The continuous-time equations of motions of the left $* = l$ and the right $* = r$ arm's end-effector frames are known as

$$\begin{aligned}\dot{\mathbf{x}}_* =& \boldsymbol{\upsilon}_*, \\ \dot{\mathbf{R}}_* =& \mathbf{R}_* \boldsymbol{\omega}_*^\times,\end{aligned}$$

where $\mathbf{x}_*$ are their positions, $\mathbf{R}_*$ are the rotation matrix representations of their orientations, $\boldsymbol{\upsilon}_*$ are their linear spatial velocities, and $\boldsymbol{\omega}_*$ are their angular body velocities. Let ΔT be the sampling period. Then, their discrete-time equations of motions are expressed as

$$\begin{aligned}\mathbf{x}_{*,k+1} =& \mathbf{x}_{*,k} + \Delta T \boldsymbol{\upsilon}_{*,k}, \\ \mathbf{R}_{*,k+1} =& \mathbf{R}_{*,k} \exp\left(\Delta T \boldsymbol{\omega}_{*,k}^\times\right)\end{aligned}$$

at any time step $k = 0, 1, 2, \cdots$, where the subscripts k and $k+1$ index their associated quantities at the time steps k and $k+1$, respectively.

With sufficiently small ΔT, the first-order approximations

$$\exp(\Delta T \boldsymbol{\omega}_{*,k}^\times) \approx \mathbf{I} + \Delta T \boldsymbol{\omega}_{*,k}^\times$$

then linearize the discrete-time equations of motions of the left $* = l$ and the right $* = r$ arms by

$$\begin{aligned}\mathbf{x}_{*,k+1} =& \mathbf{x}_{*,k} + \Delta T \boldsymbol{\upsilon}_{*,k}, \\ \mathbf{R}_{*,k+1} =& \mathbf{R}_{*,k} + \Delta T \mathbf{R}_{*,k} \boldsymbol{\omega}_{*,k}^\times.\end{aligned}$$

One can thus predict their positions $\mathbf{x}_{*,\star}$ and orientations $\mathbf{R}_{*,\star}$ at the time steps $\star = k+2, \cdots, k+N$ by

$$\begin{aligned}\mathbf{x}_{*,k+2} =& \mathbf{x}_{*,k} + \Delta T \boldsymbol{\upsilon}_{*,k} + \Delta T \boldsymbol{\upsilon}_{*,k+1}, \\ \mathbf{R}_{*,k+2} =& \mathbf{R}_{*,k} + \Delta T \mathbf{R}_{*,k} \boldsymbol{\omega}_{*,k}^\times + \Delta T \mathbf{R}_{*,k} \boldsymbol{\omega}_{*,k+1}^\times, \\ &\vdots \\ \mathbf{x}_{*,k+N} =& \mathbf{x}_{*,k} + \Delta T \boldsymbol{\upsilon}_{*,k} + \cdots + \Delta T \boldsymbol{\upsilon}_{*,k+N-1}, \\ \mathbf{R}_{*,k+N} =& \mathbf{R}_{*,k} + \Delta T \mathbf{R}_{*,k} \boldsymbol{\omega}_{*,k}^\times + \cdots + \Delta T \mathbf{R}_{*,k} \boldsymbol{\omega}_{*,k+N-1}^\times,\end{aligned}$$

where $\boldsymbol{\upsilon}_{*,\star-1}$ and $\boldsymbol{\omega}_{*,\star-1}$ denote their linear spatial and angular body velocities, respectively.

For the ease of presentation, let $\mathbf{s}_n$ be an $N \times 1$ vector of which the first $n = 1, \cdots, N$ elements are 1 and the remaining ones are 0. Then, the positions $\mathbf{x}_{*,k+n}$ and orientations $\mathbf{R}_{*,k+n}$ of the left $* = l$ and the right $* = r$ arms at any time step $k+n$ in the prediction horizon $n = 1, \cdots, N$ can be described by

$$\begin{aligned}\mathbf{x}_{*,k+n} =& \mathbf{x}_{*,k} + \Delta T (\mathbf{s}_n \otimes \mathbf{I})^\mathsf{T} \boldsymbol{\upsilon}_{*,k,N}, \\ \mathbf{R}_{*,k+n} =& \mathbf{R}_{*,k} + \Delta T \mathbf{R}_{*,k} [(\mathbf{s}_n \otimes \mathbf{I})^\mathsf{T} \boldsymbol{\omega}_{*,k,N}]^\times,\end{aligned}$$

