma$ so $T_v = 2M(g - a)$

b. $\Sigma\tau = T_vR - T_hR = I\alpha = 3MR^2(a/R)$
$T_h = (T_vR - 3MRa)/R = 2M(g - a) - 3Ma = 2Mg - 5Ma$

c. $F_f = \mu F_N = \mu(4Mg)$
$T_h - F_f = 3Ma$
$2Mg - 5Ma - 4\mu Mg = 3Ma$
$4\mu Mg = 2Mg - 8Ma$
$\mu = (2g - 8a)/4g$
plugging in given values gives $\mu = 0.1$

d. $F_f = 4\mu Mg = ma_C = 4Ma_C$
$a_C = 1\ m/s^2$

1991M2

a. $\Sigma\tau = 0$; $m_2gr_2 = m_1gr_1$; $m_2 = m_1r_1/r_2 = 6.67$ kg

b./c. $\tau = I\alpha$; $Tr_1 = (45\ kg\text{-}m^2)\alpha$
$\Sigma F = ma$; $(20\ kg)g - T = (20\ kg)a$
Combining with $a = \alpha r$ gives $\alpha = 2\ rad/s^2$ and $T = 180$ N

d. $mgh = ½ mv^2 + ½ I\omega^2 = ½ mv^2 + ½ I(v^2/r^2)$ giving $v = 1.4$ m/s

1993M3

a. $\Sigma\tau = F_a\ell - Mg\ell/2 = 0$ giving $F_a = Mg/2$

b. $\Sigma\tau = Mg\ell/2 = I\alpha = (M\ell^2/3)\alpha$; $\alpha = 3g/2\ell$

c. $a = \alpha r$ where $r = \ell/2$
$a = (3g/2\ell)(\ell/2) = 3g/4$

d. $\Sigma F = Ma$; $Mg - F_a = Ma = M(3g/4)$
$F_a = Mg/4$

e. $\Delta U = \Delta K_{rot}$
$mgh = mg(\ell/2)\sin\theta = \frac{1}{2} I\omega^2 = \frac{1}{2}(M\ell^2/3)\omega^2$
solving gives $\omega = (3g\sin\theta/\ell)^{1/2}$

1999M3

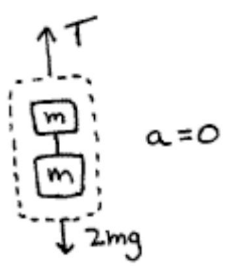
a. $\Sigma\tau = 0$ so $\tau_{cw} = \tau_{ccw}$ and $\tau_{cw} = TR$ (from the string) so we just need to find τ_{ccw} as the sum of the torques from the various parts of the system
$\Sigma\tau_{ccw} = \tau_{rod} + \tau_{block} = mgR\sin\theta_0 + 2mg(2R)\sin\theta_0 = 5mgR\sin\theta_0 = TR$ so $T = 5mg\sin\theta_0$

b. i. $I = I_{disk} + I_{rod} + I_{block} = 3mR^2/2 + 4mR^2/3 + 2m(2R)^2 = 65mR^2/6$
$\alpha = \tau/I = (5mgR\sin\theta_0)/(65mR^2/6) = 6g\sin\theta_0/13R$
ii. $a = \alpha r$ where $r = 2R$ so $a = 12g\sin\theta_0/13$

c. ΔU (from each component) $= K = \frac{1}{2} I\omega^2$
$mgR\cos\theta_0 + 2mg(2R)\cos\theta_0 = \frac{1}{2}(65mR^2/6)\omega^2$
$\omega = (12g\cos\theta_0/13R)^{1/2}$ and $v = \omega r = \omega(2R) = 4(3gR\cos\theta_0/13)^{1/2}$

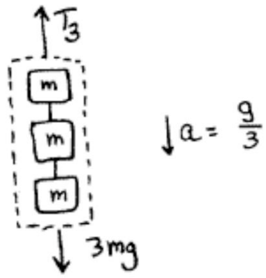
2000M3

a.



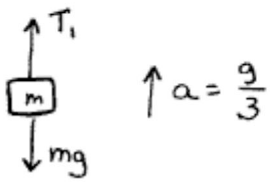
$\Sigma F = ma = 0$ so $T = 2mg$

b. i.



$\Sigma F = ma$
$3mg - T_3 = 3m(g/3)$
$T_3 = 2mg$

ii.



