MATH-UA 263 Partial Differential Equations
Recitation Summary

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Topics: verifying solution to a PDE, dispersion relations, well-posedness, general solution via integration.

1. (EPDE, Exercise 1.6) Verify the general solution to the heat equation on the real line:
$$\begin{cases} u_t = ku_{xx}, \quad -\infty < x < \infty,\, t > 0 \\ u(x,0) = g(x) \end{cases}$$
is given by
$$u(x,t) = \frac{1}{\sqrt{4\pi kt}} \int_{-\infty}^{\infty} e^{-(x-s)^2/4kt} g(s)\, ds. \tag{1}$$

2. (PDE p.51-52) Solve the heat equation with the initial condition $u(x,0) = g(x) = e^{-x}$. To do so, use (1) and the integral identity $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = 1$ to show that the solution is
$$u(x,t) = e^{kt-x}. \tag{2}$$

3. Find the dispersion relation of the linear PDE: $u_t = -u - \delta u_{xx} - u_{xxxx}$, $\delta > 0$.

4. (Well-posedness) It is straightforward to check that the solution of the PDE
$$\begin{cases} v_{tt} + v_{xx} = 0 \\ v(x,0) = 0 \\ v_t(x,0) = \epsilon \sin(\frac{x}{\epsilon}) \end{cases} \tag{3}$$
is given by
$$v(x,t) = \epsilon^2 \sin(\frac{x}{\epsilon}) \sinh(\frac{t}{\epsilon}),$$

and the solution of the PDE

$$\begin{cases} u_{tt} - u_{xx} = 0 \\ u(x,0) = 0 \\ u_t(x,0) = \epsilon \sin(\frac{x}{\epsilon}) \end{cases} \tag{4}$$

is given by

$$u(x,t) = \epsilon^2 \sin(\frac{x}{\epsilon}) \sin(\frac{t}{\epsilon}).$$

Assum ϵ is small, use "stability with respect to initial data" to argue that (3) is an ill-posed problem, but (4) is well-posed.

5. Find the general solution of the PDE: $u_{xt} + 3u_x = 1$.

2 February 9, 2018