A Proportional Integral Controller-Enhanced Non-negative Latent Factor Analysis Model Supplementary File

I. Introduction

This is the supplementary file for the paper entitled "*A Proportional Integral Controller-Enhanced Non-negative Latent Factor Analysis Model*". It mainly contains the tables and figures of experimental results.

II. Supplementary Tables

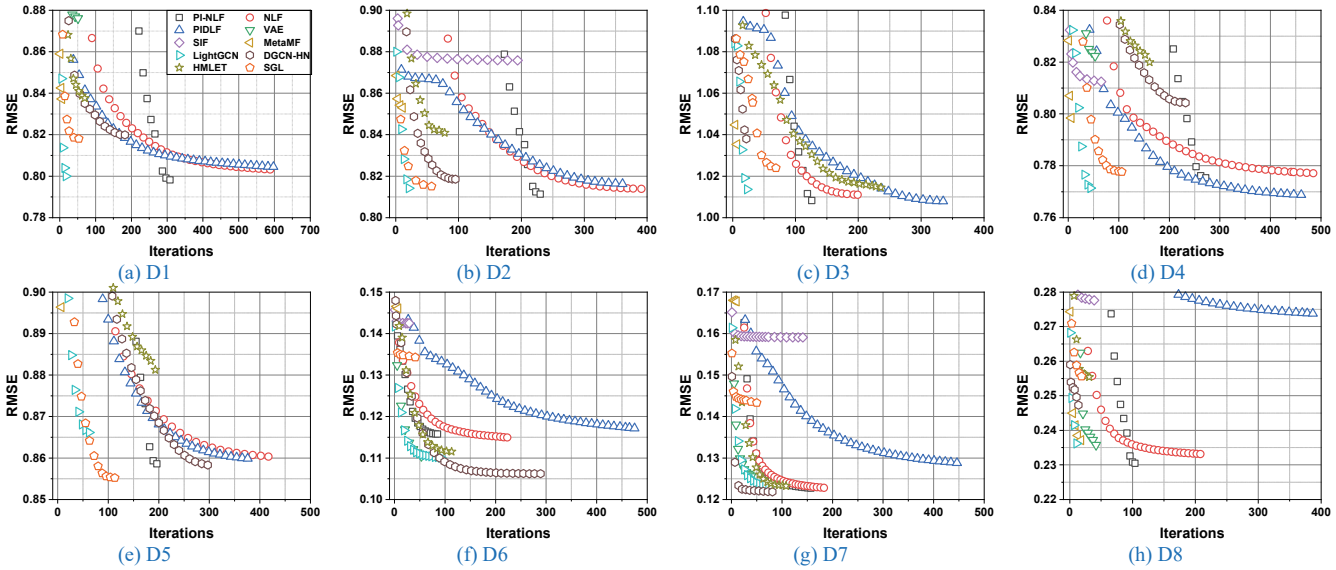
TABLE S(I). The comparison results on total time cost (Secs), where ✪ points out that PI-NLF is outperformed by its peer.

