
False Discovery Rate Control and Statistical Quality Assessment of Annotators in Crowdsourced Ranking

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Abstract

With the rapid growth of crowdsourcing platforms it has become easy and relatively inexpensive to collect a dataset labeled by multiple annotators in a short time. However due to the lack of control over the quality of the annotators, some abnormal annotators may be affected by position bias which can potentially degrade the quality of the final consensus labels. In this paper we introduce a statistical framework to model and detect annotator's position bias in order to control the false discovery rate (FDR) without a prior knowledge on the amount of biased annotators – the expected fraction of false discoveries among all discoveries being not too high, in order to assure that most of the discoveries are indeed true and replicable. The key technical development relies on some new knockoff filters adapted to our problem and new algorithms based on the Inverse Scale Space dynamics whose discretization is potentially suitable for large scale crowdsourcing data analysis. Our studies are supported by experiments with both simulated examples and real-world data. The proposed framework provides us a useful tool for quantitatively studying annotator's abnormal behavior in crowdsourcing.

1. Introduction

In applications, building good predictive models is challenging primarily due to the difficulties in obtaining annotated training data. A traditional way for data labeling is to hire a small group of experts to provide labels for the entire set of data. However, such an approach can be expensive and time consuming for large scale data. Thanks to the wide spread of crowdsourcing platforms (e.g., [MTurk](#), [Innocentive](#), [CrowdFlower](#), [CrowdRank](#), and [Allourideas](#)), a much more efficient way is to post unlabeled data to a crowdsourcing marketplace, where a big crowd of low-paid workers can be hired instantaneously to perform labeling tasks ([Sheng et al., 2008](#); [Snow et al., 2008](#); [Hsueh et al., 2009](#); [Nowak & Rüger, 2010](#); [Chen et al., 2009](#)).

Despite of its high efficiency and immediate availability, crowd labeling raises many new challenges. Since typical crowdsourced tasks are tedious and annotators usually come from a diverse pool including genuine experts, novices, biased workers, and malicious annotators, labels generated by the crowd suffer from low quality. Thus, all crowdsourcers need strategies to ensure the reliability of answers. In other words, outlier detection is a critical task in order to achieve a robust labeling results. Various methods have been developed in literature for outlier detection, of which majority voting strategy ([Gygli et al., 2013](#); [Jiang et al., 2013](#)) is the most typical one. In this setting, each pair is allocated to multiple annotators and their opinions are averaged over so as to identify and discard noisy data provided by unreliable raters. They thus require large amount of pairwise labels to be collected. More impor-

tantly as a local outlier detection method, majority voting is ineffective in identifying outliers that can cause global ranking inconsistencies (Fu et al., 2014; 2016). The work in (Xu et al., 2013) attacks this problem and formulates the outlier detection as a LASSO problem based on sparse approximations of the cyclic ranking projection of paired comparison data in Hodge decomposition. Regularization paths of the LASSO problem could provide an order on samples tending to be outliers. However, these work all treat pairwise comparison judgements as independent random outliers, which are typically defined to be data samples that have unusual deviations from the remaining data.

In this paper, instead of modeling the random effect of sample-wise outliers, we are primarily interested in the fixed effect where the annotators are influenced by positions when labeling in pairwise comparison setting. Such an annotator’s position bias (Day, 1969) is ubiquitous in uncontrolled crowdsourced ranking experiments. In our studies, annotator’s position bias typically arises from: i) **the ugly**: one typically clicks one side more often than another. As some pairs are highly confusing or annotators get too tired, in these cases, some annotators tend to click one side hoping to simply raise their record to receive more payment; while for pairs with substantial differences, they click as usual. ii) **the bad**: some extremely careless annotators, or robots pretending to be human annotators, actually do not look at the instances and click one side all the time to quickly receive pay for work. Such kinds of annotators may significantly deteriorate the quality of crowdsourcing data and increase the cost of acquiring annotations (since each raw feedback comes with a cost: the task requestor has to pay workers a pre-specified monetary reward for each labeling they provide, usually, regardless of the feedback correctness). Although it might be relatively easy to identify the bad annotators above by inspecting their inputs, it is impossible for eye inspection to pick up those ugly annotators with mixed behaviors. Therefore it is desired to design a statistical framework to quantitatively detect and eliminate annotator’s position bias for crowdsourcing platforms in market. Such a systematic study, up to the author’s knowledge, however has not been seen in literature.

In this paper, we propose a linear model with annotator’s position bias and new algorithms to find good estimates with an automatic control on the false discovery rate (FDR) – the expected fraction of false discoveries among all discoveries. To understand FDR, imagine that we have a detection method that has just made 100 discoveries. Then, if our method is known to control the FDR at the 10% level, this means that with high probability, we can expect at most 10 of these discoveries to be false and, therefore, at least 90 to be true and replicable. Such a FDR control is desired when we don’t have a prior knowledge about the amount of bad or ugly annotators and typical statistical es-

timates will lead to an over-identification of them.

Specifically, our contributions in this work are highlighted as follows:

- (A) A linear model with annotator’s position bias as fixed effects;
- (B) New algorithms to find good estimates of such position bias, etc., based on Inverse Scale Space dynamics and its discretization Linearized Bregman Iteration;
- (C) New knockoff filters for FDR control adapted to our setting, which aims to mimic the correlation structure found within the original features for position bias;
- (D) Extensive experimental validation based on one simulated and four real-world crowdsourced datasets.

2. Methodology

In this section, we systematically introduce the methodology for annotator’s position bias estimation. Specifically, we first start from a basic linear model with different types of noise models, which have been successfully used widely in literature. Then we introduce a new dynamic approach with unbiased estimator called Inverse Scale Space (ISS). Based on this, we present the modified knockoff filter for FDR control in details.

2.1. Basic Linear Model

Let $V = \{1, 2, \dots, n\}$ be the set of nodes and $E = \{(\alpha, i, j) : i, j \in V, \alpha \in U\}$ be the set of edges, where U is the set of all annotators. Suppose the pairwise ranking data is given as $Y : E \rightarrow \mathbb{R}$. $Y_{ij}^\alpha > 0$ means α prefers i to j and $Y_{ij}^\alpha \leq 0$ otherwise. The magnitude of Y_{ij}^α can represent the degree of preference and it varies in applications. It can be dichotomous choice $\{\pm 1\}$, k -point Likert scale (e.g. $k = 3, 4, 5$), or even real values.