$\Sigma F = ma$
$T_1 - mg = m(g/3)$
$T_1 = 4mg/3$

iii. $\Sigma\tau = (T_3 - T_1)R_1 = I\alpha$ and $\alpha = a/R_1 = g/3R_1$
$(2mg - 4mg/3)R_1 = I_1(g/3R_1)$
$I_1 = 2mR_1^2$

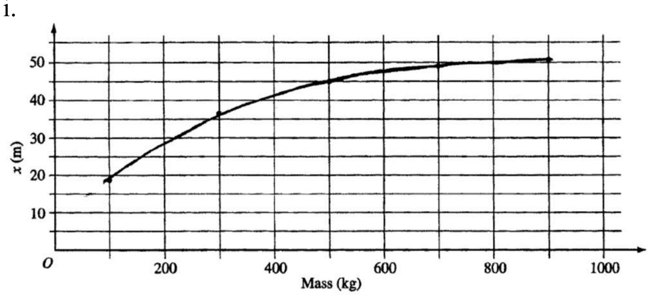
c. i. Tangential speeds are equal; $\omega_1 R_1 = \omega_2 R_2 = \omega_2(2R_1)$ therefore $\omega_2 = \omega_1/2$
ii. $L = I\omega = (16I_1)(\omega_1/2) = 8I_1\omega_1$
iii. $K = ½\, I_1\omega_1^2 + ½\, I_2\omega_2^2 = (5/2)I_1\omega_1^2$

2001M3

a. $I = \Sigma mr^2 = mL^2 + mL^2 = 2mL^2$
b. $\Sigma F = ma$; $4mg - T = 4ma$
$\Sigma\tau = I\alpha$; $Tr = I\alpha$; $T = I\alpha/r = 4mg - 4ma$ and $\alpha = a/r$, solving gives $a = 2gr^2/(L^2 + 2r^2)$
c. Equal, total energy is conserved
d. Less, the small blocks rise and gain potential energy. The total energy available is still 4mgD, therefore the kinetic energy must be less than in part c.

2003M3

a. i.



ii. x = 33 m

b. i. $y = ½ gt^2$; $t = (2y/g)^{1/2} = 1.75$ s
ii. $U_{initial} = U_{bucket} + U_{projectile} = M(9.8 \text{ m/s}^2)(3 \text{ m}) + (10 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) = 29.4M + 294$
iii. $U_{initial} = U_{final} + K$ where $U_{final} = Mg(1 \text{ m}) + (10 \text{ kg})g(15 \text{ m}) = 9.8M + 1470$
$K_{projectile} = ½\, 10v_x^2$ and $K_{bucket} = ½\, Mv_b^2$ where $v_b = v_x/6$
putting it all together gives $29.4M + 294 = 9.8M + 1470 + 5v_x^2 + (M/72)v_x^2$

$$v_x = \sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

c. i. $x = v_x t$

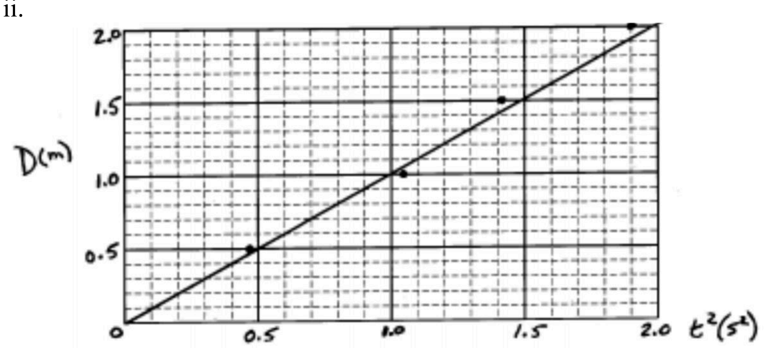
$$x = 1.75\sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

d. x(300 kg) = 39.7 m (greater than the experimental value)
possible reasons include friction at the pivot, air resistance, neglected masses not negligible

2004M2

a. $x = v_0t + ½ at^2$
$x = D$ and $v_0 = 0$ so $D = ½ at^2$ and $a = 2D/t^2$

b. i. graph D vs. t^2 (as an example)
ii.



iii. a = 2(slope) = 2.04 m/s²

c. $\Sigma\tau = TR = I\alpha$ and $\alpha = a/R$ so $I = TR^2/a$
$\Sigma F = mg - T = ma$ so $T = m(g - a)$
$I = m(g - a)R^2/a = mR^2((g/a) - 1)$

d. The string was wrapped around the pulley several times, causing the effective radius at which the torque acted to be larger than the radius of the pulley used in the calculation.