where the linear spatial $\boldsymbol{\upsilon}_{*,k,N}$ and angular body $\boldsymbol{\omega}_{*,k,N}$ velocity sequences are defined by

$$\boldsymbol{\upsilon}_{*,k,N} = \begin{bmatrix} \boldsymbol{\upsilon}_{*,k} \\ \boldsymbol{\upsilon}_{*,k+1} \\ \vdots \\ \boldsymbol{\upsilon}_{*,k+N-2} \\ \boldsymbol{\upsilon}_{*,k+N-1} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\omega}_{*,k,N} = \begin{bmatrix} \boldsymbol{\omega}_{*,k} \\ \boldsymbol{\omega}_{*,k+1} \\ \vdots \\ \boldsymbol{\omega}_{*,k+N-2} \\ \boldsymbol{\omega}_{*,k+N-1} \end{bmatrix}.$$

Let the left $* = l$ and the right $* = r$ arms plan their desired positions $\mathbf{x}_{*,d}$ and orientations $\mathbf{R}_{*,d}$ by

$$\begin{bmatrix} \mathbf{R}_{l,0} & \mathbf{x}_{l,0} \\ \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_{l,d} & \mathbf{x}_{l,d} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{r,0} & \mathbf{x}_{r,0} \\ \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_{r,d} & \mathbf{x}_{r,d} \\ \mathbf{0} & 1 \end{bmatrix}.$$

The target-reaching control objective in the prediction horizon $n = 1, \cdots, N$ is then formulated as

$$\underset{\boldsymbol{\upsilon}_{k,N},\boldsymbol{\omega}_{k,N}}{\text{minimize}} \quad V_d = \sum_{n=1}^{N} V_{d,n},$$

where linear spatial $\boldsymbol{\upsilon}_{k,N}$ and angular body $\boldsymbol{\omega}_{k,N}$ velocity sequences are defined by

$$\boldsymbol{\upsilon}_{k,N} = \begin{bmatrix} \boldsymbol{\upsilon}_{l,k,N} \\ \boldsymbol{\upsilon}_{r,k,N} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\omega}_{k,N} = \begin{bmatrix} \boldsymbol{\omega}_{l,k,N} \\ \boldsymbol{\omega}_{r,k,N} \end{bmatrix},$$

and the cost functions $V_{d,n}$ at the time steps $k+n$ are given by

$$\begin{aligned}
V_{d,n} =& \kappa_\upsilon \sum_{*=l,r} (\mathbf{x}_{*,k+n} - \mathbf{x}_{*,k})^\top (\mathbf{x}_{*,k+n} - \mathbf{x}_{*,d}) + (\Delta T)^2 \alpha_\upsilon \sum_{*=l,r} \boldsymbol{\upsilon}_{*,k+n-1}^\top \boldsymbol{\upsilon}_{*,k+n-1} \\
&+ \kappa_\omega \sum_{*=l,r} \operatorname{tr}\left(\mathbf{I} - \mathbf{R}_{*,d}^\top \mathbf{R}_{*,k+n}\right) + (\Delta T)^2 \alpha_\omega \sum_{*=l,r} \boldsymbol{\omega}_{*,k+n-1}^\top \boldsymbol{\omega}_{*,k+n-1} \\
\approx& \kappa_\upsilon \sum_{*=l,r} \left[\mathbf{x}_{*,dk} + \Delta T(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{*,k,N}\right]^\top \left[\mathbf{x}_{*,dk} + \Delta T(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{*,k,N}\right] + (\Delta T)^2 \sum_{*=l,r} \boldsymbol{\upsilon}_{*,k,N}^\top (\alpha_\upsilon \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\upsilon}_{*,k,N} \\
&+ \kappa_\omega \sum_{*=l,r} \operatorname{tr}\left(\mathbf{I} - \mathbf{R}_{*,dk}\right) - \Delta T \kappa_\omega \sum_{*=l,r} \operatorname{tr}\left(\mathbf{R}_{*,dk}[(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{*,k,N}]^\times\right) + (\Delta T)^2 \sum_{*=l,r} \boldsymbol{\omega}_{*,k,N}^\top (\alpha_\omega \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\omega}_{*,k,N} \\
=& \kappa_\upsilon \sum_{*=l,r} \mathbf{x}_{*,dk}^\top \mathbf{x}_{*,dk} + 2\Delta T \kappa_\upsilon \sum_{*=l,r} \mathbf{x}_{*,dk}^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{*,k,N} + (\Delta T)^2 \sum_{*=l,r} \boldsymbol{\upsilon}_{*,k,N}^\top \left[(\kappa_\upsilon \mathbf{s}_n \mathbf{s}_n^\top + \alpha_\upsilon \mathbb{I}_{N,n}) \otimes \mathbf{I}\right] \boldsymbol{\upsilon}_{*,k,N} \\
&+ \kappa_\omega \sum_{*=l,r} \operatorname{tr}\left(\mathbf{I} - \mathbf{R}_{*,dk}\right) + 2\Delta T \kappa_\omega \sum_{*=l,r} [\operatorname{sk}(\mathbf{R}_{*,dk})^\vee]^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{*,k,N} + (\Delta T)^2 \sum_{*=l,r} \boldsymbol{\omega}_{*,k,N}^\top (\alpha_\omega \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\omega}_{*,k,N},
\end{aligned}$$

