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# 1 Gates, Expressions, Circuits, and Analysis

## 1/13 and 1/15

Topics:

- Digital Logic Gates
- Boolean Algebra
- Combination Logic Circuits
- Sum of Products
- Karnaugh Maps

### 1.1 Logic Gates

A gate has (for example NOT gate):

1. Name
2. Schematic Diagram
  - Input, A, for example, with boolean (0 or 1)
  - Output, Y, for example, boolean (0 or 1)
3. Boolean Expressions, i.e.  $Y = \overline{A}$
4. Truth Table

### Example

We can also have two or more input gates:

- AND  $\rightarrow Y = AB$ , A and B must be true
- OR  $\rightarrow Y = A + B$ , A or B must be true
- XOR  $\rightarrow Y = A \oplus B$
- NAND  $\rightarrow Y = \overline{AB}$
- NOR  $\rightarrow Y = \overline{A+B}$
- XNOR  $\rightarrow Y = \overline{A \oplus B}$

A nice to know is that if the NOT's are the actual gate, then it would turn, for example,  $X = \overline{A} \overline{B} \neq X = \overline{AB}$ .

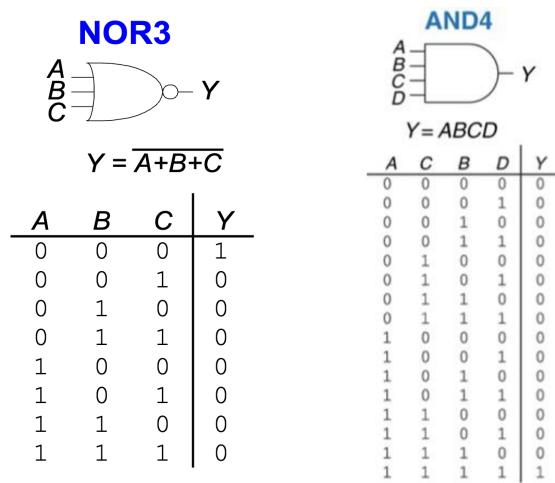


Figure 1: It can also have more than 2 inputs as seen here with their truth tables

## 1.2 Boolean Algebra

**Symbols and Boolean operators:**

$x \cdot y$ ,  $xy$ ,  $x \wedge y$ ,  $\text{AND}(x,y)$ ,  $x$  AND  $y$

$x + y$ ,  $x \vee y$ ,  $\text{OR}(x,y)$ ,  $x$  OR  $y$

$\bar{x}$ ,  $x'$ ,  $\neg x$ ,  $\text{NOT}(x)$ ,  $\text{INV}(x)$

$\overline{x \cdot y}$ ,  $\overline{x \wedge y}$ ,  $\overline{xy}$ ,  $\text{NAND}(x,y)$ ,  $x$  NAND  $y$

$\overline{x + y}$ ,  $\overline{x \vee y}$ ,  $\text{NOR}(x,y)$ ,  $x$  NOR  $y$

$x \oplus y$ ,  $\text{XOR}(x,y)$ ,  $x$  XOR  $y$

$x \overline{\oplus} y$ ,  $\overline{x \oplus y}$ ,  $\text{XNOR}(x,y)$ ,  $x$  XNOR  $y$

Figure 2: Notation before we get started

Moreover, here are some basic identities of boolean algebra

### Basic Identities of Boolean Algebra

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$\overline{X + \bar{X}} = 1$	8.	$X \cdot \bar{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \bar{X} \cdot \bar{Y}$	17.	$\overline{X \cdot Y} = \bar{X} + \bar{Y}$	DeMorgan's

Figure 3: Some basic identities

### Definition

Variable Substitution, is a way of substitution that makes it more tangible and math more easy

$$\begin{aligned} \mathbf{ABC + YZ = (ABC + Y)(ABC + Z)} \\ \text{Substitute } X \text{ for } ABC \\ \mathbf{X + YZ = (X+Y)(X+Z)} \end{aligned}$$

DeMorgan's Identity is used a lot and is very useful. As shown in these examples:

### Example

<p>16. <math>\overline{X+Y} = \overline{X} \cdot \overline{Y}</math></p> <p><b>NOR</b></p> <p><math>A \cdot \overline{B} = \overline{A} \cdot B</math></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>A</th> <th>B</th> <th>Y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	A	B	Y	0	0	1	0	1	0	1	0	0	1	1	0	<p>17. <math>\overline{X \cdot Y} = \overline{X} + \overline{Y}</math></p> <p><b>NAND</b></p> <p><math>A + \overline{B} = \overline{A} + B</math></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>A</th> <th>B</th> <th>Y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	A	B	Y	0	0	1	0	1	1	1	0	1	1	1	0
A	B	Y																													
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A	B	Y																													
0	0	1																													
0	1	1																													
1	0	1																													
1	1	0																													

Where the two equivalencies share a truth table due to this Identity

We can also see that it is like "pushing the bubble" as seen in this example:

- imagine the bubble at the output is being pushed towards the inputs
  1. it becomes a bubble at every input, and
  2. the shape of the gate changes from AND to OR, and vice versa



### 1.3 Combinational Logic Circuits

#### Definition

Stateless Digital Logic Circuits:

- Combinational logic combination of logic gates
- Change input values
- Immediate change in output values
- No Memory
- No feedback

*Remark 1.* There are specifics types of wire connections

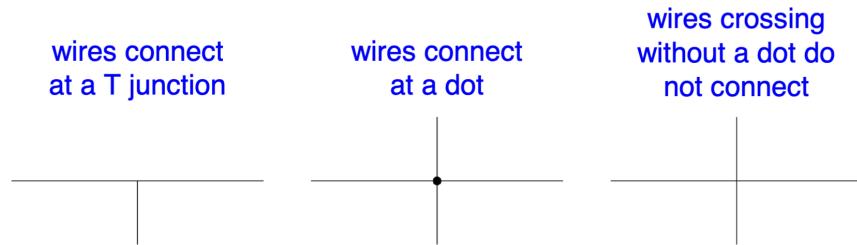
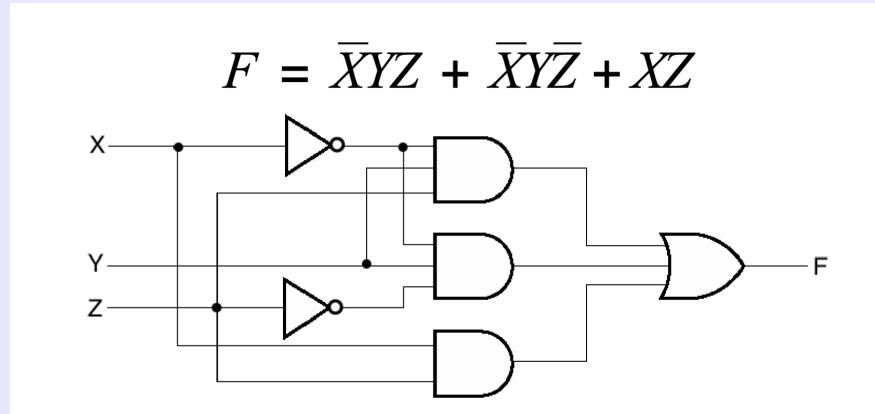


Figure 4: Here are the various ways wires can connect/not connect

## Example

Here is an example of a circuit and the resulting algebra to "solve" it and how to simplify it



$$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$$

Apply 14.  $X(Y+Z) = XY + XZ$

$$F = \bar{X}Y(Z + \bar{Z}) + XZ$$

Apply 7.  $X + \bar{X} = 1$

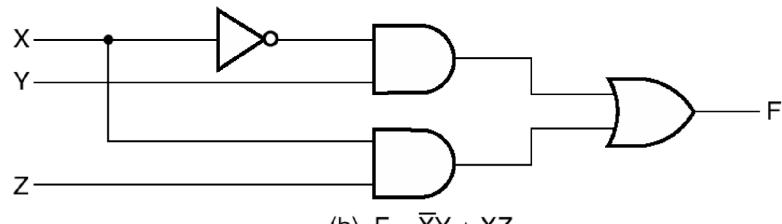
$$F = \bar{X}Y \cdot 1 + XZ$$

Apply 2.  $X \cdot 1 = X$

$$F = \bar{X}Y + XZ$$

## Fewer Gates

$$F = \bar{X}Y + XZ$$



Where output variables are either equivalent to 0 or 1 and input is the same. Moreover, simplifying this circuit and circuits in general allow for greater efficiency.

## 1.4 Standard Design Approach Sum of Products (SOP)

The three step approach:

1. Define truth table
2. Write down a Boolean expression for every row with the '1' in the output, for example,  $Y = \overline{C}BA + \overline{C}BA + CBA + CBA$
3. Wire up all of the gates

**Truth Table**

C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Figure 5: Here is the truth table given the example

## 1.5 Karnaugh Map

### Definition

Karnaugh maps, aka k-maps, are graphical representations of truth tables that use a grid with one cell for each row of the truth table

		BA 00	01	11	10	
		0	0	1	1	0
		1	0	0	1	1

C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Figure 6: An example k-map and its respective truth table!