No.	Time	D1	D2	D3	D4	D5	D6	D7	D8	✪Loss/Win
PI-NLF	$\text{Time}_{(\text{RMSE})}$	**794.6**$_{\pm 8.2}$	**484.4**$_{\pm 4.3}$	**26.7**$_{\pm 0.4}$	**72.1**$_{\pm 2.3}$	**39.8**$_{\pm 0.3}$	**13.3**$_{\pm 0.3}$	**29.5**$_{\pm 0.2}$	**59.3**$_{\pm 1.2}$	—
	$\text{Time}_{(\text{MAE})}$	**712.1**$_{\pm 6.7}$	**442.3**$_{\pm 11.7}$	**24.2**$_{\pm 0.8}$	**64.7**$_{\pm 3.17}$	**54.4**$_{\pm 1.1}$	**14.5**$_{\pm 0.5}$	**22.2**$_{\pm 0.2}$	**53.6**$_{\pm 1.8}$	
NLF	$\text{Time}_{(\text{RMSE})}$	1481.8$_{\pm 75.3}$	813.3$_{\pm 37.4}$	41.8$_{\pm 1.1}$	127.6$_{\pm 5.6}$	83.4$_{\pm 1.7}$	34.3$_{\pm 1.1}$	32.1$_{\pm 0.3}$	116.5$_{\pm 2.1}$	0/16
	$\text{Time}_{(\text{MAE})}$	1401.1$_{\pm 88.7}$	804.9$_{\pm 66.8}$	48.1$_{\pm 1.5}$	115.9$_{\pm 3.7}$	82.8$_{\pm 0.9}$	26.9$_{\pm 0.4}$	27.3$_{\pm 0.2}$	106.4$_{\pm 0.9}$	
PIDLF	$\text{Time}_{(\text{RMSE})}$	2531.7$_{\pm 98.3}$	1032.5$_{\pm 107.6}$	137.4$_{\pm 2.5}$	193.6$_{\pm 4.2}$	146.6$_{\pm 2.3}$	147.6$_{\pm 1.7}$	165.1$_{\pm 5.6}$	406.4$_{\pm 5.5}$	0/16
	$\text{Time}_{(\text{MAE})}$	2337.6$_{\pm 65.2}$	1066.8$_{\pm 97.8}$	142.3$_{\pm 3.1}$	176.8$_{\pm 5.6}$	147.8$_{\pm 1.4}$	120.9$_{\pm 2.1}$	146.5$_{\pm 3.2}$	378.2$_{\pm 6.2}$	
VAE	$\text{Time}_{(\text{RMSE})}$	5722.2$_{\pm 89.3}$	632.0$_{\pm 12.8}$	27.9$_{\pm 0.9}$	632.2$_{\pm 32.4}$	61.4$_{\pm 0.4}$	28.2$_{\pm 0.6}$	33.2$_{\pm 0.7}$	127.3$_{\pm 1.1}$	0/16
	$\text{Time}_{(\text{MAE})}$	5831.7$_{\pm 106.8}$	597.1$_{\pm 21.5}$	26.9$_{\pm 0.5}$	597.1$_{\pm 28.6}$	64.2$_{\pm 0.8}$	29.8$_{\pm 1.1}$	34.6$_{\pm 0.3}$	126.7$_{\pm 1.6}$	
SIF	$\text{Time}_{(\text{RMSE})}$	8858.7$_{\pm 99.4}$	56766.3$_{\pm 325.1}$	7582.3$_{\pm 110.7}$	4594.2$_{\pm 99.6}$	802.4$_{\pm 12.1}$	765.3$_{\pm 11.2}$	1961.3$_{\pm 33.9}$	1487.5$_{\pm 45.9}$	0/16
	$\text{Time}_{(\text{MAE})}$	8966.1$_{\pm 125.7}$	55128.1$_{\pm 227.9}$	7471.8$_{\pm 99.5}$	4722.5$_{\pm 57.9}$	794.2$_{\pm 9.9}$	778.2$_{\pm 15.1}$	1922.7$_{\pm 48.9}$	1422.8$_{\pm 85.2}$	
MetaMF	$\text{Time}_{(\text{RMSE})}$	8121.4$_{\pm 229.2}$	6511.5$_{\pm 144.6}$	236.5$_{\pm 2.3}$	529.4$_{\pm 6.2}$	375.1$_{\pm 5.5}$	224.9$_{\pm 2.8}$	446.4$_{\pm 7.7}$	785.8$_{\pm 26.5}$	0/16
	$\text{Time}_{(\text{MAE})}$	8004.2$_{\pm 167.3}$	6633.7$_{\pm 132.8}$	234.2$_{\pm 2.9}$	522.1$_{\pm 8.8}$	370.8$_{\pm 6.1}$	227.5$_{\pm 3.6}$	455.2$_{\pm 4.4}$	771.2$_{\pm 38.2}$	
LightGCN	$\text{Time}_{(\text{RMSE})}$	5264.1$_{\pm 96.1}$	2940.2$_{\pm 69.2}$	173.4$_{\pm 5.7}$	118.9$_{\pm 4.1}$	409.8$_{\pm 3.8}$	149.1$_{\pm 2.2}$	173.2$_{\pm 4.5}$	636.1$_{\pm 12.3}$	0/16
	$\text{Time}_{(\text{MAE})}$	5100.7$_{\pm 88.7}$	2998.8$_{\pm 77.1}$	171.5$_{\pm 4.9}$	120.5$_{\pm 4.2}$	416.6$_{\pm 6.3}$	142.4$_{\pm 3.6}$	175.6$_{\pm 6.2}$	622.7$_{\pm 9.9}$	
DGCN-HN	$\text{Time}_{(\text{RMSE})}$	197924.1$_{\pm 511.4}$	67741.6$_{\pm 385.2}$	150.9$_{\pm 5.2}$	1677.9$_{\pm 131.8}$	2773.4$_{\pm 111.9}$	5400.8$_{\pm 309.2}$	645.5$_{\pm 35.1}$	327.3$_{\pm 12.1}$	0/16
	$\text{Time}_{(\text{MAE})}$	192205.4$_{\pm 496.5}$	86776.4$_{\pm 578.2}$	151.1$_{\pm 4.3}$	1814.6$_{\pm 142.9}$	2612.7$_{\pm 142.5}$	2427.1$_{\pm 279.5}$	302.6$_{\pm 8.9}$	363.8$_{\pm 15.5}$	
HMLET	$\text{Time}_{(\text{RMSE})}$	146128.8$_{\pm 211.9}$	53277.9$_{\pm 510.9}$	15935.2$_{\pm 366.2}$	5923.5$_{\pm 205.3}$	4150.4$_{\pm 156.7}$	2296.2$_{\pm 172.3}$	2982.6$_{\pm 177.2}$	6939.9$_{\pm 366.7}$	0/16
	$\text{Time}_{(\text{MAE})}$	138659.2$_{\pm 389.2}$	55165.3$_{\pm 677.5}$	16262.5$_{\pm 412.8}$	5183.1$_{\pm 99.6}$	4205.7$_{\pm 205.2}$	1649.2$_{\pm 134.6}$	2544.8$_{\pm 98.5}$	8238.1$_{\pm 415.6}$	
SGL	$\text{Time}_{(\text{RMSE})}$	96874.1$_{\pm 488.7}$	13357.8$_{\pm 177.5}$	3765.8$_{\pm 125.6}$	3621.8$_{\pm 144.5}$	966.4$_{\pm 14.2}$	856.6$_{\pm 37.4}$	1080.9$_{\pm 77.5}$	1978.5$_{\pm 85.3}$	0/16
	$\text{Time}_{(\text{MAE})}$	95005.2$_{\pm 439.5}$	13728.3$_{\pm 210.9}$	3960.7$_{\pm 77.3}$	3477.9$_{\pm 132.2}$	1041.9$_{\pm 22.9}$	229.4$_{\pm 9.5}$	996.5$_{\pm 62.3}$	2223.1$_{\pm 99.7}$	

