The continuous-time equations of motion of the manipulator's end-effector frame are known by

$$\begin{aligned}\dot{\mathbf{x}} =& \boldsymbol{v},\\ \dot{\mathbf{R}} =& \mathbf{R}\boldsymbol{\omega}^{\times},\end{aligned}$$

where $\mathbf{x}$ is the position vector, $\mathbf{R}$ is the rotation matrix representation of the orientation, $\boldsymbol{v}$ is the linear spatial velocity, and $\boldsymbol{\omega}$ is the angular body velocity.

Let ΔT be the sampling time period. Then, in discrete time, the equations of motion of the manipulator's end-effector frame can be expressed as

$$\begin{aligned}\mathbf{x}_{k+1} =& \mathbf{x}_k + \Delta T \boldsymbol{v}_k,\\ \mathbf{R}_{k+1} =& \mathbf{R}_k \exp\left(\Delta T \boldsymbol{\omega}_k^{\times}\right),\end{aligned}$$

at any time step $k = 0, 1, 2, \cdots$, where $\mathbf{x}_*$ and $\mathbf{R}_*$ denote the position vectors and rotation matrices at the time steps $* = k$ and $* = k+1$, and $\boldsymbol{v}_k$ and $\boldsymbol{\omega}_k$ denote the linear spatial and the angular body velocities at the time step k, respectively.

With sufficiently small ΔT, the first-order approximation

$$\exp\left(\Delta T \boldsymbol{\omega}_k^{\times}\right) \approx \mathbf{I} + \Delta T \boldsymbol{\omega}_k^{\times}$$

then linearizes the discrete-time equations of motion of the manipulator in the Cartesian space by

$$\begin{aligned}\mathbf{x}_{k+1} =& \mathbf{x}_k + \Delta T \boldsymbol{v}_k,\\ \mathbf{R}_{k+1} \approx& \mathbf{R}_k + \Delta T \mathbf{R}_k \boldsymbol{\omega}_k^{\times}.\end{aligned}$$

Similarly, one can predict the position vectors $\mathbf{x}_*$ and rotation matrices $\mathbf{R}_*$ at the following time steps $* = k+2, \cdots, k+N$ by

$$\begin{aligned}\mathbf{x}_{k+2} =& \mathbf{x}_k + \Delta T \boldsymbol{v}_k + \Delta T \boldsymbol{v}_{k+1},\\ \mathbf{R}_{k+2} \approx& \mathbf{R}_k + \Delta T \mathbf{R}_k \boldsymbol{\omega}_k^{\times} + \Delta T \mathbf{R}_k \boldsymbol{\omega}_{k+1}^{\times},\\ &\vdots\\ \mathbf{x}_{k+N} =& \mathbf{x}_k + \Delta T \boldsymbol{v}_k + \cdots + \Delta T \boldsymbol{v}_{k+N-1},\\ \mathbf{R}_{k+N} \approx& \mathbf{R}_k + \Delta T \mathbf{R}_k \boldsymbol{\omega}_k^{\times} + \cdots + \Delta T \mathbf{R}_k \boldsymbol{\omega}_{k+N-1}^{\times},\end{aligned}$$

where $\boldsymbol{v}_{*-1}$ and $\boldsymbol{\omega}_{*-1}$ are the linear and angular velocities at the time steps $k+1, \cdots, k+N-1$, respectively.

For the ease of notation, let $\mathbf{s}_n$ be an $N \times 1$ vector of which the first $n = 1, \cdots, N$ elements are 1 and the remaining ones are 0. Then, the position $\mathbf{x}_{k+n}$ and orientation $\mathbf{R}_{k+1}$ of the manipulator's end-effector frame at any time step $k+n$ in the prediction horizon can be expressed as follows

$$\begin{aligned}\mathbf{x}_{k+n} =& \mathbf{x}_k + \Delta T (\mathbf{s}_n \otimes \mathbf{I})^{\mathsf{T}} \boldsymbol{v}_{k,N},\\ \mathbf{R}_{k+n} =& \mathbf{R}_k + \Delta T \mathbf{R}_k \left[(\mathbf{s}_n \otimes \mathbf{I})^{\mathsf{T}} \boldsymbol{\omega}_{k,N}\right]^{\times},\end{aligned}$$