In this paper, consider the following linear model:

$$Y_{ij}^\alpha = \theta_i - \theta_j + z_{ij}^\alpha \quad (1)$$

where $\theta : V \rightarrow \mathbb{R}$ is some common score on V and the residue z_{ij}^α may have interesting structures in crowdsourcing settings.

The annotators might have different effects on the residues. While for most annotators, the deviations from the common score are due to random noise; occasionally the annotators deviate from the common behavior regularly – some careless ones always choose the left or the right candidate in comparisons, but others only do this when they get too confused to decide. Such behaviors can be modeled in the following way,

$$z_{ij}^\alpha = \gamma^\alpha + \varepsilon_{ij}^\alpha, \quad (2)$$

where γ^α measures an annotator's position bias in a fixed effect, and the remainder ε_{ij}^α measures the random effect in sampling which is assumed to be sub-gaussian noise. For example, a positive value of γ^α means the annotator α is more likely to prefer the left choice. Under the random design of pairwise comparison experiments, a candidate should be placed on the left or the right randomly, so the position should not affect the choice of a careful (good) annotator. Therefore γ^α is assumed to be sparse, i.e., zero for most of annotators, and a nonzero position bias γ^α means the annotator α is either always choosing one position over the other (bad) or occasionally incurring this when they get too confused or tired (ugly).

We note that (2) should not be confused with recent studies in (Fu et al., 2014; Xu et al., 2013) on outlier detection problem, $z_{ij}^\alpha = \gamma_{ij}^\alpha + \varepsilon_{ij}^\alpha$, where γ_{ij}^α models sparse outliers for each sample (α, i, j) , which only measures the random effect of samples rather than the fixed effect of annotators. By modeling the annotator's fixed effect on position bias, one can systematically classify the annotators into the good, the ugly, and the bad according to their behaviors.

2.2. ISS/LBI

Define the gradient operator by $\delta_0 : \mathbb{R}^{|V|} \rightarrow \mathbb{R}^{|E|}$ such that $(\delta_0 \theta)(i, j, \alpha) = \theta_i - \theta_j$, and the annotator operator $A : \mathbb{R}^{|A|} \rightarrow \mathbb{R}^{|E|}$ by $(A\gamma)(i, j, \alpha) = \gamma^\alpha$, then the model above can be rewritten as:

$$Y = \delta_0 \theta + A\gamma + \varepsilon, \quad (3)$$

In this case, detecting the annotators affected by position bias can be reformulated as: learning a sparse vector γ from given data (δ_0, A, Y) . To solve such a problem, in this paper, we choose a new approach based on the following dynamics,

$$\frac{dp}{dt} = A^T(Y - \delta_0 \theta - A\gamma) \quad (4a)$$

$$0 = \delta_0^T(Y - \delta_0 \theta - A\gamma) \quad (4b)$$

$$p \in \partial \|\gamma\|_1. \quad (4c)$$

Its solution path can be easily solved by a sequence of non-negative least squares, see (Osher et al., 2016) and references therein. In this paper we use the free R-package (Xiong et al., 2016).

In (Osher et al., 2016), it has been shown that the dynamics above has several advantages over the traditional LASSO approach, which can be formulated as follows in our setting

$$\min_{\theta, \gamma} \frac{1}{2} \|Y - \delta_0 \theta - A\gamma\|_2^2 + \lambda \|\gamma\|_1. \quad (5)$$

First of all, the dynamics above is statistically equivalent to LASSO in terms of model selection consistency but may

render oracle estimator which is bias-free, while the LASSO estimator is well-known biased. In this sense the ISS path can be better than the LASSO path. Here the solution path $\hat{\gamma}(t)_{t:0 \rightarrow \infty}$ plays the same role of the regularization path of LASSO $\hat{\gamma}(\lambda)_{\lambda:\infty \rightarrow 0}$ with roughly $t = 1/\lambda$, where the important features (variables) are selected before the noisy ones. Following the tradition in image processing, such a dynamics is called *Inverse Scale Space* (ISS).

Beyond the charming statistical properties, ISS also admits an extremely simple discrete approximation, i.e., the Linearized Bregman Iteration (LBI), which has been widely used in image reconstruction with TV-regularization. Adapted to our setting, the discretized algorithm is illustrated in Algorithm 1, which is scalable, easy for parallelization, and particularly suitable for large scale crowdsourced ranking data analysis.

Algorithm 1 LBI in correspondence to (3)

Initialization: Given parameter κ and Δt , define $k = 0, w^0 = 0, \theta^0 = (\delta_0^T \delta_0)^\dagger \delta_0^T Y, \gamma^0 = 0$.

Iteration:

$$w^{k+1} = w^k + A^T(Y - \delta_0 \theta^k - A\gamma^k) \Delta t. \quad (6a)$$

$$\gamma^{k+1} = \kappa \text{shrink}(w^{k+1}). \quad (6b)$$

$$\theta^{k+1} = \theta^k + \kappa \delta_0^T(Y - \delta_0 \theta^k - A\gamma^k) \Delta t. \quad (6c)$$

Stopping: exit when $k\Delta t > t$.

where $\text{shrink}(x) := \text{sign}(x) \max\{|x| - 1, 0\}$.

2.3. FDR Control and New Knockoff Filter

A crucial question for LASSO and ISS is how to choose the regularization parameter λ and t in real-world data. After all, different parameters can give different bad or ugly annotator sets. Traditional methods either require a prior knowledge on the amount of such annotators which is often unknown in practice, or some statistically optimal choice of such regularization parameters. Extensive studies in statistics have shown that such parameter tuning typically lead to an over estimation of the sparse signal, therefore False Discovery Rate (FDR) control is necessary (Barber & Candès, 2015) which is adopted in this paper.