TABLE S(II). The comparison results on RMSE/MAE, where ✪ points out that PI-NLF is outperformed by its peer.

No.	Case	D1	D2	D3	D4	D5	D6	D7	D8	✪Loss/Win
PI-NLF	RMSE	**0.7988**$_{\pm 1.3E-4}$	**0.8124**$_{\pm 2.4E-4}$	1.0096$_{\pm 1.4E-3}$	**0.7695**$_{\pm 5.6E-4}$	**0.8589**$_{\pm 1.3E-4}$	0.1126$_{\pm 2.4E-4}$	**0.1220**$_{\pm 4.3E-4}$	**0.2352**$_{\pm 2.3E-4}$	—
	MAE	**0.6115**$_{\pm 2.1E-4}$	**0.6237**$_{\pm 3.3E-4}$	**0.7794**$_{\pm 2.2E-3}$	**0.5776**$_{\pm 4.2E-4}$	**0.6723**$_{\pm 2.5E-4}$	0.0739$_{\pm 3.1E-4}$	0.0807$_{\pm 2.3E-4}$	**0.1795**$_{\pm 1.3E-4}$	
NLF	RMSE	0.8037$_{\pm 2.1E-4}$	0.8146$_{\pm 2.2E-4}$	1.0114$_{\pm 2.1E-3}$	0.7716$_{\pm 4.1E-4}$	0.8612$_{\pm 3.3E-4}$	0.1127$_{\pm 2.9E-4}$	0.1221$_{\pm 3.7E-4}$	0.2376$_{\pm 5.3E-4}$	2/14
	MAE	0.6246$_{\pm 2.1E-4}$	0.6380$_{\pm 5.6E-4}$	0.7960$_{\pm 8.5E-4}$	0.5980$_{\pm 3.6E-4}$	0.6827$_{\pm 2.9E-4}$	✪**0.0738**$_{\pm 2.6E-4}$	✪**0.0801**$_{\pm 2.6E-4}$	0.1859$_{\pm 4.3E-4}$	
PIDLF	RMSE	0.8039$_{\pm 4.5E-4}$	0.8155$_{\pm 3.2E-4}$	✪**1.0067**$_{\pm 2.4E-3}$	0.7742$_{\pm 4.2E-4}$	0.8595$_{\pm 2.3E-4}$	0.1193$_{\pm 3.3E-4}$	0.1296$_{\pm 3.9E-4}$	0.2699$_{\pm 1.4E-4}$	1/15
	MAE	0.6253$_{\pm 1.3E-3}$	0.6400$_{\pm 5.1E-4}$	0.7907$_{\pm 6.3E-4}$	0.5907$_{\pm 2.9E-4}$	0.6817$_{\pm 4.5E-4}$	0.0742$_{\pm 3.9E-4}$	0.0824$_{\pm 5.2E-4}$	0.2204$_{\pm 2.4E-4}$	
VAE	RMSE	0.8755$_{\pm 2.6E-3}$	0.9386$_{\pm 4.5E-3}$	1.2419$_{\pm 3.4E-4}$	0.8223$_{\pm 7.6E-4}$	0.9352$_{\pm 1.7E-4}$	✪**0.1121**$_{\pm 2.1E-4}$	0.1256$_{\pm 1.5E-4}$	0.2357$_{\pm 2.6E-4}$	1/15
	MAE	0.6821$_{\pm 1.4E-3}$	0.7671$_{\pm 5.7E-4}$	1.0299$_{\pm 3.4E-4}$	0.6261$_{\pm 3.9E-4}$	0.7388$_{\pm 1.4E-4}$	0.0741$_{\pm 4.3E-4}$	0.0858$_{\pm 1.9E-4}$	0.1899$_{\pm 1.9E-4}$	
SIF	RMSE	0.8852$_{\pm 1.4E-4}$	0.8758$_{\pm 4.4E-4}$	1.1415$_{\pm 1.2E-3}$	0.8124$_{\pm 5.1E-4}$	0.9295$_{\pm 6.2E-4}$	0.1424$_{\pm 6.6E-4}$	0.1590$_{\pm 1.1E-4}$	0.2776$_{\pm 1.3E-3}$	0/16