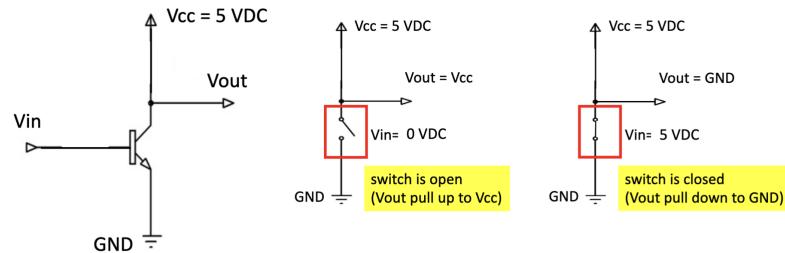
You pretty much put the 1's and 0's onto the cell given the values

Here are some rules given to the k-map

1. The grouping must be in the shape of a rectangle. There are no diagonal adjacencies allowed
2. All cells in the rectangle must contain ones. No zeros are allowed
3. The number of cells in groupings must be in powers of 2
4. Outside edges of K-maps are considered adjacent, so it may wrap around
5. Cells may be contained in more than one rectangle, but every rectangle must have at least ONE unique cell to
6. Every rectangle must be as large as possible
7. Everyone 1 must be covered by at least one rectangle

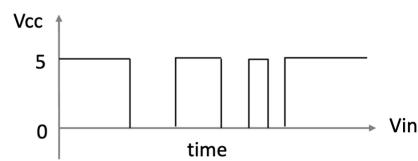
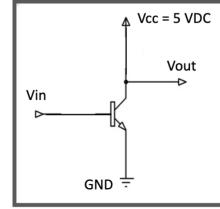
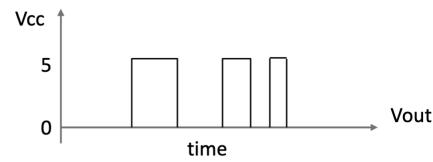
## 2 CMOS Gate Design and Analysis 1/27

The basic design of a transistor is as follows:



Basic operations:

Let's work through a simple example

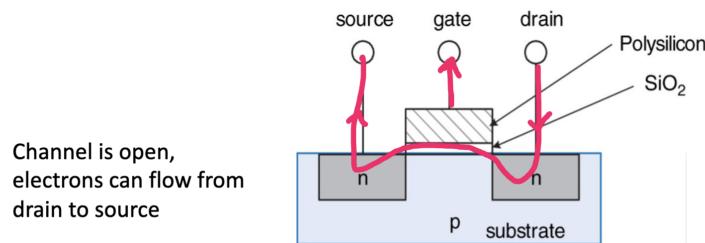
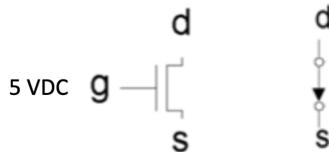


Input (Vin) voltage is "switching" the output (Vout) voltage

## 2.1 Metal Oxide Semiconductor (MOS) Transistor

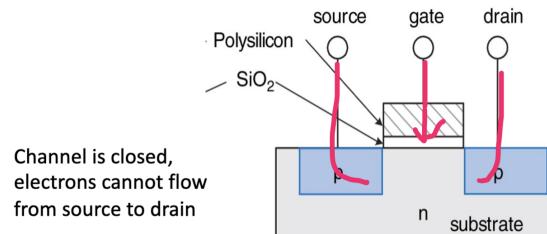
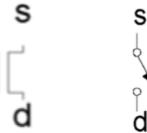
**nMOS** The n-channel Metal Oxide Semiconductor (nMOS) transistor

Switch is closed when  
gate (g) has a positive  
VDC value (e.g., 5 VDC).

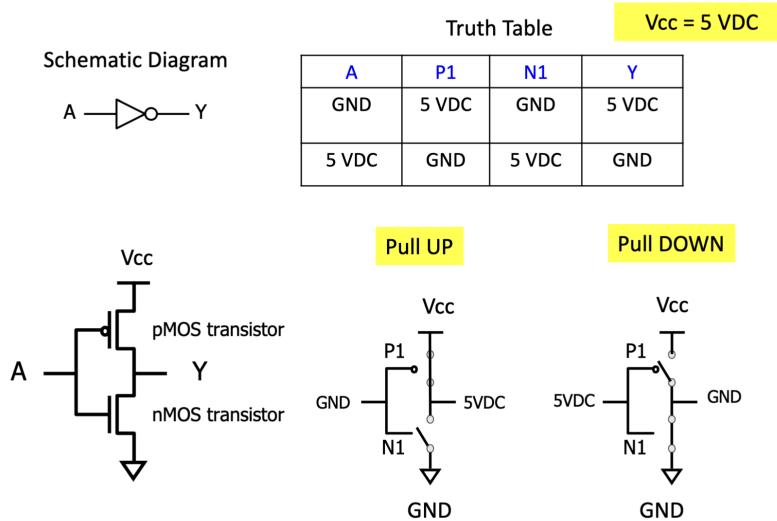


**pMOS** The p-channel Metal Oxide Semiconductor (pMOS) Transistor  
It is similar to a dam, where the analogy states, there is a lot of "water" on  
one side and then directly flows down depending on amount of "water"

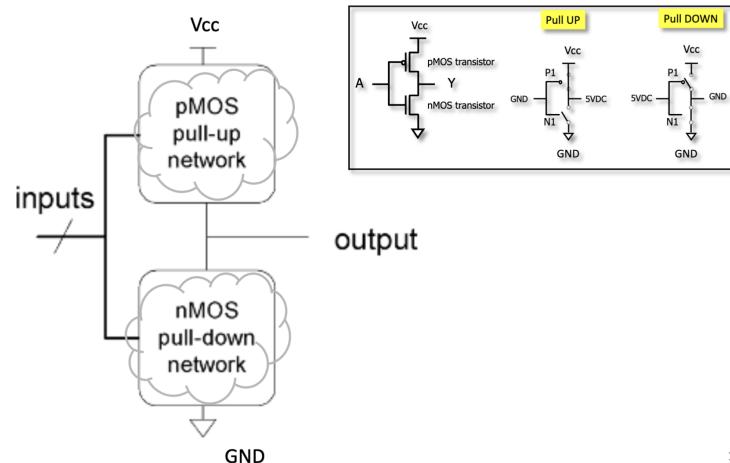
Switch is open when  
gate has a positive VDC  
value (e.g., 5 VDC).



**NOT Gate MOS Gate Design** A strong 5V and strong no 5V



Complementary MOS Designs here

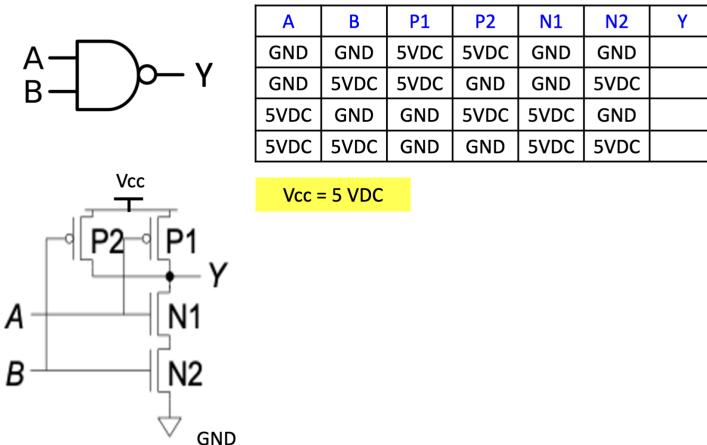


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### 3 CMOS oscillator (clock) properties and design 1/29

This lecture, we are finishing up the remaining lecture from last time...

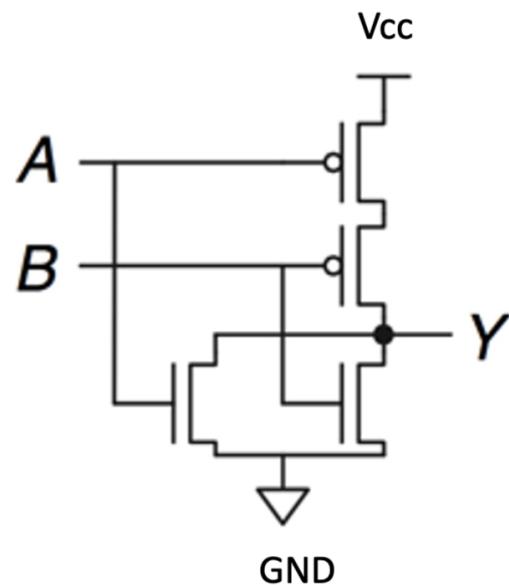
**NAND Gate: MOS Design** Only needs one or NONE of them on



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For the first three, the output should be 5V, while the last output should be 0

**NOR Gate: MOS Design** Opposite of OR, where only 5V output when all off.



### 3.1 Relationship between Voltage and Logic

Digital Abstraction

#### Definition

**Voltage** a continuous value

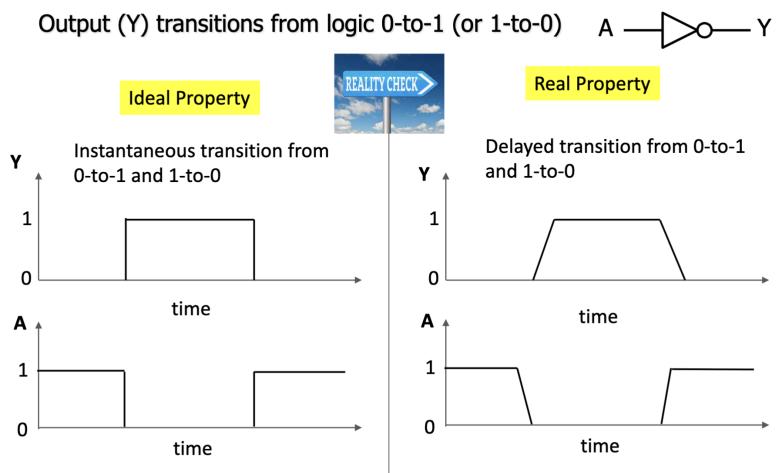
- Has a defined range of values, e.g. from 0 to 5VDC
- And any VDC value between, e.g., 0.1, 0.11, etc...
- Hardware understands voltage values

**Boolean Logic** (Logic) is a discrete value of 0 or 1

- Abstractions that humans understand
- Apply the rules of Boolean algebra
- Simplifies circuits

Continuous to Discrete conversion can be defined as having:

- Logic 1 - Has voltage range from 5 to 2 VDC
- Logic 0 - Has voltage range from 0 to 0.8 VDC
- Invalid - Less than 2 VDC, greater than 0.8 VDC, unreliable measurements

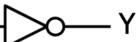


**NOT Gate: Closer Inspection** It will take time to transition from 0 to 1, and vice versa

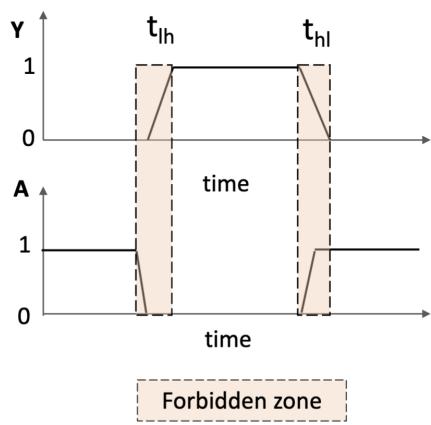
This moves onto to our definition of *Gate Delay*