where the linear spatial $\boldsymbol{v}_{k,N}$ and angular body $\boldsymbol{\omega}_{k,N}$ velocity sequences in the prediction horizon $[k, k+N]$ are defined by

$$\boldsymbol{v}_{k,N} = \begin{bmatrix} \boldsymbol{v}_k \\ \boldsymbol{v}_{k+1} \\ \vdots \\ \boldsymbol{v}_{k+N-2} \\ \boldsymbol{v}_{k+N-1} \end{bmatrix}, \quad \boldsymbol{\omega}_{k,N} = \begin{bmatrix} \boldsymbol{\omega}_k \\ \boldsymbol{\omega}_{k+1} \\ \vdots \\ \boldsymbol{\omega}_{k+N-2} \\ \boldsymbol{\omega}_{k+N-1} \end{bmatrix}.$$

Let $\mathbf{x}_d$ and $\mathbf{R}_d$ be the desired position and orientation of the manipulator's end-effector frame. The control objective in the prediction horizon $[k, k+N]$ is then formulated as

$$\underset{\boldsymbol{v}_{k,N}, \boldsymbol{\omega}_{k,N}}{\text{minimize}} \quad V = \sum_{n=1}^{N} V_{k+n},$$

where the cost functions V_{k+n} at the time steps $k+n$ are defined by

$$
\begin{aligned}
V_{k+n} =& \kappa_\upsilon (\mathbf{x}_{k+n} - \mathbf{x}_d)^\top (\mathbf{x}_{k+n} - \mathbf{x}_d) + (\Delta T)^2 \alpha_\upsilon \boldsymbol{\upsilon}_{k+n-1}^\top \boldsymbol{\upsilon}_{k+n-1} \\
&+ \kappa_\omega \mathrm{tr}\left(\mathbf{I} - \mathbf{R}_{k+n}^\top \mathbf{R}_d\right) + (\Delta T)^2 \alpha_\omega \boldsymbol{\omega}_{k+n-1}^\top \boldsymbol{\omega}_{k+n-1} \\
\approx& \kappa_\upsilon \left[\mathbf{x}_k + \Delta T (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{k,N} - \mathbf{x}_d\right]^\top \left[\mathbf{x}_k + \Delta T (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{k,N} - \mathbf{x}_d\right] + (\Delta T)^2 \boldsymbol{\upsilon}_{k,N}^\top (\alpha_\upsilon \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\upsilon}_{k,N} \\
&+ \kappa_\omega \mathrm{tr}\left(\mathbf{I} - \mathbf{R}_k^\top \mathbf{R}_d\right) + \Delta T \kappa_\omega \mathrm{tr}\left([(\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{k,N}]^\times \mathbf{R}_k^\top \mathbf{R}_d\right) + (\Delta T)^2 \boldsymbol{\omega}_{k,N}^\top (\alpha_\omega \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\omega}_{k,N} \\
=& \kappa_\upsilon (\mathbf{x}_k - \mathbf{x}_d)^\top (\mathbf{x}_k - \mathbf{x}_d) + 2\Delta T \kappa_\upsilon (\mathbf{x}_k - \mathbf{x}_d)^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\upsilon}_{k,N} + (\Delta T)^2 \boldsymbol{\upsilon}_{k,N}^\top \left[(\kappa_\upsilon \mathbf{s}_n \mathbf{s}_n^\top + \alpha_\upsilon \mathbb{I}_{N,n}) \otimes \mathbf{I}\right] \boldsymbol{\upsilon}_{k,N} \\
&+ \kappa_\omega \mathrm{tr}(\mathbf{I} - \mathbf{R}_k^\top \mathbf{R}_d) + 2\Delta T \kappa_\omega \left[\mathrm{sk}(\mathbf{R}_d^\top \mathbf{R}_k)^\vee\right]^\top (\mathbf{s}_n \otimes \mathbf{I})^\top \boldsymbol{\omega}_{k,N} + (\Delta T)^2 \boldsymbol{\omega}_{k,N}^\top (\alpha_\omega \mathbb{I}_{N,n} \otimes \mathbf{I}) \boldsymbol{\omega}_{k,N},
\end{aligned}
$$

