Reinforcement Learning

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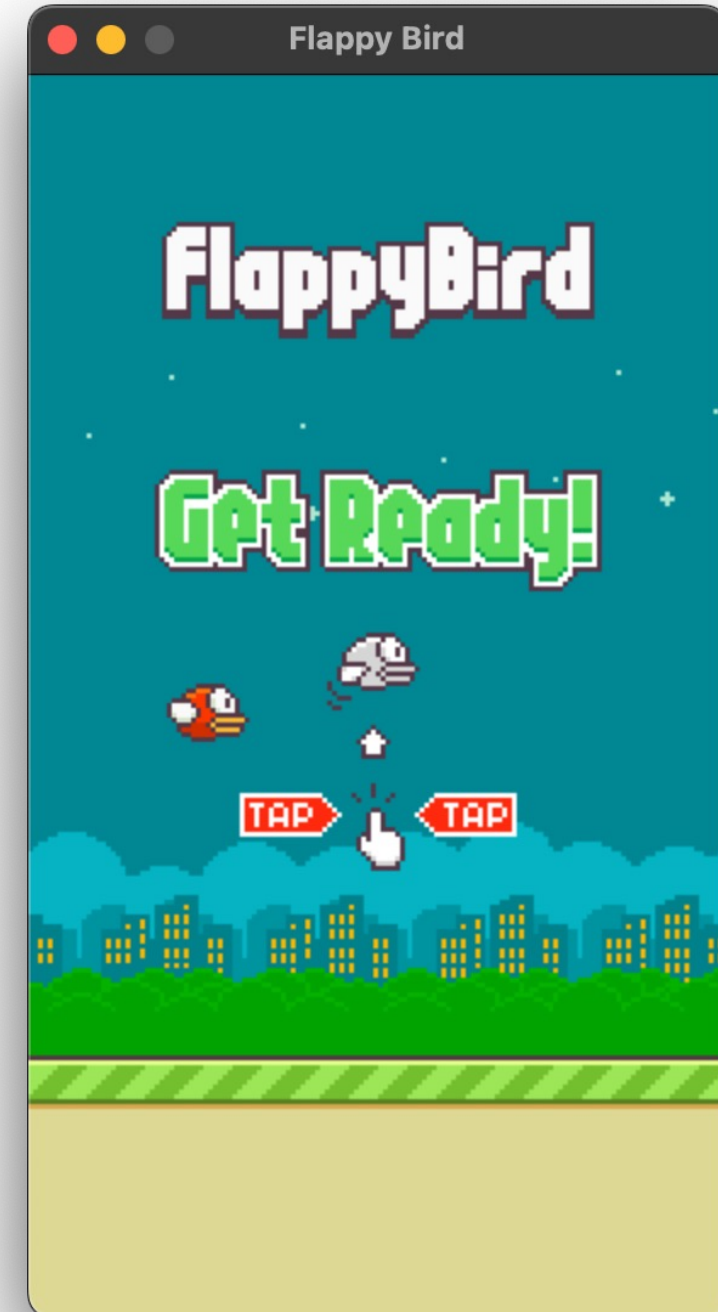
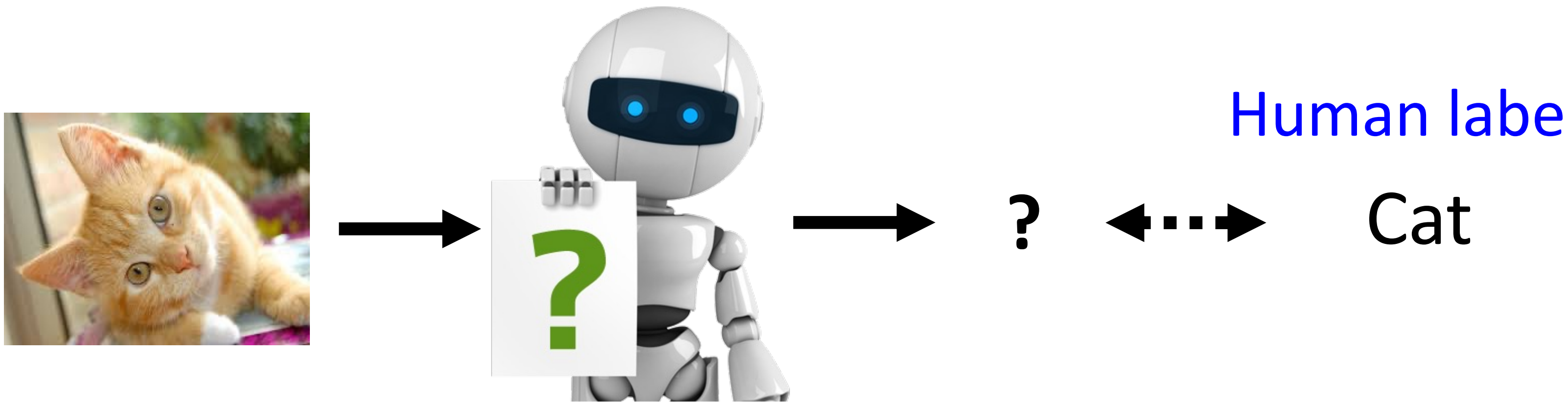
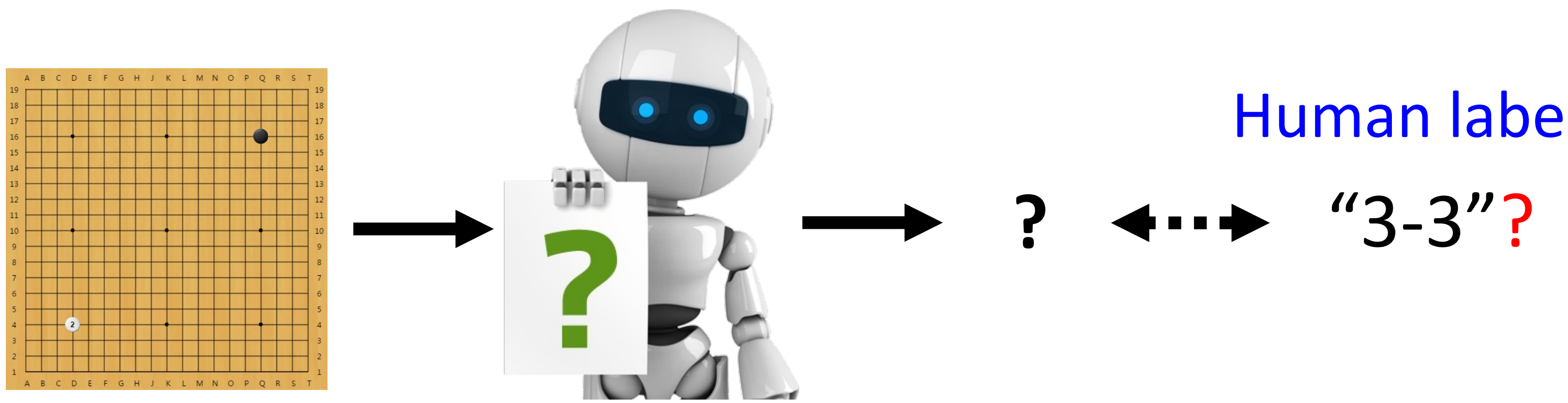
Outline

- FlappyBird Competition
- Introducing RL
- Q-learning



Supervised Learning → RL

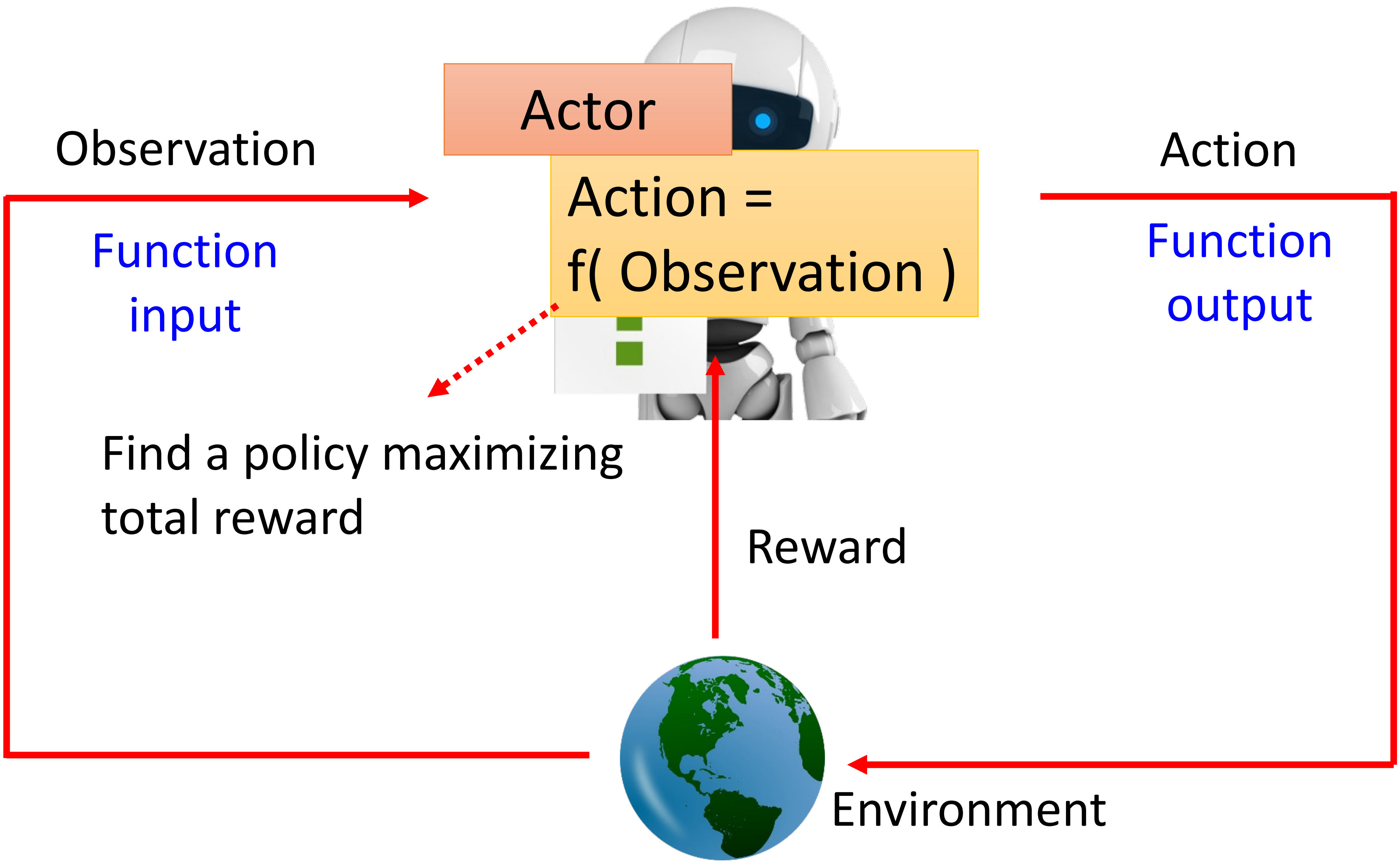




It is challenging to label data in some tasks.

…… machine can know the results are good or not.

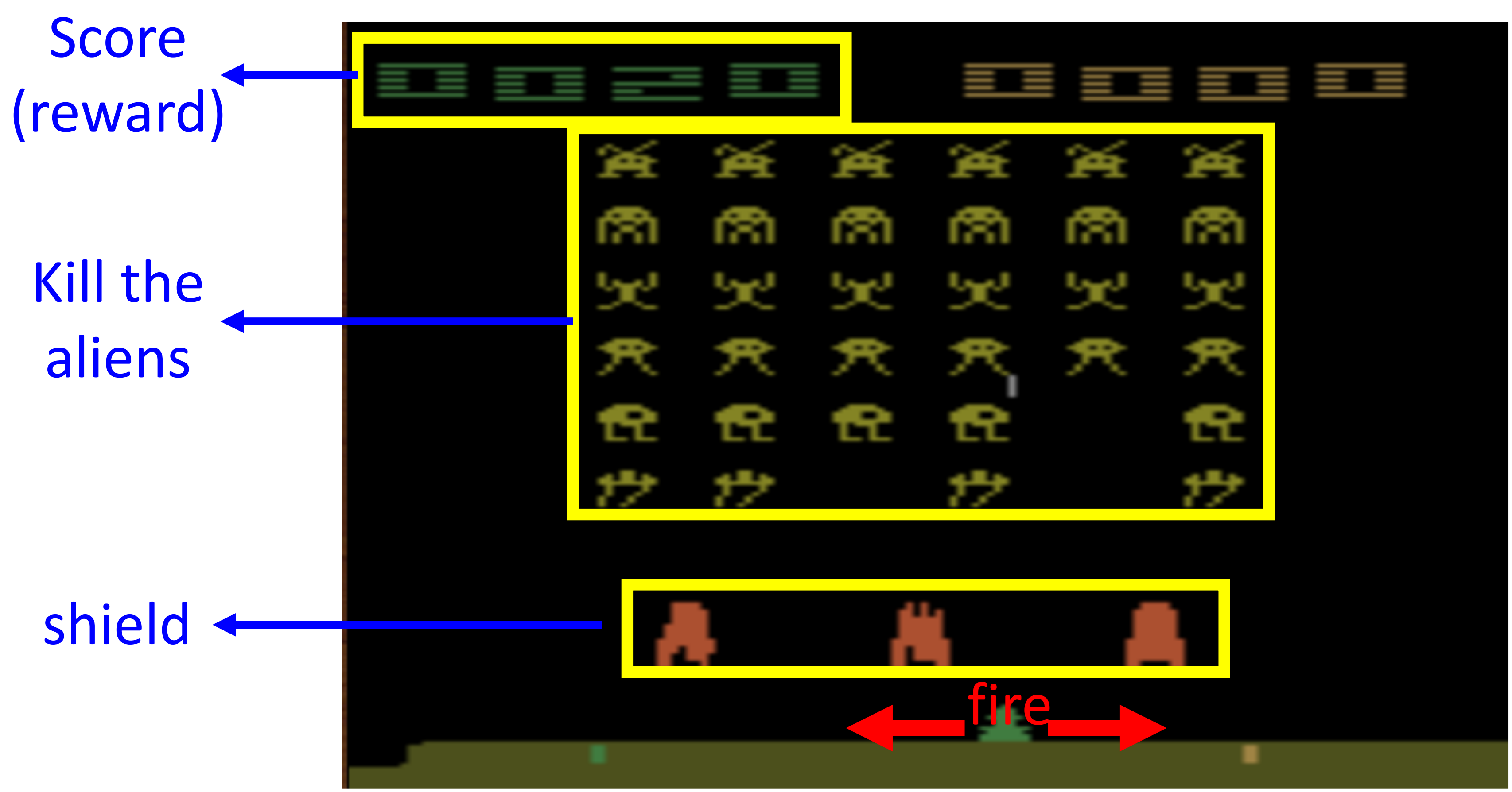
Machine Learning ≈ Looking for a Function



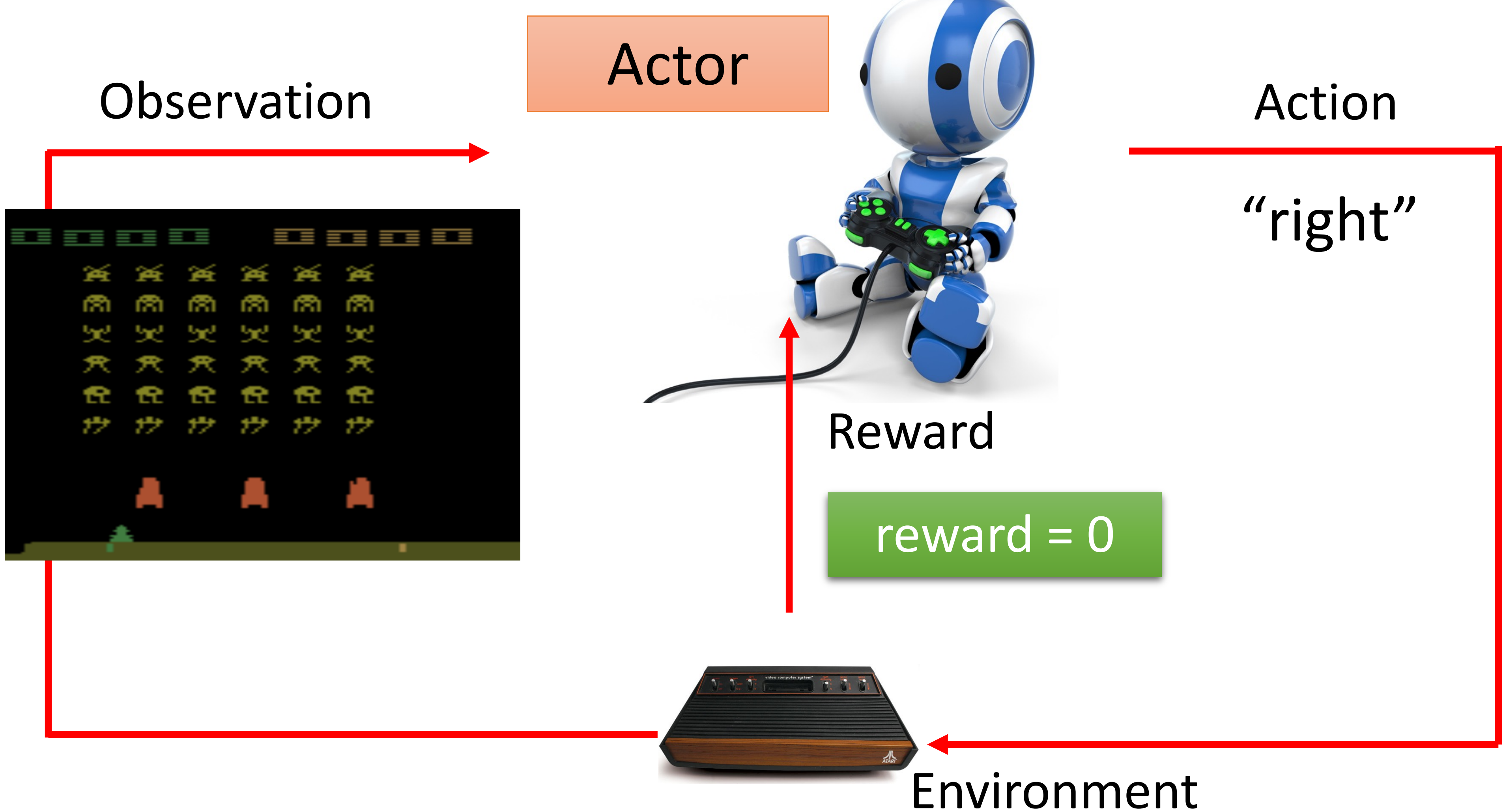
Example: Playing Video Game

- Space invader

Termination: all the aliens are killed, or your spaceship is destroyed.

