Diagram of What We're Doing

Assume we have the wealth dynamics in continuous and discrete time (work to be done here both theoretical and simulation), and assume returns are i.i.d. and bounded below (by -1). For simplicity, assume $W_0 = 1$, and note by Jensen's inequality, the second/fifth growth rate is bounded above by the third/sixth growth rate. Also note for the predictable, unconstrained cases, we need to prove that the optimal f is in fact a constant fraction.

Row:Problems Col: "growth rates"	$\frac{1}{n}\ln(wealth_n)$	$\frac{1}{n}\mathbf{E}[\ln(wealth_n)]$	$\frac{1}{n}\ln(\mathbf{E}[wealth_n])$	$\frac{1}{t}\ln(wealth_t)$	$\frac{1}{t}\mathbf{E}[\ln(wealth_t)]$	$\frac{1}{t}\ln(\mathbf{E}[wealth_t])$
Original Kelly and Thorp	$\sup_{f\in[0,1]}\lim_{n\to\infty}\frac{1}{n}\ln W_1^{n,f}$	$\sup_{f\in[0,1]}\lim_{n\to\infty}\frac{1}{n}\mathbf{E}[\ln W_1^{n,f}]$	$\sup_{f\in[0,1]}\lim_{n\to\infty}\frac{1}{n}\ln \mathbf{E}[W_1^{n,f}]$	continue		
Kelly Criterion with predictable $\{f_{n-1}\}_{n\geq 1}$, $f_{n-1}\in[0,1]\forall n$	$\sup_{\{f_{n-1}\}}\lim_{n\to\infty}\frac{1}{n}\ln W_1^{n,f}$	continue				
Kelly Criterion with drawdown constraint $\left\{\{f_{n-1}\}\middle\vert\mathbf{P}\left(\frac{W_t^f}{\sup_{t-\delta\leq s\leq t} W_s^f}\leq 1-\beta\right)\leq\epsilon,\forall t\right\}$	$\sup_{\{\cdots\leq\epsilon\}}\frac{1}{n}\lim_{n\to\infty}\ln W_1^{n,f}$	continue				

We need to work out the discrete time for the cases of a binomial tree, biased coin flips, and power laws (see Rachev and Samorodnitsky). Also, for the Kelly with constraint, recall Lagrange multipliers, $V_0 := \sup_{f(x)\leq c} g(x)$

Note our drawdown constraint is formulated from defining a relative drawdown at time t over a period of length δ corresponding to having exposure (betting with a fraction) f as $d_t^\delta(f) := \dfrac{\sup_{t-\delta\leq s\leq t} W_s^f - W_t^f}{\sup_{t-\delta\leq s\leq t} W_s^f}$.

We would like to have a high chance of the relative drawdown being no larger than some acceptable threshold β, for all $t>0$. Thus we get either a pointwise or pathwise constraint, and as of now I am not sure which one makes more sense, or how they're related, $\mathbf{P}(d_t^\delta(f)\leq\beta)\geq 1-\epsilon$, $\forall t>0$ (which is equivalent to what we have above), or $\mathbf{P}(d_t^\delta(f)\leq\beta,\forall t)\geq 1-\epsilon$