where $\kappa_\star$ and $\alpha_\star$ with $\star = \upsilon, \omega$ are positive control gains, $\mathbb{I}_{N,n}$ is an $N \times N$ matrix of which all elements are 0 except the n-th diagonal element being 1, $\mathbf{x}_{*,dk} = \mathbf{x}_{*,k} - \mathbf{x}_d$ and $\mathbf{R}_{*,dk} = \mathbf{R}_d^\top \mathbf{R}_k$. Let

$$\begin{aligned}
\mathbf{A}_{\upsilon,d} =& \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \left(\kappa_\upsilon \sum_{n=1}^{N} (\mathbf{s}_n \mathbf{s}_n^\top) \otimes \mathbf{I} + \alpha_\upsilon \mathbf{I}\right), \quad \mathbf{b}_{\upsilon,d} = \kappa_\upsilon \sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_n \otimes \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{l,dk} \\ \mathbf{x}_{r,dk} \end{bmatrix}, \quad c_{\upsilon,d} = \kappa_\upsilon \sum_{*=l,r} \mathbf{x}_{*,dk}^\top \mathbf{x}_{*,dk}, \\
\mathbf{A}_{\omega,d} =& \alpha_\omega \mathbf{I}, \quad \mathbf{b}_{\omega,d} = \kappa_\omega \sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_n \otimes \mathbf{I} \end{bmatrix} \begin{bmatrix} \operatorname{sk}(\mathbf{R}_{l,dk})^\vee \\ \operatorname{sk}(\mathbf{R}_{r,dk})^\vee \end{bmatrix}, \quad c_{\omega,d} = \kappa_\omega \sum_{*=l,r} \operatorname{tr}(\mathbf{I} - \mathbf{R}_{*,dk}),
\end{aligned}$$

the cost function V_d can be thus be written compactly as

$$\begin{aligned}
V_d \approx& (\Delta T)^2 \boldsymbol{\upsilon}_{k,N}^\top \mathbf{A}_{\upsilon,d} \boldsymbol{\upsilon}_{k,N} + 2\Delta T \mathbf{b}_{\upsilon,d}^\top \boldsymbol{\upsilon}_{k,N} + N c_{\upsilon,d} \\
&+ (\Delta T)^2 \boldsymbol{\omega}_{k,N}^\top \mathbf{A}_{\omega,d} \boldsymbol{\omega}_{k,N} + 2\Delta T \mathbf{b}_{\omega,d}^\top \boldsymbol{\omega}_{k,N} + N c_{\omega,d}.
\end{aligned}$$

When moving towards the targets, the left and the right arms need to synchronize their poses as per

$$\begin{bmatrix} \mathbf{R}_{l,0} & \mathbf{x}_{l,0} \\ \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_{l,k+n} & \mathbf{x}_{l,k+n} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{r,0} & \mathbf{x}_{r,0} \\ \mathbf{0} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_{r,k+n} & \mathbf{x}_{r,k+n} \\ \mathbf{0} & 1 \end{bmatrix}.$$

