Contents

1 Gates, Expressions, Circuits, and Analysis 1/13 and 1/15

Topics:

– Digital Logic Gates

– Boolean Algebra

– Combination Logic Circuits

– Sum of Products

– Karnaugh Maps

1.1 Logic Gates

A gate has (for example NOT gate):

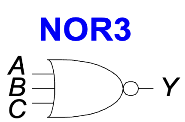
1. Name
2. Schematic Diagram
   – Input, A, for example, with boolean (0 or 1)
   – Output, Y, for example, boolean (0 or 1)
3. Boolean Expressions, i.e. $Y = \overline{A}$
4. Truth Table

Example

We can also have two or more input gates:

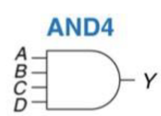
- AND $\rightarrow Y = AB$, A and B must be true
- OR $\rightarrow Y = A + B$, A or B must be true
- XOR $\rightarrow Y = A \oplus B$
- NAND $\rightarrow Y = \overline{AB}$
- NOR $\rightarrow Y = \overline{A + B}$
- XNOR $\rightarrow Y = \overline{A \oplus B}$

A nice to know is that if the NOT's are the actual gate, then it would turn, for example, $X = \overline{A}\,\overline{B} \neq X = \overline{AB}$.



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



$Y = ABCD$

A	C	B	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Figure 1: It can also have more than 2 inputs as seen here with their truth tables

1.2 Boolean Algebra

Symbols and Boolean operators:

$$x \cdot y,\ \ xy,\ \ x \wedge y,\ \ \mathrm{AND}(x,y),\ \ x\ \ \mathrm{AND}\ y$$

$$x + y,\ \ x \vee y,\ \ \mathrm{OR}(x,y),\ \ x\ \ \mathrm{OR}\ \ y$$

$$\overline{x},\ \ x',\ \ \neg x,\ \ \mathrm{NOT}(x),\ \ \mathrm{INV}(x)$$

$$\overline{x \cdot y},\ \ \overline{x \wedge y},\ \ \overline{xy},\ \ \mathrm{NAND}(x,y),\ \ x\ \ \mathrm{NAND}\ y$$

$$\overline{x + y},\ \ \overline{x \vee y},\ \mathrm{NOR}(x,y),\ \ x\ \mathrm{NOR}\ y$$

$$x \oplus y,\ \ \mathrm{XOR}(x,y),\ \ x\ \mathrm{XOR}\ y$$

$$x \overline{\oplus} y,\ \ \overline{x \oplus y},\ \ \mathrm{XNOR}(x,y),\ \ x\ \mathrm{XNOR}\ y$$

Figure 2: Notation before we get started

Moreover, here are some basic identities of boolean algebra

Basic Identities of Boolean Algebra

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Figure 3: Some basic identities

Definition

Variable Substitution, is a way of substitution that makes it more tangible and math more easy

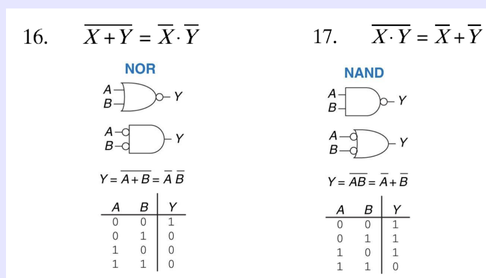
$$ABC + YZ = (ABC + Y)(ABC + Z)$$

Substitute X for ABC

$$X + YZ = (X+Y)(X+Z)$$

DeMorgan's Identity is used a lot and is very useful. As shown in these examples:

Example

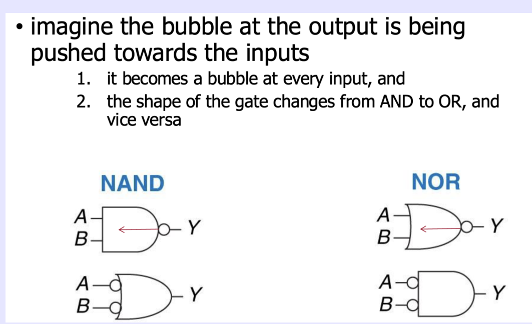


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Where the two equivalencies share a truth table due to this Identity

We can also see that it is like "pushing the bubble" as seen in this example:



1.3 Combinational Logic Circuits

Definition

Stateless Digital Logic Circuits:

- Combinational logic combination of logic gates
- Change input values
- Immediate change in output values
- No Memory
- No feedback

Remark 1. There are specifics types of wire connections

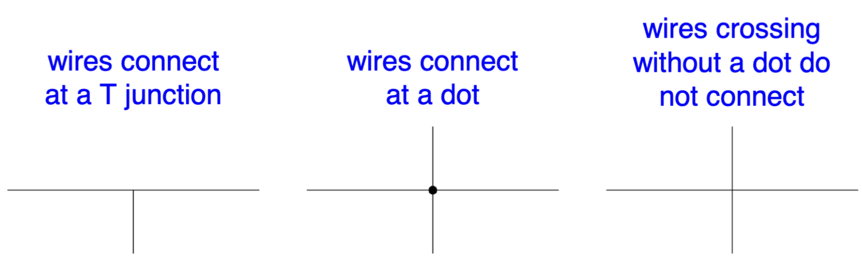
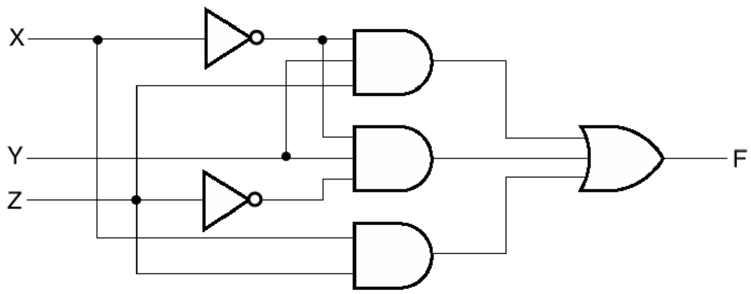


Figure 4: Here are the various ways wires can connect/not connect

Example

Here is an example of a circuit and the resulting algebra to ”solve” it and how to simplify it

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$



$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

Apply 14. $X(Y+Z) = XY + XZ$

$$F = \overline{X}Y(Z + \overline{Z}) + XZ$$

Apply 7. $X + \overline{X} = 1$

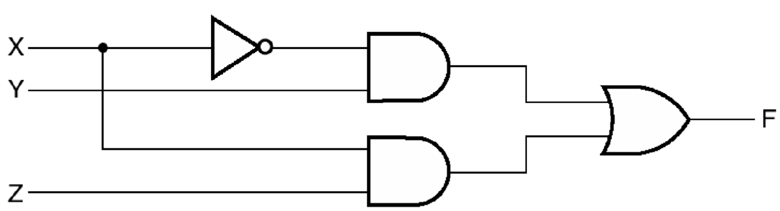
$$F = \overline{X}Y \cdot 1 + XZ$$

Apply 2. $X \cdot 1 = X$

$$F = \overline{X}Y + XZ$$

Fewer Gates

$$F = \overline{X}Y + XZ$$



(b) $F = \overline{X}Y + XZ$

Where output variables are either equivalent to 0 or 1 and input is the same. Moreover, simplifying this circuit and circuits in general allow for greater efficiency.

1.4 Standard Design Approach Sum of Products (SOP)

The three step apporach:

1. Define truth table
2. Write down a Boolean expression for every row with the '1' in the output, for example, $Y = \overline{C}\overline{B}A + \overline{C}BA + CB\overline{A} + CBA$
3. Wire up all of the gates

Truth Table

C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Figure 5: Here is the truth table given the example

1.5 Karnaugh Map

Definition

Karnaugh maps, aka k-maps, are graphical representations of truth tables that use a grid with one cell for each row of the truth table

C \ BA	00	01	11	10
0	0	1	1	0
1	0	0	1	1

C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Figure 6: An example k-map and its respective truth table!

You pretty much put the 1's and 0's onto the cell given the values

Here are some rules given to the k-map

1. The groupoing must be in the shape of a rectangle. There are no diagonal adjacencies allowed
2. All cells in the rectangle must contain ones. No zeros are allowed
3. The number of cells in groupings must be in powers of 2
4. Outside edges of K-maps are considered adjacent, so it may wrap around
5. Cells may be contained in more than one rectangle, but every rectangle must have at least ONE unique cell to
6. Every rectangle must be as large as possible
7. Everyone 1 must be covered by at least one rectangle