### Definition

**Gate Delay** is defined as the transition from logic 0-to-1 and vice versa

Output (Y) transitions from logic 0-to-1 (or 1-to-0) 

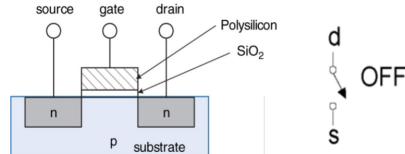
#### Real Property



$t_{Lh}$  = 0-to-1 (low to high) time delay  
 $t_{Hl}$  = 1-to-0 (high to low) time delay

The amount of time (in seconds) needed for the output value to change (**propagation delay**,  $t_d$ )

In this course, we'll assume:  
 $t_d = t_{Lh} = t_{Hl}$



Moving onto 1/29's actual lecture:

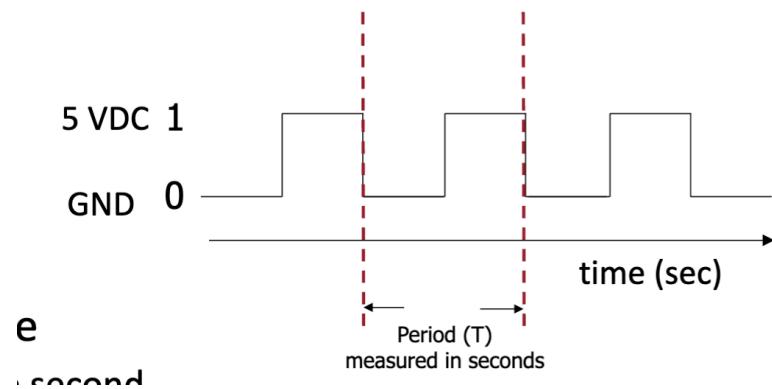
## 3.2 Clock and Clock design

### Properties: Period and Frequency

- Clock period (T)
  - One **COMPLETE** cycle
  - Typical period of 1ns
  - Measured in **seconds**
- Clock Frequency (F) or rate
  - How many cycles in 1 seconds
  - Frequency =  $F = \frac{1}{T}$

- Measured in **Hertz**

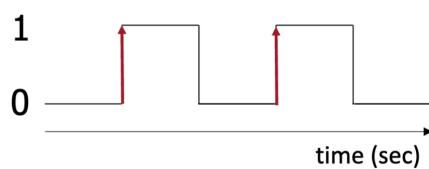
Shown below is what these ways would look like:



**Properties: Events** What are the different edges?

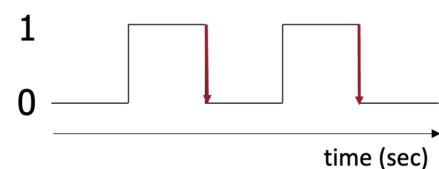
### Rising-edge

- Signal transitions from logic 0 to logic 1
- What is the amount of time (sec) between two successive rising-edge events?



### Falling-edge

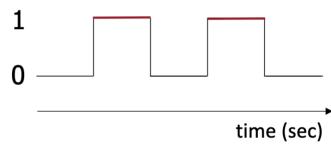
- Signal transitions from logic 1 to logic 0
- What is the amount of time (sec) between two successive falling-edge events?



**Properties: Active High and Low** Now lets look at the highs and lows

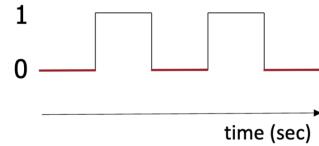
### Active high

- Signal is logic 1
- What is the amount of time (sec) in one clock period?



### Active low

- Signal is logic 0
- What is the amount of time (sec) in one clock period?



## 3.3 Clock design

**Ring Oscillator** clock that oscillates using inverter logic gates

### Definition

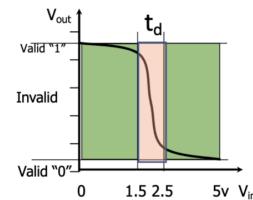
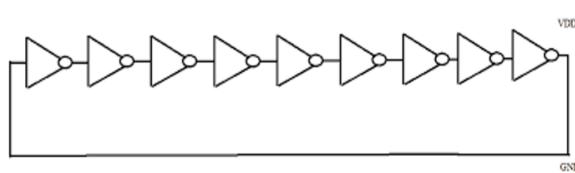
**Clock Period** is the propagation delay for a sequence of inverters

$$\text{Frequency} = 1 / (2 * \# \text{ of inverters} * t_d)$$

Can you think of a limitation?

Hint: # of inverters

- $t_d$  = propagation delay for a single inverter
- propagation delay = amount of time (sec) for the output value to change when given an input value.

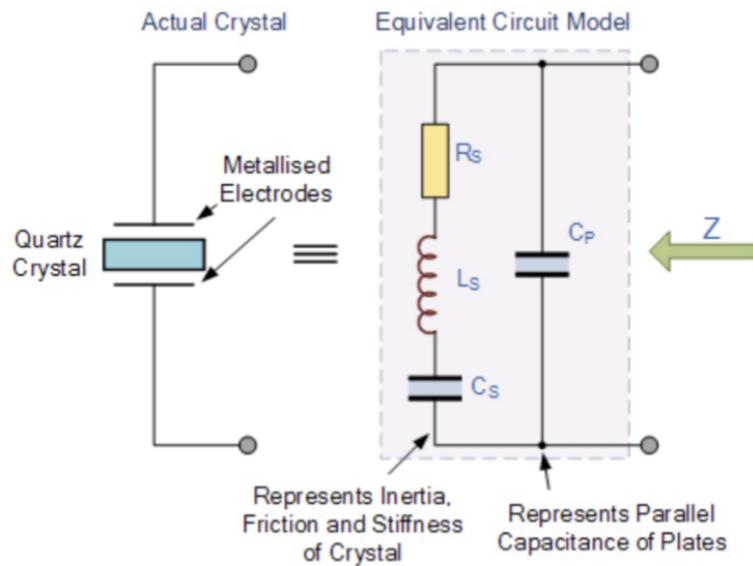


### Quartz Crystal

Used in modern processor

Depending on the crystal's physical thickness and size, it can control:

- Frequency of oscillations
- Inversely proportional to its physical thickness between 2 metallic surfaces



## 4 Common components and analysis 2/3

There exists a Don't care (x)

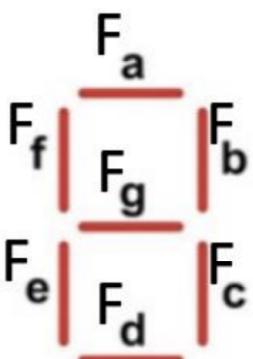
One example of this is our Lab 1

Example

Where if we don't need to display support letters A-F, we can put x ("don't care") in those rows

Where we can show it as:

**7-Segment Display Layout**



The diagram shows a 7-segment display layout with segments labeled F<sub>a</sub> through F<sub>g</sub>. The segments are arranged as follows: F<sub>a</sub> is the top horizontal bar; F<sub>b</sub> is the right vertical bar; F<sub>c</sub> is the bottom right vertical bar; F<sub>d</sub> is the bottom horizontal bar; F<sub>e</sub> is the bottom left vertical bar; F<sub>f</sub> is the left vertical bar; and F<sub>g</sub> is the middle horizontal bar.

**Inputs      Outputs**

A	B	C	D	F <sub>a</sub>	F <sub>b</sub>	F <sub>c</sub>	F <sub>d</sub>	F <sub>e</sub>	F <sub>f</sub>	F <sub>g</sub>	
0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0							
0	0	1	0	0	1						
0	0	1	1	0	1						
0	1	0	0	0	0						
0	1	0	1	0	1						
0	1	1	0	0	1						
0	1	1	1	0	1						
1	0	0	0	0	1						
1	0	0	1	0	1						
1	0	1	0	?	X	X	X	X	X	X	
1	0	1	1	?	X	X	X	X	X	X	
1	1	0	0	?	X	X	X	X	X	X	X
1	1	0	1	?	X	X	X	X	X	X	X
1	1	1	0	?	X	X	X	X	X	X	X
1	1	1	1	?	X	X	X	X	X	X	X

Don't care K-map rules:

- Because  $x$  denotes either 0 or 1, we treat it as a 0 or 1 in a K-map
- We circle an  $x$  if it helps us cover the 1's with larger or fewer rectangles, i.e.  $x$  is treated as 1
- We don't circle an  $x$  if it is not helpful for covering 1's, in this case we treat it as a 0

## Example

## K-map example 1

- Complete the K-map by drawing rectangle(s) that satisfy all K-map rules. Optionally, write the simplified Boolean expression.

AB\CD	00	01	11	10
00	0	0	x	1
01	0	1	x	1
11	0	1	x	x
10	0	0	x	x

$$C + BD$$

## K-map example 2

- Complete the K-map by drawing rectangle(s) that satisfy all K-map rules. Optionally, write the simplified Boolean expression.