FDR is defined as the expected proportion of false discoveries among the discoveries. Putting in a mathematical way, here we consider

$$FDR = \mathbb{E} \left[\frac{\#\{\alpha : \gamma^\alpha = 0, \hat{\gamma}^\alpha \neq 0\}}{\#\{\alpha : \hat{\gamma}^\alpha \neq 0\} \wedge 1} \right].$$

To control the FDR means to control the accuracy of the bad/ugly annotators we detected to see if they are reasonable ones.

Recently, a new method called knockoff filter (Barber & Candès, 2015) is proposed to automatically control FDR in standard linear regression without a prior knowledge on the sparsity. In this paper, such an approach will be extended to our linear model (3) and the algorithms, where both non-sparse θ and sparse γ co-exist in the model. The extended method consists of the same three steps as in (Barber & Candès, 2015), where the key difference lies in the knockoff feature construction adapted to our setting.

1. Construct knockoff features: let \tilde{A} be knockoff features that satisfy

$$\tilde{A}^T \tilde{A} = A^T A, \quad A^T \tilde{A} = A^T A - \text{diag}(s), \quad \delta_0^T \tilde{A} = \delta_0^T A \quad (7)$$

where s is positive and can be solved by SDP:

$$\begin{aligned} \max_s \quad & \sum_j s_j \\ \text{s.t.} \quad & 0 \leq s_j \leq 1 \\ & \text{diag}(s) \preceq 2A^T(I - H)A, \end{aligned}$$

with $H := \delta_0(\delta_0^T \delta_0)^\dagger \delta_0^T$. Let $Q \in \mathbb{R}^{|E| \times |A|}$ be an orthonormal matrix such that $\delta_0^T Q = 0$, $A^T Q = 0$, which requires $|E| \geq 2|A| + |V|$ easily met in crowdsourcing. Then (7) can be satisfied by defining

$$\tilde{A} := A - (I - H)A(A^T(I - H)A)^{-1} \text{diag}(s) + QC$$

where $C \in \mathbb{R}^{|A| \times |A|}$ satisfies $C^T C = 2\text{diag}(s) - \text{diag}(s)(A^T(I - H)A)^{-1} \text{diag}(s)$.

Now define the extended design matrix $A_{ko} = [A, \tilde{A}]$ and $\gamma_{ko} = [\gamma, \tilde{\gamma}]^T$, then replace A with A_{ko} and γ with γ_{ko} in (5), (4) or Alg. 1, we can get solution path $\hat{\gamma}_{ko}(\lambda)$ (or $\hat{\gamma}_{ko}(t)$).

2. Generate knockoff statistics for every original feature: define Z_j to be the first entering time for A_j , i.e., $Z_j = \sup\{\lambda : \hat{\gamma}_j(\lambda) \neq 0\}$ for LASSO (or $\sup\{1/t : \hat{\gamma}_j(t) \neq 0\}$ for ISS/LBI) and \tilde{Z}_j can be defined similarly. Then the knockoff statistics becomes

$$W_j = \max(Z_j, \tilde{Z}_j) \text{sign}(Z_j - \tilde{Z}_j) \quad (8)$$

3. Choose variables based on the knockoff statistics: define the selected variable set $\hat{S} = \{j : W_j \geq T_{0/1}\}$, where

$$T_{0/1} = \min\{t : \frac{0/1 + \#\{j : W_j \leq -t\}}{\#\{j : W_j \geq t\}} \leq q\}.$$

T_0 is knockoff cut and T_1 is knockoff+ cut.

It can be shown that the new knockoff filter above indeed controls FDR in the following sense, whose proof is similar to that of (Barber & Candès, 2015) (collected in Supplementary Materials for completeness).

Theorem 1 *If ϵ is i.i.d $N(0, \sigma^2)$ and $|E| \geq 2|A| + |V|$, then for any $q \in [0, 1]$, the knockoff filter with ISS/LBI (or LASSO) satisfies*

$$\mathbb{E} \left[\frac{\#\{j : \gamma_j = 0 \text{ and } j \in \hat{S}\}}{\#\{j : j \in \hat{S}\} + q^{-1}} \right] \leq q$$

and the knockoff+ method satisfies

$$\mathbb{E} \left[\frac{\#\{j : \gamma_j = 0 \text{ and } j \in \hat{S}\}}{\#\{j : j \in \hat{S}\}} \right] \leq q$$

Remark 1 *There is an equivalent reformulation of (3) to eliminate the non-sparse structure variable θ and convert it to a standard LASSO. Let δ_0 have a full SVD decomposition $\delta_0 = U\Sigma V^T$ and $U = [U_1, U_2]$, where U_1 is an orthonormal basis of the column space $\text{col}(\delta_0)$ and U_2 becomes an orthonormal basis for $\ker(\delta_0^T)$. Then*

$$U_2^T Y = U_2^T A \gamma + U_2^T \epsilon. \quad (9)$$

Let $y = U_2^T Y$, $X = U_2^T A$, $e = U_2^T \epsilon$, then e is i.i.d $N(0, \sigma^2)$

$$y = X \gamma + e. \quad (10)$$

Based on this, we can use the original knockoff filter \tilde{X} in (Barber & Candès, 2015) to select the position-biased annotators.

A shortcoming of this approach lies in the full SVD decomposition which might be too expensive for large scale problem. The former approach will not suffer from this. However, one can see in the following theorem both approaches are in fact equivalent. Therefore such a reformulation provides us a conceptual insight in understanding the construction of knockoff filters.

Theorem 2 *The approach in Remark 1 is equivalent to what we proposed above in the following sense:*

- The knockoff features of (10) satisfies $\tilde{X} = U_2^T \tilde{A}$ and $\tilde{A} = U_2 \tilde{X} + U_1 U_1^T A$;
- The knockoff statistics constructed by ISS (or LASSO) for both procedures are exactly the same.