	MAE	0.6871$_{\pm 2.6E-4}$	0.6985$_{\pm 5.1E-4}$	0.9265$_{\pm 4.8E-4}$	0.6177$_{\pm 4.6E-4}$	0.7361$_{\pm 4.5E-4}$	0.0941$_{\pm 4.3E-4}$	0.1098$_{\pm 1.2E-4}$	0.2210$_{\pm 1.1E-3}$	
MetaMF	RMSE	0.8373$_{\pm 2.1E-4}$	0.8513$_{\pm 3.3E-4}$	1.0336$_{\pm 4.1E-4}$	0.7988$_{\pm 2.9E-4}$	0.8964$_{\pm 3.3E-4}$	0.1461$_{\pm 2.9E-4}$	0.1678$_{\pm 3.1E-4}$	0.2372$_{\pm 1.4E-4}$	0/16
	MAE	0.6595$_{\pm 1.9E-4}$	0.6618$_{\pm 3.7E-4}$	0.8026$_{\pm 2.7E-4}$	0.6029$_{\pm 4.2E-4}$	0.7041$_{\pm 1.1E-4}$	0.0950$_{\pm 2.2E-4}$	0.1137$_{\pm 1.9E-4}$	0.1845$_{\pm 2.1E-4}$	
LightGCN	RMSE	0.7999$_{\pm 1.9E-4}$	0.8141$_{\pm 3.4E-4}$	1.0136$_{\pm 2.7E-4}$	0.7714$_{\pm 3.7E-4}$	0.8661$_{\pm 1.8E-4}$	✪**0.1101**$_{\pm 2.6E-4}$	0.1232$_{\pm 2.1E-4}$	0.2362$_{\pm 1.4E-4}$	3/13
	MAE	0.6137$_{\pm 2.1E-4}$	0.6266$_{\pm 1.8E-4}$	0.7859$_{\pm 4.1E-4}$	0.5803$_{\pm 2.6E-4}$	0.6798$_{\pm 1.5E-4}$	✪**0.0676**$_{\pm 2.8E-4}$	✪**0.0791**$_{\pm 3.3E-4}$	0.1813$_{\pm 3.6E-4}$	
DGCN-HN	RMSE	0.8195$_{\pm 1.2E-4}$	0.8180$_{\pm 1.5E-4}$	1.0379$_{\pm 3.1E-4}$	0.8090$_{\pm 2.2E-4}$	0.8592$_{\pm 1.3E-4}$	✪**0.1124**$_{\pm 1.5E-4}$	0.1229$_{\pm 3.3E-4}$	0.2416$_{\pm 1.3E-4}$	2/14
	MAE	0.6291$_{\pm 1.3E-4}$	0.6341$_{\pm 2.3E-4}$	0.8118$_{\pm 3.1E-4}$	0.6041$_{\pm 2.9E-4}$	0.6726$_{\pm 1.5E-4}$	✪**0.0735**$_{\pm 1.1E-4}$	0.0809$_{\pm 3.2E-4}$	0.1870$_{\pm 1.2E-4}$	
HMLET	RMSE	0.8357$_{\pm 0.7E-4}$	0.8395$_{\pm 1.5E-4}$	1.0137$_{\pm 1.6E-4}$	0.8271$_{\pm 2.6E-4}$	0.8818$_{\pm 2.7E-4}$	0.1141$_{\pm 1.9E-4}$	0.1245$_{\pm 2.9E-4}$	0.2484$_{\pm 2.3E-4}$	0/16
	MAE	0.6399$_{\pm 1.5E-4}$	0.6466$_{\pm 1.8E-4}$	0.7895$_{\pm 1.8E-4}$	0.6211$_{\pm 2.2E-4}$	0.6912$_{\pm 2.2E-4}$	0.0741$_{\pm 1.6E-4}$	0.0844$_{\pm 3.5E-4}$	0.1905$_{\pm 2.5E-4}$	
SGL	RMSE	0.8177$_{\pm 1.7E-4}$	0.8141$_{\pm 2.5E-4}$	1.0223$_{\pm 2.3E-4}$	0.7832$_{\pm 1.4E-4}$	0.8607$_{\pm 1.2E-4}$	0.1351$_{\pm 2.9E-4}$	0.1442$_{\pm 0.6E-4}$	0.2505$_{\pm 3.3E-4}$	0/16
	MAE	0.6274$_{\pm 1.4E-4}$	0.6259$_{\pm 3.1E-4}$	0.7958$_{\pm 2.9E-4}$	0.5891$_{\pm 2.2E-4}$	0.6744$_{\pm 1.1E-4}$	0.0974$_{\pm 4.1E-4}$	0.1042$_{\pm 0.7E-4}$	0.1942$_{\pm 2.1E-4}$	