The string slipped on the pulley, allowing the block to accelerate faster than it would have otherwise, resulting in a smaller experimental moment of inertia.

Friction is not a correct answer, since the presence of friction would make the experimental value of the moment of inertia too large

SECTION C – Rolling

NOTE: Rolling problems may be solved considering rotation about the center of mass or the point of contact. The solutions below will only show one of the two methods, but for most, if not all cases, the other method is applicable. When considering rotation about the point of contact, remember to use the parallel axis theorem for the moment of inertia of the rolling object.

1974M2

a. Torque provided by friction; at minimum μ, $F_f = \mu F_N = \mu Mg \cos\theta$
$\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma = \mu Mg \cos\theta$ giving $a = (5/2)\mu g \cos\theta$
$\Sigma F = Ma$; $Mg \sin\theta - \mu Mg \cos\theta = Ma = (5/2)\mu Mg \cos\theta$ giving $\mu = (2/7)\tan\theta$

b. Energy at the bottom is the same in both cases, however with $\mu = 0$, there is no torque and no energy in rotation, which leaves more (all) energy in translation and velocity is higher

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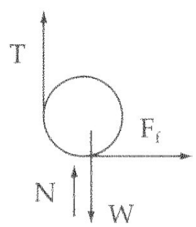
1977M2

a.



b. $\Sigma F_y = 0$; $T + N = W$; $N = W - T = Mg - (3/5)Mg = (2/5)Mg$
$\Sigma F_x = ma$; $F_f = ma$; $\mu N = ma$; $½ (2/5)Mg = Ma$; $a = g/5$

c. $\Sigma\tau = I\alpha$; $(T - F_f)R = ½ MR^2\alpha$
$(3/5)Mg - (1/5)Mg = ½ MR\alpha$
$(2/5)g = ½ R\alpha$
$\alpha = 4g/5R$

d. The cylinder is slipping on the surface and does not meet the condition for pure rolling

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1980M3

a. $\Sigma F = ma$; $F_f = \mu F_N$;
$-\mu Mg = Ma$
$a = -\mu g$
$v = v_0 + at$
$v = v_0 - \mu gt$

b. $\tau = I\alpha$ where the torque is provided by friction $F_f = \mu Mg$
$\mu MgR = (2MR^2/5)\alpha$
$\alpha = (5\mu g/2R)$
$\omega = \omega_0 + \alpha t = (5\mu g/2R)t$

c. Slipping stops when the tangential velocity si equal to the velocity of the center of mass, or the condition for pure rolling has been met: $v(t) = \omega(t)R$
$v_0 - \mu gt = R(5g/2R)t$, which gives $T = (2/7)(v_0/\mu g)$

d. Since the line of action of the frictional force passes through P, the net torque about point P is zero. Thus, the time rate of change of the angular momentum is zero and the angular momentum is constant.

1986M2

a. U = K

$Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$

i. $K_{trans} = ½ Mv^2 = (5/7)Mgh$

ii. $K_{rot} = ½ I\omega^2 = (2/7)Mgh$ (or $Mgh – K_{trans}$)

b. i. $\tau = F_fR = I\alpha = I(a/R)$

$F_fR = (2/5)MR^2(a/R)$

$F_f = (2/5)Ma$

$\Sigma F = ma$

$Mg \sin \theta – F_f = Ma$

$Mg \sin \theta – (2/5)Ma = Ma$

$g \sin \theta = (7/5) a$

$a = (5/7) g \sin \theta$

ii. $F_f = (2/5)Ma = (2/5)M(5/7)g \sin \theta = (2/7)Mg \sin \theta$

c. $K_{tot} = Mgh$

d. Greater, the moment of inertia of the hollow sphere is greater and will be moving slower at the bottom of the incline. Since the translational speed is less, the translational KE is taking a smaller share of the same total energy as the solid sphere.