where $\alpha > 0$ is the damping gain, and $\mathbb{I}_{N,n}$ is an $N \times N$ matrix of which all elements are zero except the n-th diagonal element being 1. Because $\mathbf{x}_k - \mathbf{x}_d$ and $\mathbf{R}_k \mathbf{R}_d^\top$ are constant in the prediction horizon $[k, k+N]$, one can thus simplify the objective of the model predictive control problem to

$$
\underset{\boldsymbol{\upsilon}_{k,N}, \boldsymbol{\omega}_{k,N}}{\text{minimize}} \quad \Delta T \boldsymbol{\upsilon}_{k,N}^\top \mathbf{A}_\upsilon \boldsymbol{\upsilon}_{k,N} + 2\mathbf{b}_\upsilon^\top \boldsymbol{\upsilon}_{k,N} + \Delta T \boldsymbol{\omega}_{k,N}^\top \mathbf{A}_\omega \boldsymbol{\omega}_{k,N} + 2\mathbf{b}_\omega^\top \boldsymbol{\omega}_{k,N},
$$

where the matrices $\mathbf{A}_*$ and vectors $\mathbf{b}_*$ with $* = \upsilon, \omega$ are defined by

$$
\mathbf{A}_\upsilon = \kappa_\upsilon \sum_{n=1}^N (\mathbf{s}_n \mathbf{s}_n^\top) \otimes \mathbf{I} + \alpha_\upsilon \mathbf{I}, \quad \mathbf{b}_\upsilon = \kappa_\upsilon \sum_{n=1}^N (\mathbf{s}_n \otimes \mathbf{I})(\mathbf{x}_k - \mathbf{x}_d), \quad \mathbf{A}_\omega = \alpha_\omega \mathbf{I}, \quad \mathbf{b}_\omega = \kappa_\omega \sum_{n=1}^N (\mathbf{s}_n \otimes \mathbf{I}) \mathrm{sk}(\mathbf{R}_d^\top \mathbf{R}_k)^\vee.
$$

Let $\mathbf{x}_{eo}$ and $\mathbf{R}_{eo}$ denote the position and orientation of the object frame relative to the end-effector frame of the manipulator. The position and orientation of the object frame in the world frame is then expressed as

$$
\begin{aligned}
\mathbf{x}_o =& \mathbf{x} + \mathbf{R} \mathbf{x}_{eo}, \\
\mathbf{R}_o =& \mathbf{R} \mathbf{R}_{eo}.
\end{aligned}
$$

Let the forces exerted by the tray on the object via contact points $i = 1, \cdots, 4$ be expressed in the object's body frame by $\mathbf{f}_i = \begin{bmatrix} f_{xi} & f_{yi} & f_{zi} \end{bmatrix}^\top$. The rigid-body dynamics of the object are described by

$$
\begin{aligned}
m \dot{\boldsymbol{\upsilon}}_o + m \mathbf{g} &= \sum_{i=1}^4 \mathbf{R}_o \mathbf{f}_i, \\
\boldsymbol{\mathcal{I}} \dot{\boldsymbol{\omega}}_o + \boldsymbol{\omega}_o^\times \boldsymbol{\mathcal{I}} \boldsymbol{\omega}_o &= \sum_{i=1}^4 \mathbf{r}_i^\times \mathbf{f}_i,
\end{aligned}
$$

where $\boldsymbol{\upsilon}_o$ and $\boldsymbol{\omega}_o$ are the linear spatial and angular body velocities of the object, i.e.,