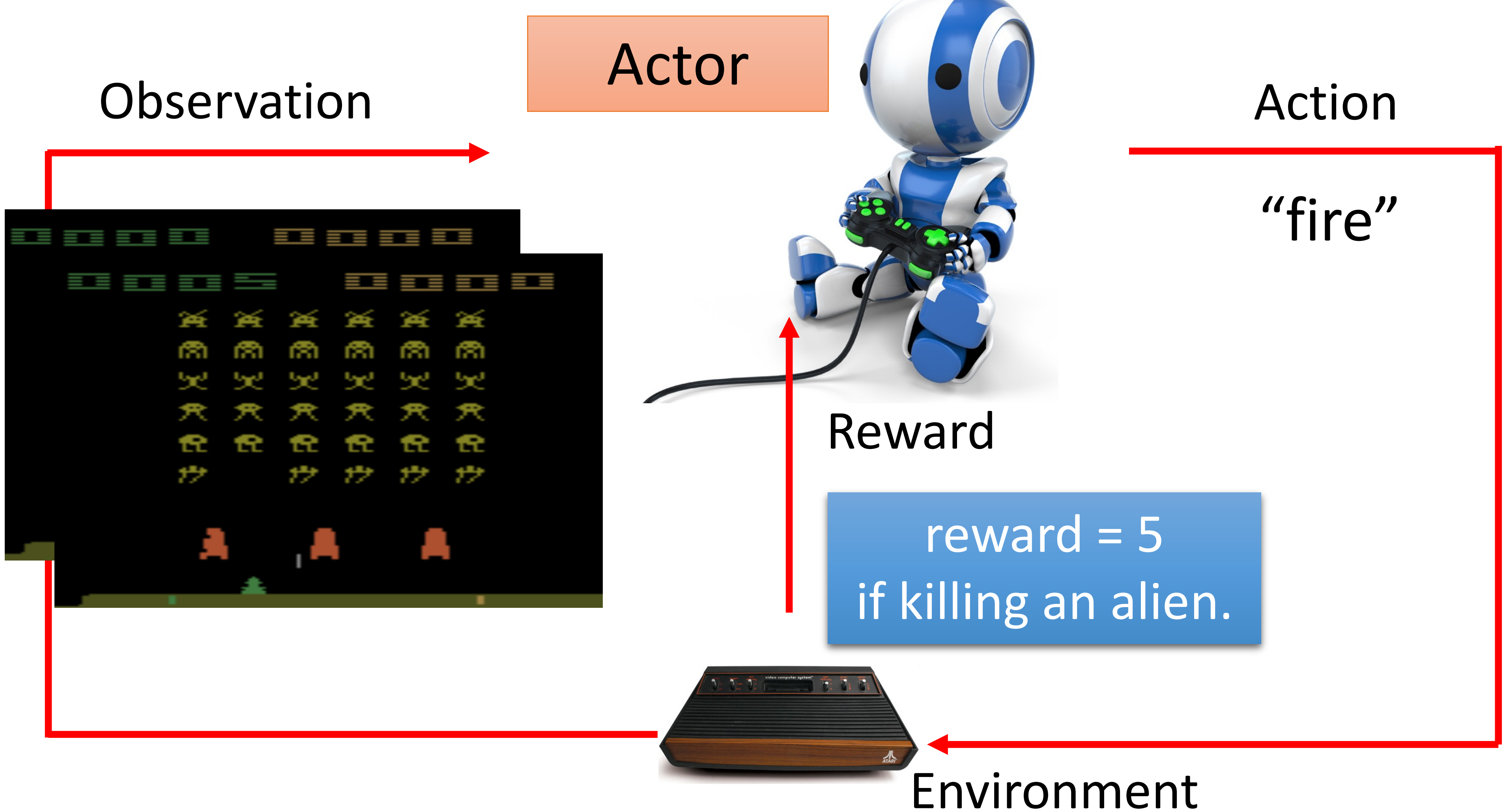


Example: Playing Video Game

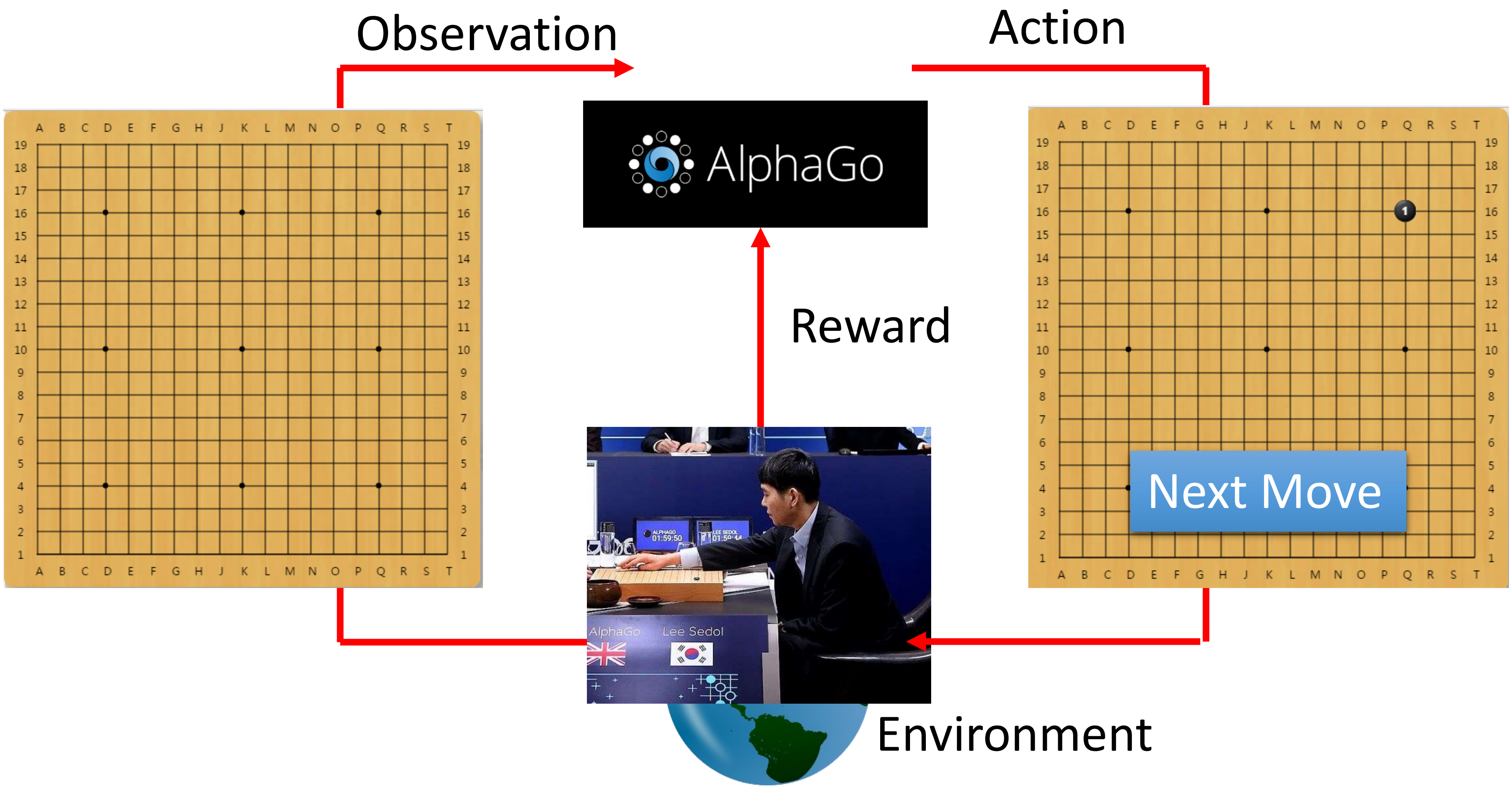


Example: Playing Video Game

Find an actor maximizing expected reward.

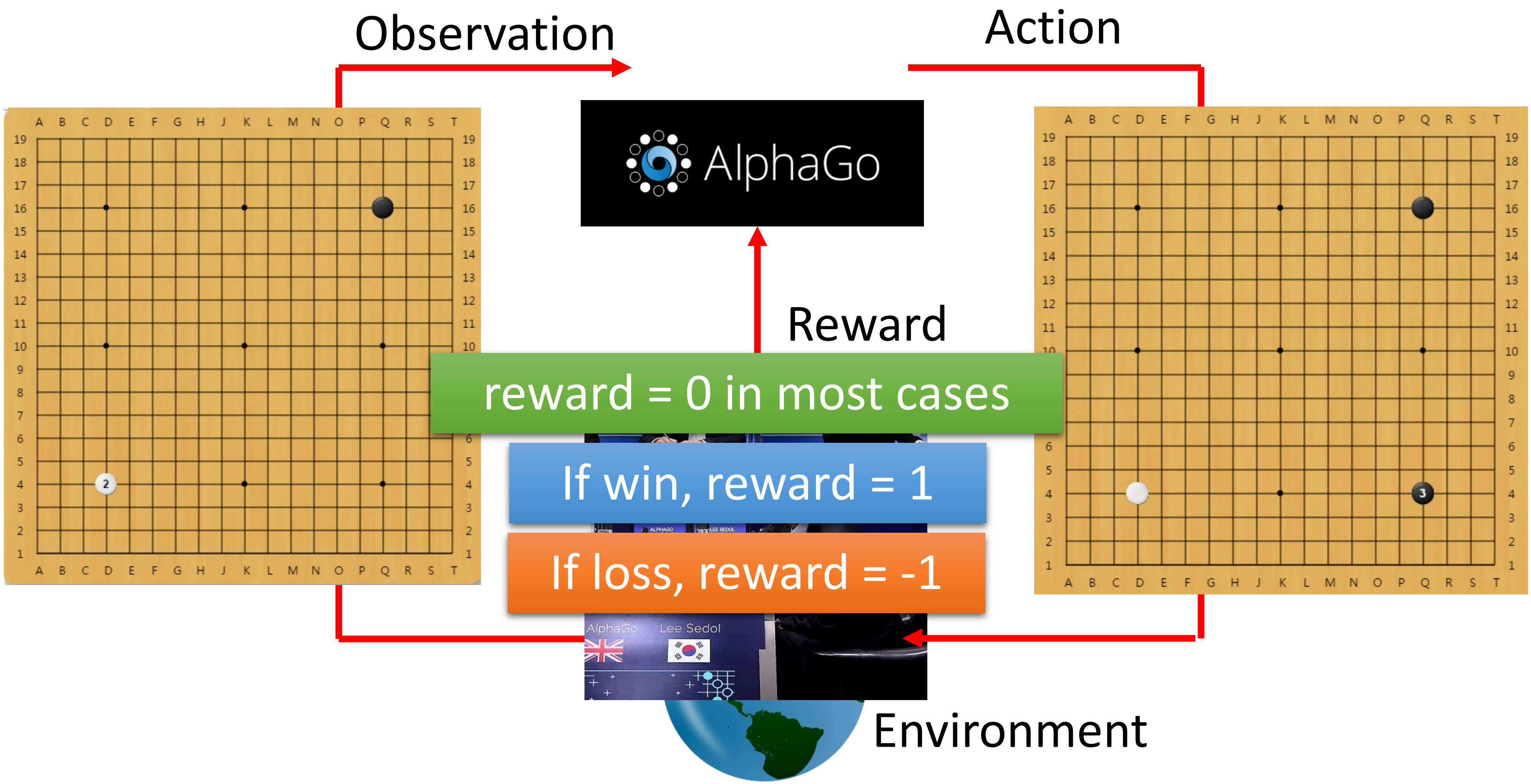


Example: Learning to play Go



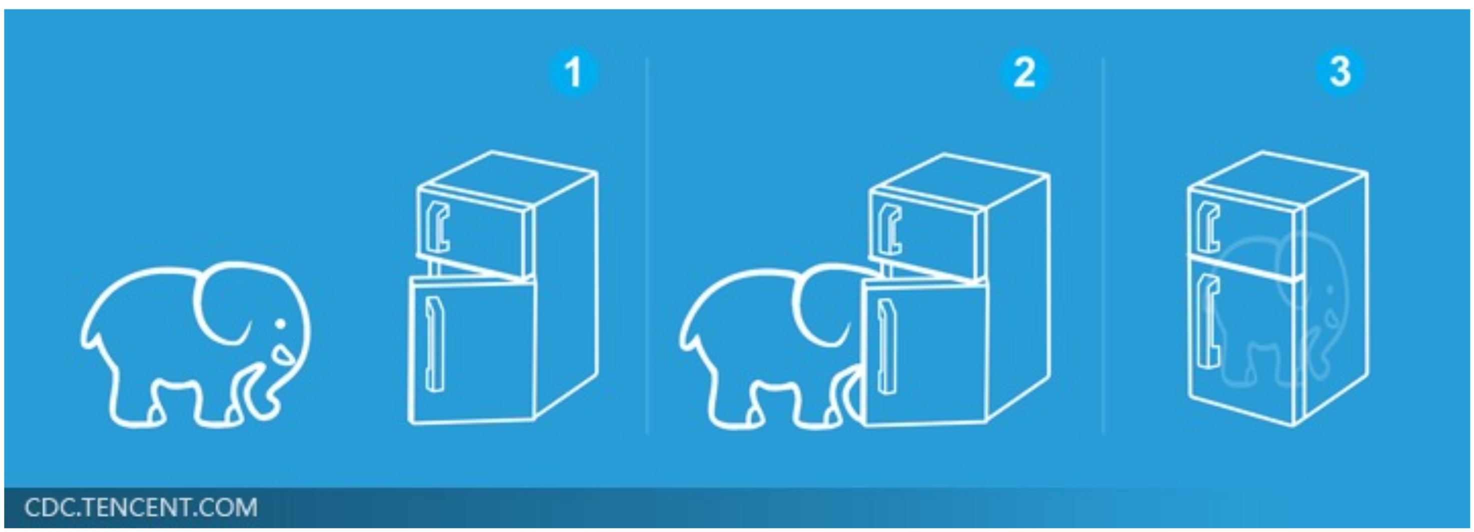
Example: Learning to play Go

Find an actor maximizing expected reward.

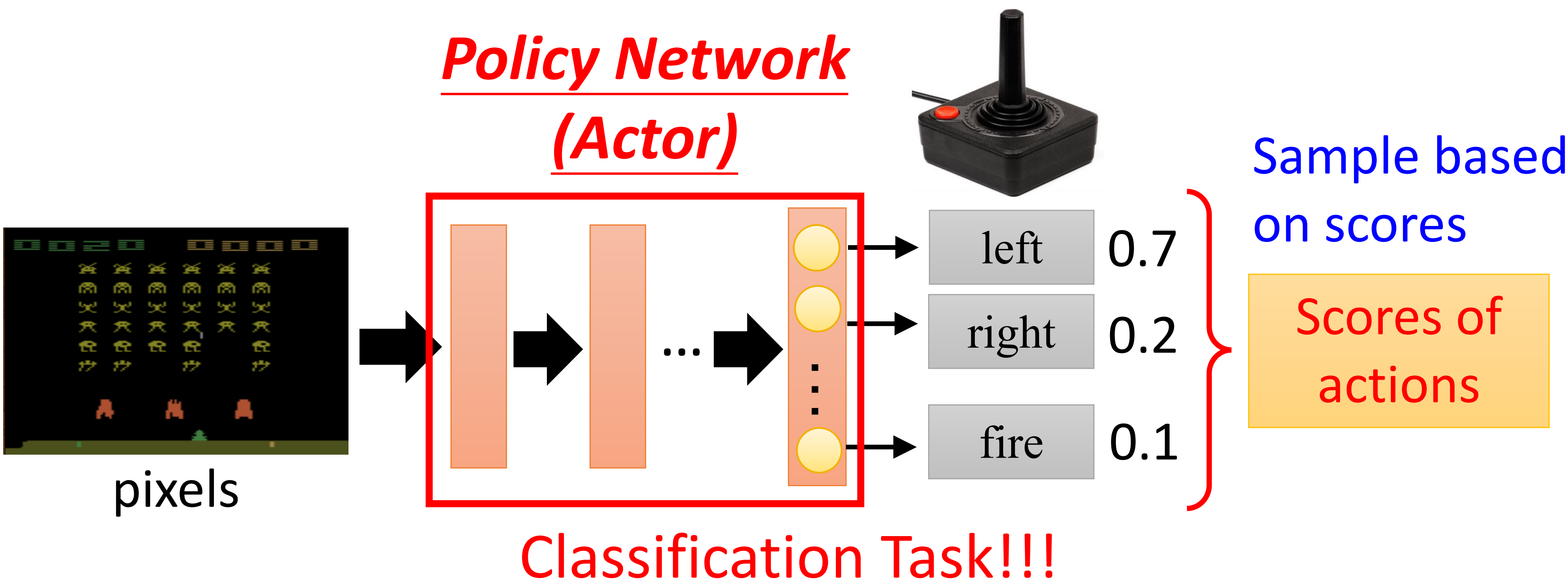


Machine Learning is so simple ……

Step 1: function with unknown → Step 2: define loss from training data → Step 3: optimization