Namely, at any time steps $k+n$ in the prediction horizon $n = 1, \cdots, N$, the deviations of the poses of both arms from their initial poses are supposed to be identical as follows

$$\begin{aligned}
\mathbf{R}_{l,0}^\top \mathbf{R}_{l,k+n} =& \mathbf{R}_{r,0}^\top \mathbf{R}_{r,k+n}, \\
\mathbf{R}_{l,0}^\top (\mathbf{x}_{l,k+n} - \mathbf{x}_{l,0}) =& \mathbf{R}_{r,0}^\top (\mathbf{x}_{r,k+n} - \mathbf{x}_{r,0}).
\end{aligned}$$

The synchronization control objective in then formulated as

$$\underset{\boldsymbol{\upsilon}_{k,N},\boldsymbol{\omega}_{k,N}}{\text{minimize}} \quad V_s = \sum_{n=1}^{N} V_{s,n},$$

where the cost functions $V_{s,n}$ are given by

$$
\begin{aligned}
V_{s,n} =& \rho_\upsilon \left[\mathbf{R}_{l,0}^\top(\mathbf{x}_{l,k+n} - \mathbf{x}_{l,0}) - \mathbf{R}_{r,0}^\top(\mathbf{x}_{r,k+n} - \mathbf{x}_{r,0})\right]^\top \left[\mathbf{R}_{l,0}^\top(\mathbf{x}_{l,k+n} - \mathbf{x}_{l,0}) - \mathbf{R}_{r,0}^\top(\mathbf{x}_{r,k+n} - \mathbf{x}_{r,0})\right] \\
&+ \rho_\omega \mathrm{tr}\left(\mathbf{I} - \mathbf{R}_{l,k+n}^\top \mathbf{R}_{l,0} \mathbf{R}_{r,0}^\top \mathbf{R}_{r,k+n}\right) \\
\approx& \rho_\upsilon \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k} + \Delta T\left[\mathbf{R}_{l,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{l,k,N} - \mathbf{R}_{r,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{r,k,N}\right]\right)^\top \\
&\cdot \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k} + \Delta T\left[\mathbf{R}_{l,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{l,k,N} - \mathbf{R}_{r,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{r,k,N}\right]\right) \\
&+ \rho_\omega \mathrm{tr}\Big[\mathbf{I} - \left(\mathbf{I} + \Delta T[(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{l,k,N}]^\times\right)^\top \mathbf{R}_{l,0k}^\top \mathbf{R}_{r,0k}\left(\mathbf{I} + \Delta T[(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{r,k,N}]^\times\right)\Big] \\
=& (\Delta T)^2 \rho_\upsilon \left[\mathbf{R}_{l,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{l,k,N} - \mathbf{R}_{r,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{r,k,N}\right]^\top \left[\mathbf{R}_{l,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{l,k,N} - \mathbf{R}_{r,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{r,k,N}\right] \\
&+ 2\Delta T \rho_\upsilon \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right)^\top \left[\mathbf{R}_{l,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{l,k,N} - \mathbf{R}_{r,0}^\top(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{r,k,N}\right] \\
&+ \rho_\upsilon \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right)^\top \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right) + \rho_\omega \mathrm{tr}\left(\mathbf{I} - \mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k}\right) \\
&+ (\Delta T)^2 \rho_\omega \boldsymbol{\omega}_{l,k,N}^\top (\mathbf{s}_n \otimes \mathbf{I})\left[\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k} - \mathrm{tr}\left(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k}\right)\mathbf{I}\right](\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{r,k,N} \\
&+ 2\Delta T \rho_\omega \left[\mathrm{sk}(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k})^\vee\right]^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{l,k,N} - 2\Delta T \rho_\omega \left[\mathrm{sk}(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k})^\vee\right]^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{r,k,N},
\end{aligned}
$$

where ρ_υ and ρ_ω are positive control gains, and $\mathbf{x}_{*,0k} = \mathbf{x}_{*,k} - \mathbf{x}_{*,0}$ and $\mathbf{R}_{*,0k} = \mathbf{R}_{*,0}^\top \mathbf{R}_{*,k}$ for $* = l, r$. Let