AB\CD	00	01	11	10
00	1	1	x	1
01	1	0	x	1
11	1	1	x	x
10	1	0	x	x

$$\bar{D} + \bar{A} \bar{B} + AB$$

## 4.1 Multiplexer (MUX)

S bits to "select" which input (A or B) becomes the output (Y)

$$S = 1 \rightarrow Y = B$$

$$S = 0 \rightarrow Y = A$$

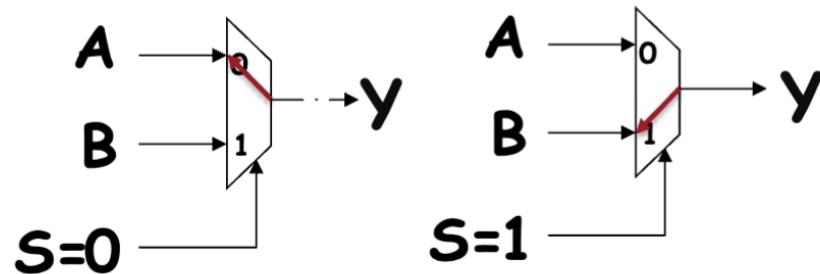


Figure 7: Example of the multiplexer

**Number of inputs** = 2 to the power of whatever number of select bits  
 If we have 4 inputs, A, B, C, D, then we would need 2 signal bits for example

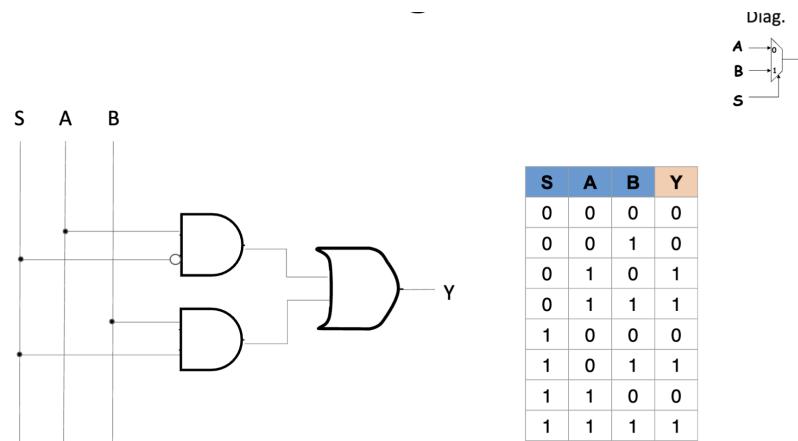


Figure 8: Truth table and circuit with two inputs of MUX

## 4.2 Demultiplexer (DeMUX)

S bits to "select" which input (A or B) becomes the output (Y)

$$S = 1 \rightarrow A = Y$$

$$S = 0 \rightarrow B = Y$$

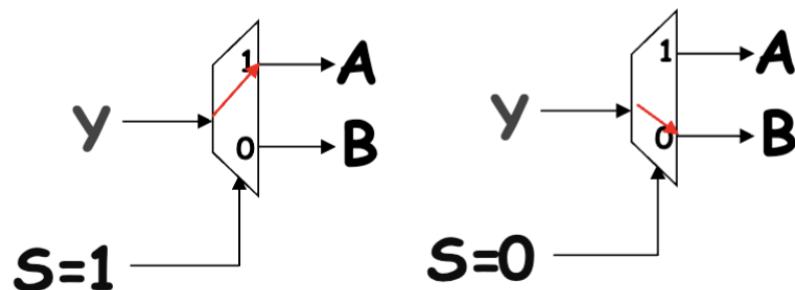


Figure 9: Example of the Demultiplexer

Number select bits =  $\log_2(\text{number of output bits})$

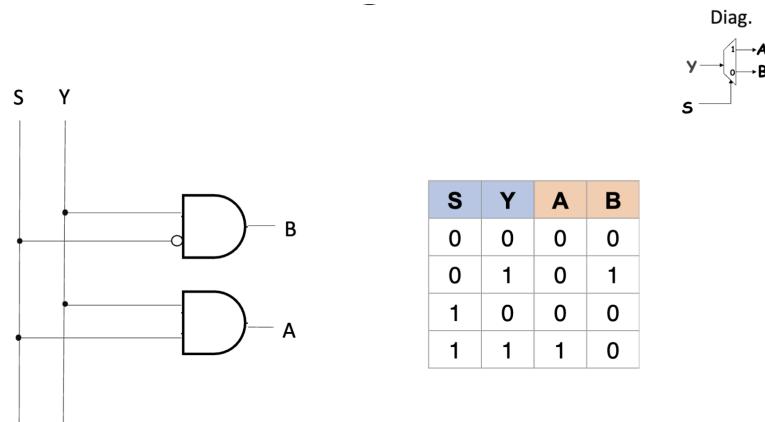


Figure 10: Truth table and circuit for DeMUX

### 4.3 Decoder

S input bits are used to "select" which output bits (A) are turned on (logic 1)

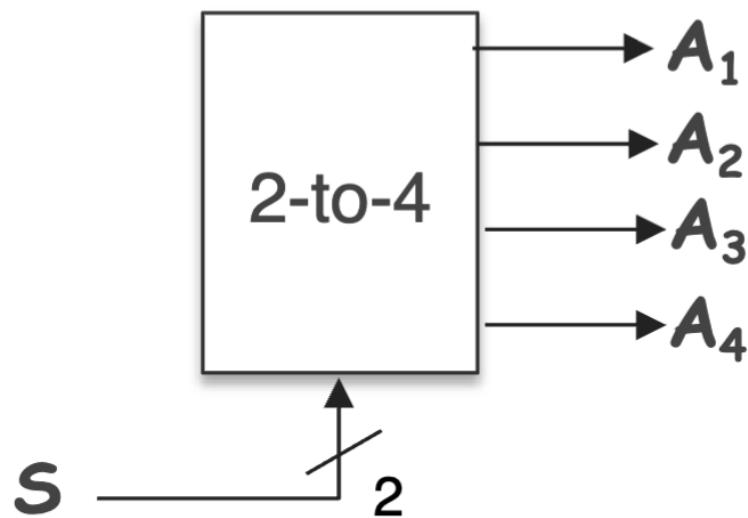


Figure 11: Decoder example

The number of outputs = 2 to the power of number of select bits

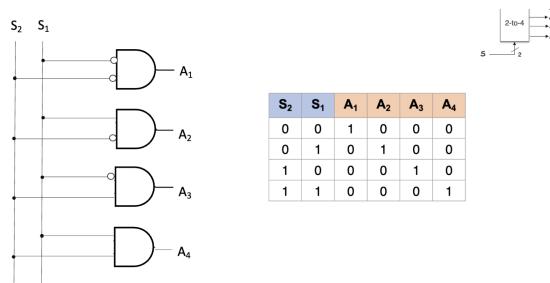


Figure 12: Truth table and circuit for Decoder

#### 4.4 Encoder

Input bit (A) are used to select which output bits (S) are turned on (logic 1)

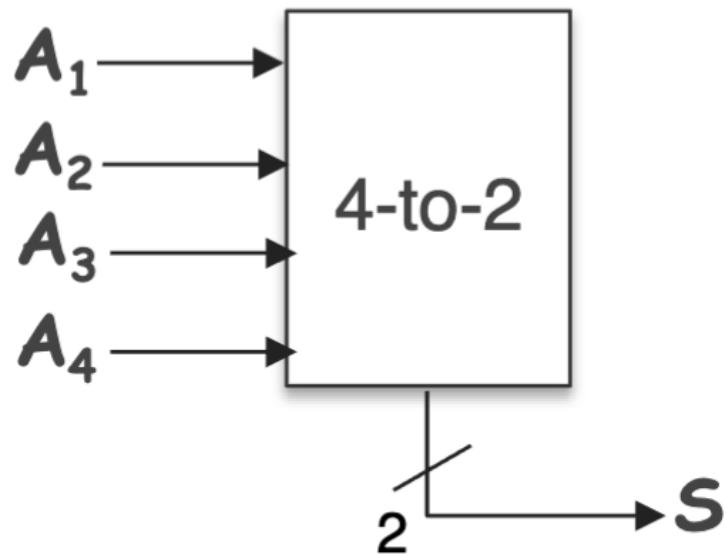


Figure 13: Encoder example

$$\text{Number of output bits} = \log_2(\text{number of input bits})$$

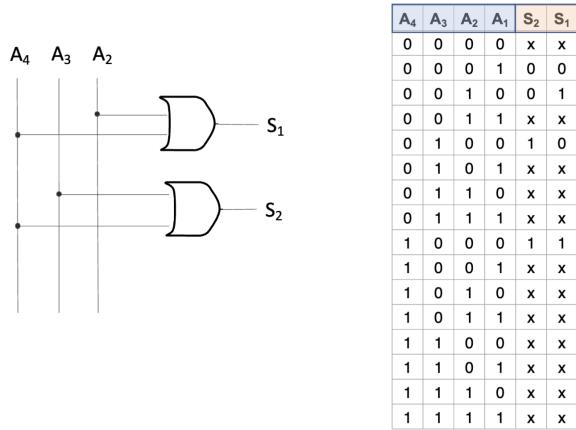


Figure 14: Truth table and circuit for Encoder

## 5 ALU component design and analysis 2/5

What does the ALU do? It allows computer to add, divide, etc. also allows for bit shifts and what not

### 5.1 Add and Subtract

**Binary Addition** not a single operation

Where  $A + B = \text{sum}$  and carry-out  
 $A, B, \text{Sum } (S)$  and Carry-out ( $C_0$ ) are one bit binary values as seen below:

**Four possibilities (A and B):**

$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 0 \end{array}$$

**Binary Half Adder Circuit Design** where there are two input bits, A and B holding 1 bit each. And two output bits, C and S, holding 1 bit each

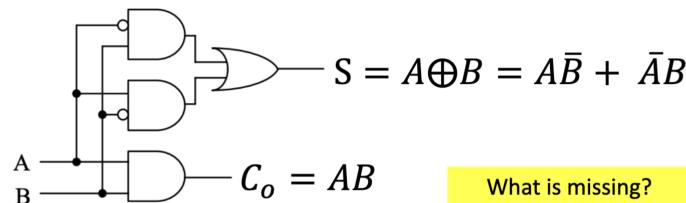


Figure 15: Half Adder Circuit

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Figure 16: Truth table for half adder circuit

**Binary Full Adder Circuit Design** Similar to the half adder, where we have three input bits:  $C_{in}$ , A, B, where they have 1 bit each

And two output bits:  $C_0$  and S, that have 1 bit each.