Both knockoff filters above can choose variables with FDR control but the estimator $(\hat{\theta}, \hat{\gamma}_{ko})$ consists of knockoff features, so we need to reestimate $\hat{\theta}, \hat{\gamma}$ after bad annotator detection by passing to a least square while only keeping those nonzero parameters and features. Suppose that \hat{S} is the set of bad or ugly annotators given by knockoff filters, then one can find the final estimators by

$$(\hat{\theta}, \hat{\gamma}) = \arg \min_{\theta, \gamma_{\hat{S}}} \|Y - \delta_0 \theta - A_{\hat{S}} \gamma_{\hat{S}}\|_2^2. \quad (11)$$

Table 1. Knockoff with $q = 10\%$ via ISS.
(a) Control of Actual FDR

	p2=40%	p2=50%	p2=60%	p2=70%
p1=10%	0.0959	0.0833	0.1198	0.1229
p1=20%	0.0917	0.0989	0.0935	0.1006
p1=30%	0.0919	0.0991	0.0921	0.0854
p1=40%	0.1062	0.1034	0.0998	0.1184

(b) Number of True Discoveries

	p2=40%	p2=50%	p2=60%	p2=70%
p1=10%	49.95	50.00	50.00	50.00
p1=20%	49.90	50.00	50.00	50.00
p1=30%	49.80	50.00	50.00	50.00
p1=40%	49.75	49.95	50.00	50.00

 Table 2. Knockoff with $q = 10\%$ via LASSO.
(a) Control of Actual FDR

	p2=40%	p2=50%	p2=60%	p2=70%
p1=10%	0.0711	0.1326	0.1433	0.1256
p1=20%	0.0998	0.0954	0.0970	0.0780
p1=30%	0.1044	0.0918	0.1093	0.1061
p1=40%	0.0843	0.1035	0.1063	0.0941

(b) Number of True Discoveries

	p2=40%	p2=50%	p2=60%	p2=70%
p1=10%	49.95	50.00	50.00	50.00
p1=20%	49.90	50.00	50.00	50.00
p1=30%	49.90	50.00	50.00	50.00
p1=40%	49.85	50.00	50.00	50.00

3. EXPERIMENTS

In this section, five examples are exhibited with both simulated and real-world data to illustrate the validity of the analysis above and applications of the methodology proposed. The first example is with simulated data while the latter four exploit real-world data collected by crowdsourcing.

3.1. Simulated Study

Settings We first validate the proposed algorithm on simulated binary data labeled by 150 annotators. Of the 150 annotators we have 100 *good* annotators (annotators 1 to 100 without position bias) and 50 *bad/ugly* annotators (annotators 101 to 150 with position bias). We note that for good annotators, it does not mean that each worker always present the correct labels. Instead, it means that they also have the probability to make incorrect judgements due to certain reasons, rather than position effect.

Specifically, we first create a random total order on n candidates V as the ground-truth and add paired comparison edges $(i, j) \in E$ to graph $G = (V, E)$ until a complete graph, with the preference direction following the ground-truth order. Here we choose $n = |V| = 16$, which is consistent with the third real-world dataset with smallest node size. Then, for good annotators, they make judgements with an incorrect probability p_1 (i.e., $p_1\%$ of E is reversed in preference direction), while for bad/ugly annotators, they are with a probability of p_2 disturbed by position effect.

Evaluation metrics Two metrics are employed to evaluate the performance of the proposed algorithms. The first one is *Control of Actual FDR*, the second is *Number of True Discoveries*.

Experimental results With different choices of p_1 and p_2 , the mean *Control of Actual FDR* and *Number of True Discoveries*

Table 3. Position biased annotators detected in Human age dataset, together with the click counts of each side (i.e., Left and Right).

ID	Left	Right	ID	Left	Right
40	40	0	50	60	3
51	63	0	59	213	66
94	0	30	64	5	14
12	90	270	70	191	9
18	74	25	72	5	24
34	32	48	77	11	1
38	110	15	81	4	28
43	79	1	91	79	5
46	40	10			

coveries with $q = 10\%$ over 100 runs are shown in Table 1 to measure the performance of knockoff filter via ISS in position biased annotator detection. It can be seen that via a knockoff filter, ISS can provide an accurate detection of position biased annotators (indicated by *control of actual FDR* around 10% and *Number of True Discoveries* around 50). Comparable results of LASSO with $q = 10\%$ can be found in Table 2. It can be seen that via knockoff filter, both LASSO and ISS can provide an accurate detection of position biased annotators. This result is consistent with the theoretical comparison between LASSO and ISS discussed in (Osher et al., 2016), where ISS/LBI has similar theoretical guarantees as LASSO, but with bias-free and simpler implementation (the 3 line algorithm in Sec. 2.2) properties.

3.2. Real-world Datasets

As there is no ground-truth for position biased annotators in real-world data, one can not compute *control of actual FDR* and *Number of True Discoveries* as in simulated data to evaluate the detection performance here. In this subsection, we inspect the annotators returned by knockoff filter via ISS/LASSO under $q = 10\%$ to see if they are reasonably good position biased workers.

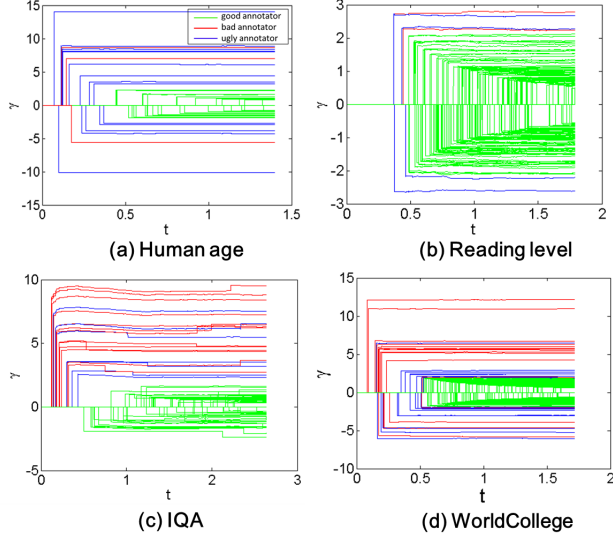


Figure 1. ISS regularization path of four real-world datasets (Green: the good; Red: the bad; Blue: the ugly).