$$
\begin{aligned}
\mathbf{A}_{\upsilon,s} =& \rho_\upsilon \sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{s}_n \otimes \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{l,0} \\ \mathbf{R}_{r,0} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{l,0} \\ \mathbf{R}_{r,0} \end{bmatrix}^\top \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{s}_n \otimes \mathbf{I} \end{bmatrix}^\top, \\
\mathbf{b}_{\upsilon,s} =& \rho_\upsilon \sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{s}_n \otimes \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{l,0} \\ \mathbf{R}_{r,0} \end{bmatrix} \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right), \\
c_{\upsilon,s} =& \rho_\upsilon \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right)^\top \left(\mathbf{R}_{l,0}^\top \mathbf{x}_{l,0k} - \mathbf{R}_{r,0}^\top \mathbf{x}_{r,0k}\right), \\
\mathbf{A}_{\omega,s} =& \frac{\rho_\omega}{2} \sum_{n=1}^{N} \begin{bmatrix} \mathbf{0} & (\mathbf{s}_n \mathbf{s}_n^\top) \otimes \left[\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k} - \mathrm{tr}(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k})\mathbf{I}\right] \\ (\mathbf{s}_n \mathbf{s}_n^\top) \otimes \left[\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k} - \mathrm{tr}(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k})\mathbf{I}\right]^\top & \mathbf{0} \end{bmatrix}, \\
\mathbf{b}_{\omega,s} =& \rho_\omega \sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_n \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{s}_n \otimes \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \mathrm{sk}(\mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k})^\vee, \\
c_{\omega,s} =& \rho_\omega \mathrm{tr}\left(\mathbf{I} - \mathbf{R}_{r,0k}^\top \mathbf{R}_{l,0k}\right),
\end{aligned}
$$

the cost function V_s can thus be written compactly as

$$
\begin{aligned}
V_s \approx& (\Delta T)^2 \boldsymbol{\upsilon}_{k,N}^\top \mathbf{A}_{\upsilon,s} \boldsymbol{\upsilon}_{k,N} + 2\Delta T \mathbf{b}_{\upsilon,s}^\top \boldsymbol{\upsilon}_{k,N} + N c_{\upsilon,s} \\
&+ (\Delta T)^2 \boldsymbol{\omega}_{k,N}^\top \mathbf{A}_{\omega,s} \boldsymbol{\omega}_{k,N} + 2\Delta T \mathbf{b}_{\omega,s}^\top \boldsymbol{\omega}_{k,N} + N c_{\omega,s}.
\end{aligned}
$$

Let $\mathbf{x}_{*o}$ and $\mathbf{R}_{*o}$ denote the positions and orientations of the object frame relative to the end-effector frames of the left $* = l$ and the right $* = r$ arms, respectively. One can then describe the position $\mathbf{x}_o$ and orientation $\mathbf{R}_o$ of the object in the world frame by

$$
\begin{aligned}
\mathbf{x}_o =& \mathbf{x}_* + \mathbf{R}_* \mathbf{x}_{*o}, \\
\mathbf{R}_o =& \mathbf{R}_* \mathbf{R}_{*o}.
\end{aligned}
$$

Let the trays exert forces $\mathbf{f}_i = \begin{bmatrix} f_{xi} & f_{yi} & f_{zi} \end{bmatrix}^\top$ on the object via their contact points $i = 1, \cdots, 4$ in the object's body frame. The object of mass m and inertia $\boldsymbol{\mathcal{I}}$ then moves as per

$$
m\dot{\boldsymbol{\upsilon}}_o + m\mathbf{g} = \sum_{i=1}^{4} \mathbf{R}_o \mathbf{f}_i,
$$

$$
\boldsymbol{\mathcal{I}}\dot{\boldsymbol{\omega}}_o + \boldsymbol{\omega}_o^\times \boldsymbol{\mathcal{I}} \boldsymbol{\omega}_o = \sum_{i=1}^{4} \mathbf{r}_i^\times \mathbf{f}_i,
$$

where $\mathbf{g} = \begin{bmatrix} 0 & 0 & 9.81 \end{bmatrix}^\mathsf{T}$, and $\boldsymbol{\upsilon}_o$ and $\boldsymbol{\omega}_o$ are the linear spatial velocity and the angular body velocity of the object, respectively. Namely,