**S:** 

bit each

**S:** 

bit each

C <sub>i</sub>	A	B	C <sub>o</sub>	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$C_o = C_i(A + B) + AB = C_i(A \oplus B) + AB$$

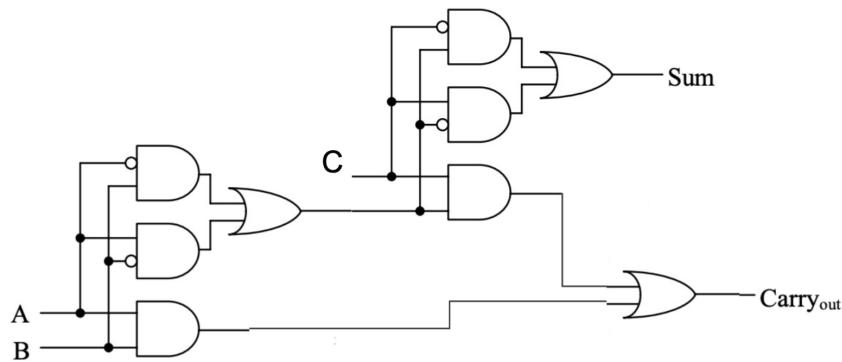
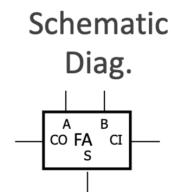
$$S = C_i \oplus A \oplus B$$

Figure 17: The truth table and equations for the Binary Full Adder

Moreover, we can take a look at the full adder circuit itself:

# Full Adder Circuit

Two HA circuits plus one or gate



## 5.2 FA Component Properties

We can assume that the propagation delay ( $t_d$ ) for each logic gate is 1 nanosecond (ns).

**Sum bit analysis** We can get the value of the output (S) and we can also get the delay,  $t_d$

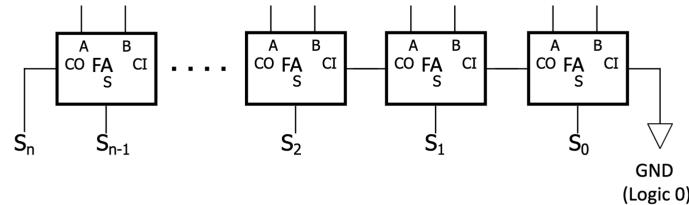
Therefore, we can also get worst case analysis, where it is the max(Sum  $t_d$ , Carry-out  $t_d$ )

In the case of the full adder, we know that the sum is 2ns for worst case, and carry-out is 3ns for worst case.

Thus, all component outputs will be stable in 3ns

### 5.3 Add and subtract Circuit

Using FA circuits, extend to arbitrary number of bits



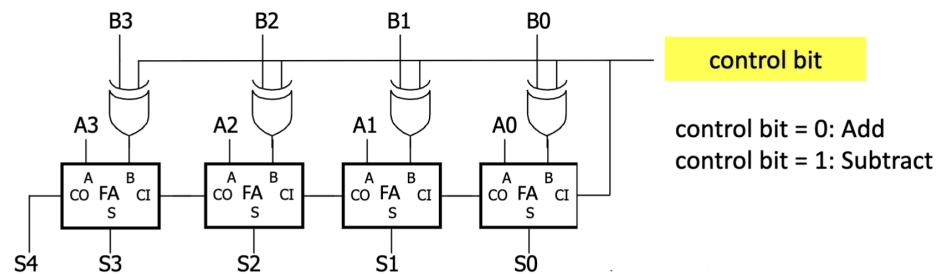
"Ripple-Carry Adder"

- carries ripple through from right to left
- longest chain of carries has length  $n$

### Subtract A-B: 2's complement Operation

$\sim = \text{bit-wise complement}$

2's complement:  $-B = \sim B + 1$

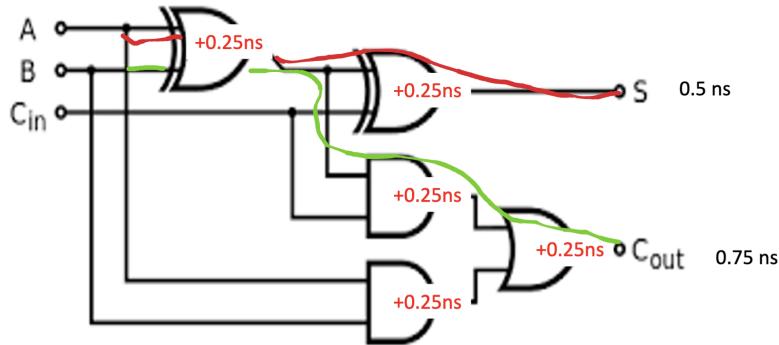


Carry-out bit    We'll see how this is used very soon!

Figure 18: Here is a 4 bit example of addition and subtract

Looking back at the Full Adder, we see that the worst case per component is this:

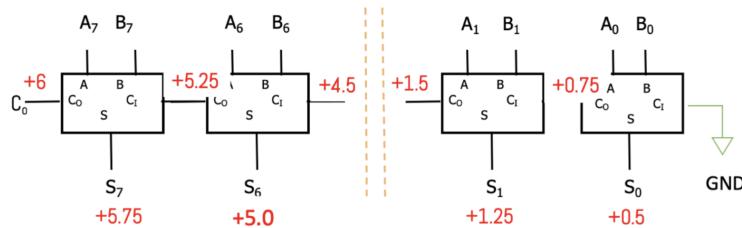
### Worst case analysis (i.e., upper bound)



Therefore, we when we take a look at the 8-bit full add circuit:

If the clock period is 1 ns, then how many clock cycles (worst case) are needed?

- Carry out delay ( $t_d$ ) = 0.75 ns
- Total delay =  $t_d \times \text{number of bits} = 0.75 \times 8 = 6 \text{ ns}$  (**6 clock cycles are needed**).

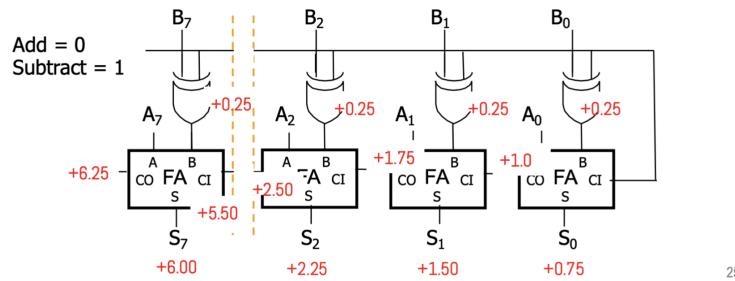


As a side note, the reason why  $S_1$  is 1.5 ns is due to the fact that S and  $C_{out}$  is different in structure and has different delays

**Full Add and Subtract Circuit** Here we have a 8-bit full add and subtract circuit that is actually in parallel.

If the clock period is 1 ns, then how many clock cycles (worst case) are needed?

- LSB ( $C_0$ ) carry-out delay ( $t_{d0}^0$ ) = 1.0 ns
- FA carry-out delay ( $t_d$ ) = 0.75 ns
- Total delay =  $t_{d0}^0 + t_d \times (\text{number of bits} - 1) = 1.0 + 0.75 \times 7 = 6.25 \text{ ns}$   
**(7 clock cycles are needed)**



25

## 5.4 Bit Shift Circuit

Recall

Here is a **reminder** of what bit shifts are!

**Left Shift:** shifts in a 0 from the right end

- $(X << 1) = 00101000_2 = 40_{10}$

$$X = 20_{10} = 00010100_2$$

**"Logic" Right Shift:** shifts in a 0 from the left end

- $(X >> 1) = 00001010_2 = 10_{10}$

**"Arithmetic" Right Shift:** maintains the sign bit

- $-X = -20_{10} = 2\text{'s complement of } X = 11101100_2$
- $(-20_{10} >>> 1) = (11101100_2 >>> 1) = 11110110_2 = -10_{10}$

Note:

- shift right arithmetic notation ( $>>>$ )
- shift right logic notation ( $>>$ )

**Bit Shift Component Design** But how do we do this with a circuit? We can do it here!

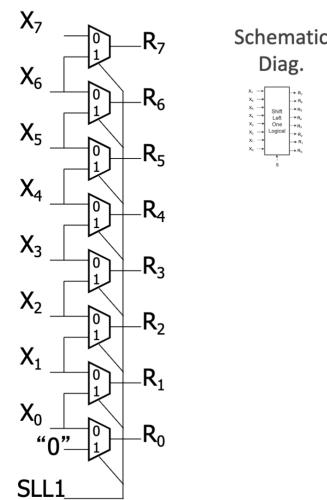
### Example Shift Left Circuit

If SLL1 is true (logic 1)

- Shifts the input X one bit to the left
- $R \leftarrow X \ll 1$

If SLL1 is false (logic 0)

- Do not shift X
- $R \leftarrow X$



Shift left by 2

- Rewire the multiplexors so each  $X_i$  feeds into  $R_{i+2}$
- Similarly: shift left by 4, etc.

Shift right circuits have similar circuitry

- shift right logical: each  $X_i$  feeds into a lower numbered  $R_j$
- shift right arithmetic: sign bit stays the same

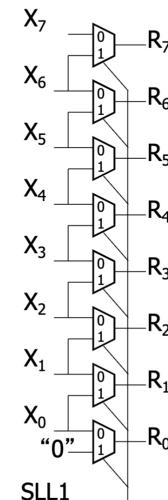


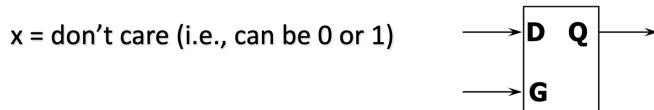
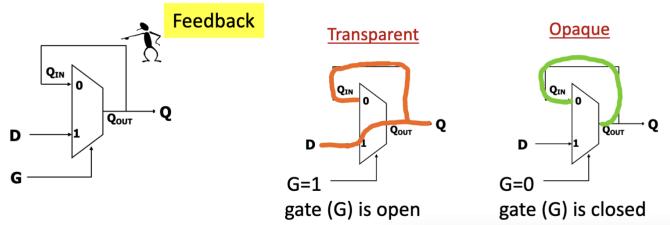
Figure 19: Here is an example of the bitshift with this circuit

## 6 Latches, D Flip-Flops (introduce J/K and T Flip-Flops), and memory/register design 2/24

### 6.1 Latch Component

stores 1 bit

- Store 1 bit (0 or 1)
- Storage = stateful = has memory!
- 2-to-1 MUX with Feedback
- Inputs D,  $Q_{in}$  (or Q), G (switch or gate bit)
- Output  $Q_{out}$
- Q has both an input and output