3.2.1. HUMAN AGE

In this dataset, 30 images from human age dataset FG-NET¹ are annotated by a group of volunteer users on [ChinaCrowds](#) platform. The groundtruth age ranking is known to us. The annotator is presented with two images and given a binary choice of which one is older. Totally, we obtain 14,011 pairwise comparisons from 94 annotators. By adopting the knockoff-based algorithm we proposed, LASSO and ISS identify exactly the same set of abnormal annotators (i.e., 17 users) at $q=10\%$, as is shown in Table 3.

It is easy to see that these annotators can be divided into two types: (1) *the bad*: click one side all the time (with ID in red); (2) *the ugly*: click one side with high probability (with ID in blue). Besides, the regularization paths of ISS can be found in Figure 1(a), where the position biased annotators detected mostly lie outside the majority of the paths. Note that since we allow a small percentage of false positives, some ugly annotators might be good in reality as well.

To see the effect of position biased annotators on global ranking scores, Table 4 shows the outcomes of two ranking algorithms, namely original and corrected. The original is calculated by least squares problems on all of the pairwise comparisons, while the corrected is obtained by the correction step via knockoff illustrated in Section 2.3. It is easy to see that the removal of position biased annotators often changes the orders of some competitive images, such as ID=11 and ID=21, ID=30 and ID=8, etc.

To see which ranking is more reasonable, Table 5 shows

Table 4. Comparison of original vs. corrected rankings on Human age dataset. The integer represents the ranking position and the number in parenthesis represents the global ranking score returned by the corresponding algorithm.

ID	Original.	Corrected.	ID	Original.	Corrected.
28	1 (0.7780)	1 (0.7573)	23	16 (0.0208)	16 (0.0099)
3	2 (0.6661)	2 (0.6771)	8	17 (0.0086)	18 (-0.0024)
14	3 (0.5653)	3 (0.5647)	30	18 (-0.0025)	17 (0.0055)
29	4 (0.4482)	4 (0.4490)	12	19 (-0.0201)	19 (-0.0632)
21	5 (0.4087)	6 (0.4086)	13	20 (-0.1961)	21 (-0.2111)
11	6 (0.4059)	5 (0.4343)	15	21 (-0.2160)	23 (-0.2791)
7	7 (0.3873)	7 (0.4017)	25	22 (-0.2166)	20 (-0.2099)
5	8 (0.3634)	8 (0.3478)	16	23 (-0.2551)	24 (-0.2887)
27	9 (0.3582)	9 (0.3377)	2	24 (-0.3710)	22 (-0.2785)
24	10 (0.2064)	10 (0.1722)	9	25 (-0.4158)	25 (-0.3949)
6	11 (0.0932)	13 (0.1084)	1	26 (-0.6135)	27 (-0.6376)
4	12 (0.0914)	12 (0.1207)	18	27 (-0.6249)	26 (-0.6180)
22	13 (0.0896)	11 (0.1032)	19	28 (-0.6653)	28 (-0.6390)
17	14 (0.0872)	14 (0.1232)	10	29 (-0.6969)	29 (-0.7040)
20	15 (0.0816)	15 (0.0559)	26	30 (-0.7660)	30 (-0.7509)

Table 5. Groundtruth ranking of the competitive images highlighted with red color in Table 4.

11 > 21
22 > 4 > 6
30 > 8
25 > 13 > 16 > 2 > 15
18 > 1

the *groundtruth* ranking of these competitive images. We can find from this table that, compared with the original ranking, the corrected one is in more agreement with the groundtruth ranking, which further shows that: i) position biased annotators may disturb the ranking to a departure from the real ranking. ii) pairs with little differences are more likely to lead to position biased annotations. From this viewpoint, we can see that the knockoff-based FDR-controlling method indeed effectively selects the position biased annotators.

3.2.2. READING LEVEL

The second dataset is a subset of reading level dataset (Chen et al., 2013), which contains 490 documents. 8,000 pairwise comparisons are collected from 346 annotators using [CrowdFlower](#) crowdsourcing platform. More specifically, each annotator is asked to provide his/her opinion on which text is more challenging to read and understand. Table 6 shows the position biased annotators detected from this dataset, together with the ISS regularization path shown in Figure 1(b). It is easy to see that LASSO and ISS picked out the same 6 annotators as position biased ones. In terms of the small number of bad annotators detected, we can say that the overall quality of annotators on this task is relatively high.

¹<http://www.fgnet.rsunit.com/>

Table 6. Position biased annotators detected in Reading level.

ID	Left	Right
50	5	0
69	6	0
122	19	3
148	4	19
167	22	8
275	7	22

Table 7. Position biased annotators detected in reference image 1.

ID	Left	Right	ID	Left	Right
2	55	0	300	11	0
23	42	0	317	20	0
29	58	0	334	90	0
99	29	0	33	15	1
177	77	0	34	8	1
190	36	0	103	74	4
228	14	0	133	20	11
241	22	0	207	46	2
259	96	0	260	49	2
287	34	0	304	17	1

3.2.3. IMAGE QUALITY ASSESSMENT

The third dataset is a pairwise comparison dataset for subjective image quality assessment (IQA), which contains 15 reference images and 15 distorted versions of each reference, for a total of 240 images which come from two publicly available datasets LIVE, (LIV, 2008) and IVC (IVC, 2005). Totally, 342 observers, each of whom performs a varied number of comparisons via Internet, provide 52,043 paired comparisons for crowdsourced subjective image quality assessment. Note that the number of responses each reference image received is different in this dataset.

To validate whether the annotators we detected are good position biased annotators or not, we randomly take reference image 1 as an illustrative example while other reference images exhibit similar results. Table 7 shows the annotators with position bias picked by knockoff filter and the ISS regularization path is shown in Figure 1(c). In this dataset, the abnormal annotators picked out by LASSO and ISS are also exactly the same. It is easy to see that annotators picked out are mainly those clicking on one side almost all the time. Besides, it is interesting to see that all these bad annotators highlighted with red color in Table 7 click the left side all the time. We then go back to the crowdsourcing platform and find out that the reason behind this is a default choice on the left button thus induces some lazy annotators cheat for the annotation task.