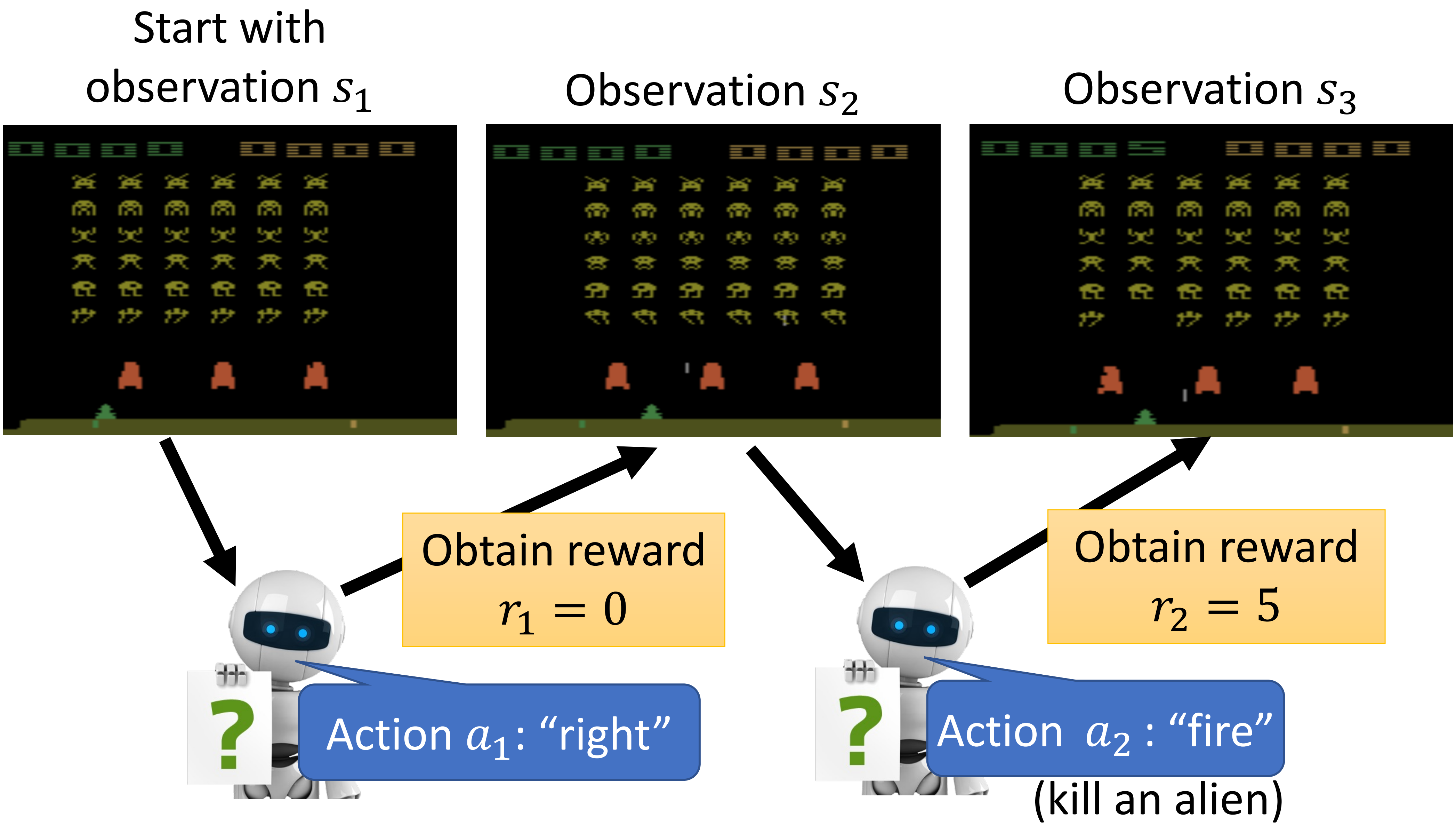


Step 1: Function with Unknown



- Input of neural network: the observation of machine represented as a vector or a matrix
- Output neural network : each action corresponds to a neuron in output layer

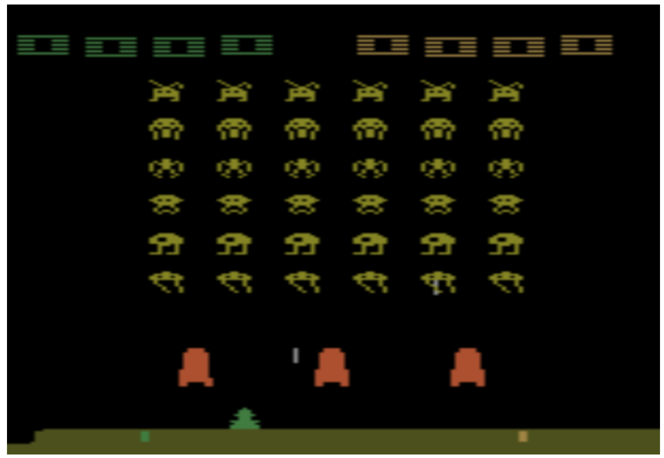
Step 2: Define "Loss"



Step 2: Define "Loss"

Start with observation s_1

Observation s_2



Observation s_3

After many turns

Game Over
(spaceship destroyed)

Obtain reward r_T

Action a_T

This is an ***episode***.

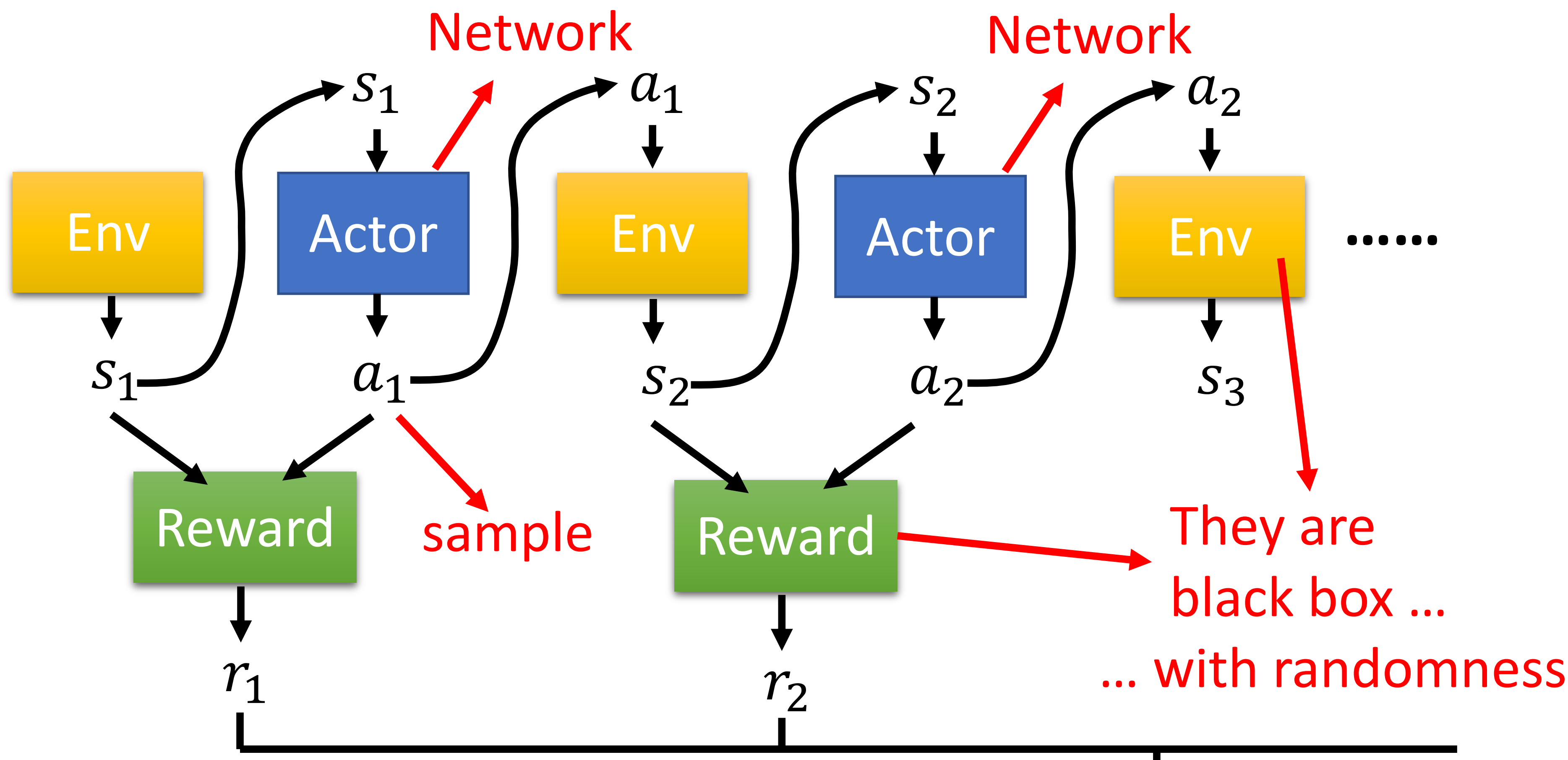
Total reward (*return*): $R = \sum_{t=1}^{T} r_t$

What we want to maximize

Step 3: Optimization

Trajectory

$\tau = \{s_1, a_1, s_2, a_2, \cdots\}$



$R(\tau) = \sum_{t=1}^{T} r_t$

How to do the optimization here is the main challenge in RL.

c.f. GAN

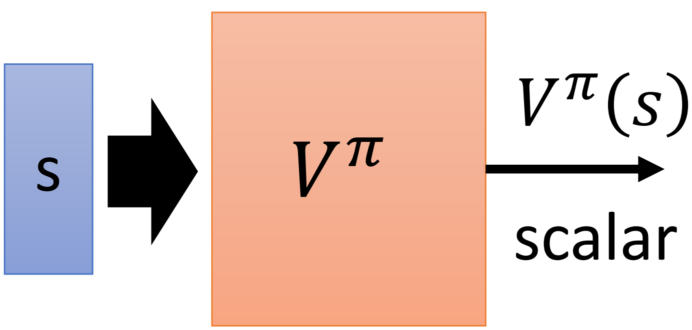
Outline

- Introduction of Q-Learning
- Tips of Q-Learning
- Q-Learning for Continuous Actions

Critic

The output values of a critic depend on the actor evaluated.

- A critic does not directly determine the action.
- Given an actor π, it evaluates how good the actor is
- State value function $V^\pi(s)$
  - When using actor π, the *cumulated* reward expects to be obtained after visiting state s



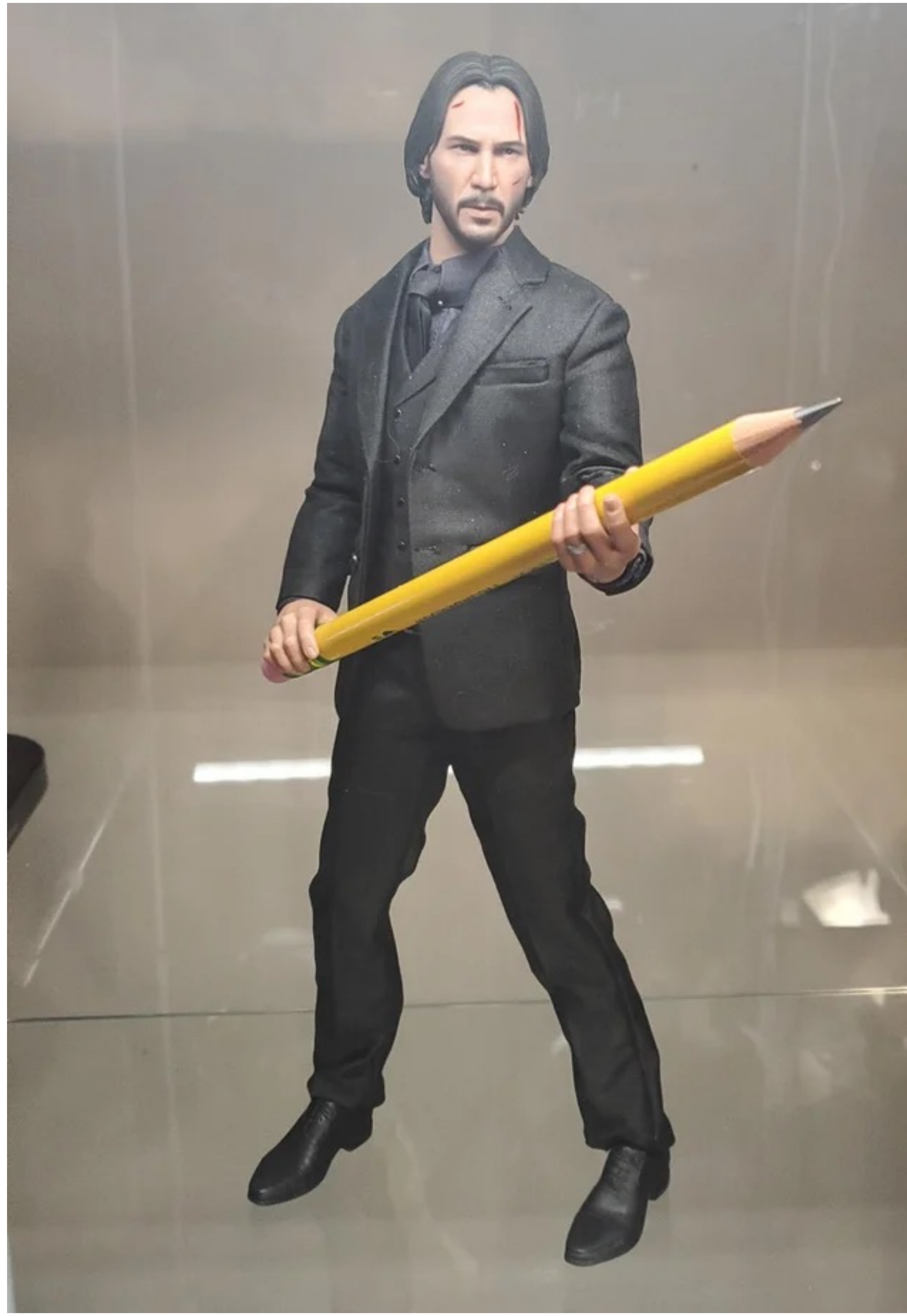
$V^\pi(s)$ is large

$V^\pi(s)$ is smaller

Critic

$V^{you}(Pencil) = \text{bad}$

$V^{John\,Wick}(Pencil) = \text{good}$

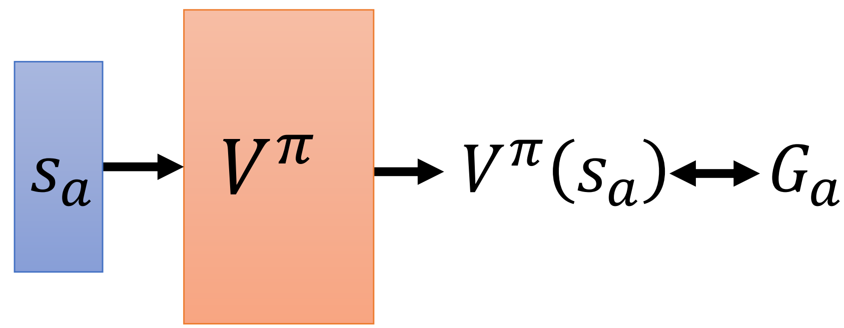


How to estimate $V^{\pi}(s)$

- **Monte-Carlo (MC) based approach**
  - The critic watches π playing the game

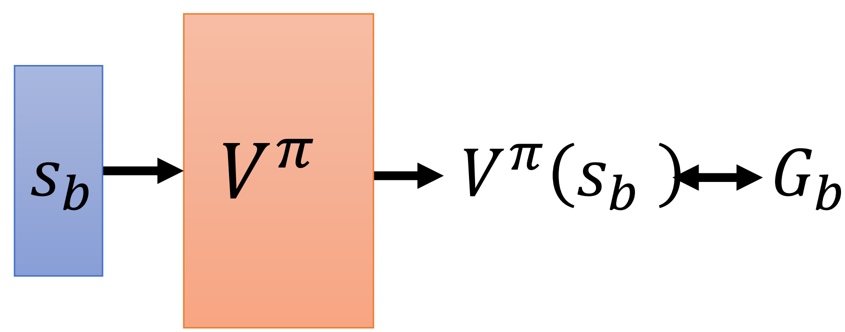
After seeing s_a,

Until the end of the episode, the cumulated reward is G_a



After seeing s_b,

Until the end of the episode, the cumulated reward is G_b

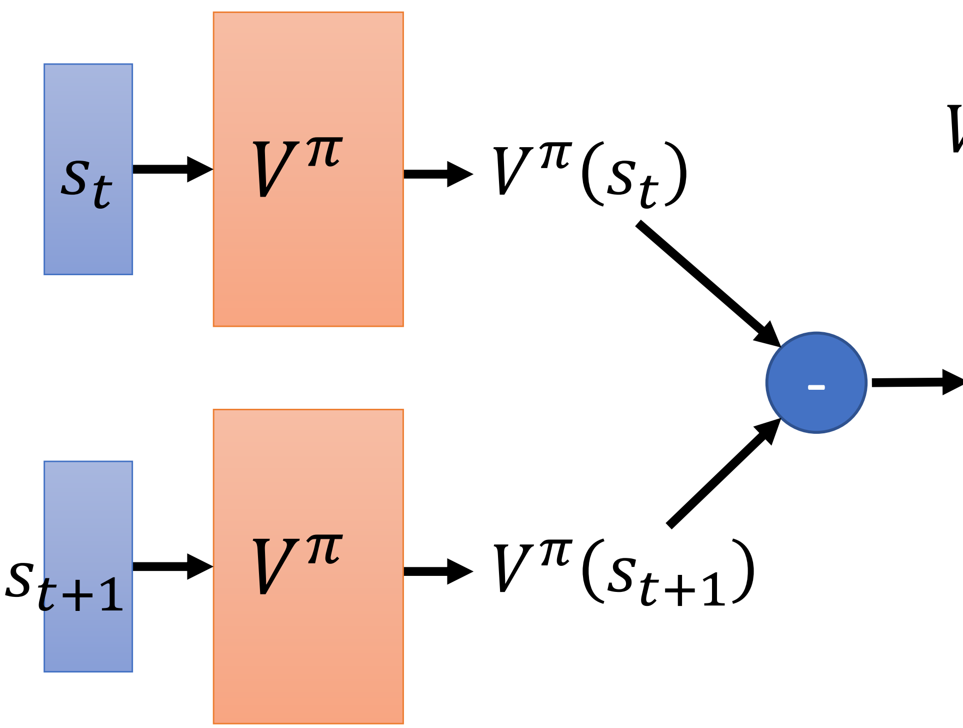


How to estimate $V^\pi(s)$

- **Temporal-difference (TD) approach**

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$V^\pi(s_t) = V^\pi(s_{t+1}) + r_t$$