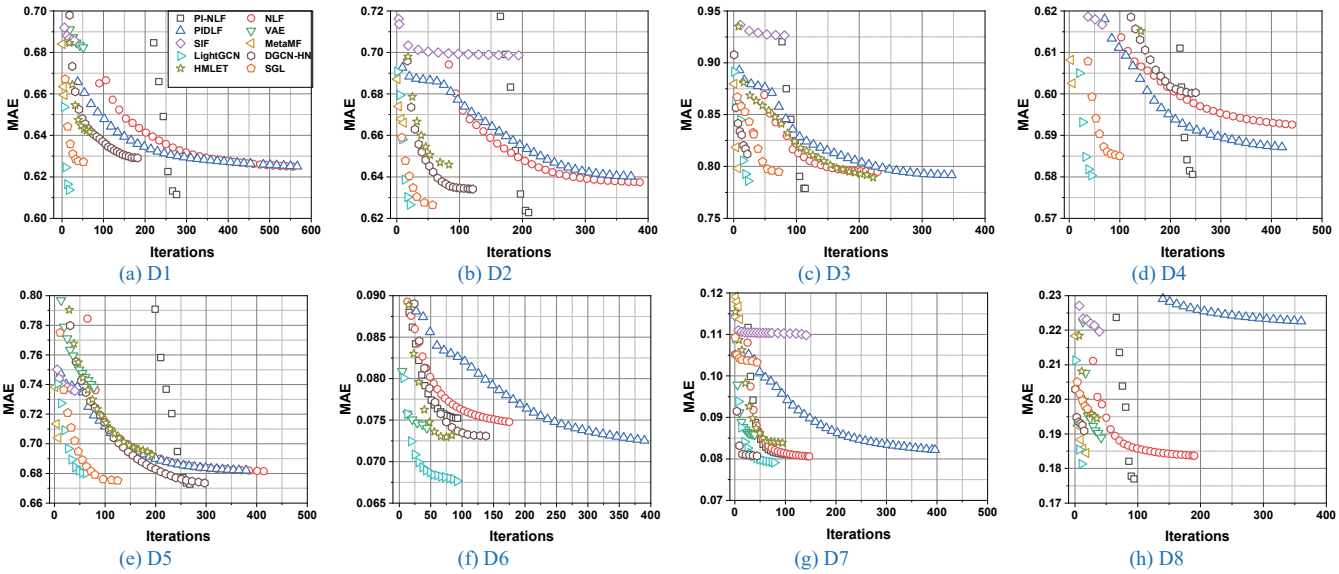
III. SUPPLEMENTARY FIGURES



(a) D1 (b) D2 (c) D3 (d) D4

(e) D5 (f) D6 (g) D7 (h) D8

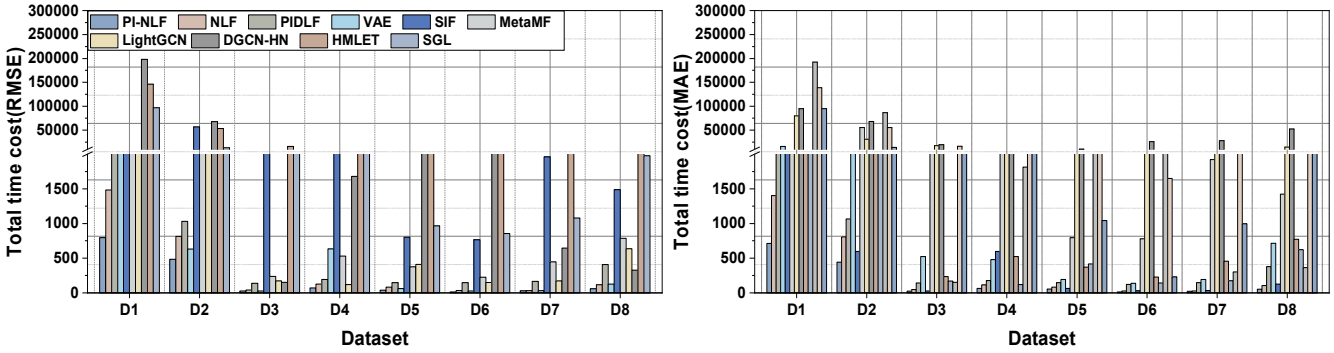
Fig. S1. Convergence curves in RMSE; all panels' legends are the same with panel (a)'s.



(a) D1 (b) D2 (c) D3 (d) D4

(e) D5 (f) D6 (g) D7 (h) D8

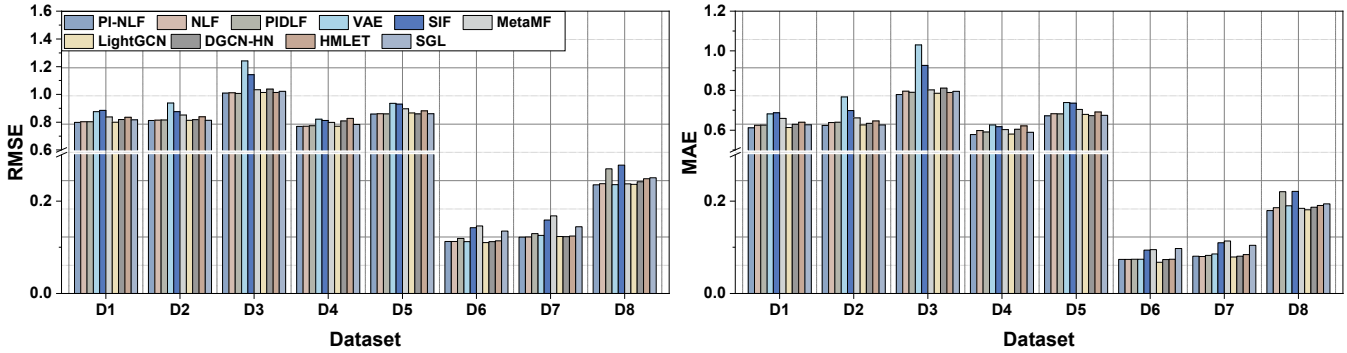