3.2.4. WORLD COLLEGE RANKING

We now apply the knockoff filter to the WorldCollege dataset, which is composed of 261 colleges. Using the Al-lourideaus crowdsourcing platform, a total of 340 distinct annotators from various countries (e.g., USA, Canada, Spain, France, Japan) are shown randomly with pairs of

Table 8. Position biased annotators detected in WorldCollege.

ID	Left	Right	ID	Left	Right
56	17	0	25	17	6
75	0	3	59	9	29
101	26	0	87	11	62
115	34	0	122	13	9
145	0	27	134	20	7
166	35	0	140	12	4
209	127	0	156	189	67
222	0	2	189	2	12
245	0	34	191	23	7
256	0	21	202	2	8
267	45	0	207	23	10
268	148	0	208	10	2
275	1	0	239	11	2
289	35	0	258	2	13
299	31	0	270	20	70
321	33	0	276	16	54
323	35	0	320	253	324
338	0	21	330	4	10

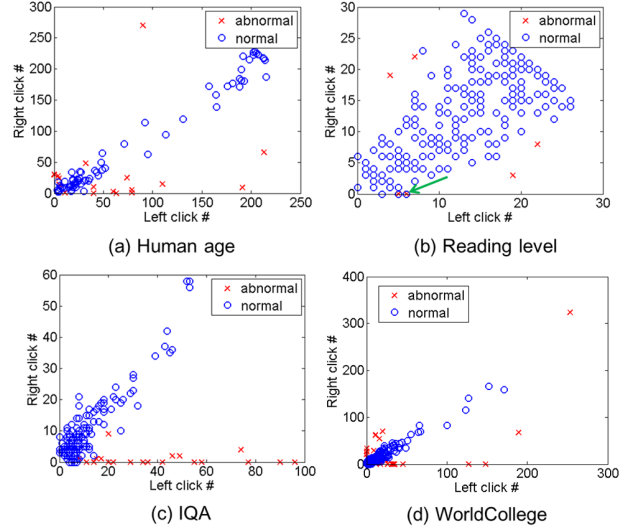


Figure 2. Number of left clicks vs. right clicks of abnormal and normal annotators on four real-world datasets.

these colleges, and asked to decide which of the two universities is more attractive to attend. Finally, we obtain a total of 8,823 pairwise comparisons. We then apply knock-off filter to the resulting dataset and find out that both LASSO and ISS selected 36 annotators as position biased ones, as is shown in Table 8 and Figure 1(d). It is easy to see that similar to the human age dataset, the annotators picked out are either clicking one side all the time, or clicking one side with high probability.

3.3. Discussion

Someone may argue that setting a threshold on the ratio of left/right answers can be an easy way to detect position biased annotators. To illustrate why simply setting a threshold does not work, Figure 2 shows the click counts of each side (i.e., X-axis: number of left clicks; Y-axis: number of right clicks), where each color \circ/\times represents

one annotator. It is easy to see that there are indeed some overlaps between abnormal and normal annotators. For example, in reading level dataset, annotators with ID=69 and ID=57 both provide 6:0 on the ratio of left/right clicks. However, ID=69 is detected as abnormal annotator, while ID=57 as normal one. To figure out the reason behind this, we further compute the Match Ratio (MR) of these two annotators with the global ranking scores obtained by all pairwise comparisons and find that $MR_{ID=69} = 3/6$ and $MR_{ID=57} = 5/6$. This indicates that the position biased annotator (i.e., ID=69) we picked out is the one not only with one-side click but also with a large deviation with the majority. Similar results can be easily found in other three datasets.

4. Related Work

4.1. Outlier Detection

Outliers are often referred to as abnormalities, discordants, deviants, or anomalies in data. Generally speaking, there can be two types of outliers: (1) samples as outliers; (2) subjects as outliers. Hawkins formally defined in (Hawkins, 1980) the concept of an outlier as follows: “An outlier is an observation which deviates so much from the other observations as to arouse suspicions that it was generated by a different mechanism.” Outliers are rare events, but once they have occurred, they may lead to a large instability of models estimated from the noisy data. For type (1), many methods have been developed for outlier detection, such as distribution-based (Barnett & Lewis, 1994), depth-based (Johnson et al., 1998), distance-based (Knorr & Ng, 1999; Knorr et al., 2000), density-based (Breunig et al., 2000), and clustering-based (Jain et al., 1999) methods. For subject-based outlier detection, some sophisticated methods have been proposed to model annotators’ judgments. Recently, (Chen et al., 2013) propose a Crowd-BT algorithm to detect spammers and malicious annotators: spammers assign random labels while malicious annotators assign the wrong label most of the time. Besides, (Raykar & Yu, 2011) defines a score to rank the annotators for crowdsourced labeling tasks. Furthermore, (Raykar & Yu, 2012) presents an empirical Bayesian algorithm called SpEM to eliminate the spammers and estimate the consensus labels based only on the good annotators. However, a phenomenon that has annoyed researchers who have used paired comparison tests is position bias or testing order effects. Until now, little work have been found for such kind of position biased annotator detection, which is our main focus in this paper.

4.2. FDR Control and Knockoff Method

Most variable selection techniques in statistics such as LASSO suffer from over-selection as picking up too many

false positives by leaving out few true positives. In order to offer guarantees on the accuracy of the selection, it is desired to control the false discovery rate (FDR) among all the selected variables. The Benjamini-Hochberg (BH) procedure (Benjamini & Hochberg, 1995) is a typical method known to control FDR under independence scenarios. Recently, (Barber & Candès, 2015) developed a new knockoff filter method for FDR control for general dependent features as long as the sample size is larger than that of parameters. In this paper, we extend this method to our setting with mixed parameters of both nonsparse and sparse ones to achieve the same FDR control.

4.3. Inverse Scale Space and Linearized Bregman Iteration

Linearized Bregman Iteration (LBI) has been widely used in image processing and compressed sensing (Osher & Yin, 2005; Yin et al., 2008) even before its limit form as Inverse Scale Space (ISS) dynamics (Burger et al., 2005). ISS/LBI at least have two advantages over the popular LASSO in variable selection: (1) ISS may give unbiased estimator (Osher et al., 2016), under nearly the same condition for model selection consistency as LASSO whose estimators are however always biased (Fan & L, 2001). (2) LBI, regarded as a discretization of ISS dynamics, is an extremely simple algorithm which combines an iterative gradient descent algorithm together with a soft thresholding. It only runs in a single path and regularization is achieved by early stopping like boosting algorithms (Osher et al., 2016), which may save the computational cost greatly and thus suitable for large scale implementation (Yuan et al., 2013).