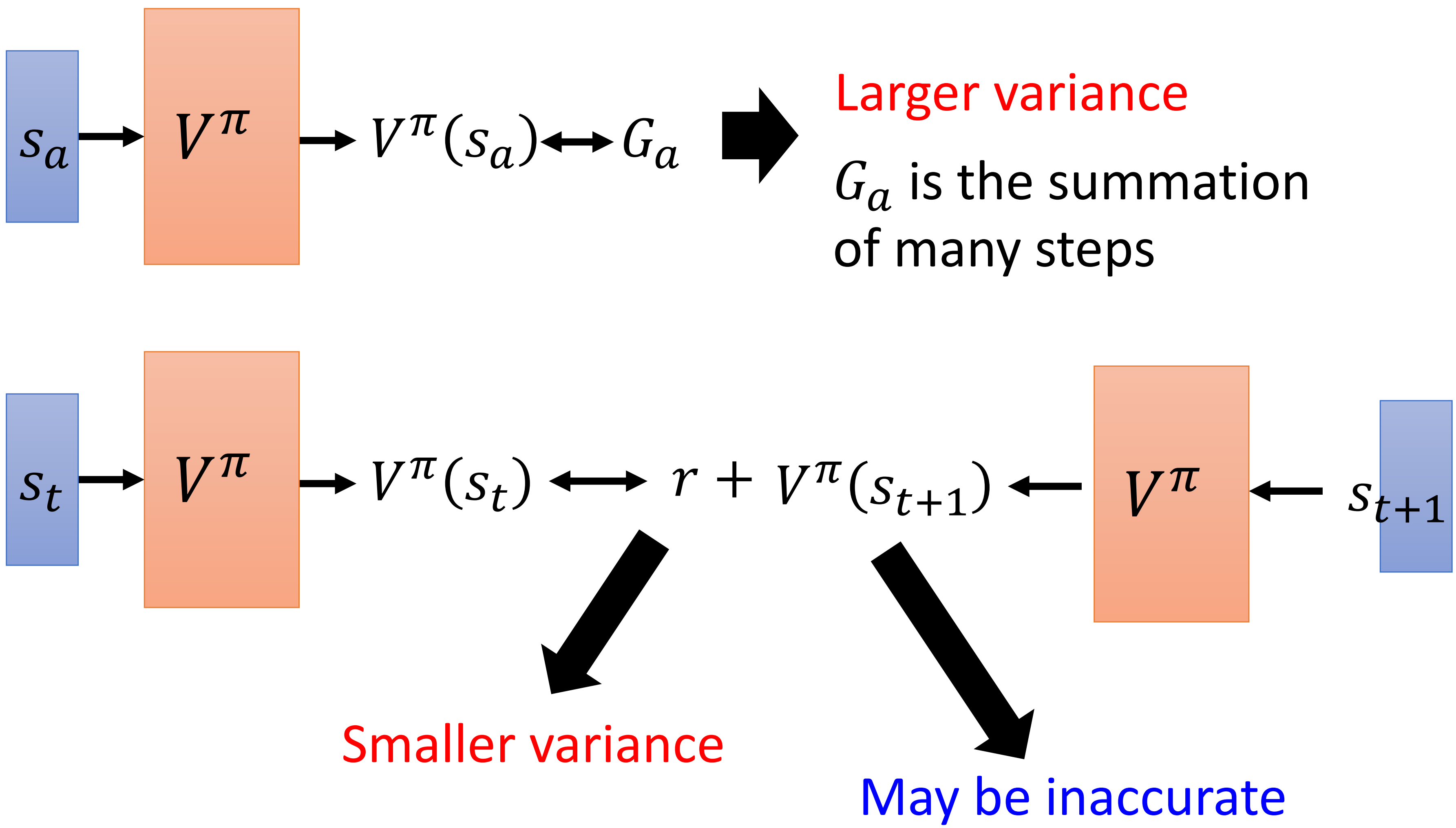


$$V^\pi(s_t) - V^\pi(s_{t+1}) \leftrightarrow r_t$$

Some applications have very long episodes, so that delaying all learning until an episode's end is too slow.

$$Var[kX] = k^2 Var[X]$$

MC v.s. TD



MC v.s. TD

[Sutton, v2, Example 6.4]

- The critic has the following 8 episodes
  - $s_a, r = 0, s_b, r = 0$, END
  - $s_b, r = 1$, END
  - $s_b, r = 1$, END
  - $s_b, r = 1$, END
  - $s_b, r = 1$, END
  - $s_b, r = 1$, END
  - $s_b, r = 1$, END
  - $s_b, r = 0$, END

$$V^\pi(s_b) = 3/4$$

$V^\pi(s_a) = ?$ 0? 3/4?

Monte-Carlo: $V^\pi(s_a) = 0$

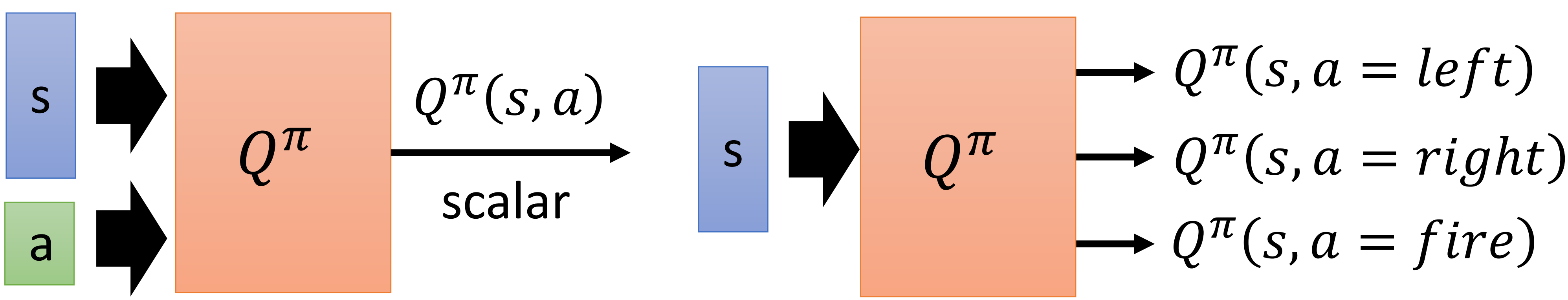
Temporal-difference:

$$\underbrace{V^\pi(s_a)}_{3/4} = \underbrace{V^\pi(s_b)}_{3/4} + \underbrace{r}_{0}$$

(The actions are ignored here.)

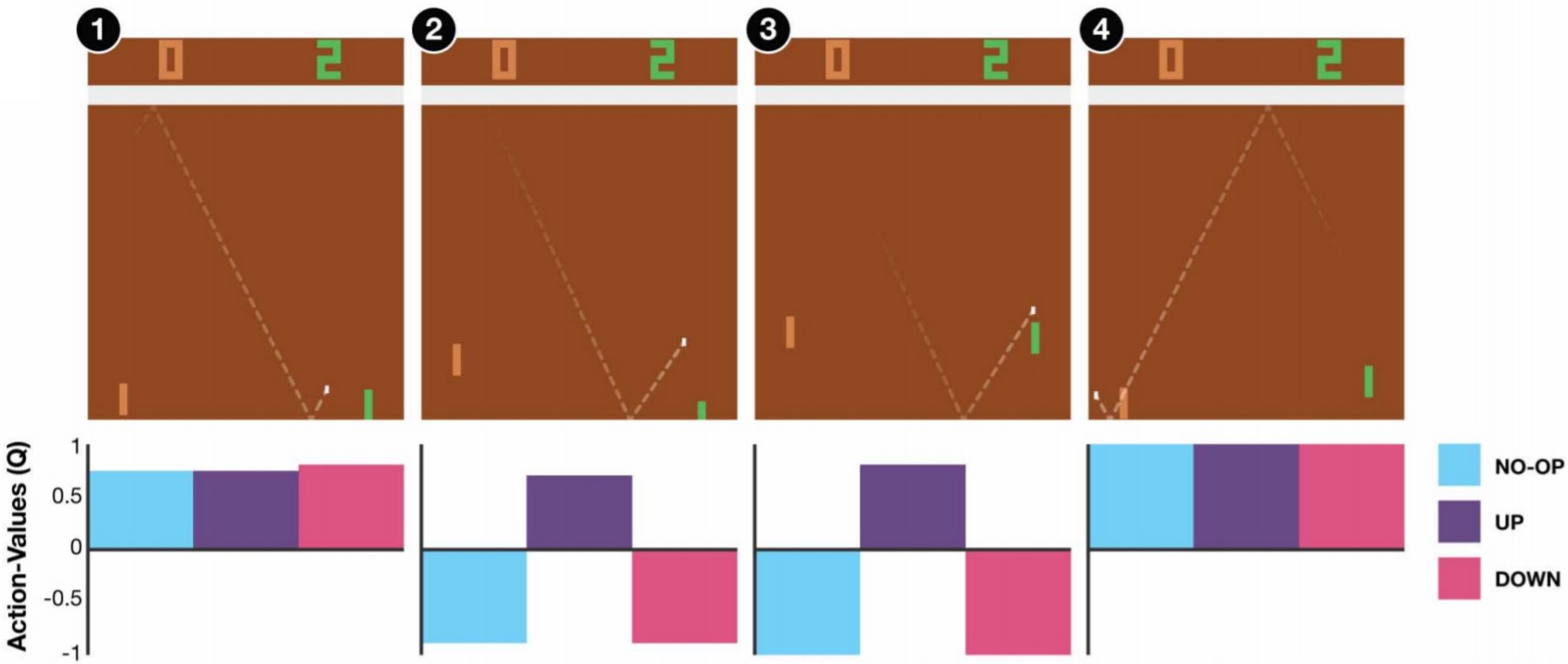
Another Critic

- State-action value function $Q^\pi(s,a)$
  - When using actor π, the *cumulated* reward expects to be obtained after taking a at state s



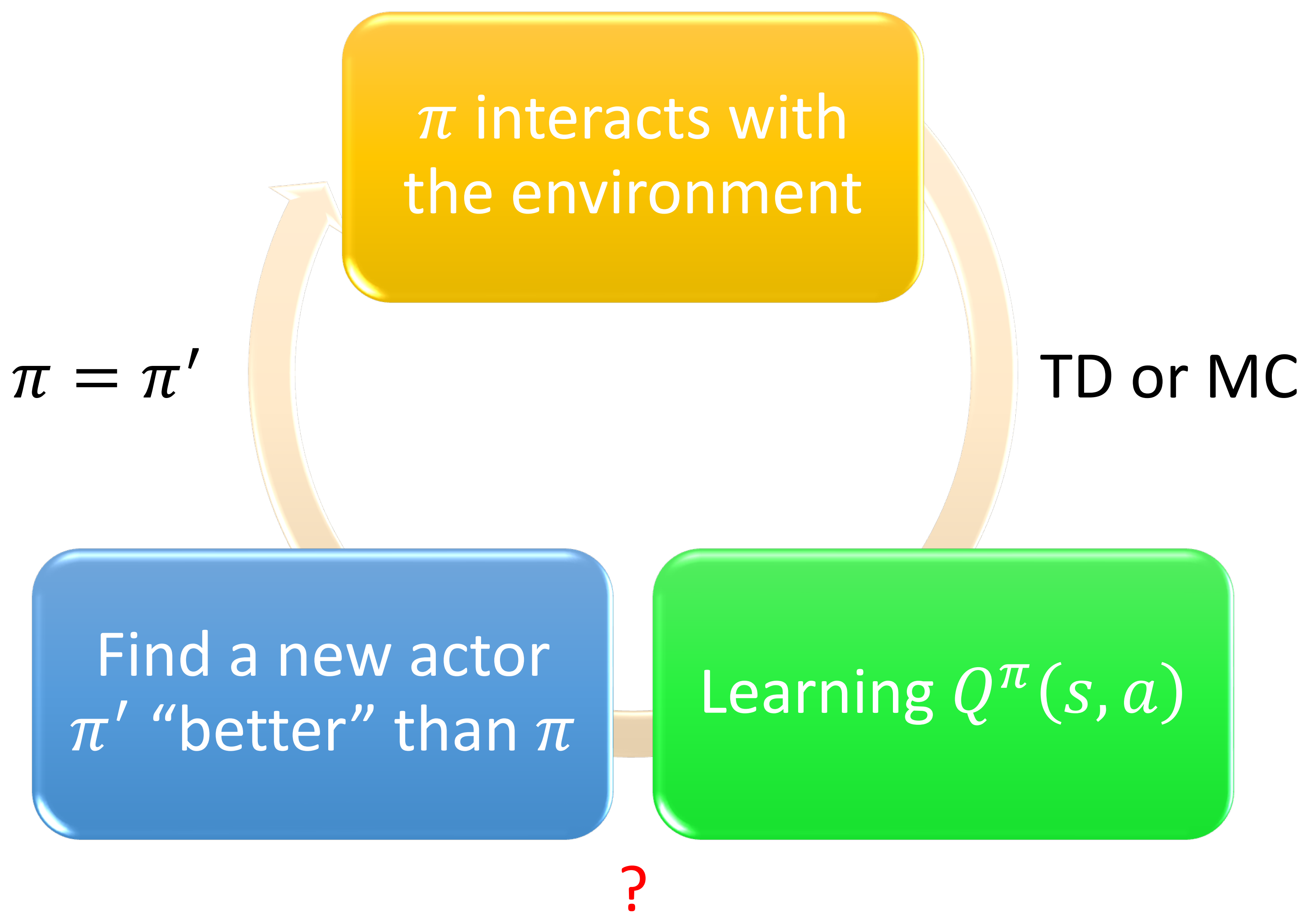
for discrete action only

State-action value function

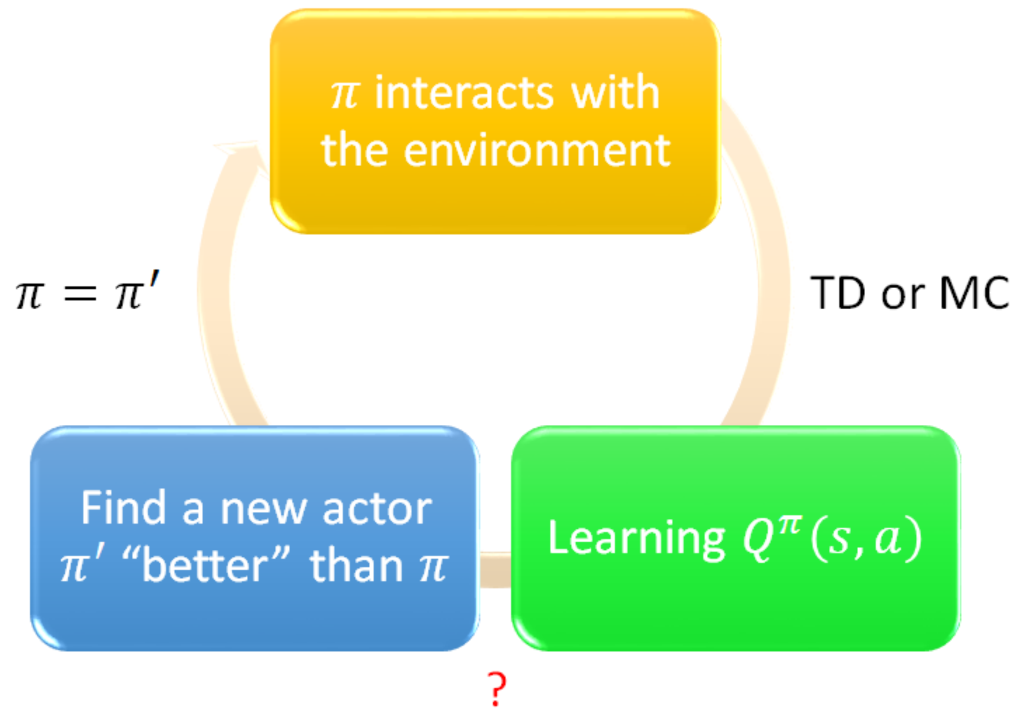


https://web.stanford.edu/class/psych209/Readings/MnihEtAlHassibis15NatureControlDeepRL.pdf

Another Way to use Critic: Q-Learning



Q-Learning



- Given $Q^\pi(s,a)$, find a new actor π' "better" than π
  - "Better": $V^{\pi'}(s) \geq V^\pi(s)$, for all state s

$$\pi'(s) = arg\max_a Q^\pi(s,a)$$

➢ π' does not have extra parameters. It depends on Q

➢ Not suitable for continuous action a (solve it later)

Q-Learning

$$\pi'(s) = arg\max_a Q^\pi(s,a)$$

$V^{\pi'}(s) \geq V^\pi(s)$, for all state s

$$V^\pi(s) = Q^\pi\big(s,\pi(s)\big)$$

$$\leq \max_a Q^\pi(s,a) \; = Q^\pi\big(s,\pi'(s)\big)$$

$$V^\pi(s) \leq Q^\pi\big(s,\pi'(s)\big)$$

$$= E[r_{t+1} + V^\pi(s_{t+1})|s_t = s, a_t = \pi'(s_t)]$$

$$\leq E[r_{t+1} + Q^\pi\big(s_{t+1},\pi'(s_{t+1})\big)|s_t = s, a_t = \pi'(s_t)]$$