$$
\begin{aligned}
\boldsymbol{\upsilon}_o =& \dot{\mathbf{x}}_o = \dot{\mathbf{x}} + \dot{\mathbf{R}} \mathbf{x}_{eo} = \boldsymbol{\upsilon} + \mathbf{R} \boldsymbol{\omega}^\times \mathbf{x}_{eo} = \boldsymbol{\upsilon} - \mathbf{R} \mathbf{x}_{eo}^\times \boldsymbol{\omega}, \\
\boldsymbol{\omega}_o^\times =& \mathbf{R}_o^\top \dot{\mathbf{R}}_o = \mathbf{R}_o^\top \dot{\mathbf{R}} \mathbf{R}_{eo} = \mathbf{R}_o^\top \mathbf{R} \boldsymbol{\omega}^\times \mathbf{R}_{eo} = (\mathbf{R}_{eo}^\top \boldsymbol{\omega})^\times.
\end{aligned}
$$

One can thus derive that, at the acceleration level,

$$
\begin{aligned}
\dot{\boldsymbol{\upsilon}}_o =& \dot{\boldsymbol{\upsilon}} + \mathbf{R} (\boldsymbol{\omega}^\times)^2 \mathbf{x}_{eo} - \mathbf{R} \mathbf{x}_{eo}^\times \dot{\boldsymbol{\omega}}, \\
\dot{\boldsymbol{\omega}}_o =& \mathbf{R}_{eo}^\top \dot{\boldsymbol{\omega}}.
\end{aligned}
$$

Let $\boldsymbol{f} = \begin{bmatrix} \mathbf{f}_1^\top & \mathbf{f}_2^\top & \mathbf{f}_3^\top & \mathbf{f}_4^\top \end{bmatrix}$ convert the rigid-body dynamics of the object into

$$
\begin{aligned}
m \dot{\boldsymbol{\upsilon}} - m \mathbf{R} \mathbf{x}_{eo}^\times \dot{\boldsymbol{\omega}} + m \mathbf{R} (\boldsymbol{\omega}^\times)^2 \mathbf{x}_{eo} + m \mathbf{g} =& \mathbf{R} \mathbf{R}_{eo} \mathbf{G}_l \boldsymbol{f}, \\
\boldsymbol{\mathcal{I}} \mathbf{R}_{eo}^\top \dot{\boldsymbol{\omega}} + (\mathbf{R}_{eo}^\top \boldsymbol{\omega})^\times \boldsymbol{\mathcal{I}} (\mathbf{R}_{eo}^\top \boldsymbol{\omega}) =& \mathbf{G}_a \boldsymbol{f},
\end{aligned}
$$

where $\mathbf{G}_l = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix}$ and $\mathbf{G}_a = \begin{bmatrix} \mathbf{r}_1^\times & \mathbf{r}_2^\times & \mathbf{r}_3^\times & \mathbf{r}_4^\times \end{bmatrix}$.

Thus, at the k-th time step, the robot manipulator can approximate the motions of the object at the following steps $k+n$ by

$$
\begin{aligned}
m(\boldsymbol{\upsilon}_{k+n}-\boldsymbol{\upsilon}_{k+n-1}) - m\mathbf{R}_k\mathbf{x}_{eo}^{\times}(\boldsymbol{\omega}_{k+n}-\boldsymbol{\omega}_{k+n-1}) + \Delta T m\mathbf{R}_k(\boldsymbol{\omega}_k^{\times})^2\mathbf{x}_{eo} + \Delta T m\mathbf{g} &= \Delta T\mathbf{R}_k\mathbf{R}_{eo}\mathbf{G}_l\boldsymbol{f}_{k+n-1},\\
\boldsymbol{\mathcal{I}}\mathbf{R}_{eo}^{\top}(\boldsymbol{\omega}_{k+n}-\boldsymbol{\omega}_{k+n-1}) + \Delta T(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k)^{\times}\boldsymbol{\mathcal{I}}(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k) &= \Delta T\mathbf{G}_a\boldsymbol{f}_{k+n-1},
\end{aligned}
$$

where $n=1,2,\cdots,N$. In other words, the predicted contact forces $\boldsymbol{f}_{k,N}$ constrain the linear spatial $\boldsymbol{\upsilon}_{k,N}$ and angular body $\boldsymbol{\omega}_{k,N}$ velocities of the manipulator as per