5. Conclusion

Annotator’s position bias is ubiquitous in crowdsourced ranking data, which, up to our knowledge, has not been systematically addressed in literature. In this paper, we propose a statistical model for annotator’s position bias with pairwise comparison data on graphs, together with new algorithms to reach statistically good estimates with a FDR control based on some new design of knockoff filters. FDR control here does not need a prior knowledge on the sparsity of position bias, i.e., the amount of bad or ugly annotators. Such a framework is valid for both traditional LASSO estimator and the new dynamic approach based on ISS/LBI with debiased estimator and scalable implementations which is desired for crowdsourcing experiments. Experimental studies are conducted with both simulated examples and real-world datasets. Our results suggest that the proposed methodology is an effective tool to investigate annotator’s abnormal behavior in modern crowdsourcing data.

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References

- Subjective quality assessment ircryn/ivc database. <http://www2.ircryn.ec-nantes.fr/ivcdb/>, 2005. 7
- LIVE image & video quality assessment database. <http://live.ece.utexas.edu/research/quality/>, 2008. 7
- Barber, R. and Candès, E. Controlling the false discovery rate via knockoffs. *The Annals of Statistics*, 43(5):2055–2085, 2015. 3, 4, 8, 1
- Barnett, V. and Lewis, T. *Outliers in statistical data*, volume 3. Wiley New York, 1994. 8
- Benjamini, Y. and Hochberg, Y. Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society. Series B (Methodological)*, 57(1):289–300, 1995. 8
- Breunig, M., Kriegel, H., Ng, R., and Sander, J. LOF: identifying density-based local outliers. In *Proceedings of the ACM International Conference on Management of Data*, volume 29, pp. 93–104, 2000. 8
- Burger, M., Osher, S., Xu, J., and Gilboa, G. *Nonlinear inverse scale space methods for image restoration*. Springer, 2005. 8
- Chen, K., Wu, C., Chang, Y., and Lei, C. A crowdsourcable QoE evaluation framework for multimedia content. In *ACM International Conference on Multimedia*, pp. 491–500, 2009. 1
- Chen, X., Bennett, P., Collins-Thompson, K., and Horvitz, E. Pairwise ranking aggregation in a crowdsourced setting. In *International Conference on Web Search and Data Mining*, pp. 193–202, 2013. 6, 8
- Day, R. Position bias in paired product tests. *Journal of Marketing Research*, 6(1):98–100, 1969. 2
- Fan, J. and L, R. Variable selection via nonconcave penalized likelihood and its oracle properties. *Journal of American Statistical Association*, 96(456):1348–1360, 2001. 8
- Fu, Y., Hospedales, T., Xiang, T., Gong, S., and Yao, Y. Interestingness prediction by robust learning to rank. In *European Conference on Computer Vision*, pp. 488–503. 2014. 2, 3
- Fu, Y., Hospedales, T., Xiang, T., Xiong, J., Gong, S., Wang, Y., and Yao, Y. Robust subjective visual property prediction from crowdsourced pairwise labels. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 38(3):563–577, 2016. 2
- Gygli, M., Grabner, H., Riemenschneider, H., Nater, F., and Gool, L. The interestingness of images. In *IEEE International Conference on Computer Vision*, pp. 1633–1640, 2013. 1
- Hawkins, D. *Identification of Outliers*, volume 11. Springer, 1980. 8
- Hsueh, P., Melville, P., and Sindhwani, V. Data quality from crowdsourcing: a study of annotation selection criteria. In *NAACL HLT Workshop on Active Learning for Natural Language Processing*, pp. 27–35, 2009. 1
- Jain, A., Murty, M., and Flynn, P. Data clustering: a review. *ACM Computing Surveys*, 31(3):264–323, 1999. 8
- Jiang, Y., Wang, Y., Feng, R., Xue, X., Zheng, Y., and Yang, H. Understanding and predicting interestingness of videos. In *AAAI Conference on Artificial Intelligence*, volume 1, pp. 2, 2013. 1
- Johnson, T., Kwok, I., and Ng, R. Fast computation of 2-dimensional depth contours. In *ACM International Conference on Knowledge Discovery and Data Mining*, pp. 224–228, 1998. 8
- Knorr, E. and Ng, R. Finding intensional knowledge of distance-based outliers. In *International Conference on Very Large Data Bases*, pp. 211–222, 1999. 8
- Knorr, E., Ng, R., and Tucakov, V. Distance-based outliers: algorithms and applications. *International Journal on Very Large Data Bases*, 8(3-4):237–253, 2000. 8
- Nowak, S. and Rüger, S. How reliable are annotations via crowdsourcing: a study about inter-annotator agreement for multi-label image annotation. In *International Conference on Multimedia Information Retrieval*, pp. 557–566, 2010. 1