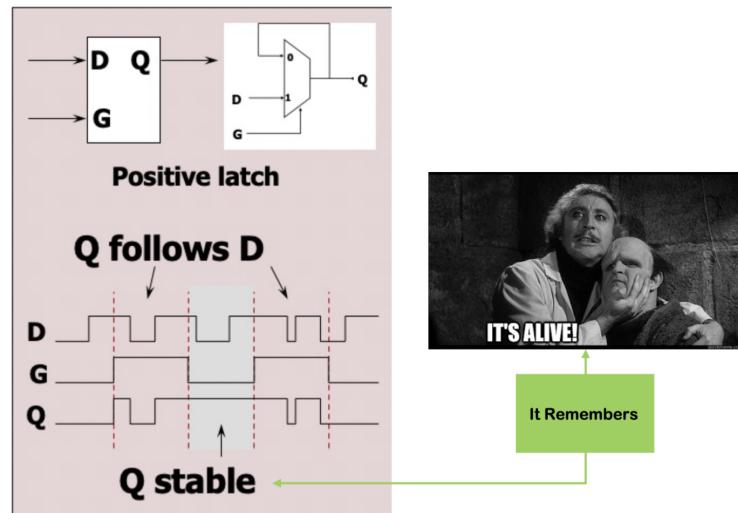


G	D	$Q_{IN}$	$Q_{OUT}$
opaque	0	X	0
	0	X	1
transparent	1	0	X
	1	1	X

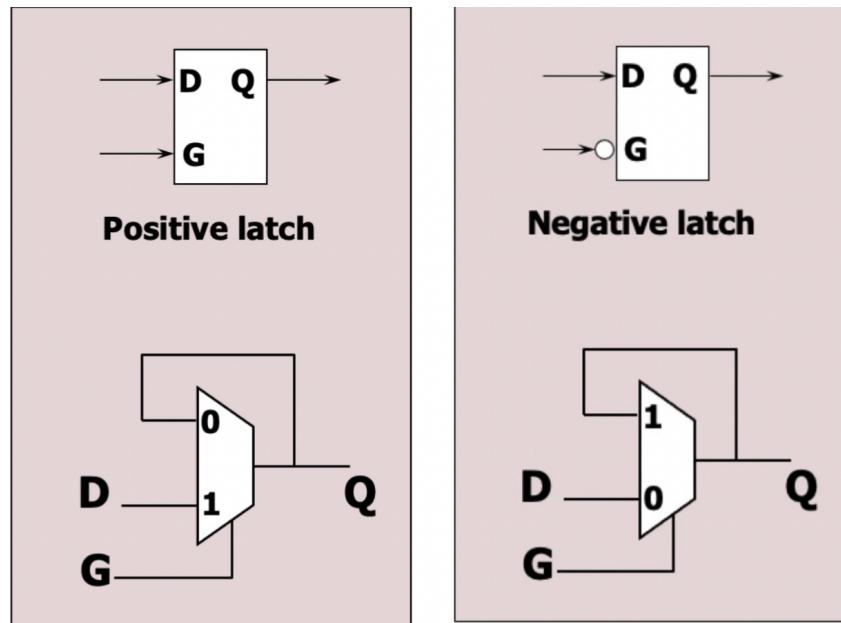
stable  
unstable

Figure 20: Truth table for Latch

**Timing Waveform for a Latch** Amount of time before stable



For the negative latch, we see that there is an inverter before G.



## 6.2 Latch Consideration: Transparency & Escapement

Latch is transparent when  $G=1$ , meaning that  $Q$  is unstable!

We must minimize the unstable properties, therefore we must make  $G$  well-defined and reliable.

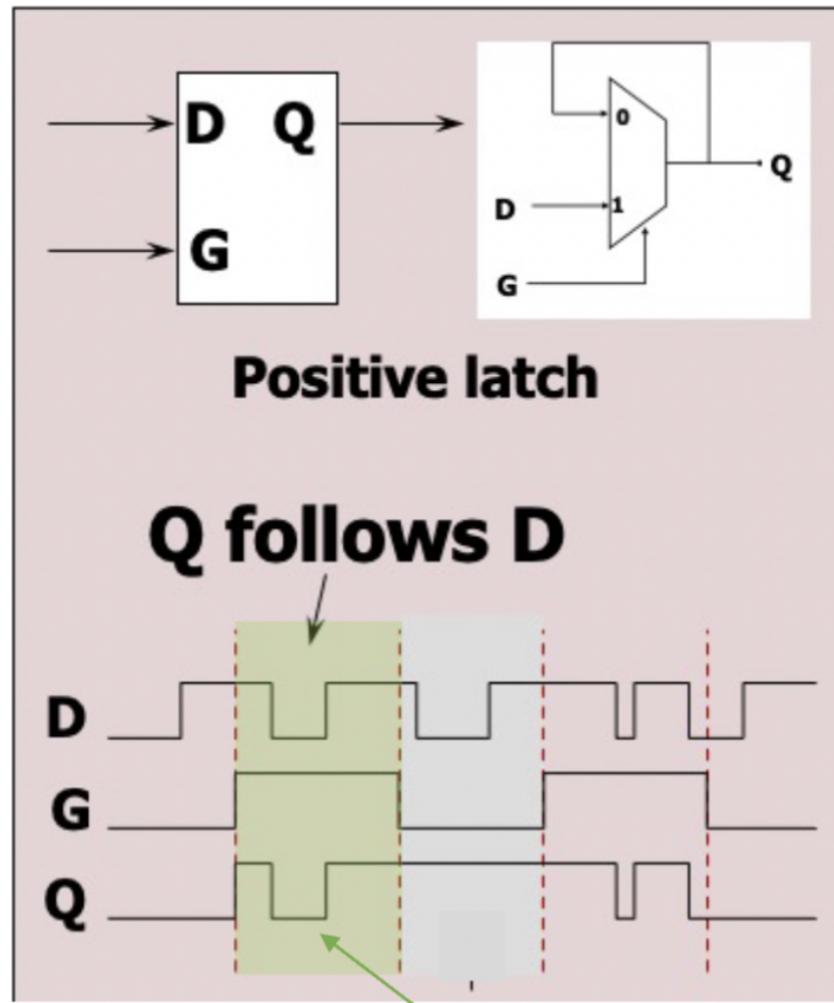


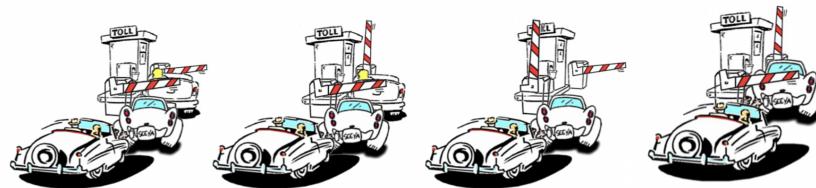
Figure 21: Transparent, unstable is not a desirable property

### 6.2.1 Principle of Escapement

Think of the cars and gate analogy:

If we have unreliable timing, then multiple cars would be able to get through.

With reliable timing, we only let one car at a time through a 2 gate approach...

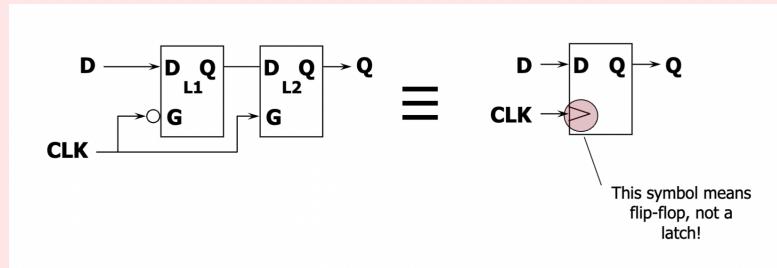


### 6.3 Flip Flop Component: Storing 1-bit with reliable timing

#### Definition

Flip Flops are two latches combined to form one flip-flop.

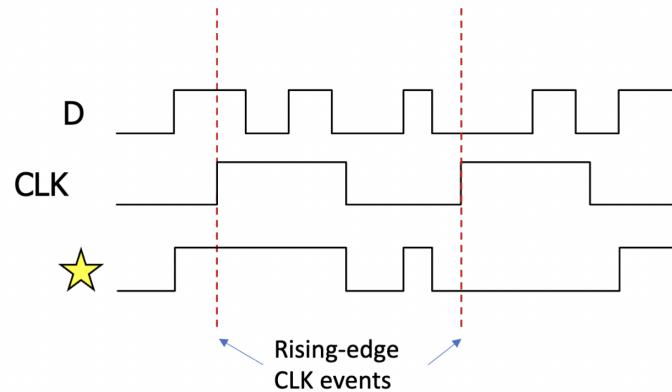
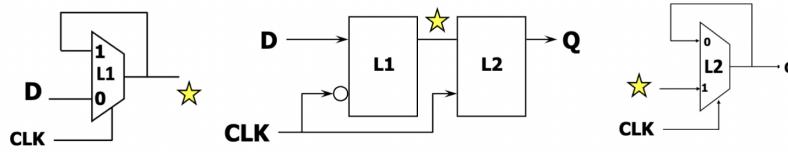
Specifically, it is one negative latch connected to one positive latch



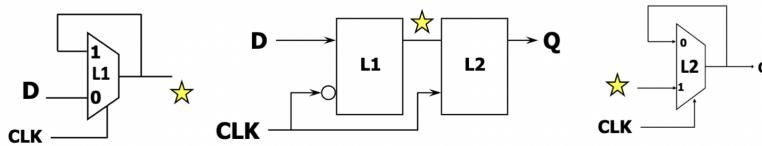
**Logical Escapement for D Flip-flop** → where only 1 latch is transparent at a time.