1990M2

a. K = U

$½ mv_0^2 = mgH$; $H = v_0^2/2g$

b. $K + W_f = U$ where $W_f = –F_fd$ and $F_f = \mu mg \cos \theta$ and $d = h/\sin \theta$

$½ mv_0^2 – (\mu mg \cos \theta)(h/\sin \theta) = mgh$

$½ mv_0^2 = mgh(\mu \cot \theta + 1)$

$h = v_0^2/(2g(\mu \cot \theta + 1)) = H/(\mu \cot \theta + 1)$

c. $K_{trans} + K_{rot} = U$ where $K_{rot} = ½ I \omega^2 = ½ (mR^2)(v/R)^2 = ½ mv_0^2$

$½ mv_0^2 + ½ mv_0^2 = mgh'$

$h' = v_0^2/g = 2H$

d. Rotational energy will not change therefore $½ mv_0^2 = mgh''$ and $h'' = v_0^2/2g = H$

1994M2

a. $K_{tot} = K_{trans} + K_{rot} = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$K_{tot} = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10 = 1750$ J

b. i. $K_{total,bottom} = K_{top} + U_{top} = 7Mv_{top}^2/10 + Mgh$; $v_{top} = 7.56$ m/s

ii. It is directed parallel to the incline: 25º

c. $y = y_0 + v_{oy}t + ½ a_yt^2$

$0 \text{ m} = 3 \text{ m} + (7.56 \text{ m/s})(\sin 25º)t + ½ (–9.8 \text{ m/s}^2)t^2$ which gives t = 1.16 s (positive root)

$x = v_xt = (7.56 \text{ m/s})(\cos 25º)(1.16 \text{ s}) = 7.93$ m

d. The speed would be less than in b.

The gain in potential energy is entirely at the expense of the translational kinetic energy as there is no torque to slow the rotation.

1997M3

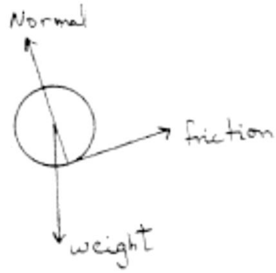
a. U = K

$MgH = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$MgH = ½ Mv^2 + ½ (½)MR^2(v/R)^2 = ½ Mv^2 + ¼ Mv^2 = 3Mv^2/4$

$v = (4gH/3)^{1/2}$

b.



c. For a change of pace, we can use kinematics:
$v_f^2 = v_i^2 + 2ad$
$4gH/3 = 0 + 2a(H/\sin\theta)$
$a = (2/3)g \sin\theta$

d. $\Sigma F = Ma$
$Mg \sin\theta - F_f = Ma = M(2/3)g \sin\theta$
$Mg \sin\theta - \mu Mg \cos\theta = (2/3)Mg \sin\theta$
$\mu \cos\theta = (1/3) \sin\theta$
$\mu = (1/3) \tan\theta$

e. i. The translational speed is greater, less energy is transferred to the rotational motion so more goes into the translational motion. Additionally, with a smaller frictional force, the translational acceleration is greater.
ii. Total kinetic energy is less. Energy is dissipated as heat due to friction.

2002M2

a. For each tire: $I = ½ ML^2 = ½ (m/4)r^2$
$I_{total} = 4 \times I = ½ mr^2$

b. U = K; total mass = 2m
$2mgh = ½ (2m)v^2 + ½ I\omega^2$ and $\omega = v/R$
$2mgh = mv^2 + ½ (½)mr^2(v/r)^2 = ½ mv^2 + (¼)mv^2 = 5mv^2/4$
$v = (8gh/5)^{1/2}$

c. $U_g = U_s$
$2mgh = ½ kx_m^2$; $x_m = 2(mgh/k)^{1/2}$

d. In an inelastic collision, energy is lost. With less energy after the collision, the spring is compressed less.

2006M3

a. $\Sigma\tau = I\alpha$
$F_fR = I\alpha = MR^2(a/R)$; $F_f = Ma$
$\Sigma F = ma$
$Mg \sin\theta - F_f = Ma$
$Mg \sin\theta - Ma = Ma$
$a = ½ g \sin\theta$

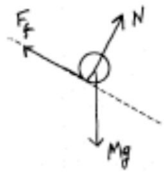
b. $v_f^2 = 2aL = gL \sin\theta$
$v_f = (gL \sin\theta)^{1/2}$

c. $H = ½ gt^2$; $t = (2H/g)^{1/2}$
$d = v_xt = (gL \sin\theta)^{1/2}(2H/g)^{1/2} = (2LH \sin\theta)^{1/2}$

d. Greater. A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

2010M2

a.