- Osher, S., Burger M. Goldfarb D. Xu J. and Yin, W. An iterative regularization method for total variation-based image restoration. *SIAM Journal on Multiscale Modeling and Simulation*, 4(2):460–489, 2005. 8
- Osher, S., Ruan, F., Xiong, J., Yao, Y., and Yin, W. Sparse recovery via differential inclusions. *Applied and Computational Harmonic Analysis*, 2016. doi: 10.1016/j.acha.2016.01.002. 3, 5, 8
- Raykar, V. and Yu, S. Ranking annotators for crowdsourced labeling tasks. In *Advances in Neural Information Processing Systems*, pp. 1809–1817, 2011. 8
- Raykar, V. and Yu, S. Eliminating spammers and ranking annotators for crowdsourced labeling tasks. *The Journal of Machine Learning Research*, 13(1):491–518, 2012. 8
- Sheng, V., Provost, F., and Ipeirotis, P. Get another label? improving data quality and data mining using multiple, noisy labelers. In *ACM International Conference on Knowledge Discovery and Data Mining*, pp. 614–622, 2008. 1
- Snow, R., O’Connor, B., Jurafsky, D., and Ng, A. Cheap and fast—but is it good?: evaluating non-expert annotations for natural language tasks. In *Conference on Empirical Methods in Natural Language Processing*, pp. 254–263, 2008. 1
- Xiong, J., Ruan, F., and Yao, Y. *A Tutorial on Libra: R package for the Linearized Bregman Algorithm in high dimensional statistics*, 2016. URL <https://cran.r-project.org/web/packages/Libra>. arXiv:1604.05910. 3
- Xu, Q., Xiong, J., Huang, Q., and Yao, Y. Robust evaluation for quality of experience in crowdsourcing. In *ACM International Conference on Multimedia*, pp. 43–52, 2013. 2, 3
- Yin, W., Osher, S., D., Jerome, and G., Donald. Bregman iterative algorithms for compressed sensing and related problems. *SIAM Journal on Imaging Sciences*, 1(1):143–168, 2008. 8
- Yuan, K., Ling, Q., Yin, W., and Ribeiro, A. A Linearized Bregman Algorithm for Decentralized Basis Pursuit. *European Signal Processing Conference*, pp. 1–5, 2013. 8

Supplementary Material

Sketchy Proof of Theorem 1.

Similar to the treatment in (Barber & Candès, 2015), we only need to prove that the knockoff statistics W_j satisfy the following two properties:

- *sufficiency property*:
 $W = f([\delta_0, A_{ko}]^T [\delta_0, A_{ko}], [\delta_0, A_{ko}]^T Y)$, which indicates W depends only on $[\delta_0, A_{ko}]^T [\delta_0, A_{ko}]$ and $[\delta_0, A_{ko}]^T Y$.
- *antisymmetry property*:
 Swapping A_j and \tilde{A}_j has the effect of switching the sign of W_j .

The second property is obvious because W_j is constructed using entering time difference. Now we go to prove the first property.

For ISS and LBI, the whole path is only determined by

$$A_{ko}^T(Y - \delta_0\theta - A_{ko}\gamma_{ko}) = A_{ko}^T Y - A_{ko}^T[\delta_0, A_{ko}][\theta^T, \gamma_{ko}^T]^T, \text{ this is actually (4b). So plugging } \hat{\theta} \text{ in (5), we get}$$

$$\delta_0^T(Y - \delta_0\theta - A_{ko}\gamma_{ko}) = \delta_0^T Y - \delta_0^T[\delta_0, A_{ko}][\theta^T, \gamma_{ko}^T]^T,$$

which is only based on $[\delta_0, A_{ko}]^T [\delta_0, A_{ko}]$ and $[\delta_0, A_{ko}]^T Y$, so is the entering time Z_j

The same reasoning holds for LASSO since

$$\min_{\theta, \gamma} \frac{1}{2} \|Y - [\delta_0, A_{ko}][\theta^T, \gamma_{ko}^T]^T\|_2^2 + \lambda \|\gamma_{ko}\|_1$$

is equivalent to

$$\min_{\theta, \gamma} \frac{1}{2} (\|Y\|_2^2 + [\theta^T, \gamma_{ko}^T][\delta_0, A_{ko}]^T [\delta_0, A_{ko}][\theta^T, \gamma_{ko}^T]^T - 2[\theta^T, \gamma_{ko}^T][\delta_0, A_{ko}]^T Y) + \lambda \|\gamma_{ko}\|_1$$

So the entire path is determined by $[\delta_0, A_{ko}]^T [\delta_0, A_{ko}]$ and $[\delta_0, A_{ko}]^T Y$.

Proof of Theorem 2.

Suppose \tilde{X} is the knockoff statistics for (10), then it satisfies

$$\tilde{X}^T \tilde{X} = X^T X, X^T \tilde{X} = X^T X - \text{diag}(s). \quad (12)$$

Let $B = A + U_2(\tilde{X} - X)$, then $\tilde{X} = U_2^T B$ and it can be verified

$$B^T B = A^T A, A^T B = A^T A - \text{diag}(s), \delta_0^T B = \delta_0^T A$$

which means B is a valid knockoff feature matrix for (3).

On the reverse, let \tilde{A} be knockoff features for (3), it is also easy to verify $\tilde{X} = U_2^T \tilde{A}$ satisfies condition (12). This establishes an injection between \tilde{X} and \tilde{A} .

The equivalence of knockoff statistics comes from the equivalence of solution paths in both approaches. To see this, (4b) actually means $\hat{\theta} = (\delta_0^T \delta_0)^\dagger \delta_0^T (Y - A_{ko} \gamma_{ko})$, plugging $\hat{\theta}$ in (4a), we get

$$\begin{aligned} \frac{dp}{dt} &= A_{ko}^T (Y - \delta_0 \hat{\theta} - A_{ko} \gamma_{ko}) \\ &= A_{ko}^T (U_2 U_2^T (Y - A_{ko} \gamma_{ko})) \\ &= (U_2^T A_{ko})^T (U_2^T Y - U_2^T A_{ko} \gamma_{ko}) \end{aligned}$$

This is equivalent to the ISS for the second procedure model (9) in Remark 1. So in both approaches, the two ISS solution paths are identical.

The same reasoning holds for LASSO, the derivative of (5) w.r.t. θ is zero at the optimal estimator which means

$$0 = \delta_0^T (Y - \delta_0 \hat{\theta} - A_{ko} \gamma_{ko})$$

$$\begin{aligned} &\|Y - \delta_0 \theta - A_{ko} \gamma_{ko}\|_2^2 \\ &= \|(I - \delta_0 (\delta_0^T \delta_0)^\dagger \delta_0^T)^T (Y - A_{ko} \gamma_{ko})\|_2^2 \\ &= \|U_2 U_2^T (Y - A_{ko} \gamma_{ko})\|_2^2 \\ &= \|U_2^T Y - U_2^T A_{ko} \gamma_{ko}\|_2^2. \end{aligned}$$

This is in fact the l_2 loss for the second procedure in Remark 1.

Finally identical paths lead to the same knockoff statistics which ends the proof.