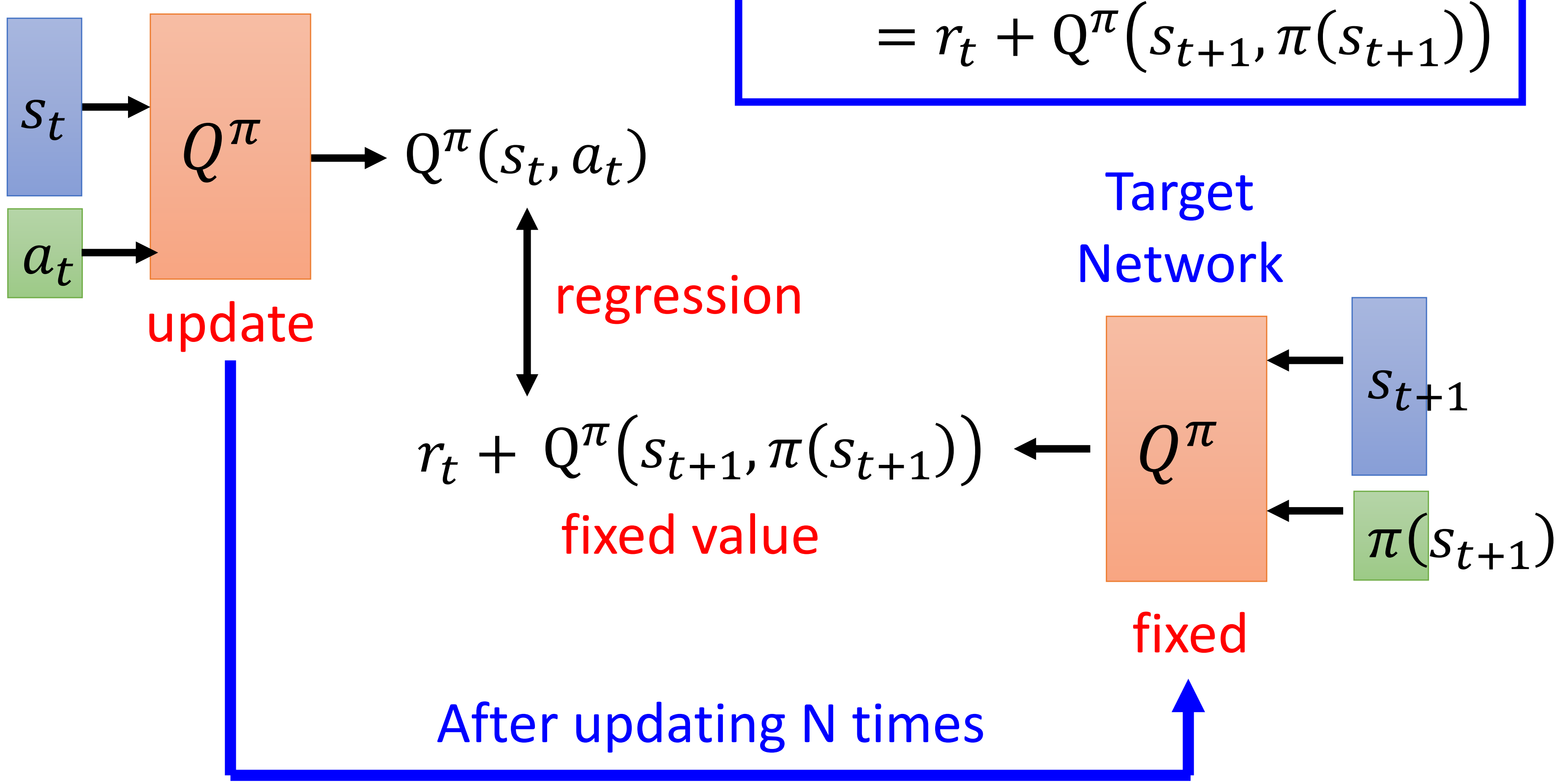
$$= E[r_{t+1} + r_{t+2} + V^\pi(s_{t+2})| \ldots]$$

$$\leq E[r_{t+1} + r_{t+2} + Q^\pi\big(s_{t+2},\pi'(s_{t+2})\big)| \ldots] \ldots \leq V^{\pi'}(s)$$

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$Q^\pi(s_t, a_t) = r_t + Q^\pi\big(s_{t+1}, \pi(s_{t+1})\big)$$



Exploration

$$
\begin{tikzcd}
 & a_1 \\
s \arrow[ur] \arrow[r] \arrow[dr] & a_2 \\
 & a_3
\end{tikzcd}
$$

a_1 $Q(s,a) = 0$ Never explore

a_2 $Q(s,a) = \boxed{1}$ Always sampled

a_3 $Q(s,a) = 0$ Never explore

- The policy is based on Q-function

$$a = arg\max_a Q(s,a)$$

This is not a good way for data collection.

Epsilon Greedy

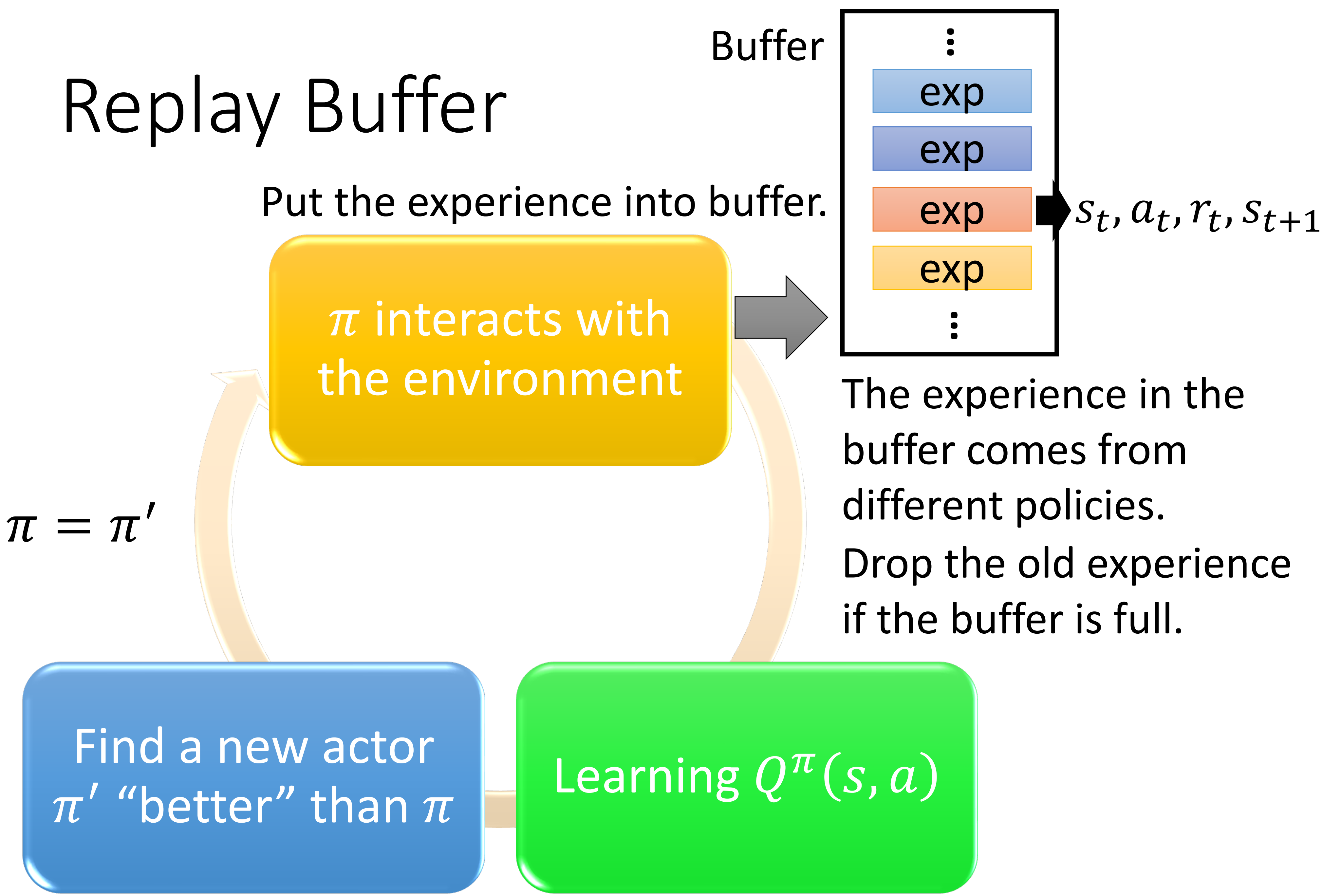
ε would decay during learning

$$a = \begin{cases} arg\max_a Q(s,a), & with\ probability\ 1-\varepsilon \\ random, & otherwise \end{cases}$$

Boltzmann Exploration

$$P(a|s) = \frac{exp\left(Q(s,a)\right)}{\sum_a exp\left(Q(s,a)\right)}$$

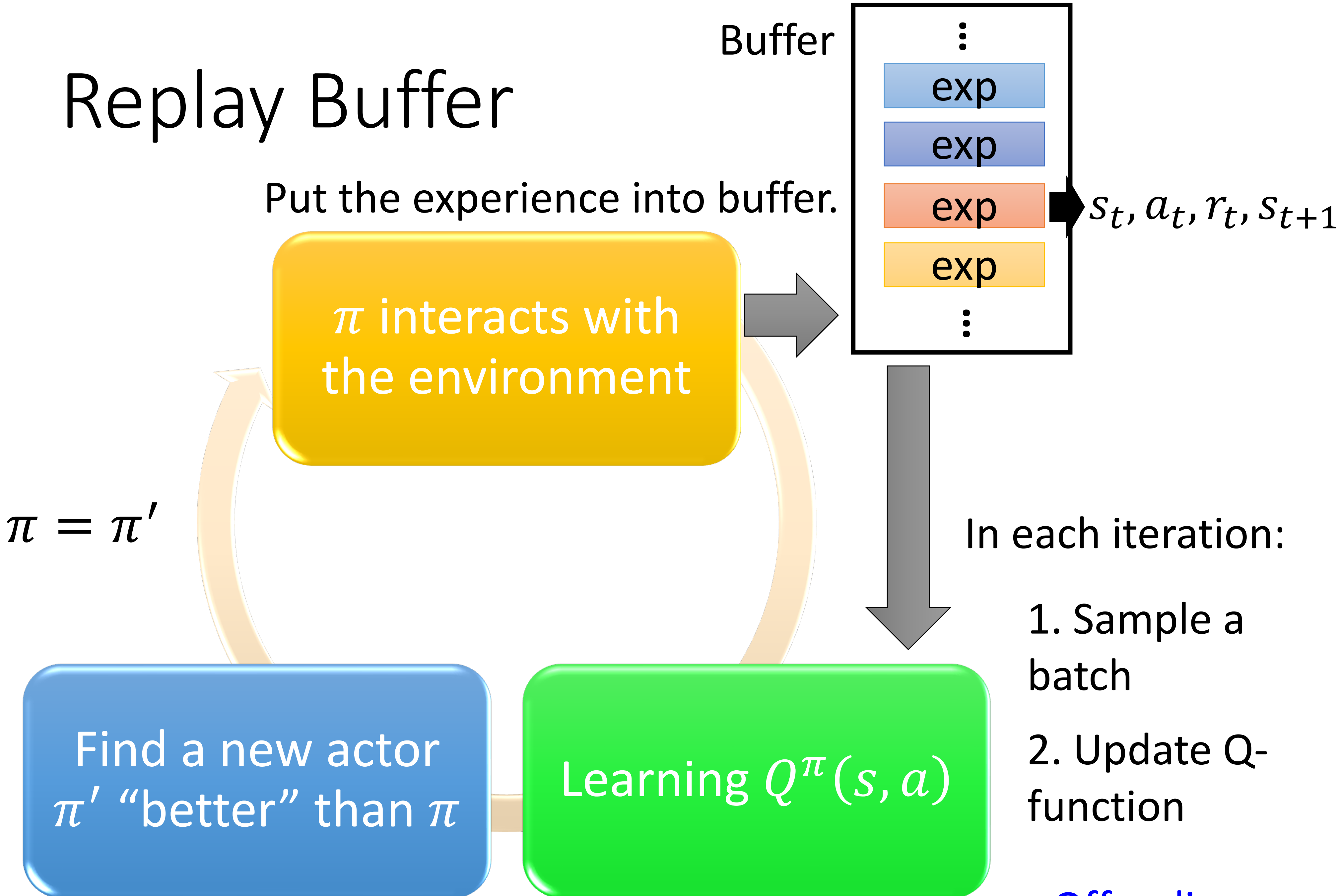
Replay Buffer



The experience in the buffer comes from different policies.

Drop the old experience if the buffer is full.

Replay Buffer



In each iteration:

1. Sample a batch
2. Update Q-function

Off-policy

Typical Q-Learning Algorithm

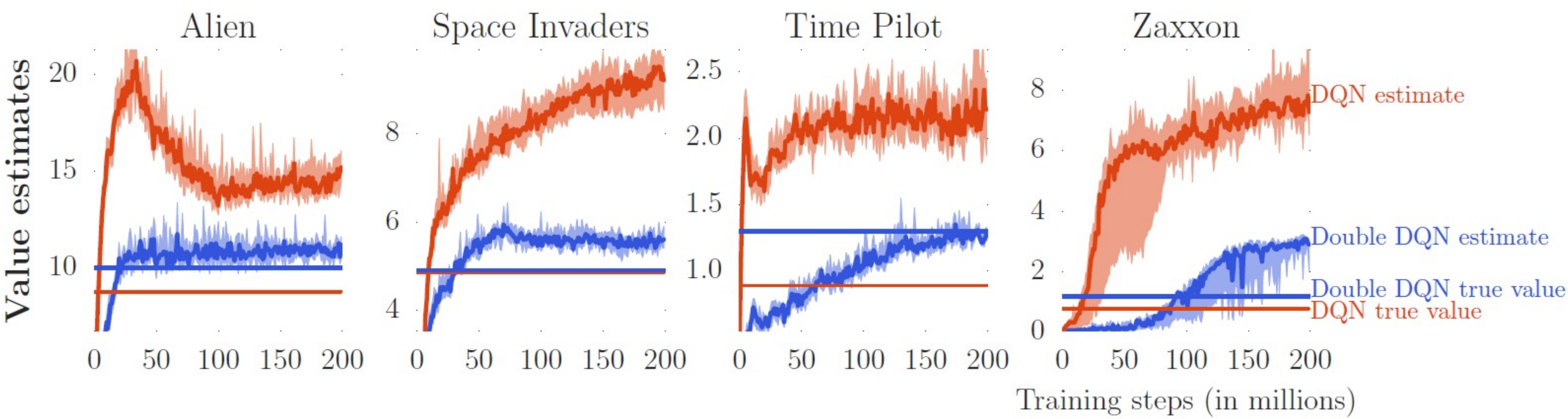
- Initialize Q-function Q, target Q-function $\hat{Q} = Q$
- In each episode
  - For each time step t
    - Given state s_t, take action a_t based on Q (epsilon greedy)
    - Obtain reward r_t, and reach new state s_{t+1}
    - Store (s_t, a_t, r_t, s_{t+1}) into buffer
    - Sample (s_i, a_i, r_i, s_{i+1}) from buffer (usually a batch)
    - Target $y = r_i + \max_a \hat{Q}(s_{i+1}, a)$
    - Update the parameters of Q to make $Q(s_i, a_i)$ close to y (regression)
    - Every C steps reset $\hat{Q} = Q$

Outline

- Introduction of Q-Learning
- Tips of Q-Learning
- Q-Learning for Continuous Actions

Double DQN

- Q value is usually over-estimated



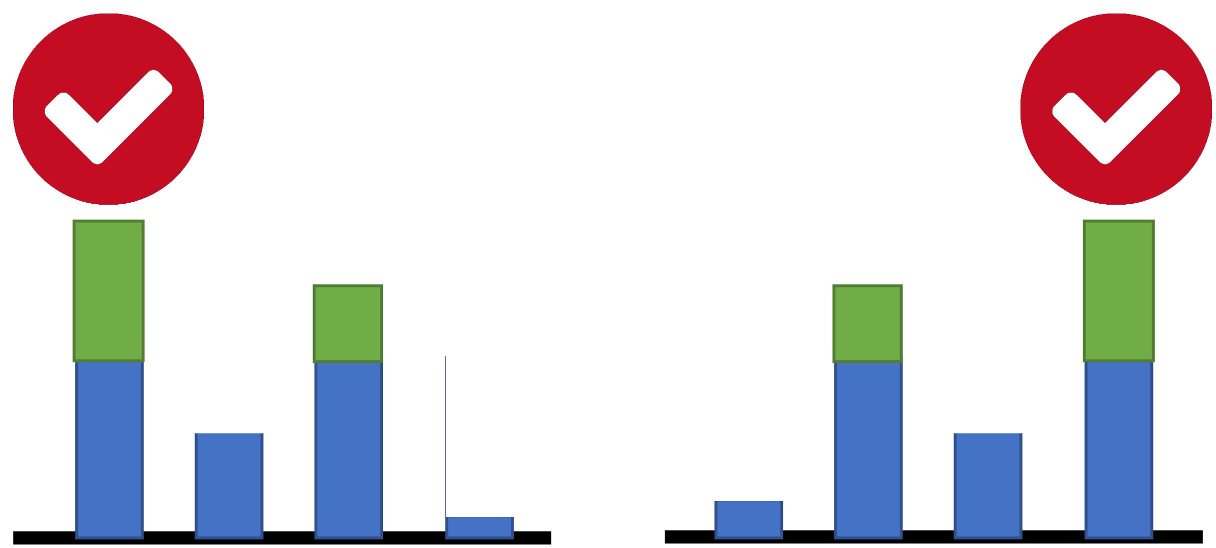
Double DQN

- Q value is usually over estimate

$$Q(s_t, a_t) \longleftrightarrow r_t + \max_a Q(s_{t+1}, a)$$

Tend to select the action that is over-estimated

$Q(s_{t+1}, a)$



Double DQN

- Q value is usually over estimate

$$Q(s_t, a_t) \longleftrightarrow r_t + \max_a Q(s_{t+1}, a)$$

- Double DQN: two functions Q and Q' Target Network

$$Q(s_t, a_t) \longleftrightarrow r_t + Q'\left(s_{t+1}, arg \max_a Q(s_{t+1}, a)\right)$$

If Q over-estimate a, so it is selected. Q' would give it proper value.

