

MATH-UA 263 Partial Differential Equations

Recitation Summary

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Topics: verifying solution to a PDE, dispersion relations, well-posedness, general solution via integration.

1. (EPDE, Exercise 1.6) Verify the general solution to the heat equation on the real line:

$$\begin{cases} u_t = ku_{xx}, & -\infty < x < \infty, t > 0 \\ u(x, 0) = g(x) \end{cases}$$

is given by

$$u(x, t) = \frac{1}{\sqrt{4\pi kt}} \int_{-\infty}^{\infty} e^{-(x-s)^2/4kt} g(s) ds. \quad (1)$$

2. (PDE p.51-52) Solve the heat equation with the initial condition $u(x, 0) = g(x) = e^{-x}$. To do so, use (1) and the integral identity $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx = 1$ to show that the solution is

$$u(x, t) = e^{kt-x}. \quad (2)$$

3. Find the dispersion relation of the linear PDE: $u_t = -u - \delta u_{xx} - u_{xxxx}$, $\delta > 0$.

4. (Well-posedness) The solution of the PDE

$$\begin{cases} v_{tt} + v_{xx} = 0 \\ v(x, 0) = 0 \\ v_t(x, 0) = \epsilon \sin(\frac{x}{\epsilon}) \end{cases} \quad (3)$$

is given by

$$v(x, t) = \epsilon^2 \sin(\frac{x}{\epsilon}) \sinh(\frac{t}{\epsilon}).$$

The solution of the PDE

$$\begin{cases} u_{tt} - u_{xx} = 0 \\ u(x, 0) = 0 \\ u_t(x, 0) = \epsilon \sin(\frac{x}{\epsilon}) \end{cases} \quad (4)$$

is given by

$$u(x, t) = \epsilon^2 \sin\left(\frac{x}{\epsilon}\right) \sin\left(\frac{t}{\epsilon}\right).$$

Argue that (3) is an ill-posed problem, but (4) is well-posed.

5. Find the general solution of the PDE: $u_{xt} + 3u_x = 1$.

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