Flip-flop appears to be triggered by reliable timing event, specifically a rising-edge clock event

## L1: Timing Waveform (D, CLK, $\star$ )



## L2: Timing Waveform ( $\star$ , CLK, Q)



L2 is a positive latch

- Transparent (unstable) when  $\text{CLK} = 1$
- Opaque (stable) when  $\text{CLK} = 0$

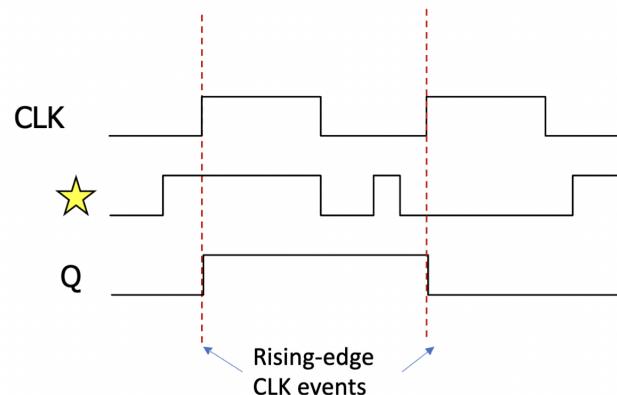


Figure 22: First and Second latch respectively in D flip-flop

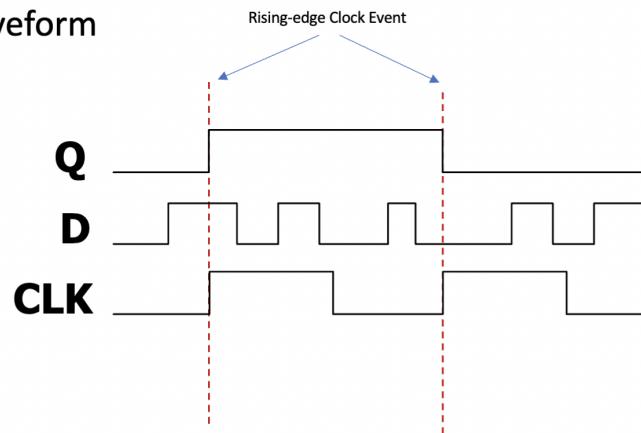
### 6.3.1 D flip-flop timing waveform & Truth Table

So lets combine both we get:

**Do not consider signal between L1 and L2 ( ★ )**

#### Flip-Flop timing waveform

- Output Q
- Input D
- Clock (CLK)



*Remark 2.* This simplifies it down, making it so that whenever the clock has a rising-edge event, output Q will change to the last value from input D

$Q_p$  = Previous Q value (before clock event)

CLK	D	$Q_p$	Q
Rising-edge	0	0	0
Rising-edge	0	1	0
Rising-edge	1	0	1
Rising-edge	1	1	1
Falling-edge, Active High, or Active low	x	0	0
Falling-edge, Active High, or Active low	x	1	1

Figure 23: D Flip-flop Truth Table with specific clock value

However, this truth table can be drastically simplified down by looking at ONLY the rising-edge enabled truth-table:

For example,

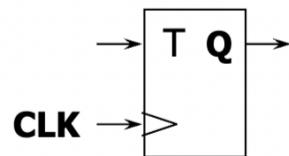
- If  $Q_p = 0$  and  $D=0$ , then  $Q=0$
- If  $Q_p = 1$  and  $D=0$ , then  $Q=0$
- If  $Q_p = 0$  and  $D=1$ , then  $Q=1$
- If  $Q_p = 1$  and  $D=1$ , then  $Q=1$

D	$Q_p$	Q
0	0	0
0	1	0
1	0	1
1	1	1

In short, on a rising-edge event, Q follows D.

### 6.3.2 T flip-flop

T (toggle) flip-flop will toggle if T is 1. Meaning that unless there is another rising-edge event, Q output will not change.



T	$Q_p$	Q
0	0	0
0	1	1
1	0	1
1	1	0

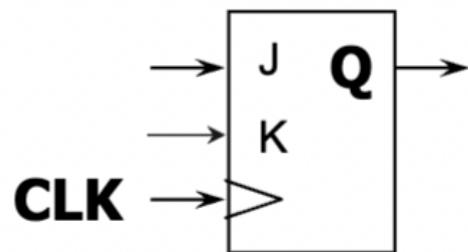
Rising-Edge Enabled

T=0, Q stays the same

T=1, Q toggles (0->1 or 1->0)

### 6.3.3 J/K flip-flop

Set (J) and Reset (K), giving us T and D capabilities!



J	K	$Q_p$	Q
0	x	0	0
1	x	0	1
x	1	1	0
x	0	1	1

Rising-Edge Enabled

J=1, K=0, Q=1 (set)

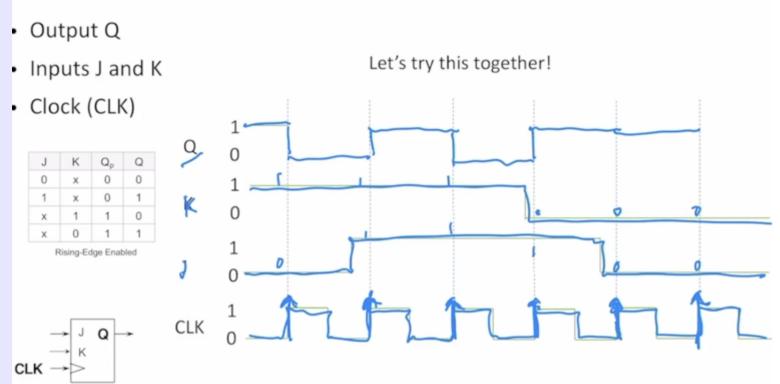
K=1, J=0, Q=0 (reset)

J=K=1, Q toggles

J=K=0, Q stays the same

### Example

Here is an example of this timing waveform:



## 6.4 Register Design and Timing

TO BE FINISHED...

## 7 Simple as possible (SAP) Computer: Intro and Instruction Set

### 7.1 Simple as possible Computer

#### 7.1.1 ALU

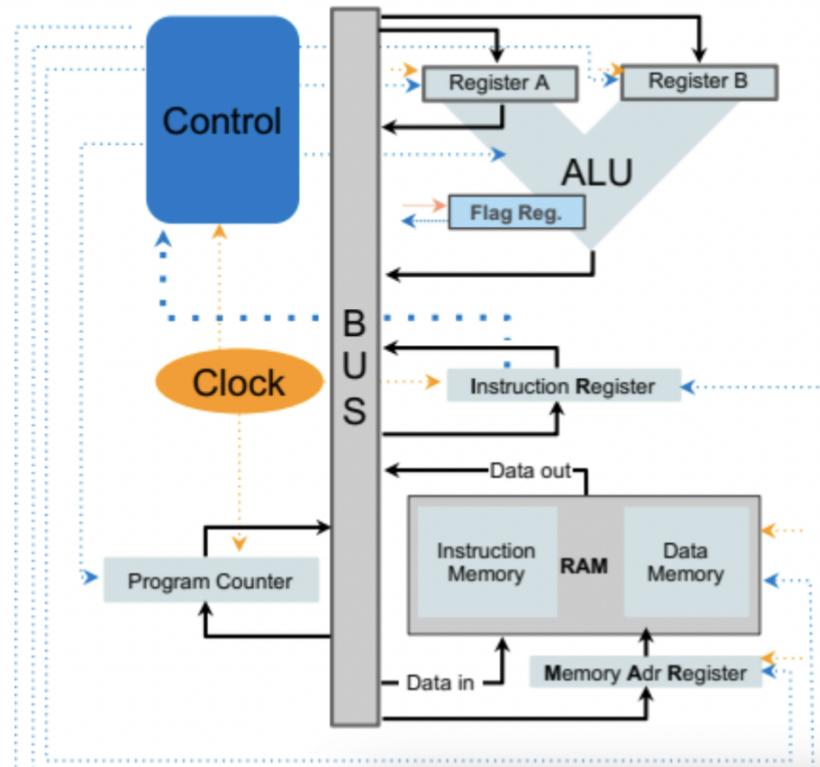


Figure 24: SAP and ALU

- ALU only executes add and subtract instructions
- Inputs → Data in Reg. A and B & control bits
- Outputs → Data to flag register, add/sub to BUS

BUS is the "highway" for the computer to talk to each other...

### 7.1.2 Clock System

Moreover the system clock allows for:

- Synchronize timing of multiple components
- Clock events input to components

### 7.1.3 Registers

- Instruction Register (IR) → Read/write data from/to BUS
- Memory Address (MAR) → Read data from BUS, write data to RAM
- Program Count (PC) → Read/write data from/to BUS
- Register A → Read/write data from/to BUS
- Register B → Read data from the BUS
- Flag Register → Read data from ALU, write data to Control

### 7.1.4 RAM

Random access memory:

- Main Memory
  - Addressable
  - Instructions
  - Data
- Input → Data from BUS (Data In) + Data from MAR
- Output → Data to BUS (Data Out)

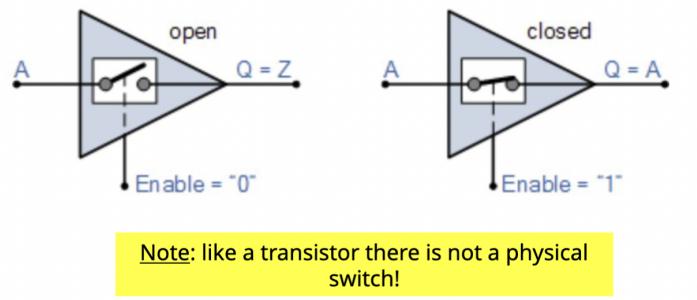
### 7.1.5 Control

- Configure components, dictates what is done in the SAP computer
  - Registers
  - RAM
  - ALU

- Input → IR (ROM instruction decode) + Flag Register
- Bus Negotiation → Components that can perform BUS read/write operations

## 7.2 SAP BUS

### 7.2.1 Tri-State Buffer Component



You can sort of picture of it as a switch

Z is also a new state where it is neither 0 or 1

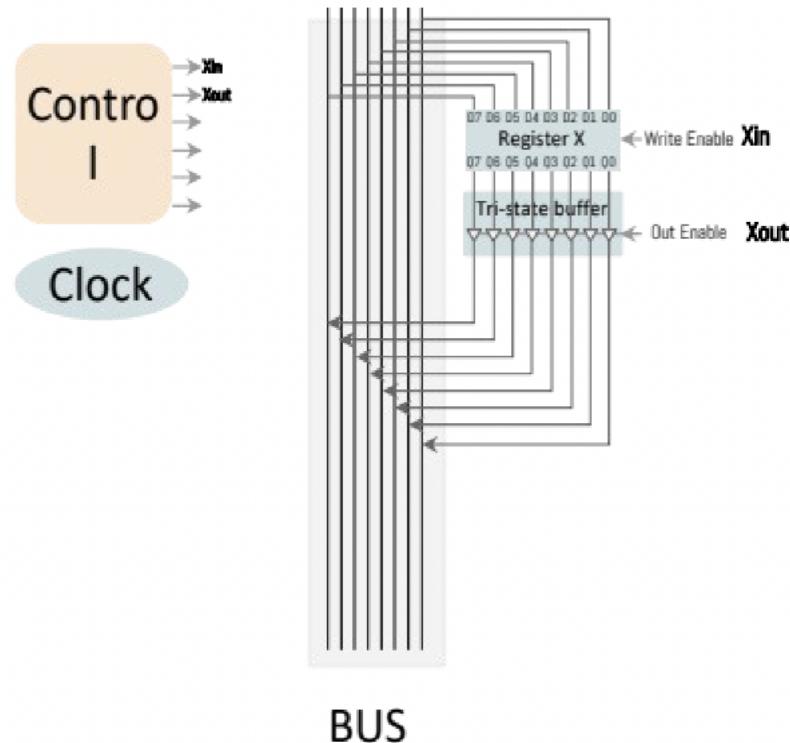
At any given time, components (e.g., CPU, RAM, etc.) can:

- Read or write bits values onto the BUS
- BUS read/write is facilitated by the Tri-state buffer.