$$
\begin{aligned}
m(\boldsymbol{\Gamma}\otimes\mathbf{I})\boldsymbol{\upsilon}_{k,N} - m\left[\boldsymbol{\Gamma}\otimes(\mathbf{R}_k\mathbf{x}_{eo}^{\times})\right]\boldsymbol{\omega}_{k,N} - \Delta T\left[\mathbf{I}\otimes(\mathbf{R}_k\mathbf{R}_{eo}\mathbf{G}_l)\right]\boldsymbol{f}_{k,N} &= m\boldsymbol{\eta}_k - \Delta T m\left(\mathbf{1}\otimes\left[\mathbf{R}_k(\boldsymbol{\omega}_k^{\times})^2\mathbf{x}_{eo}+\mathbf{g}\right]\right),\\
\left[\boldsymbol{\Gamma}\otimes\left(\boldsymbol{\mathcal{I}}\mathbf{R}_{eo}^{\top}\right)\right]\boldsymbol{\omega}_{k,N} - \Delta T(\mathbf{I}\otimes\mathbf{G}_a)\boldsymbol{f}_{k,N} &= \boldsymbol{\lambda}_k - \Delta T\left(\mathbf{1}\otimes\left[(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k)^{\times}\boldsymbol{\mathcal{I}}(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k)\right]\right),
\end{aligned}
$$

where $\mathbf{1}$ is a vector of all ones, and

$$
\boldsymbol{\Gamma} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0\\ -1 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & 0\\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix},\quad
\boldsymbol{f}_{k,N} = \begin{bmatrix} \boldsymbol{f}_k\\ \boldsymbol{f}_{k+1}\\ \vdots\\ \boldsymbol{f}_{k+N-2}\\ \boldsymbol{f}_{k+N-1} \end{bmatrix},\quad
\boldsymbol{\eta}_k = \begin{bmatrix} \boldsymbol{\upsilon}_k - \mathbf{R}_k\mathbf{x}_{eo}^{\times}\boldsymbol{\omega}_k\\ \mathbf{0}\\ \vdots\\ \mathbf{0}\\ \mathbf{0} \end{bmatrix},\quad
\boldsymbol{\lambda}_k = \begin{bmatrix} \boldsymbol{\mathcal{I}}\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k\\ \mathbf{0}\\ \vdots\\ \mathbf{0}\\ \mathbf{0} \end{bmatrix}.
$$

To further ease the presentation, let

$$
\mathbf{H}_{\upsilon} = \begin{bmatrix} m(\boldsymbol{\Gamma}\otimes\mathbf{I})\\ \mathbf{0} \end{bmatrix},\quad
\mathbf{H}_{\omega} = \begin{bmatrix} m[\boldsymbol{\Gamma}\otimes(\mathbf{R}_k\mathbf{x}_{eo}^{\times})]\\ -\boldsymbol{\Gamma}\otimes(\boldsymbol{\mathcal{I}}\mathbf{R}_{eo}^{\top}) \end{bmatrix},\quad
\mathbf{H}_f = \begin{bmatrix} \mathbf{I}\otimes(\mathbf{R}_k\mathbf{R}_{eo}\mathbf{G}_l)\\ \mathbf{I}\otimes\mathbf{G}_a \end{bmatrix},
$$

$$
\mathbf{h} = \begin{bmatrix} m\boldsymbol{\eta}_k - \Delta T m\left(\mathbf{1}\otimes\left[\mathbf{R}_k(\boldsymbol{\omega}_k^{\times})^2\mathbf{x}_{eo}+\mathbf{g}\right]\right)\\ \boldsymbol{\lambda}_k - \Delta T\left(\mathbf{1}\otimes\left[(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k)^{\times}\boldsymbol{\mathcal{I}}(\mathbf{R}_{eo}^{\top}\boldsymbol{\omega}_k)\right]\right) \end{bmatrix}
$$