b. Torque provided by friction; $F_f = \mu F_N = \mu Mg \cos\theta$
$\tau = F_fR = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma$; $Ma = (5/2)F_f$
$\Sigma F = Ma$
$Mg \sin\theta - F_f = (5/2)F_f$
$F_f = (2/7)Mg \sin\theta = 8.4$ N

c. $Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$
$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$
$v^2 = 10gh/7$; $v = 5.3$ m/s

d. The horizontal speed of the wagon is due to the horizontal component of the ball in the collision:
$M_iv_{ix} = M_fv_{fx}$; where $M_f = M_{ball} + M_{wagon} = 18$ kg
$(6\text{ kg})(5.3\text{ m/s})(\cos 30°) = (18\text{ kg})v_f$
$v_f = 1.5$ m/s

SECTION D – Angular Momentum

1975M2

a. $L_i = L_f$
$m_0v_0R \sin\theta = I\omega$
$\omega = m_0v_0R \sin\theta/I$; $I = (M + m_0)R^2$
$\omega = m_0v_0 \sin\theta/(M + m_0)R$

b. $K_i = ½ m_0v_0^2$
$K_f = ½ I\omega^2 = ½ (M + m_0)R^2(m_0v_0 \sin\theta/(M + m_0)R)^2$
$K_f/K_i = m_0 \sin^2\theta/(M + m_0)$

1978M2

a. $L_i = L_f$
$I\omega = mvr$
$(1/3)M_1\ell^2\omega = M_2v\ell$
$v = M_1\ell\omega/3M_2$

b. $p_{system} = p_{cm\ of\ rod} = M_1v_{cm} = M_1\omega(\ell/2)$

c. $P_f = M_2v_f = M_1\omega\ell/3M_2$

d. There is a net external force on the system from the axis at point P.

e. Since the net external force acts at point P (the pivot), the net torque about point P is zero, hence angular momentum is conserved.

1981M3

a. $m_2v = m_2(-v/2) + M_1v'$
$v' = 3m_2v/2M_1$

b. $L_i = L_f$
$m_2v(L/3) = m_2(-v/2)(L/3) + (1/12)M_1L^2\omega$
$\omega = 6m_2v/M_1L$

c. $\Delta K = K_f - K_i = \frac{1}{2} m_2(-v/2)^2 + \frac{1}{2} M_1 v'^2 + \frac{1}{2} I\omega^2 - \frac{1}{2} m_2 v^2$
$= -3m_2v^2/8 + 21m_2^2v^2/8M_1$

<u>1982M3</u>

a. $L = I\omega$ where $I = \Sigma mr^2 = (2m)\ell^2 + m(2\ell)^2 = 6m\ell^2$
$L = 6m\ell^2\omega$

b. $F_f = \mu mg$
$\Sigma\tau = -(\mu(2m)g\ell + \mu mg(2\ell)) = -4\mu mg\ell$

c. $\alpha = \tau/I = -4\mu mg\ell/6m\ell^2 = -2\mu g/3\ell$
$\omega = \omega_0 + \alpha t$; setting $\omega = 0$ and solving for T gives $T = 3\omega_0\ell/2\mu g$

<u>1987M3</u>

a. $K = U$
$\frac{1}{2} I\omega^2 = mgh_{cm}$
$\frac{1}{2} (m\ell^2/3)\omega^2 = mg(\ell/2)$ which gives $\omega = 5$ rad/s

b. $K_i = K_f$
$\frac{1}{2} m_0v_0^2 = \frac{1}{2} m_0v^2 + \frac{1}{2} I\omega^2$
$v = 8$ m/s

c. $L = mvr = (1\text{ kg})(10\text{ m/s})(1.2\text{ m}) = 12\text{ kg-m}^2/\text{s}$

d. $L_i = L_f$
$12\text{ kg-m}^2/\text{s} = m_0(v_\perp)\ell + I\omega = m_0(v\cos\theta)\ell + I\omega$
$\theta = 60^\circ$