Fig. S2. Convergence curves in MAE; all panels' legends are the same with panel (a)'s.



(a) Total time cost Time corresponding to RMSE

(b) Total time cost Time corresponding to MAE

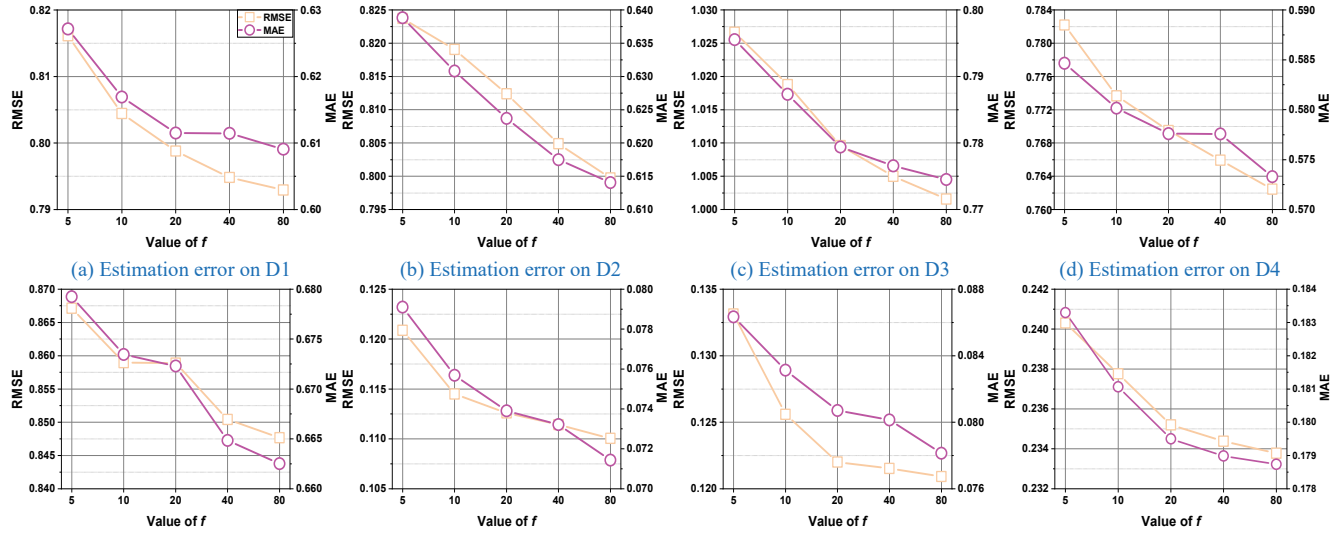
Fig. S3. Total time cost of compared models; all panels' legends are the same with panel (a)'s.