$$\begin{aligned}
\boldsymbol{\upsilon}_o =& \dot{\mathbf{x}}_o = \dot{\mathbf{x}}_* + \dot{\mathbf{R}}_* \mathbf{x}_{*o} = \boldsymbol{\upsilon}_* + \mathbf{R}_* \boldsymbol{\omega}_*^\times \mathbf{x}_{*o} = \boldsymbol{\upsilon}_* - \mathbf{R}_* \mathbf{x}_{*o}^\times \boldsymbol{\omega}_*, \\
\boldsymbol{\omega}_o^\times =& \mathbf{R}_o^\mathsf{T} \dot{\mathbf{R}}_o = \mathbf{R}_o^\mathsf{T} \dot{\mathbf{R}}_* \mathbf{R}_{*o} = \mathbf{R}_o^\mathsf{T} \mathbf{R}_* \boldsymbol{\omega}_*^\times \mathbf{R}_{*o} = (\mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_*)^\times.
\end{aligned}$$

One can further derive the accelerations of the object as follows

$$\begin{aligned}
\dot{\boldsymbol{\upsilon}}_o =& \dot{\boldsymbol{\upsilon}}_* - \mathbf{R}_* \mathbf{x}_{*o}^\times \dot{\boldsymbol{\omega}}_* + \mathbf{R}_* (\boldsymbol{\omega}_*^\times)^2 \mathbf{x}_{*o}, \\
\dot{\boldsymbol{\omega}}_o =& \mathbf{R}_{*o}^\mathsf{T} \dot{\boldsymbol{\omega}}_*.
\end{aligned}$$

Let $\boldsymbol{f} = \begin{bmatrix} \mathbf{f}_1^\mathsf{T} & \mathbf{f}_2^\mathsf{T} & \mathbf{f}_3^\mathsf{T} & \mathbf{f}_4^\mathsf{T} \end{bmatrix}$ convert the equations of motion of the object into

$$\begin{aligned}
m\dot{\boldsymbol{\upsilon}}_* - m\mathbf{R}_* \mathbf{x}_{*o}^\times \dot{\boldsymbol{\omega}}_* + m\mathbf{R}_* (\boldsymbol{\omega}_*^\times)^2 \mathbf{x}_{*o} + m\mathbf{g} =& \mathbf{R}_* \mathbf{R}_{*o} \mathbf{G}_\upsilon \boldsymbol{f}, \\
\boldsymbol{\mathcal{I}} \mathbf{R}_{*o}^\mathsf{T} \dot{\boldsymbol{\omega}}_* + (\mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_*)^\times \boldsymbol{\mathcal{I}} (\mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_*) =& \mathbf{G}_\omega \boldsymbol{f},
\end{aligned}$$

where $\mathbf{G}_\upsilon = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix}$ and $\mathbf{G}_\omega = \begin{bmatrix} \mathbf{r}_1^\times & \mathbf{r}_2^\times & \mathbf{r}_3^\times & \mathbf{r}_4^\times \end{bmatrix}$.

Thus, at any time steps $k = 0, 1, 2, \cdots$, the left $* = l$ and the right $* = r$ arms can approximate the motions of the transported object at the following time steps $k + n$ by

$$\begin{aligned}
m\frac{\boldsymbol{\upsilon}_{*,k+n} - \boldsymbol{\upsilon}_{*,k+n-1}}{\Delta T} - m\mathbf{R}_{*,k} \mathbf{x}_{*o}^\times \frac{\boldsymbol{\omega}_{*,k+n} - \boldsymbol{\omega}_{*,k+n-1}}{\Delta T} + m\mathbf{R}_{*,k} (\boldsymbol{\omega}_{*,k}^\times)^2 \mathbf{x}_{*o} + m\mathbf{g} =& \mathbf{R}_{*,k} \mathbf{R}_{*o} \mathbf{G}_\upsilon \boldsymbol{f}_{k+n-1}, \\
\boldsymbol{\mathcal{I}} \mathbf{R}_{*o}^\mathsf{T} \frac{\boldsymbol{\omega}_{*,k+n} - \boldsymbol{\omega}_{*,k+n-1}}{\Delta T} + (\mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_{*,k})^\times \boldsymbol{\mathcal{I}} (\mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_{*,k}) =& \mathbf{G}_\omega \boldsymbol{f}_{k+n-1},
\end{aligned}$$