The constraints then become

$$
\mathbf{H}_{\upsilon}\boldsymbol{\upsilon}_{k,N} - \mathbf{H}_{\omega}\boldsymbol{\omega}_{k,N} - \Delta T\mathbf{H}_f\boldsymbol{f}_{k,N} = \mathbf{h}.
$$

To prevent the object from sliding, the contact forces $\mathbf{f}_i$ need to respect the friction-cone constraints

$$
f_{xi}^2 + f_{yi}^2 - \mu^2 f_{zi}^2 \leqslant 0
$$

for the contact points $i=1,\cdots,4$, where $\mu>0$ is the friction coefficient. Also, the object keeps in contact with the tray by making their contact forces unilateral in the normal direction, i.e.,

$$
f_{zi} \geqslant \epsilon
$$

for some $\epsilon>0$, where $i=1,\cdots,4$. At the k-th time step, the friction cone constraints turn into

$$
\begin{aligned}
\boldsymbol{f}_{k,N}^{\top}\left(\mathbb{I}_{N,n}\otimes\mathbb{I}_{4,i}\otimes\boldsymbol{\Lambda}\right)\boldsymbol{f}_{k,N} &\leqslant 0,\\
\left(\mathbf{1}\otimes\mathbf{I}\otimes\begin{bmatrix}0 & 0 & 1\end{bmatrix}\right)\boldsymbol{f}_{k,N} &\geqslant \epsilon\mathbf{1},
\end{aligned}
$$

for the contact points $i=1,\cdots,4$ in the prediction horizon $n=1,\cdots,N$, where $\mathbb{I}_{4,i}$ is a 4×4 matrix of which all elements are zero except the i-th diagonal element being 1, and

$$
\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -\mu^2 \end{bmatrix}.
$$

The problem then becomes a second-order cone program as follows

$$
\begin{aligned}
\underset{\boldsymbol{\upsilon}_{k,N},\boldsymbol{\omega}_{k,N}}{\text{minimize}}\quad & \Delta T\begin{bmatrix}\boldsymbol{\upsilon}_{k,N}\\ \boldsymbol{\omega}_{k,N}\end{bmatrix}^{\top}\begin{bmatrix}\mathbf{A}_{\upsilon} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_{\omega}\end{bmatrix}\begin{bmatrix}\boldsymbol{\upsilon}_{k,N}\\ \boldsymbol{\omega}_{k,N}\end{bmatrix} + 2\begin{bmatrix}\mathbf{b}_{\upsilon}\\ \mathbf{b}_{\omega}\end{bmatrix}^{\top}\begin{bmatrix}\boldsymbol{\upsilon}_{k,N}\\ \boldsymbol{\omega}_{k,N}\end{bmatrix},\\
\text{subject to}\quad & \mathbf{1}\otimes\begin{bmatrix}\check{\boldsymbol{\upsilon}}\\ \check{\boldsymbol{\omega}}\end{bmatrix} \leqslant \begin{bmatrix}\boldsymbol{\upsilon}_{k,N}\\ \boldsymbol{\omega}_{k,N}\end{bmatrix} \leqslant \mathbf{1}\otimes\begin{bmatrix}\hat{\boldsymbol{\upsilon}}\\ \hat{\boldsymbol{\omega}}\end{bmatrix},\\
& \mathbf{H}_{\upsilon}\boldsymbol{\upsilon}_{k,N} - \mathbf{H}_{\omega}\boldsymbol{\omega}_{k,N} - \Delta T\mathbf{H}_f\boldsymbol{f}_{k,N} = \mathbf{h},\\
& \boldsymbol{f}_{k,N}^{\top}\left(\mathbb{I}_{N,n}\otimes\mathbb{I}_{4,i}\otimes\boldsymbol{\Lambda}\right)\boldsymbol{f}_{k,N} \leqslant 0,\\
& \left(\mathbf{1}\otimes\mathbf{I}\otimes\begin{bmatrix}0 & 0 & 1\end{bmatrix}\right)\boldsymbol{f}_{k,N} \geqslant \epsilon\mathbf{1}.
\end{aligned}
$$