(a) RMSE

(b) MAE

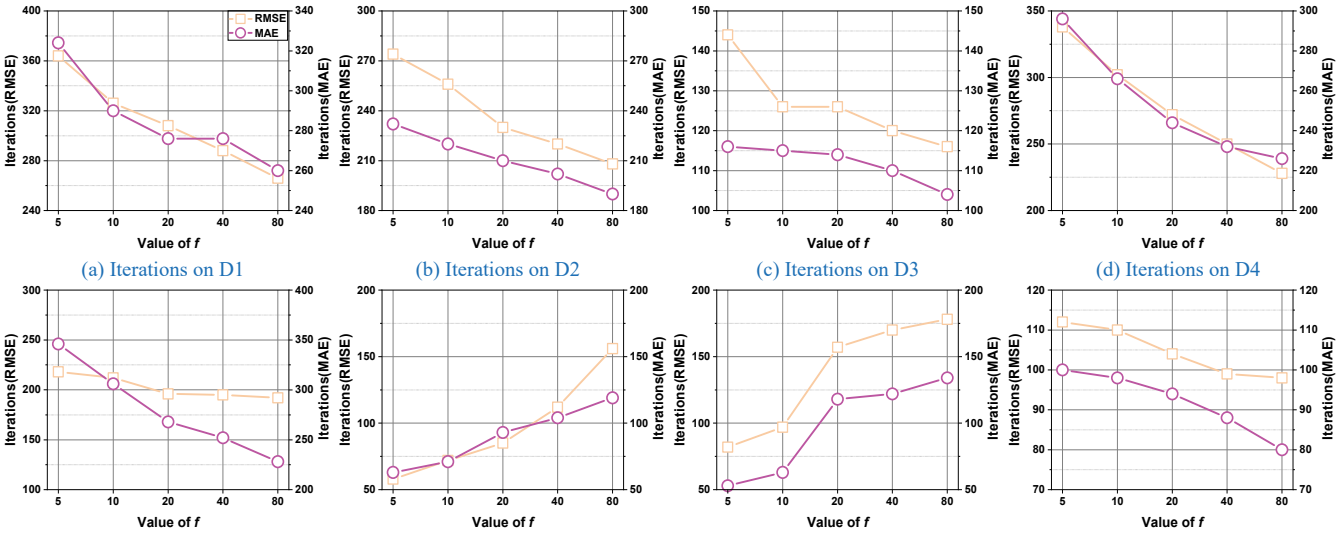
Fig. S4. Lowest RMSE/MAE of compared models; all panels' legends are the same with panel (a)'s.



(a) Estimation error on D1 (b) Estimation error on D2 (c) Estimation error on D3 (d) Estimation error on D4

(e) Estimation error on D5 (f) Estimation error on D6 (g) Estimation error on D7 (h) Estimation error on D8