where $n = 1, \cdots, N$. Let

$$\mathbf{H}_\upsilon = \begin{bmatrix} m(\boldsymbol{\Gamma} \otimes \mathbf{I}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m(\boldsymbol{\Gamma} \otimes \mathbf{I}) \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{H}_\omega = \begin{bmatrix} m[\boldsymbol{\Gamma} \otimes (\mathbf{R}_{l,k} \mathbf{x}_{lo}^\times)] & \mathbf{0} \\ -\boldsymbol{\Gamma} \otimes (\boldsymbol{\mathcal{I}} \mathbf{R}_{lo}^\mathsf{T}) & \mathbf{0} \\ \mathbf{0} & m[\boldsymbol{\Gamma} \otimes (\mathbf{R}_{r,k} \mathbf{x}_{ro}^\times)] \\ \mathbf{0} & -\boldsymbol{\Gamma} \otimes (\boldsymbol{\mathcal{I}} \mathbf{R}_{ro}^\mathsf{T}) \end{bmatrix},$$

$$\mathbf{H}_f = \begin{bmatrix} \mathbf{I} \otimes (\mathbf{R}_{l,k} \mathbf{R}_{lo} \mathbf{G}_\upsilon) & \mathbf{0} \\ \mathbf{I} \otimes \mathbf{G}_\omega & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \otimes (\mathbf{R}_{r,k} \mathbf{R}_{ro} \mathbf{G}_\upsilon) \\ \mathbf{0} & \mathbf{I} \otimes \mathbf{G}_\omega \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} m\boldsymbol{\eta}_{l,k} - \Delta T m\big(\mathbf{1} \otimes [\mathbf{R}_{l,k} (\boldsymbol{\omega}_{l,k}^\times)^2 \mathbf{x}_{lo} + \mathbf{g}]\big) \\ \boldsymbol{\lambda}_{l,k} - \Delta T\big(\mathbf{1} \otimes [(\mathbf{R}_{lo}^\mathsf{T} \boldsymbol{\omega}_{l,k})^\times \boldsymbol{\mathcal{I}} (\mathbf{R}_{lo}^\mathsf{T} \boldsymbol{\omega}_{l,k})]\big) \\ m\boldsymbol{\eta}_{r,k} - \Delta T m\big(\mathbf{1} \otimes [\mathbf{R}_{r,k} (\boldsymbol{\omega}_{r,k}^\times)^2 \mathbf{x}_{ro} + \mathbf{g}]\big) \\ \boldsymbol{\lambda}_{r,k} - \Delta T\big(\mathbf{1} \otimes [(\mathbf{R}_{ro}^\mathsf{T} \boldsymbol{\omega}_{r,k})^\times \boldsymbol{\mathcal{I}} (\mathbf{R}_{ro}^\mathsf{T} \boldsymbol{\omega}_{r,k})]\big) \end{bmatrix}$$

where $\mathbf{1}$ is a vector of all ones and

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}, \quad \boldsymbol{f}_{k,N} = \begin{bmatrix} \boldsymbol{f}_k \\ \boldsymbol{f}_{k+1} \\ \vdots \\ \boldsymbol{f}_{k+N-2} \\ \boldsymbol{f}_{k+N-1} \end{bmatrix}, \quad \boldsymbol{\eta}_{*,k} = \begin{bmatrix} \boldsymbol{\upsilon}_{*,k} - \mathbf{R}_{*,k} \mathbf{x}_{*o}^\times \boldsymbol{\omega}_{*,k} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \boldsymbol{\lambda}_{*,k} = \begin{bmatrix} \boldsymbol{\mathcal{I}} \mathbf{R}_{*o}^\mathsf{T} \boldsymbol{\omega}_{*,k} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}.$$

The linear spatial $\boldsymbol{\upsilon}_{k,N}$ and angular body $\boldsymbol{\omega}_{k,N}$ velocity sequences of the dual-arm system are then constrained by the predicted contact force sequences $\boldsymbol{f}_{k,N}$ as follows

$$\mathbf{H}_\upsilon \boldsymbol{\upsilon}_{k,N} - \mathbf{H}_\omega \boldsymbol{\omega}_{k,N} - \Delta T \mathbf{H}_f \boldsymbol{f}_{k,N} = \mathbf{h}.$$