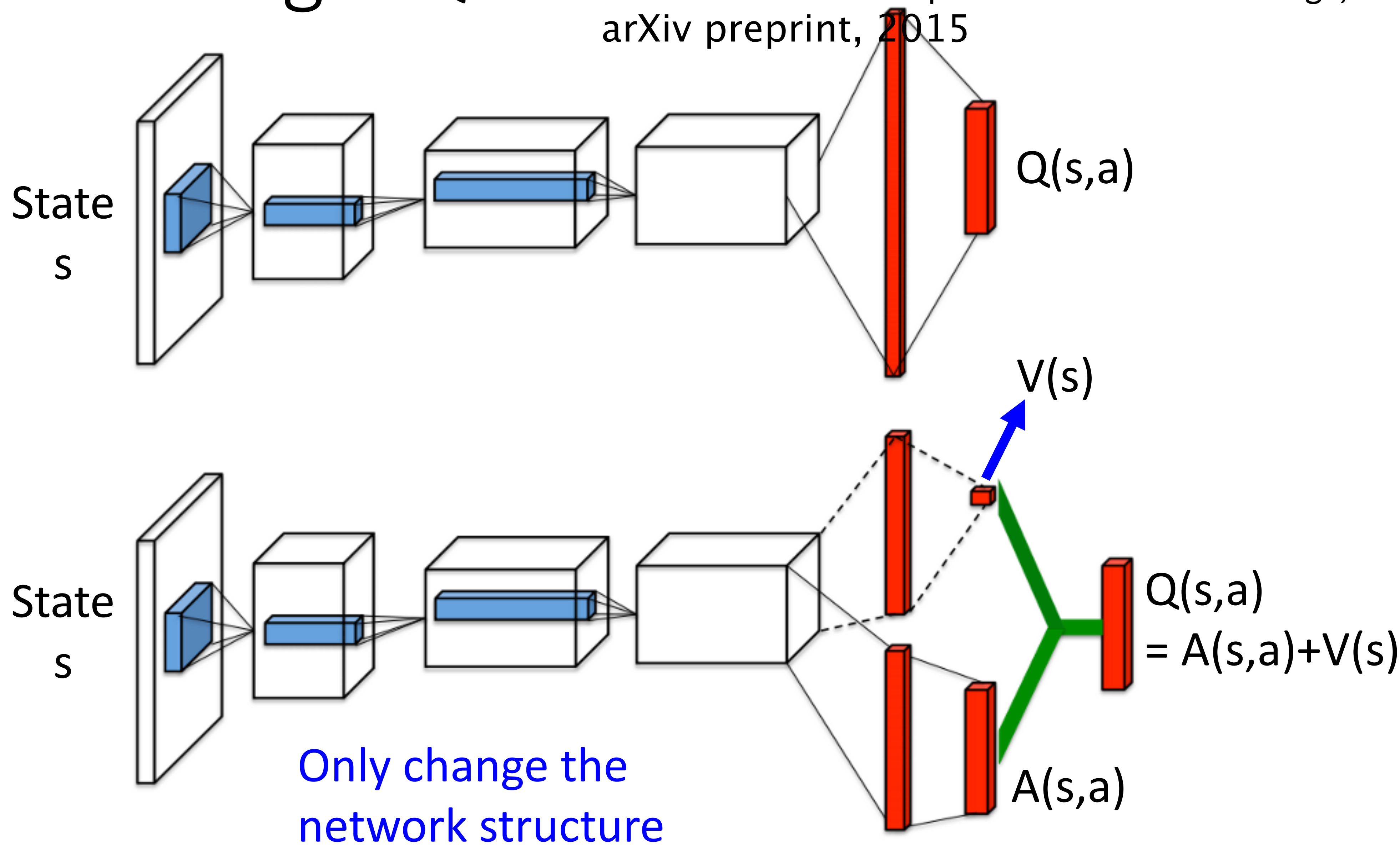
How about Q' overestimate? The action will not be selected by Q.

Hado V. Hasselt, "Double Q-learning", NIPS 2010

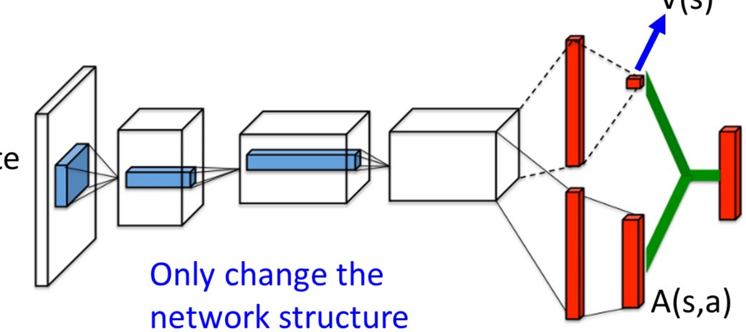
Hado van Hasselt, Arthur Guez, David Silver, "Deep Reinforcement Learning with Double Q-learning", AAAI 2016

Dueling DQN

Ziyu Wang, Tom Schaul, Matteo Hessel, Hado van Hasselt, Marc Lanctot, Nando de Freitas, “Dueling Network Architectures for Deep Reinforcement Learning”, arXiv preprint, 2015



Dueling DQN



state

Q(s,a) action

3	~~3~~ **4**	3	1
1	~~-1~~ **0**	6	1
2	~~-2~~ **-1**	3	1

=

V(s) Average of column

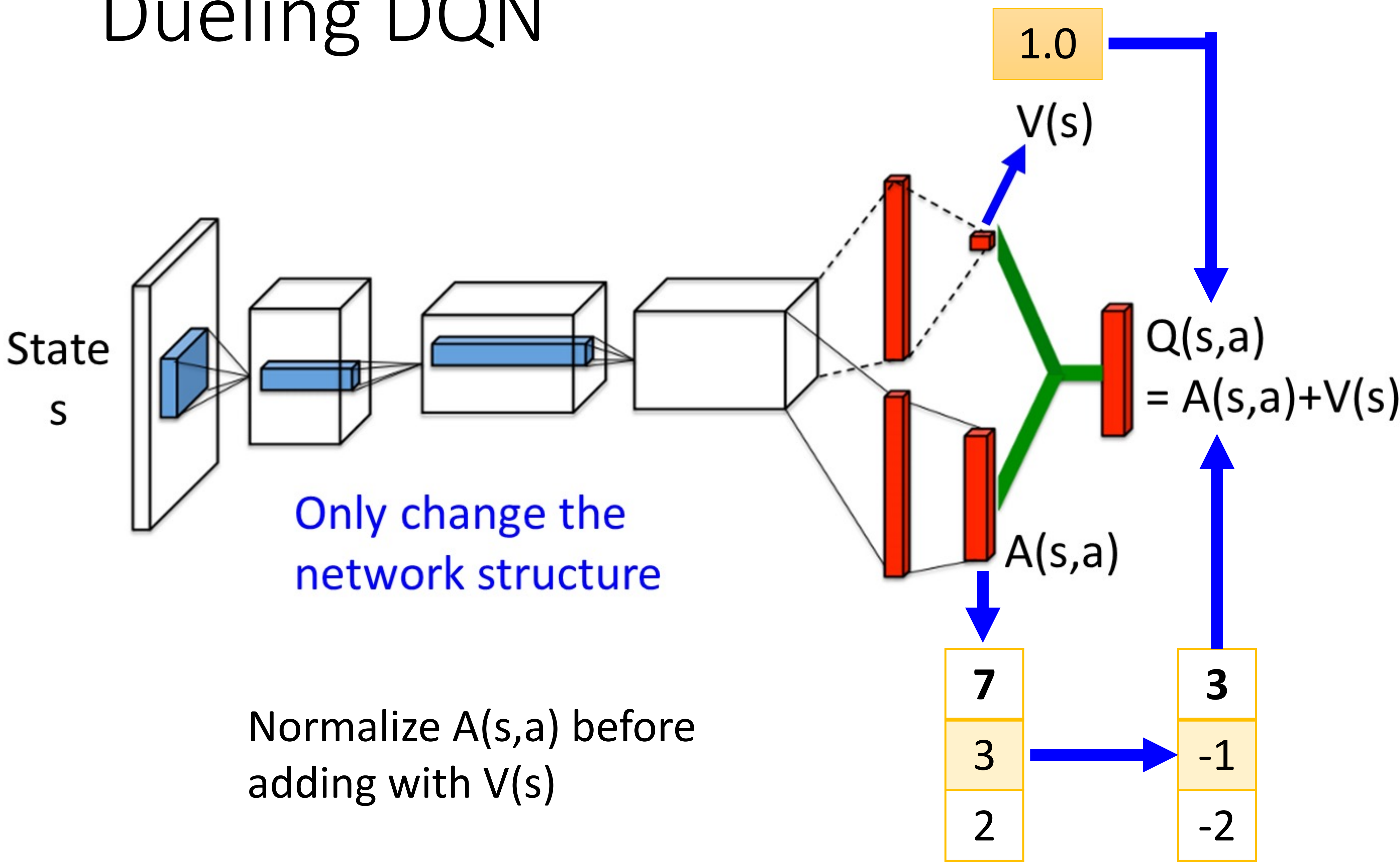
2	~~0~~ **1**	4	1

+

A(s,a) sum of column = 0

1	3	-1	0
-1	-1	2	0
0	-2	-1	0

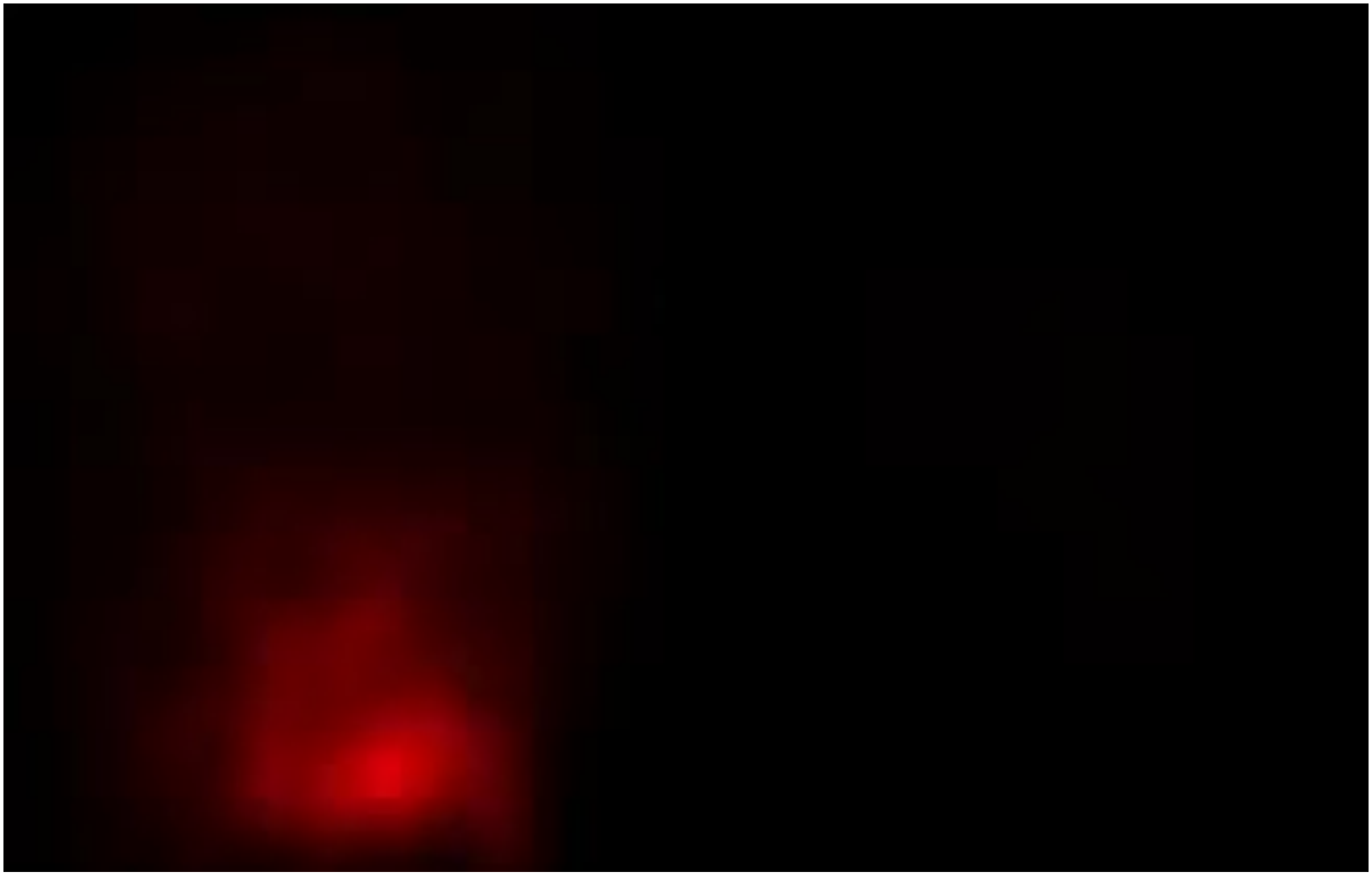
Dueling DQN



Only change the network structure

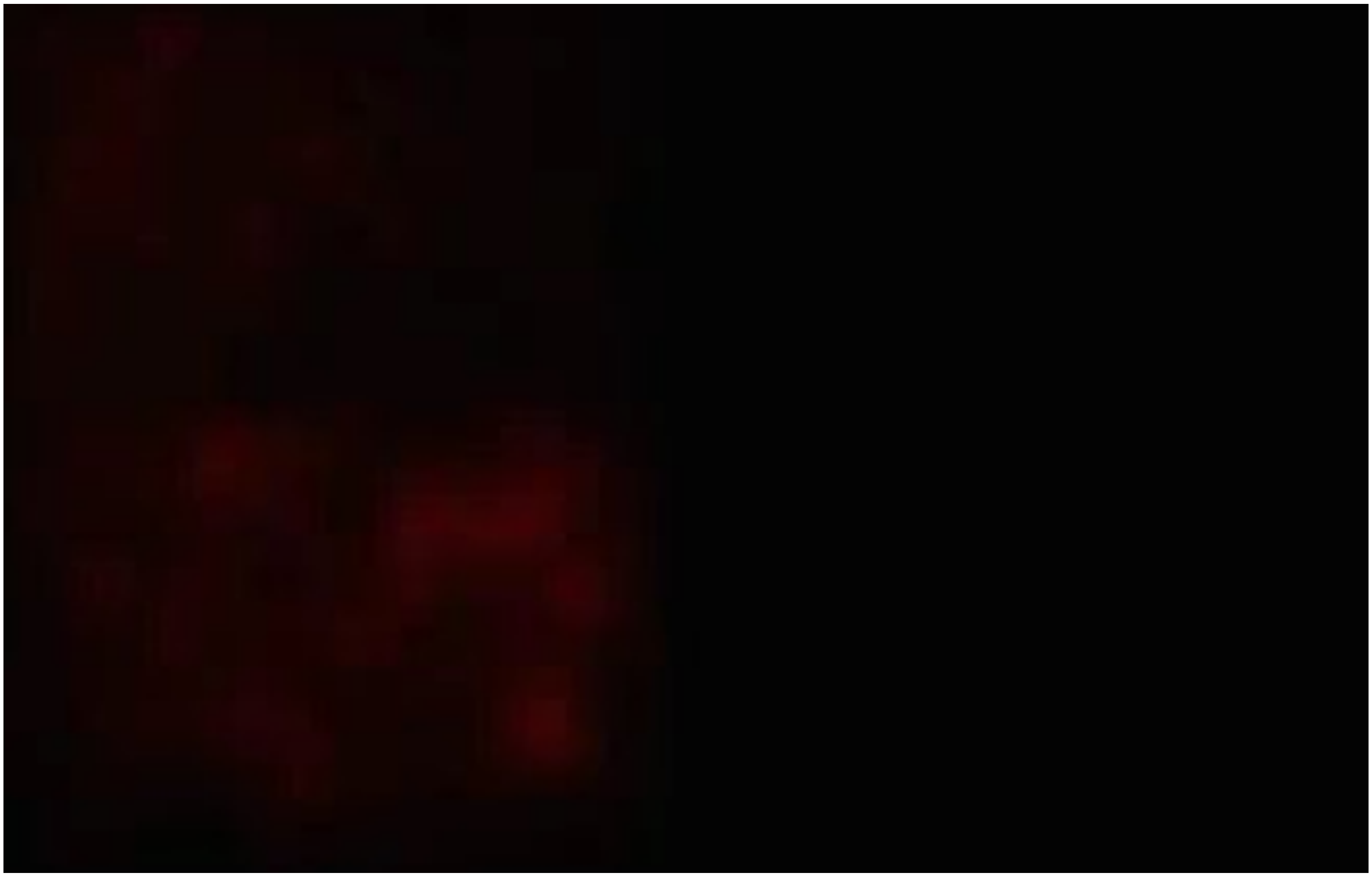
Normalize A(s,a) before adding with V(s)

Dueling DQN - Visualization



(from the link of the original paper)