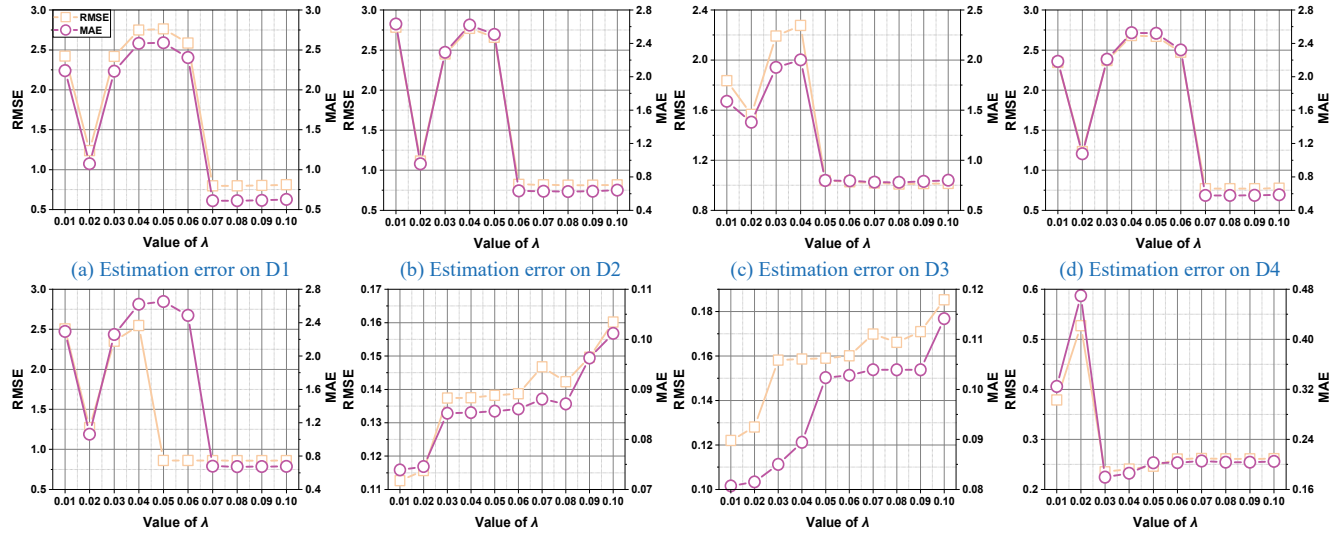
Fig. S5. PI-NLF's estimation error as *f* varies; all panels' legends are the same with panel (a)'s.



(a) Iterations on D1 (b) Iterations on D2 (c) Iterations on D3 (d) Iterations on D4

(e) Iterations on D5 (f) Iterations on D6 (g) Iterations on D7 (h) Iterations on D8

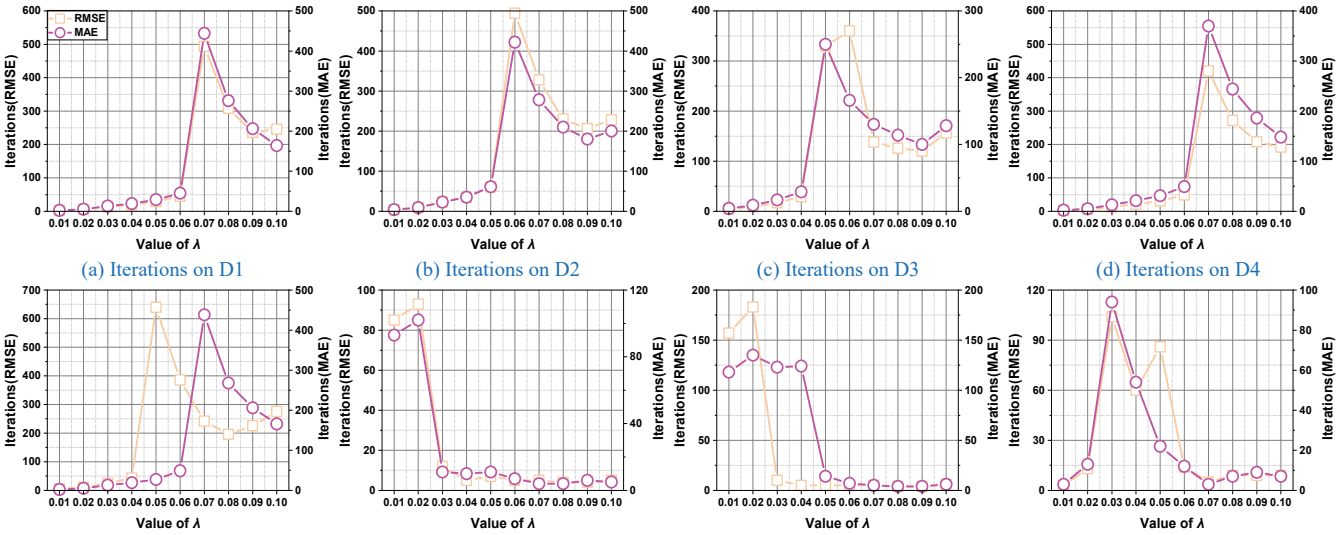
Fig. S6. PI-NLF's iterations as *f* varies; all panels' legends are the same with panel (a)'s.



(a) Estimation error on D1 (b) Estimation error on D2 (c) Estimation error on D3 (d) Estimation error on D4

(e) Estimation error on D5 (f) Estimation error on D6 (g) Estimation error on D7 (h) Estimation error on D8

Fig. S7. PI-NLF's estimation error as λ varies; all panels' legends are the same with panel (a)'s.



(a) Iterations on D1 (b) Iterations on D2 (c) Iterations on D3 (d) Iterations on D4

(e) Iterations on D5 (f) Iterations on D6 (g) Iterations on D7 (h) Iterations on D8

Fig. S8. PI-NLF's iterations as λ varies; all panels' legends are the same with panel (a)'s.