$\ln(1+x) \sim x \Rightarrow$ 凑个 $1+x$

幂指函数

$x\to 0$ $4^x\to 1$ $9^x \to 1$ $\quad \frac{4^x+9^x}{2} \to 1$

$$\lim_{x\to 0}\left(\frac{4^x+9^x}{2}\right)^{\frac{1}{x}} = 1 \qquad \frac{\left(1+\left(\frac{9}{4}\right)^x\right)^{\frac{1}{x}}}{\left(2^{1-2x}\right)^{\frac{1}{x}}} = \frac{\left[1+\left(\frac{9}{4}\right)^x\right]^{\frac{1}{\left(\frac{9}{4}\right)^x}\cdot\left(\frac{9}{4}\right)^x\cdot\frac{1}{x}}}{2^{\frac{1}{x}-2}} \quad \frac{e^{\frac{1}{x}\left(\frac{9}{4}\right)^x}}{2^{\frac{1}{x}-2}} =$$

$$= \lim_{x\to 0} e^{\ln\left(\frac{4^x+9^x}{2}\right)^{\frac{1}{x}}} = \lim_{x\to 0} e^{\frac{1}{x}\ln\left(\frac{4^x+9^x}{2}\right)} = e^{\lim_{x\to 0}\frac{1}{x}\ln\left(\frac{4^x+9^x}{2}\right)} = e^{\lim_{x\to 0}\frac{1}{x}\ln\left(1+\underbrace{\frac{4^x+9^x}{2}-1}\right)}$$

$\frac{4^x+9^x-2}{2} \Rightarrow$ 必须确保该式为无穷小

$$= e^{\lim_{x\to 0}\frac{1}{x}\ln\left(1+\frac{4^x+9^x-2}{2}\right)} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{4^x+9^x-2}{2}\right)} = \lim_{x\to 0}\frac{4^x-1}{2x} + \lim_{x\to 0}\frac{9^x-1}{2x}$$

$$= \frac{\ln 4}{2} + \frac{\ln 9}{2} = e^{\ln 6} = 6$$

$(\sqrt{1+x})' = x$

实际上正确的写法为

$\ln(1+x+x^2) \sim x+x^2+o(x+x^2) \Rightarrow$

$\ln(1-x+x^2) \sim -x+x^2+o(-x+x^2)$

$\because \frac{o(x+x^2)}{x} = \lim_{x\to 0}\frac{x+x^2}{x} = \lim_{x\to 0}(1+x) = 1$

$o(x+x^2)$ 为 $o(x)$ 的同阶无穷小 可替换

但 $\frac{o(x+x^2)}{x^2} = \lim_{x\to 0}\frac{x+x^2}{x^2} = \lim_{x\to 0}\frac{1}{x}+1 = \infty$ 低阶无穷小

$\frac{o(-x+x^2)}{x} = -1$ 同阶无穷小，也可替换，最后将 $o(x)$ 消去。 不可替换。

$\frac{o(x)}{x^2}$ 极限无法算

不能 $\sim x+x^2 \Rightarrow$ 应该为 $x+x^2+o(x^2) \Rightarrow -x+x^2+o(x^2)$?

$$\lim_{x\to 0}\frac{\ln(1+x+x^2)+\ln(1-x+x^2)}{\sec x-\cos x} = \frac{\ln[(1+x+x^2)(1-x+x^2)]}{} \Rightarrow$$ 依然变换成 $\ln[1+f(x)]$ 验证 $f(x)$ 是否为无穷小后，可用等价无穷小替换

$= \ln[(x^2+1)^2-x^2]$

$\ln(1-\cancel{x}+x^2+\cancel{x}-\cancel{x^2}+\cancel{x^3}+x^2-x^3+x^4)$

$\ln(1+x^2+x^4) \sim x^2+x^4$

$x^2+x^4 \to 0$

类比 $\tan x-\sin x \sim \frac{1}{2}x^3$

$$\frac{1}{\cos x}-\cos x = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \frac{x^2}{\cos x \to 1} = x^2$$

$$= \lim_{x\to 0}\frac{x^2+x^4}{x^2} = x^2+1.$$

$$= \frac{\sin^2 x}{\cos x} \sim \frac{x^2}{\cos x}$$

$1-\cos^2 x \sim x^2$

$= \sin^2 x$

$\frac{\sin^2 x}{e^x}$ 与 $\frac{x^2}{e^{x}}$ 同样收敛到0.

$$\lim_{x\to 0}\frac{\ln(\sin^2 x+e^x)-x}{\ln(x^2+e^{2x})-2x} = \frac{\ln(\sin^2 x+e^x)-\ln e^x}{\ln(x^2+e^{2x})-\ln e^{2x}} = \frac{\ln\left(\frac{\sin^2 x+e^x}{e^x}\right)}{\ln\left(\frac{x^2+e^{2x}}{e^{2x}}\right)} = \frac{\frac{\sin^2 x}{e^x}}{\frac{x^2}{e^{2x}}} = e^x = 1$$

产生"1"的原因

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