Dueling DQN - Visualization

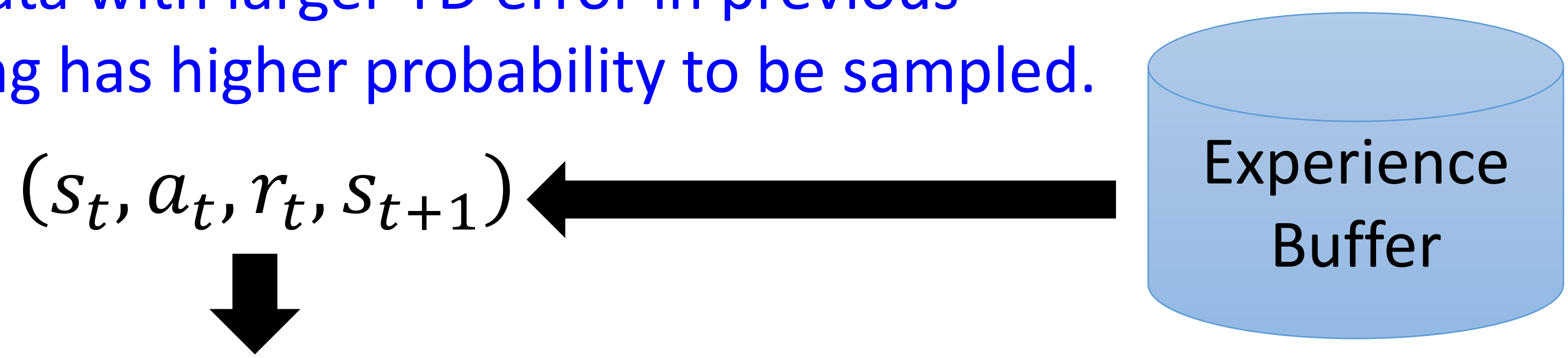


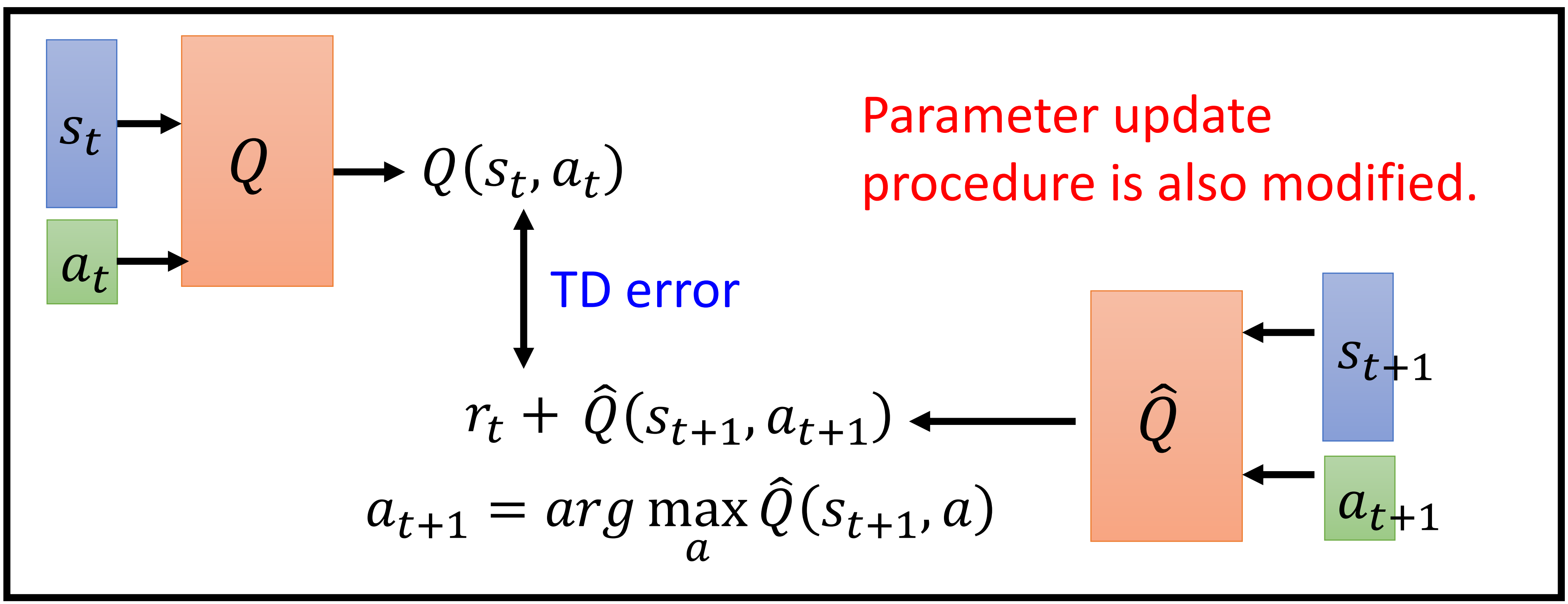
(from the link of the original paper)

https://arxiv.org/abs/1511.05952?context=cs

Prioritized Reply

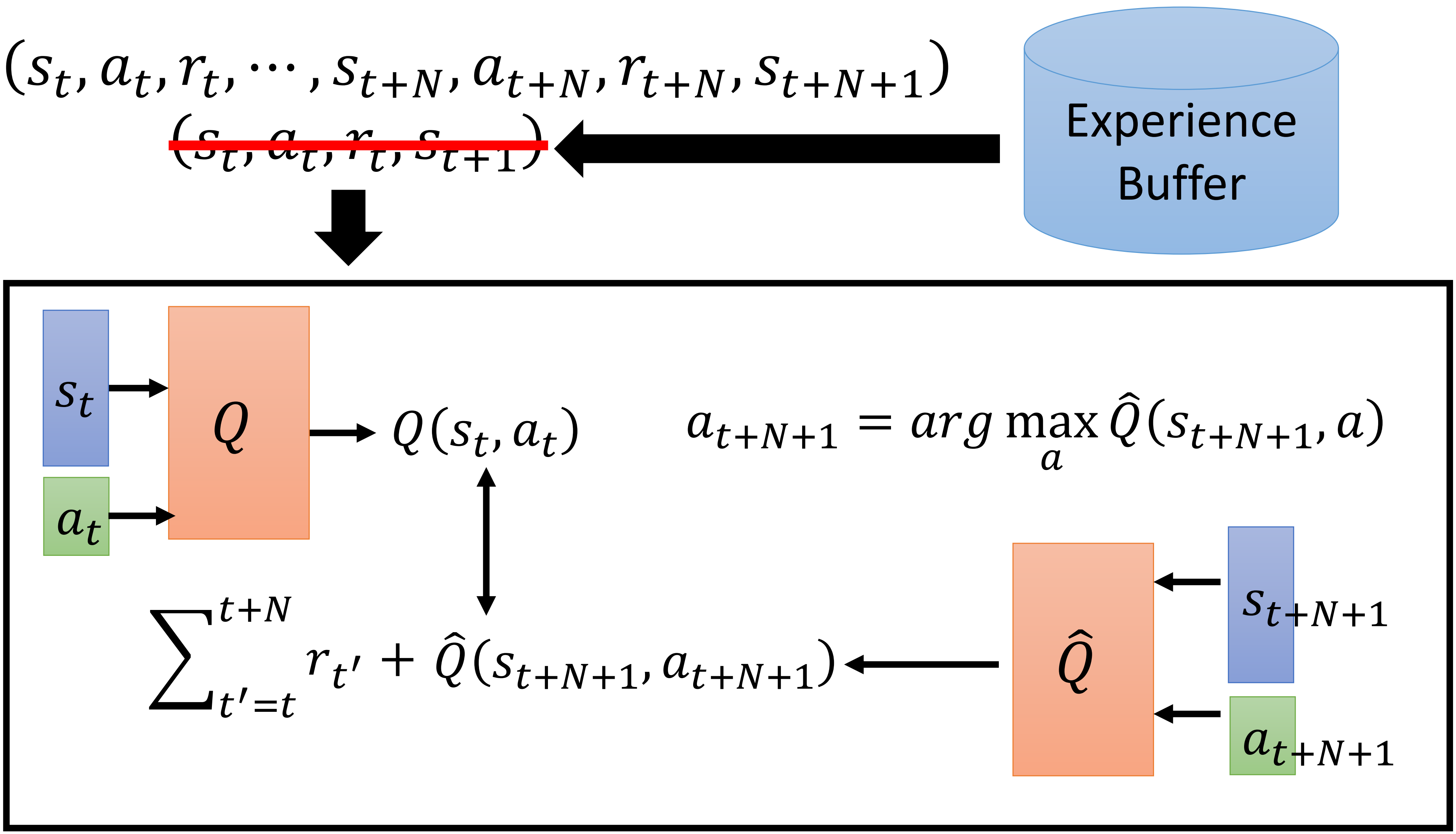
The data with larger TD error in previous training has higher probability to be sampled.





Multi-step

Balance between MC and TD



Noisy Net

https://arxiv.org/abs/1706.01905
https://arxiv.org/abs/1706.10295

- Noise on Action (Epsilon Greedy)

$$a = \begin{cases} arg \max_a Q(s,a), & with\ probability\ 1-\varepsilon \\ random, & otherwise \end{cases}$$

- Noise on Parameters

Inject noise into the parameters of Q-function **at the beginning of each episode**

$$a = arg \max_a \tilde{Q}(s,a)$$

$$Q(s,a) \xrightarrow{\text{Add noise}} \tilde{Q}(s,a)$$

The noise would **NOT** change in an episode.

Noisy Net

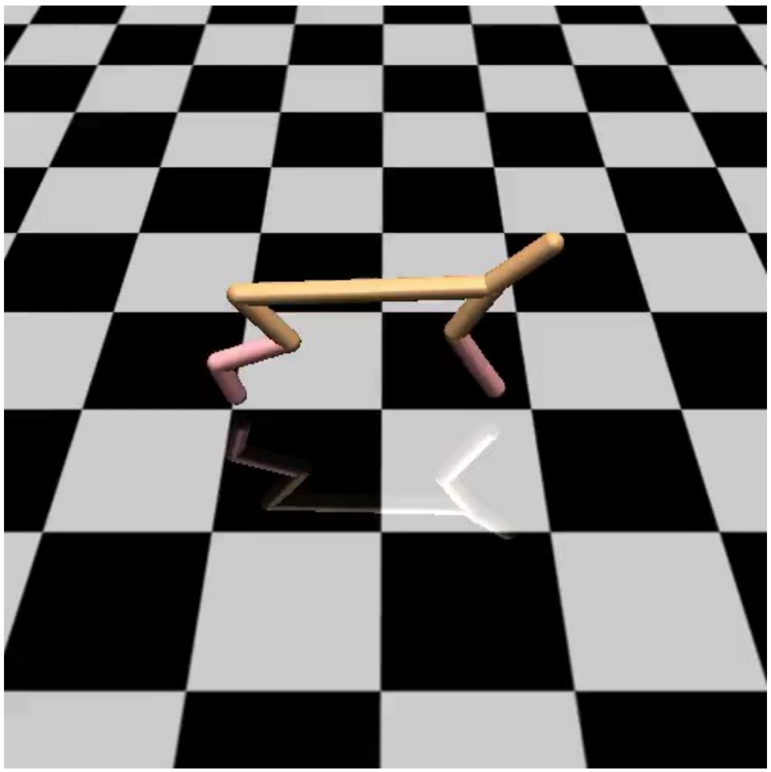
- Noise on Action
  - Given the same state, the agent may takes different actions.
  - No real policy works in this way
- Noise on Parameters
  - Given the same (similar) state, the agent takes the same action.
    - $\longrightarrow$ State-dependent Exploration
  - Explore in a *consistent* way

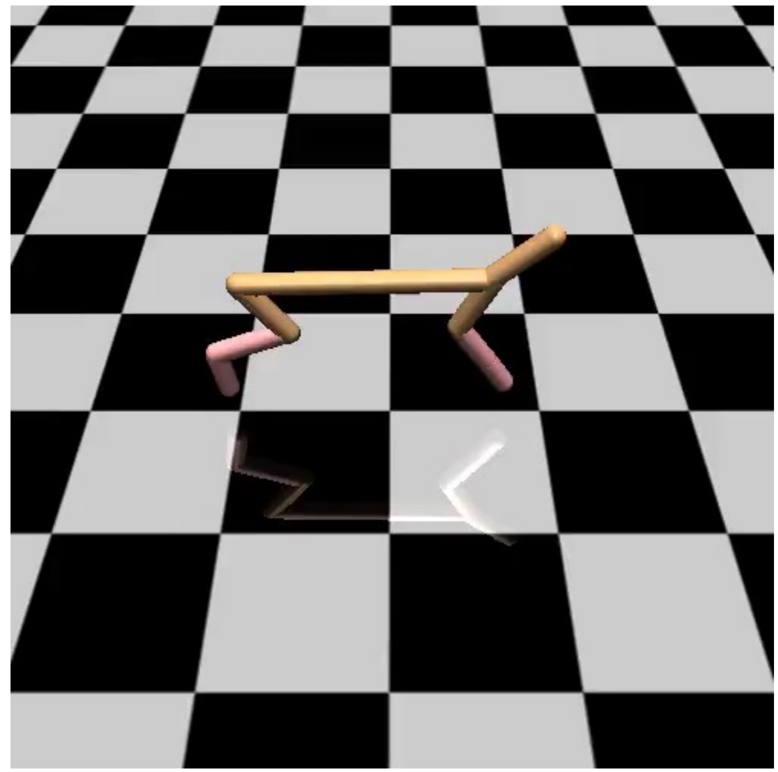
Random Testing

Systematically…

Demo

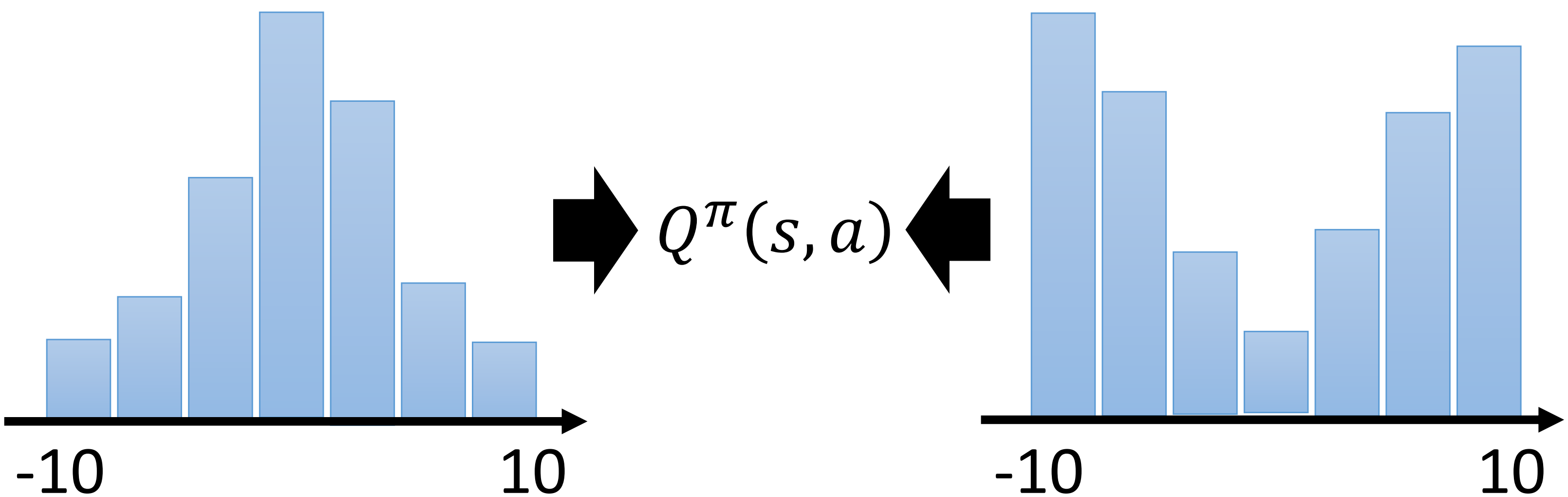
https://blog.openai.com/better-exploration-with-parameter-noise/





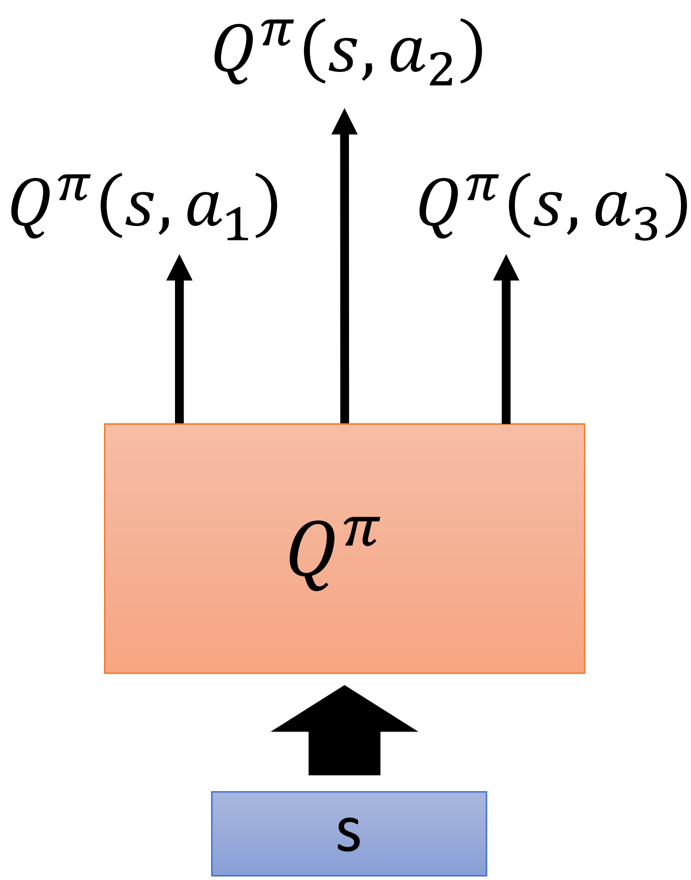
Distributional Q-function

- State-action value function $Q^\pi(s,a)$
  - When using actor π, the *cumulated* reward expects to be obtained after seeing observation s and taking a

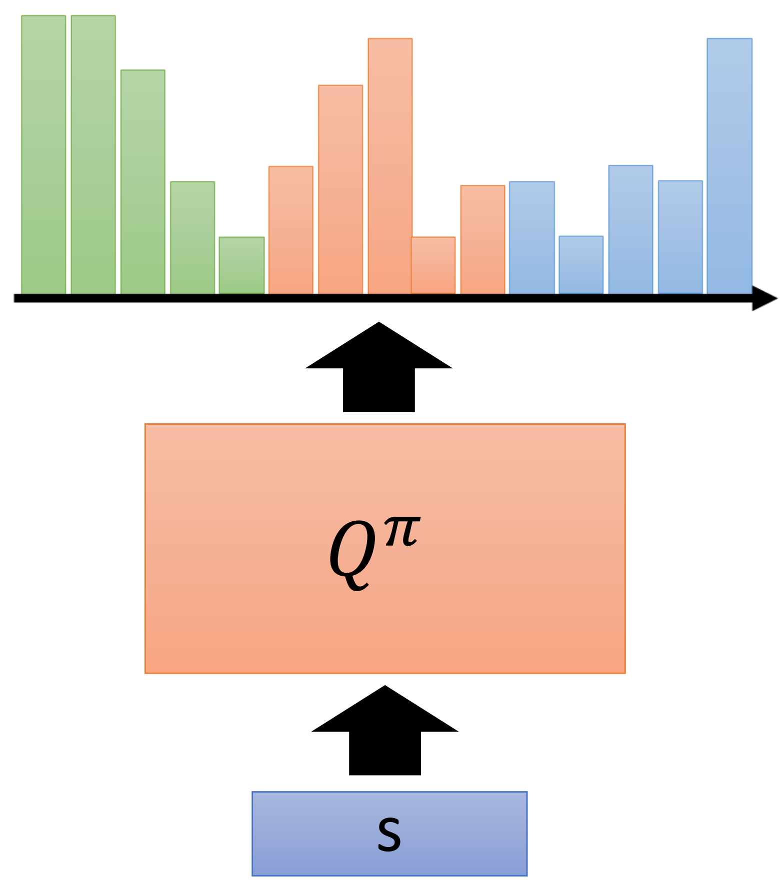


Different distributions can have the same values.