To prevent the object from sliding, the contact forces $\mathbf{f}_i$ need to respect the friction-cone constraints

$$f_{xi}^2 + f_{yi}^2 - \mu^2 f_{zi}^2 \leqslant 0$$

for the contact points $i = 1, \cdots, 4$, where $\mu > 0$ is the friction coefficient. Also, the object keeps in contact with the tray by making their contact forces unilateral in the normal direction, i.e.,

$$f_{zi} \geqslant \epsilon$$

for some $\epsilon > 0$, where $i = 1, \cdots, 4$. At the k-th time step, the friction cone constraints turn into

$$\begin{aligned} \boldsymbol{f}_{k,N}^{\mathsf{T}} \left(\mathbb{I}_{N,n} \otimes \mathbb{I}_{4,i} \otimes \boldsymbol{\Lambda}\right) \boldsymbol{f}_{k,N} &\leqslant 0, \\ \left(\mathbf{1} \otimes \mathbf{I} \otimes \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\right) \boldsymbol{f}_{k,N} &\geqslant \epsilon \mathbf{1}, \end{aligned}$$

for the contact points $i = 1, \cdots, 4$ in the prediction horizon $n = 1, \cdots, N$, where $\mathbb{I}_{4,i}$ is a 4×4 matrix of which all elements are zero except the i-th diagonal element being 1, and

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\mu^2 \end{bmatrix}.$$

Because the sampling period ΔT, the length of the prediction horizon N and $c_{*,\star}$ with $* = \upsilon, \omega$ and $\star = d, s$ are constant, minimizing the overall cost function $V = V_d + V_s$ under the constraints can be formulated as the following second-order cone program

$$\begin{aligned} \underset{\boldsymbol{\upsilon}_{k,N}, \boldsymbol{\omega}_{k,N}}{\text{minimize}} \quad & \Delta T \begin{bmatrix} \boldsymbol{\upsilon}_{k,N} \\ \boldsymbol{\omega}_{k,N} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{A}_\upsilon & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_\omega \end{bmatrix} \begin{bmatrix} \boldsymbol{\upsilon}_{k,N} \\ \boldsymbol{\omega}_{k,N} \end{bmatrix} + 2 \begin{bmatrix} \mathbf{b}_\upsilon \\ \mathbf{b}_\omega \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \boldsymbol{\upsilon}_{k,N} \\ \boldsymbol{\omega}_{k,N} \end{bmatrix}, \\ \text{subject to} \quad & \mathbf{1} \otimes \begin{bmatrix} \check{\boldsymbol{\upsilon}} \\ \check{\boldsymbol{\omega}} \end{bmatrix} \leqslant \begin{bmatrix} \boldsymbol{\upsilon}_{k,N} \\ \boldsymbol{\omega}_{k,N} \end{bmatrix} \leqslant \mathbf{1} \otimes \begin{bmatrix} \hat{\boldsymbol{\upsilon}} \\ \hat{\boldsymbol{\omega}} \end{bmatrix}, \\ & \mathbf{H}_\upsilon \boldsymbol{\upsilon}_{k,N} - \mathbf{H}_\omega \boldsymbol{\omega}_{k,N} - \Delta T \mathbf{H}_f \boldsymbol{f}_{k,N} = \mathbf{h}, \\ & \boldsymbol{f}_{k,N}^{\mathsf{T}} \left(\mathbb{I}_{N,n} \otimes \mathbb{I}_{4,i} \otimes \boldsymbol{\Lambda}\right) \boldsymbol{f}_{k,N} \leqslant 0, \\ & \left(\mathbf{1} \otimes \mathbf{I} \otimes \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\right) \boldsymbol{f}_{k,N} \geqslant \epsilon \mathbf{1}, \end{aligned}$$

where $n = 1, \cdots, N$ and $i = 1, \cdots, 4$ index the predicted time steps and contact points, $\mathbf{A}_* = \mathbf{A}_{*,d} + \mathbf{A}_{*,s}$, $\mathbf{b}_* = \mathbf{b}_{*,d} + \mathbf{b}_{*,s}$, and $\check{*}$ and $\hat{*}$ denote the lower and the upper bounds of the linear spatial $* = \upsilon$ and angular body $* = \omega$ velocities, respectively.