Tri-State buffer basic operation:

- If enable bit = 1, then switch is closed, OUT = IN(i.e. Q = A)
- If enable bit = 0, then switch is open, OUT = HIGH Impedance (i.e. Q=Z, where Z is high Impedance)

### 7.2.2 Bus Operation: One Register

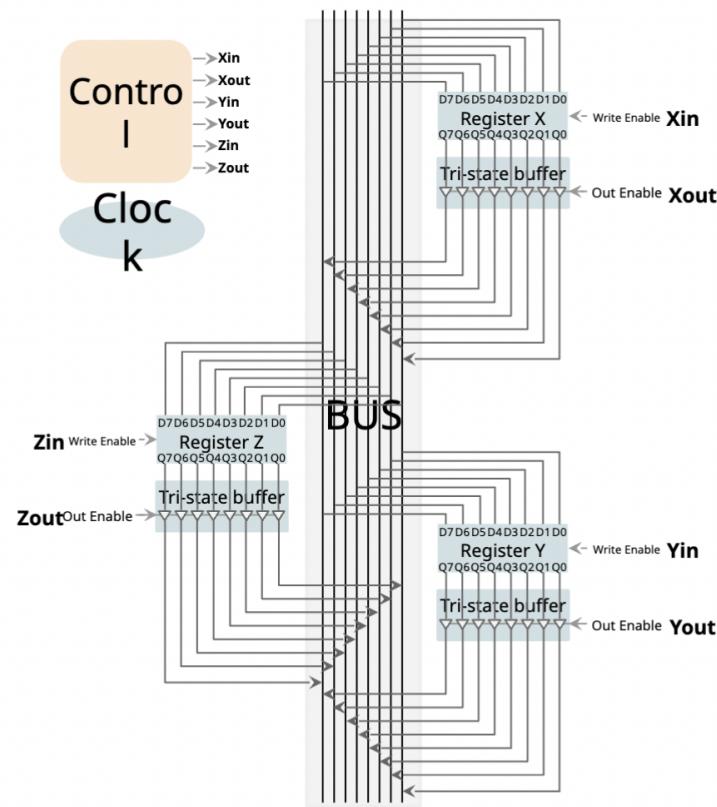


- Controller:
  - $X_{in}$  output (1 bit)
  - $X_{out}$  input (1 bit)
- 8-Bit register X three inputs
  - Data on the Bus
  - CLK
  - $X_{in}$  control bit
- X can read data from BUS and write data to the register
- X can write data on to the BUS when  $X_{out}$  control bit = 1

### 7.2.3 Bus Operation: More than 1 register

Only 1 register's **OUT** control bit can set to 1

More than one register's **IN** control bit can be set to 1

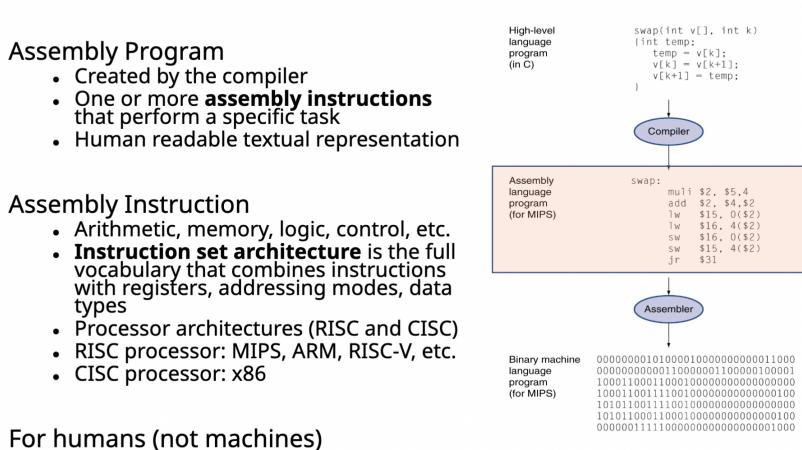


Op Code	Instruction	Arg	Function
0000	NOP		
0001	LDA	Data Address	$A = \text{Mem(arg)}$
0010	ADD	Data Address	$A = A + \text{Mem(arg)}$
0011	SUB	Data Address	$A = A - \text{Mem(arg)}$
0100	STA	Data Address	$\text{Mem(Arg)} = A$
0101	LDI	Constant value	$A = \text{arg}$
0110	JMP	Instruction Address	$PC = \text{arg}$
0111	JC	Instruction Address	if FC=1 then PC=arg else go on
1000	JZ	Instruction Address	if FZ=1 then PC=arg else go on
1110	OUT		Display = OUT = A
1111	HLT		

Figure 25: SAP instruction set

### 7.3 Language Concepts

Assembly Review:



Machine Review:

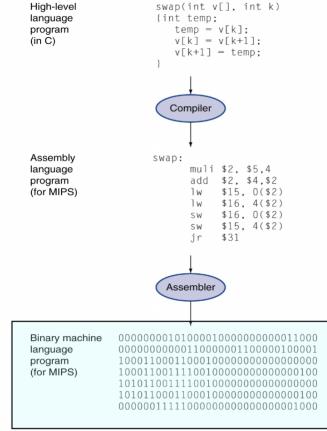
### Program

- One or more **machine instructions** that perform a specific task
- Created by the assembler
- Not human readable binary representation

### Instructions

- One-to-one correspondence with assembly instruction
- Specific to processor architecture (hardware)!

For machines (not humans)



## 7.4 SAP ALU

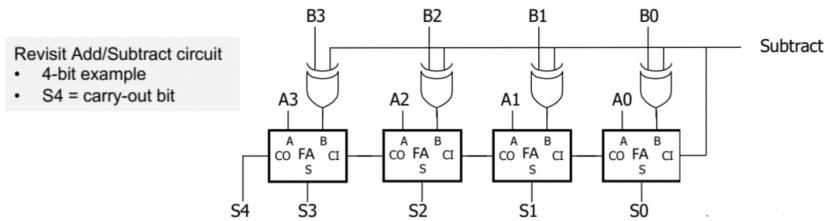
### 7.4.1 Condition Flags

#### Z (zero)

- S3, S2, S1, S0 are all 0

#### C (carry-out)

- S4 is 1
- Most significant bit (MSB) produced a carry-out



The carry out bit is the MSB

### 7.4.2 Comparing Unsigned Numbers

To compare A and B:

1. Compute A - B

2. Then check ALU flag bits Z and C

### Unsigned comparison:

EQ	==	Z
NE	!=	$\sim Z$
LTU	<	$\sim C$
LEU	$\leq$	$\sim C + Z$
GEU	$\geq$	C
GTU	>	$\sim(\sim C + Z)$

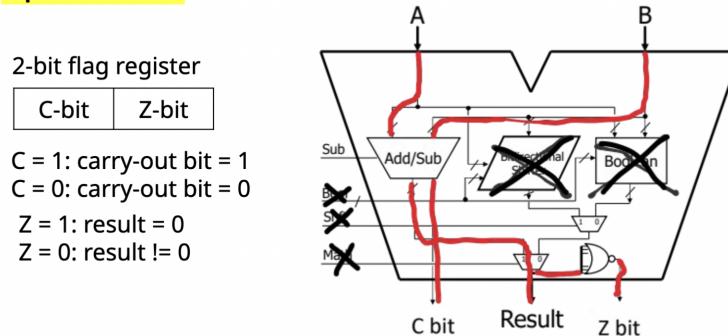
#### Example

- LTU (less than unsigned numbers)
- A < B is given by  $\sim C$
- Others in table

*Remark 3. SAP DOES NOT SUPPORT LESS THAN SIGNED NUMBER COMPARISON*

### 7.4.3 Flag Register and Operations

**Our ALU design with Z and C flags Note: SAP does not include Shift and Boolean operations**



## 7.5 SAP Assembly Programming

↓

Op Code	Instruction	Arg	Function
0000	NOP		
0001	LDA	Data Address	A = Mem(arg)
0010	ADD	Data Address	A = A + Mem(arg)
0011	SUB	Data Address	A = A - Mem(arg)
0100	STA	Data Address	Mem(Arg) = A
0101	LDI	Constant value	A = arg
0110	JMP	Instruction Address	PC = arg
0111	JC	Instruction Address	if FC=1 then PC=arg else go on if FZ=1 then PC=arg else go on
1000	JZ	Instruction Address	PC=arg else go on
1110	OUT		Display = OUT = A
1111	HLT		

Three or two letter acronym that represents a binary instruction.

- SAP has 11 instructions in total (OUT will not be used)

### 7.5.1 Binary Opcode

4-bit representation of mnemonic:

These must be: unique, i.e. two or more opcode values cannot be the same, and everyone instruction must have an opcode

### 7.5.2 Arg

Instruction ARGument, typically base-10 or base-16 representation.

Usages: address in RAM that stores data, Address in RAM that stores an instruction, immediate (constant) data, and NOTHING

### 7.5.3 LDA function

A = mem(arg) means that: at address *arg* in RAM load data in register A