Distributional Q-function

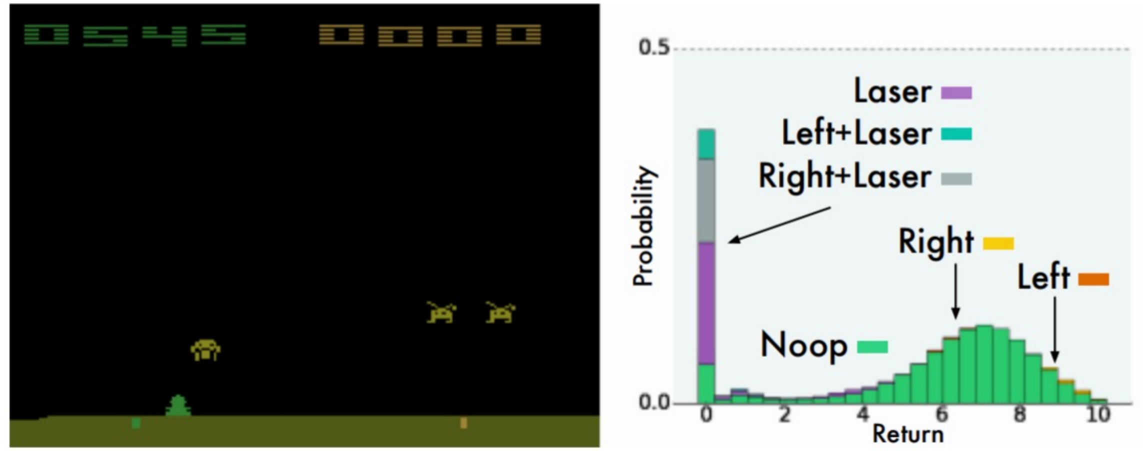


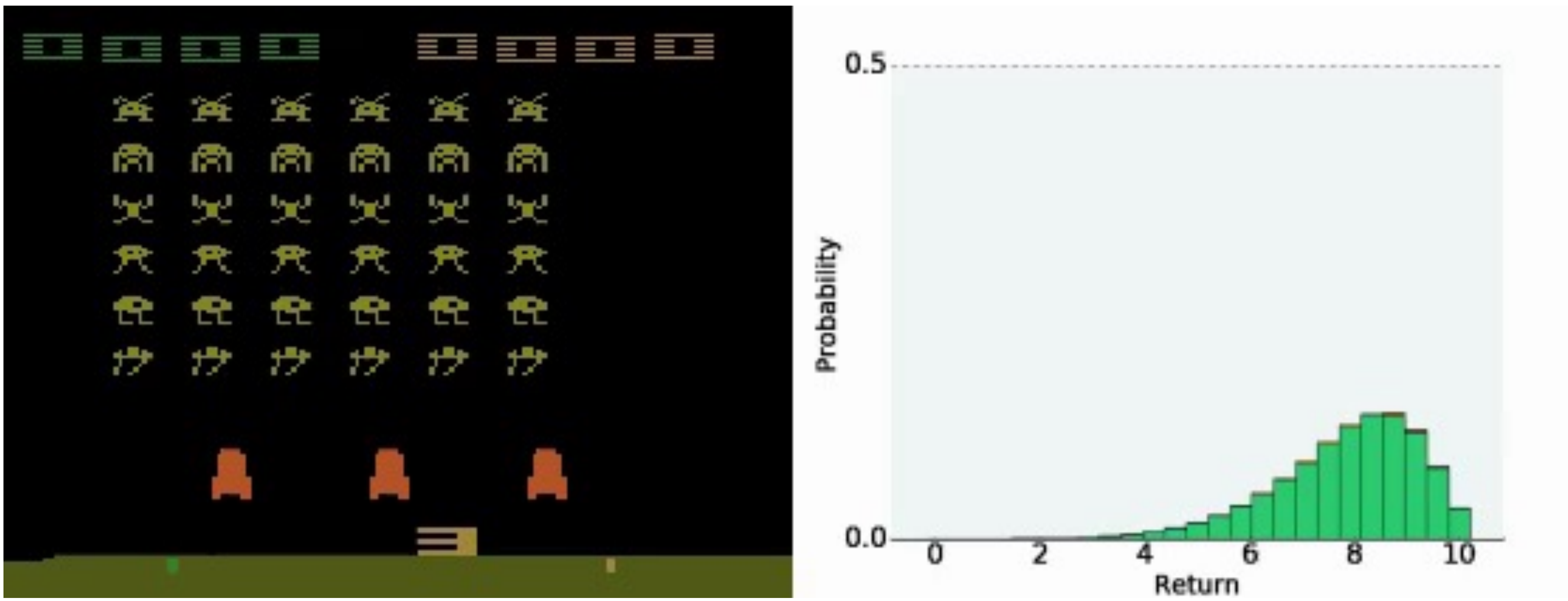
A network with 3 outputs



A network with 15 outputs
(each action has 5 bins)

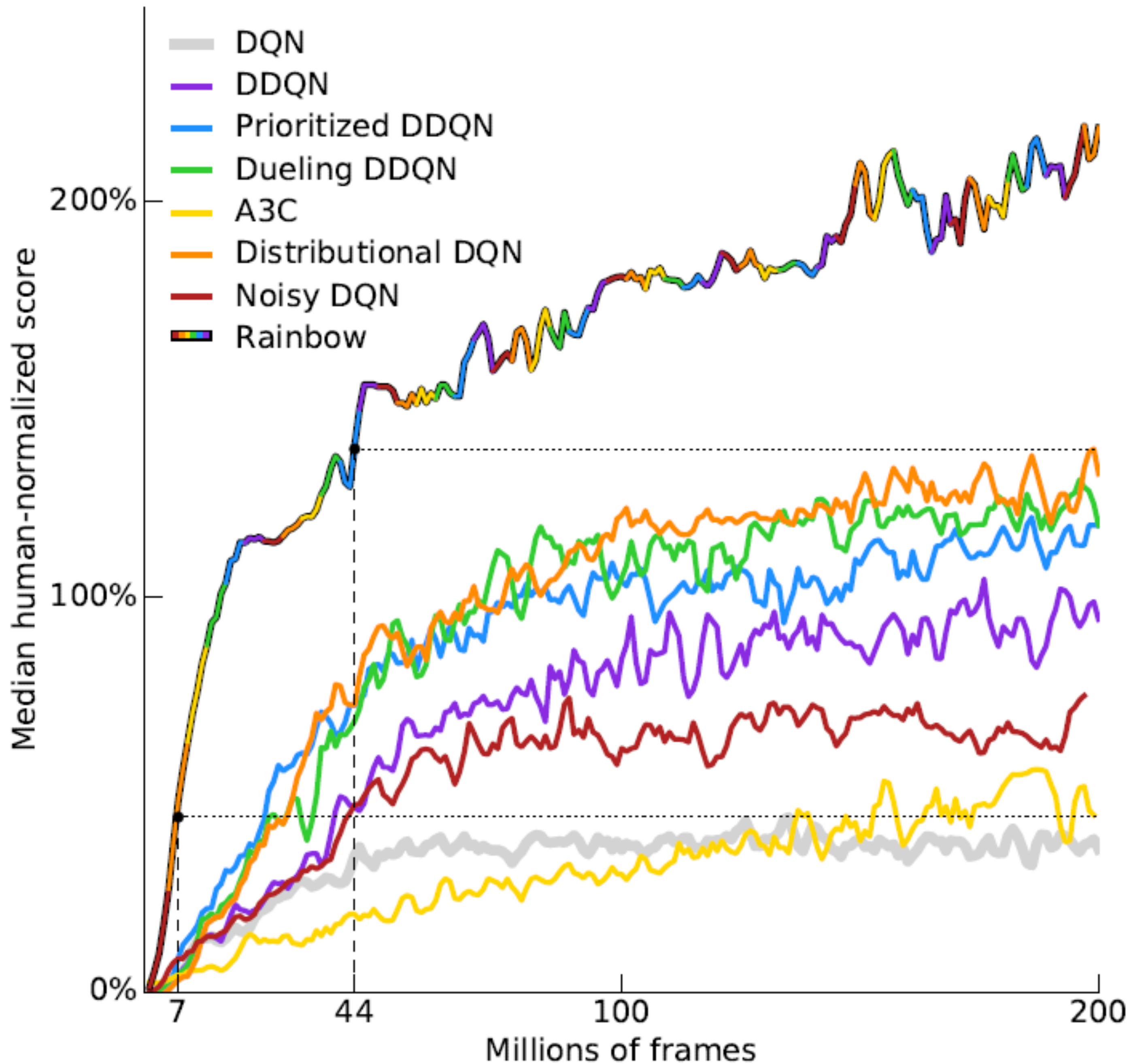
Demo





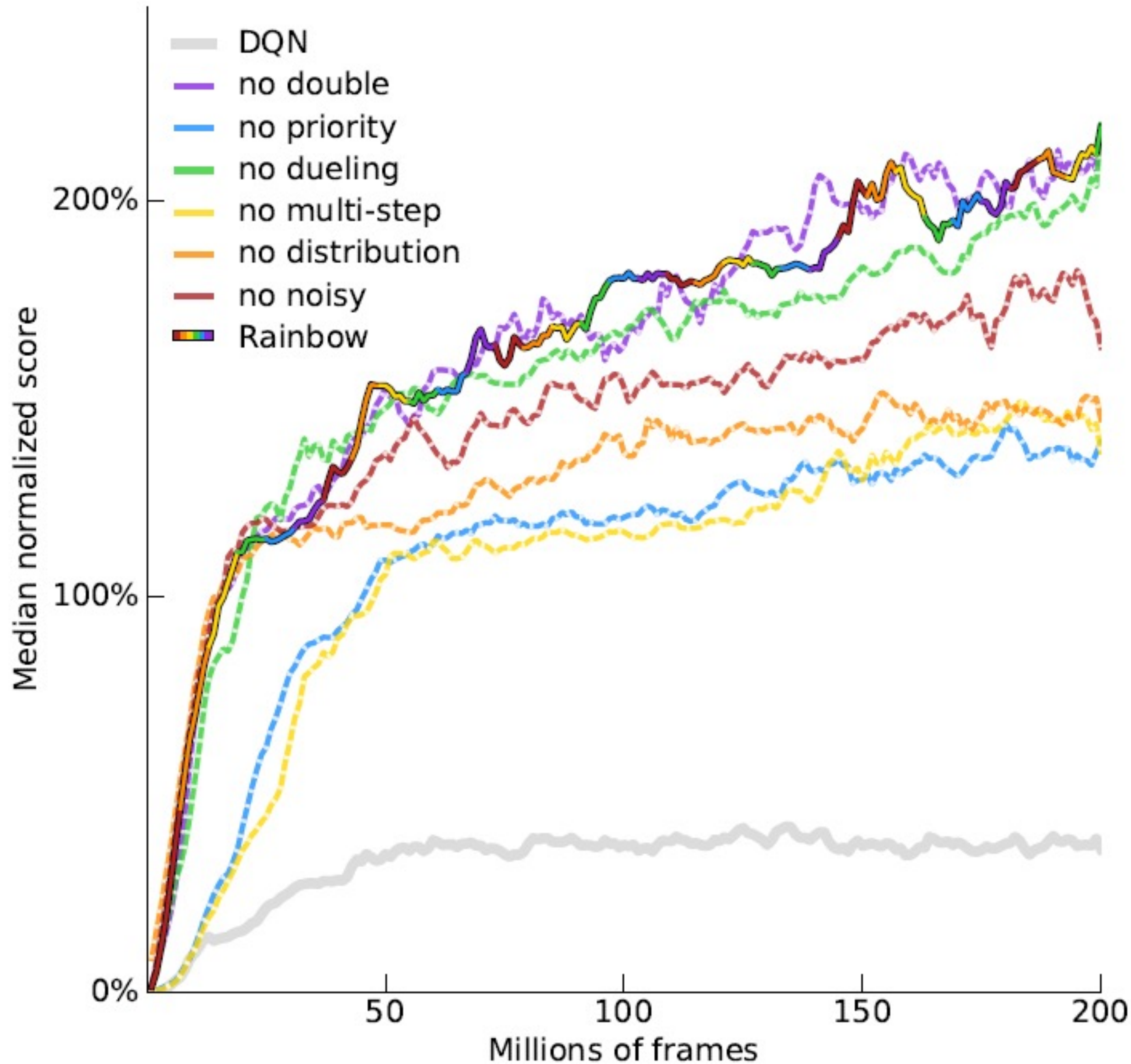
https://youtu.be/yFBwyPuO2Vg

Rainbow



https://arxiv.org/abs/1710.02298

Rainbow



https://arxiv.org/abs/1710.02298

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Continuous Actions

- Action a is a *continuous vector*

$$a = arg\max_{a} Q(s, a)$$

Solution 1

Sample a set of actions: $\{a_1, a_2, \cdots, a_N\}$

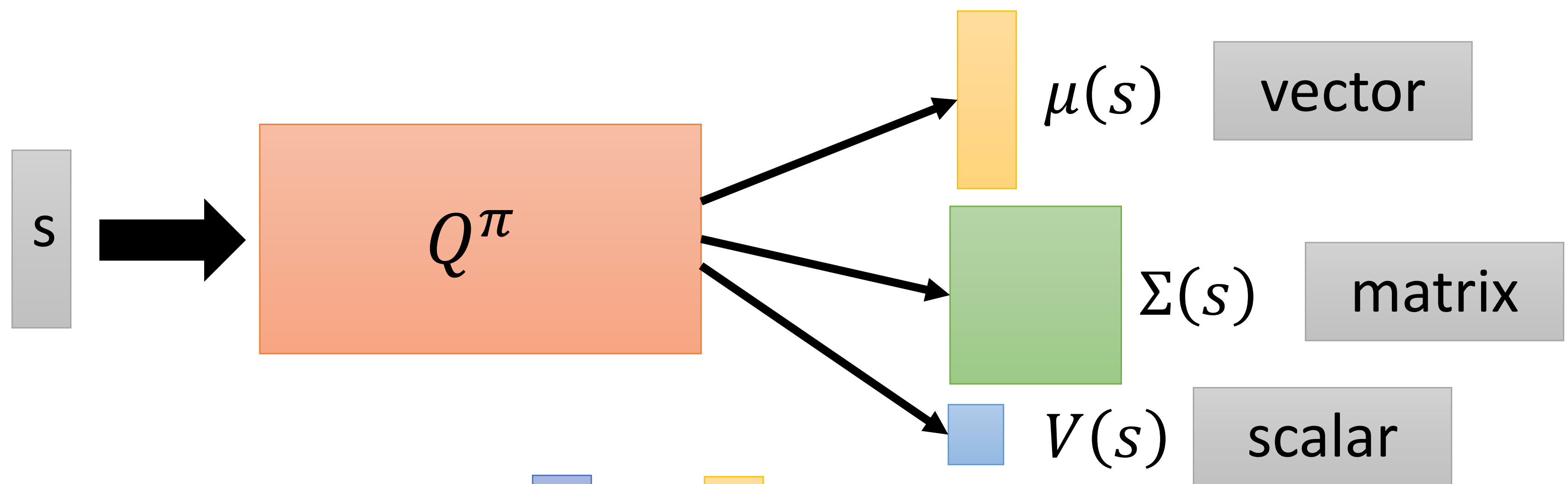
See which action can obtain the largest Q value

Solution 2

Using gradient ascent to solve the optimization problem.

Continuous Actions

Solution 3 Design a network to make the optimization easy.



$$Q(s,a) = -\left(a-\mu(s)\right)^T \Sigma(s)\left(a-\mu(s)\right) + V(s)$$

$$\mu(s) = arg\max_a Q(s,a)$$

https://www.youtube.com/watch?v=ZhsEKTo7V04

Continuous Actions

Solution 4 Don’t use Q-learning