<u>1992M2</u>

a. $\Sigma\tau = (3M + M)g\ell - Mg\ell = 3Mg\ell$

b. $I = \Sigma mr^2 = 4M\ell^2 + M\ell^2 = 5m\ell^2$
$\alpha = \tau/I = 3Mg\ell/5m\ell^2 = 3g/5\ell$

c. $\Delta U_{bug} + \Delta U_{left\ sphere} + \Delta U_{right\ sphere} = \Delta K_{rot}$
since $\Delta U_{left\ sphere} = -\Delta U_{right\ sphere}$, we only need to consider ΔU_{bug}
$3Mg\ell = \frac{1}{2} I\omega^2 = \frac{1}{2} (5M\ell^2)\omega^2$
$\omega = (6g/5\ell)^{1/2}$

d. $L = I\omega = 5M\ell^2(6g/5\ell)^{1/2} = (30M^2g\ell^3)^{1/2}$

e. Let T be the force we are looking for
$\Sigma F = ma_c$
$T - 3Mg = M\omega^2\ell$
$T = 3Mg + 3M(6g/5\ell)\ell = 33Mg/5$

<u>1996M3</u>

a.

$$I = \quad r^2 dm$$

$$dm = \frac{M}{l}dr$$

$$I = {}_{-\frac{l}{2}}^{\frac{l}{2}} \frac{M}{l} r^2 dr$$

$$I = \frac{M}{l}\frac{r^3}{3}\Big|_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

b. $I = \Sigma I = M\ell^2/12 + M(\ell/2)^2 = M\ell^2/3$

c./d./e.

$\Sigma F = ma$

for cat: $Mg - T = Ma$

$\Sigma\tau = I\alpha$ where $\alpha = a/r = a/(\ell/2)$

for hoop: $T\ell/2 = (M\ell^2/3)(a/(\ell/2))$ which gives $a = 3T/4M$

substituting gives $Mg - T = 3T/4$

$T = 4Mg/7$

$\alpha = T\ell/2I = 6g/7\ell$

$a = \alpha(\ell/2) = 3g/7$

f. $L = Mv(\ell/2)$ where v is found from $v^2 = v_0^2 + 2aH = 2(3g/7)(5\ell/3) = 10g\ell/7$

$L = ½ M\ell(10g\ell/7)^{1/2}$

1998M2

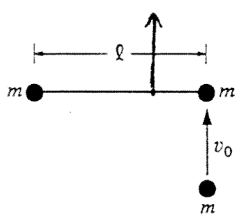
a. i. $mv_0 = (3m)v_f$; $v_f = v_0/3$

$K_f = ½ (3m)(v_0/3)^2 = mv_0^2/6$

ii. $\Delta K = K_f - K_i = mv_0^2/6 - ½ mv_0^2 = -mv_0^2/3$

b. i. $r_{cm} = \Sigma m_i r_i/\Sigma m = m(0) + 2m(\ell)/(m + 2m) = (2/3)\ell$

ii.



iii. $p_i = p_f$

$mv_0 = (3m)v_f$; $v_f = v_0/3$

iv. $L_i = L_f$

$mv_0R \sin\theta = mv_0(\ell/3) = I\omega$ where $I = \Sigma mr^2 = m(2\ell/3)^2 + 2m(\ell/3)^2 = (2/3)m\ell^2$

solving yields $\omega = v_0/2\ell$

v. $K_i = ½ mv_0^2$

$K_f = ½ mv_f^2 + ½ I\omega^2 = ½ (3m)(v_0/3)^2 + ½ (2/3)m\ell^2(v_0/2\ell)^2 = ¼ mv_0^2$

$\Delta K = -¼ mv_0^2$

2005M3

a. $L = I\omega = (1/3)M_1d^2\omega$

b. $L_f = L_i$

$M_2vd = (1/3)M_1d^2\omega$

$v = M_1d\omega/3M_2$

c. $K_f = K_i$

$½ M_2v^2 = ½ I\omega^2$

$M_2v^2 = I\omega^2$

$M_2(M_1d\omega/3M_2) = (1/3)M_1d^2\omega^2$ $\quad$ $M_2(1/9)(M_1/M_2)^2d^2\omega^2 = (1/3)M_1d^2\omega^2$ $\quad$ $(1/9)(M_1^2/M_2) = M_1/3$

$M_1/M_2 = 3$

d. $L_f = L_i$

$M_1vx = (1/3)M_1d^2\omega$

$v = d^2\omega/3x$

$½ M_1v^2 = ½ I\omega^2$ $\quad$ $M_1v^2 = I\omega^2 = (1/3)M_1d^2\omega^2$

$v^2 = d^2\omega^2/3$ $\quad$ substituting from above $\quad$ $(d^2\omega/3x)^2 = d^2\omega^2/3$

solving for x gives $x = d/\sqrt{3}$