COMP/PHYS 447 Midterm Formulas

Single Qubits

$|v\rangle = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$ $\langle v| = \begin{pmatrix} \overline{a_1} & \dots & \overline{a_n} \end{pmatrix}$ v and $v^\dagger$ are also very similar to these two on top as well.

Probability of passing through filter

$(\hat{p} * \hat{P})^2$ Where $\hat{p}$ is the polarization of **photon** and $\hat{P}$ is the orientation of **polaroid**

Linear independence

- No linear combination of one another

Spherical Coordinates

$x = r \sin\theta \cos\phi$ $y = r \sin\theta \sin\phi$ $z = r\cos\theta$

Inner Product

1. $|A\rangle = c_1 |v_1\rangle + c_2 |v_2\rangle \implies \langle B|A\rangle = c_1 \langle B|v_1\rangle + c_2 \langle B|v_2\rangle$
2. $\langle A|B\rangle = \langle B|A\rangle^*$
3. $\langle A|A\rangle \geq 0$ unless $|A\rangle = 0$ then $\langle A|A\rangle = 0$

Normalization

Any basis has two orthogonal UNIT vector:
Given $|v\rangle = \alpha |u\rangle + \beta |u^\perp\rangle$, the normalized Vector MUST: $\langle v|v\rangle = |\alpha|^2 + |\beta|^2 = 1$
Given some orthonormal $|v\rangle$ then, $\langle e_i|v\rangle = 0+0+\cdots+c_i+\cdots+0 = c_i$
Given some sort of projection, say $Proj_{|\Psi\rangle} = a|0\rangle \otimes |0\rangle + c|1\rangle \otimes |0\rangle$ then we can get the following normalized version of it:
$\frac{1}{\sqrt{|a|^2+|c|^2}}(a|0\rangle \otimes |0\rangle + c|1\rangle \otimes |0\rangle)$
Giving us the probability of: $|a|^2 + |b|^2$

Euler's Identity

$e^{i\theta} = \cos\theta + i\sin\theta$

Bloch Sphere

- X-axis $\to |-\rangle$ & $|+\rangle$
  - $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
  - $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Y-axis $\to |-i\rangle$ & $|i\rangle$
  - $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$
  - $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$
- Z-axis $\to |0\rangle$ & $|1\rangle$

Quantum Gates & Multi-qubit systems

X Gate: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Y Gate: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Z Gate: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ CNOT: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

CNOT $\to$ Similar to XOR gate, if x is 0 then y will remain the same, if x is 1 then y will flip
Toffoli $\to$ A CNOT that has two control qubits and one target, where the third qubits output is: $|q_3 \oplus q_1 q_2\rangle$
$A \oplus (A \oplus B) = B$

Tensor Product

$(|u\rangle + |v\rangle) \otimes (|w\rangle + |x\rangle) =$ normal distribution
$(\langle u| + \langle v|) \otimes (|w\rangle + |x\rangle) = \langle u|w\rangle + \langle v|x\rangle$

Bell State

$|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$