Consider the Inverse Model,

$$X_y = \mu + \Gamma\nu_y + \varepsilon,$$

where $X_y \in \mathbb{R}^p, \nu_y \in \mathbb{R}^d, d < p$, the basis $\Gamma \in \mathbb{R}^{p\times d}$ with $\Gamma^T\Gamma = I_d$, and $\varepsilon \sim N_p(0, \sigma^2 I_p)$.

The following proposition states under the assumption of inverse model, Γ is actually a sufficient reduction. See Cook(2007) for more general case.

Proposition: Under the inverse model, the distribution of $Y|X$ is the same as the distribution of $Y|\Gamma^T X$.

Proof: Firstly, $X|Y = y \sim N_p(\mu + \Gamma\nu_y, \sigma^2 I_p)$. By Bayesian formula, we have

$$\begin{aligned} f_{Y|X}(y|x) &\propto f_{X|Y}(x|y) f_Y(y) \\ &\propto exp(-\frac{1}{2\sigma^2}\|x - \mu - \Gamma\nu_y\|^2) f_Y(y) \\ &\propto exp(-\frac{1}{2\sigma^2}(\nu_y^T\nu_y - 2\nu_y^T\Gamma^T(x-\mu))) f_Y(y) \end{aligned}$$

The last line is given by the orthogonality of Γ. Similarly, since $\Gamma^T X|Y = y \sim N_d(\Gamma^T\mu + \nu_y, \sigma^2 I_d)$, we have

$$\begin{aligned} f_{Y|\Gamma^T X}(y|\Gamma^T x) &\propto f_{\Gamma^T X|Y}(\Gamma^T x|y) f_Y(y) \\ &\propto exp(-\frac{1}{2\sigma^2}\|\Gamma^T x - \Gamma^T\mu - \nu_y\|^2) f_Y(y) \\ &\propto exp(-\frac{1}{2\sigma^2}(\nu_y^T\nu_y - 2\nu_y^T\Gamma^T(x-\mu))) f_Y(y) \end{aligned}$$

Therefore, the kernel of $Y|X$ and $Y|\Gamma^T X$ are the same, which implies the result.