Macroscopic Heat Transfer Model for Additive Manufacturing Testbed Problem

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1 Macroscopic model

We model the macroscopic heat transfer in the rectangular domain Ω

$$\frac{\partial T}{\partial t} = \nabla \cdot (K \nabla T) + L_m \frac{\partial f_l(T)}{\partial t} \quad \text{in } \Omega \tag{1}$$

with boundary conditions

$$(-K\nabla T) \cdot \hat{n} = q_s + h(T - T_e) + \epsilon\sigma(T^4 - T_e^4) \quad \text{on } \Gamma_{top} \tag{2}$$

$$(-K\nabla T) \cdot \hat{n} = 0 \quad \text{on } \Gamma \backslash \Gamma_{top} \tag{3}$$

where K is the heat conductivity, L_m is the latent heat, h is the convective heat transfer coefficient, ϵ is the thermal radiation coefficient, σ is the Stefan-Boltzmann constant, and L is the latent heat. The fluid mass fraction f_l is modeled as

$$f_l(T) = \begin{cases} 1 & T > T_l \\ \frac{T - T_s}{T_l - T_s} & T_s \leq T \leq T_l \\ 0 & T < T_s \end{cases} \tag{4}$$

where T_l and T_s are the liquidus and solidus temperature, respectively. The heat source q_S is modeled as a moving Gaussian

$$q_s(x, t) = \frac{2Q\eta}{\pi r_b^2} \exp\left(-\frac{2(x - V_s t)^2}{r_b^2}\right), \tag{5}$$

where Q is the source of heat power, η is the absorption coefficient, r_b is the radius of heat source and V_s is the scanning speed.

2 Discretization

We discretize the domain Ω using $(N + 1) \times (M + 1)$ grid with meshwidth h. Let $T_{ij}^n = T(ih, jh, t_n)$ for $i = 0, \dots, N$ and $j = 0, \dots, M$.

$$\frac{T_{i,j}^n - T_{i,j}^{n-1}}{\Delta t} = K \frac{T_{i,j+1}^n + T_{i,j-1}^n + T_{i-1,j}^n + T_{i+1,j}^n - 4T_{i,j}^n}{h^2} + L_m \frac{\partial f_l(T)}{\partial t}\bigg|_{t=t_n} \tag{6}$$

At the top boundary ($j = M$), we treat the convection and radiation term explicitly,

$$-K\frac{T^n_{i,M+1} - T^n_{i,M-1}}{2h} = q_s(ih, t_n) + h(T^{n-1}_{i,M} - T_e) + \epsilon\sigma((T^{n-1}_{i,M})^4 - T_e^4), \ i = 0, \dots, N \quad (7)$$

Bao: 1. Can you fill the details of the other boundaries? Basically, no flux BCs.

Bao: 2. How do we treat the latent heat term? Implicit or explicit?

Eventually, the linear system of equation should look like

$$(\mathbf{I} - \alpha\mathbf{L})\mathbf{T}^n - \mathbf{N}(\mathbf{T}^n) = \mathbf{T}^{n-1} + \text{BC terms} \quad (8)$$

where $\mathbf{L}$ is the discrete 2D laplacian with Neumann BCs and $\mathbf{N}$ is a nonlinear function due the implicit treatment of latent heat term. If we treat the latent heat term explicitly, we need two starting values initially.

Bao: 3. Can you find out what L, N and BC terms are?

Bao: 4. I am afraid we have to solve nonlinear system of equation due to the latent heat term.